The Parker spiral magnetic field

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THE PARKER SPIRAL

The magnetic field of interplanetary space

- The spiral pattern of magnetic field lines in the solar system results from a combination of effects:
  - Outward expansion of the solar wind
  - Solar rotation
  - High-electrical conductivity of the solar-wind plasma

  This is an excellent demonstration of Alfven's "frozen-flux theorem"

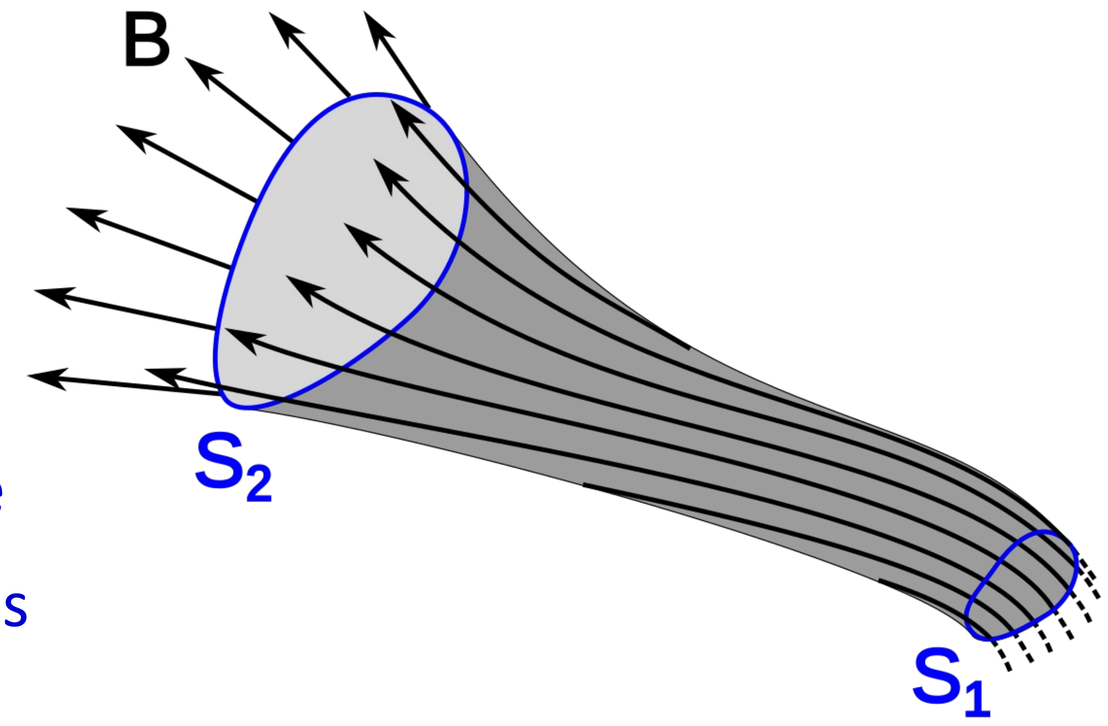
- The Parker spiral field plays a critical role in the transport of solar-energetic particles
- It may also play a role in the shape of the Heliosphere

Alfven's frozen (magnetic) flux theorem

- In ideal MHD, the evolution of the magnetic field (***B***) is governed by the magnetic induction equation

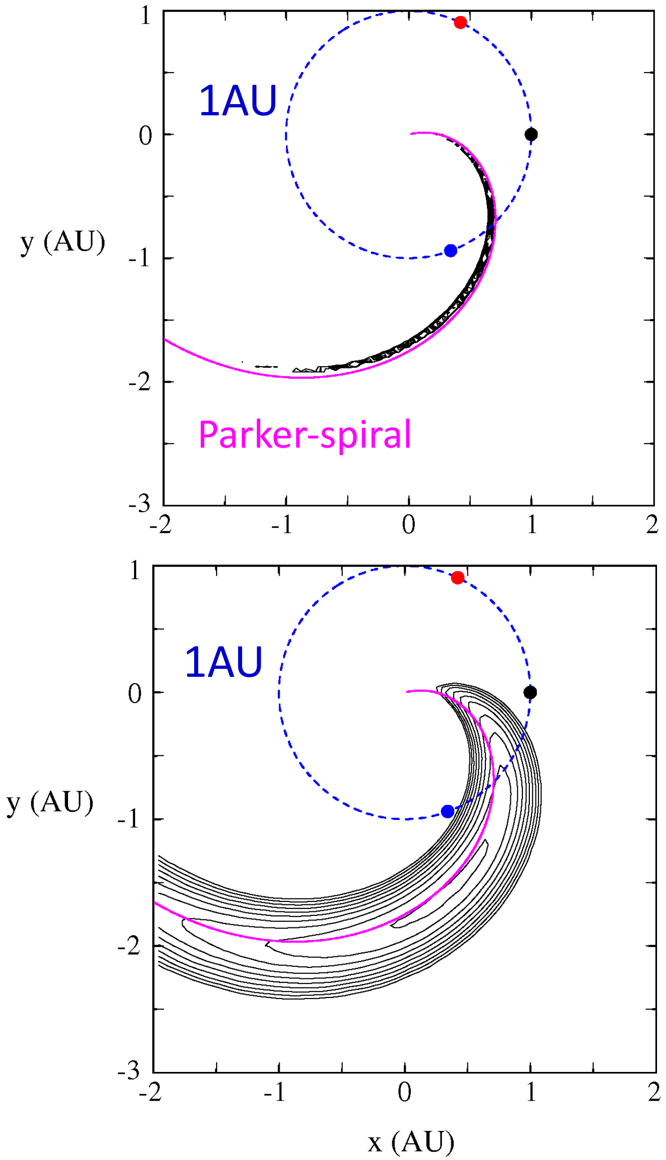
$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{U} \times \vec{B})$$

Where ***U*** is the plasma velocity



- One consequence of this equation is that the magnetic flux passing through a surface remains constant even as the surface changes due to plasma motions
- As a result, magnetic field lines are forced to move with the plasma.
- The plasma velocity may be effected by the field since the field provides a force on the plasma, but the field lines are always frozen into the flow in the limit of infinite electrical conductivity
- The Parker spiral is an excellent demonstration of the theorem

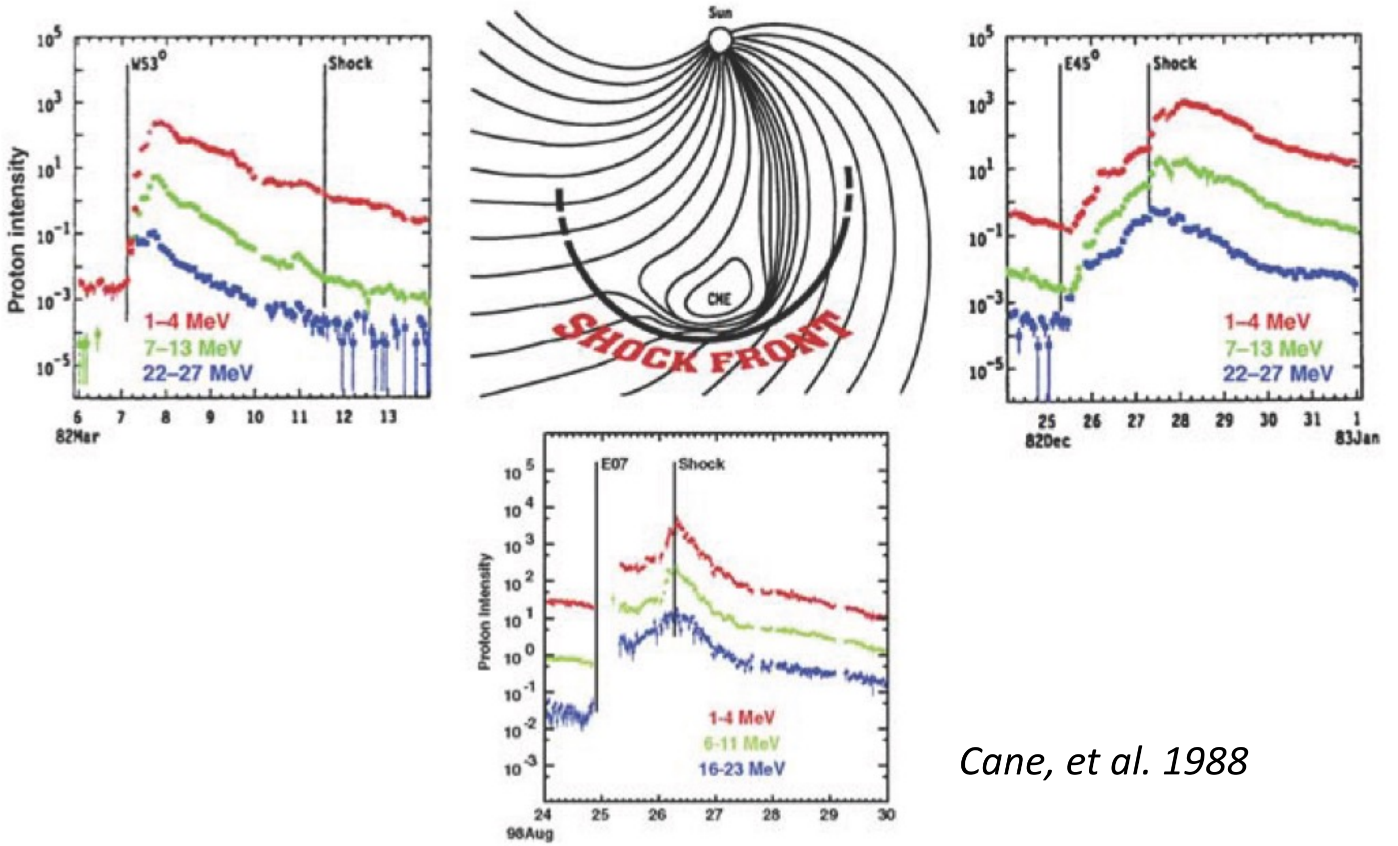
The Parker spiral has a critical role in determining how energetic particles from the Sun arrive to Earth, and beyond



Numerical simulation of the distribution function (contours) of SEPs associated with a solar flare several hours after the release of the particles

The two different plots are for two different values of cross-field diffusion

CME-related events depend on the direction the CME is moving relative to the observer



Cane, et al. 1988

The Heliosphere

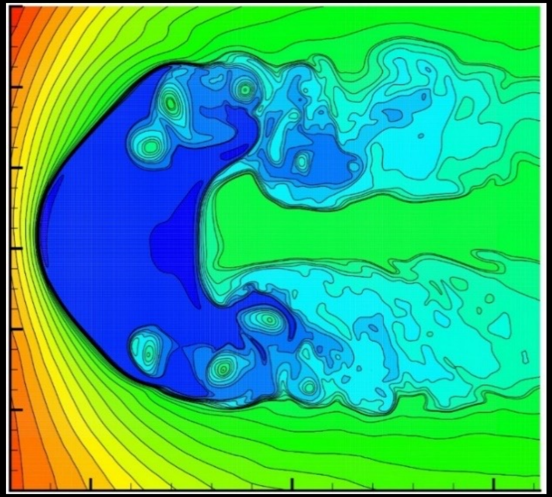
The heliosphere is the cavity carved out of the local interstellar medium by the solar wind

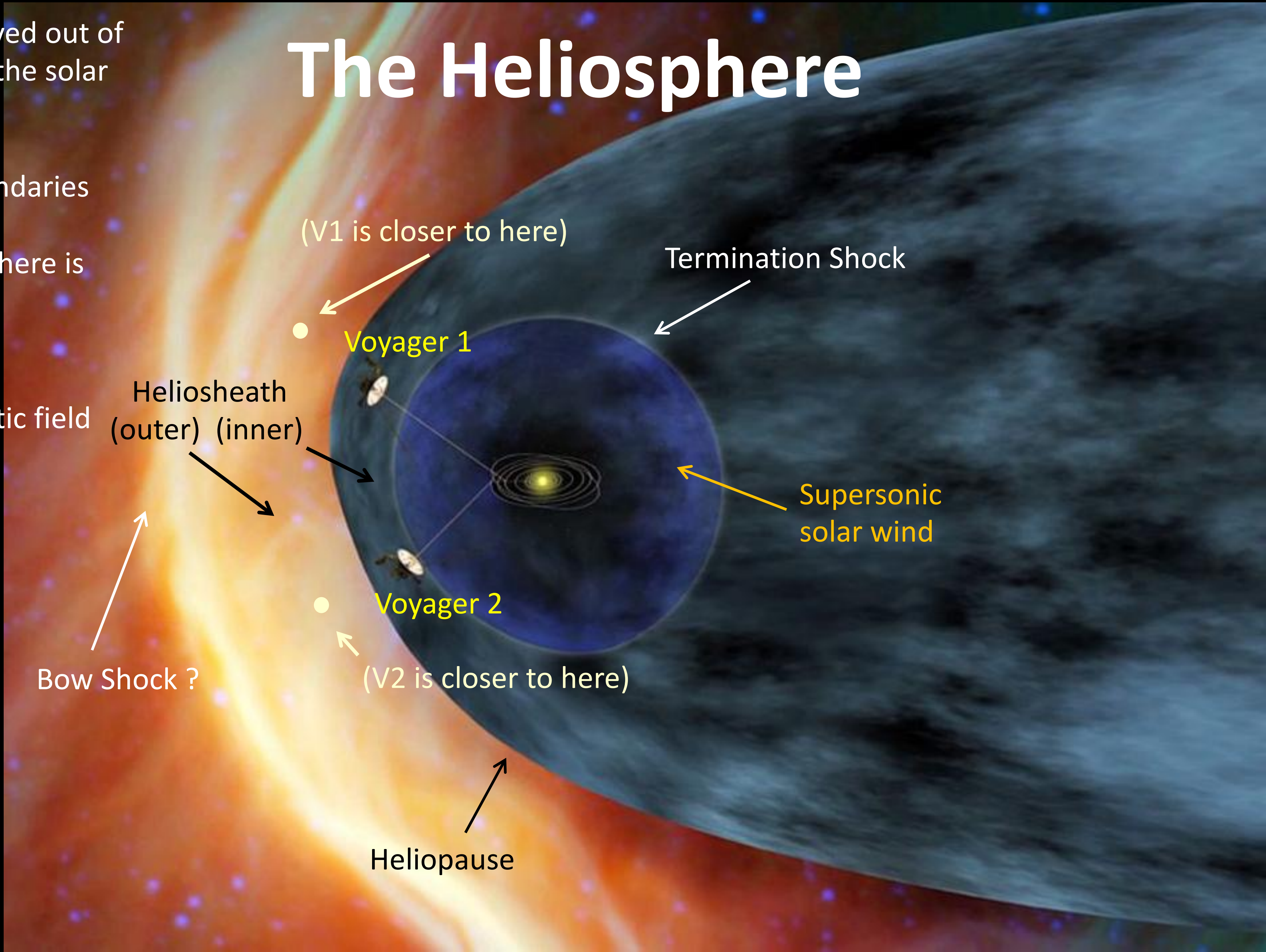
There are several important boundaries

The general shape of the heliosphere is under debate

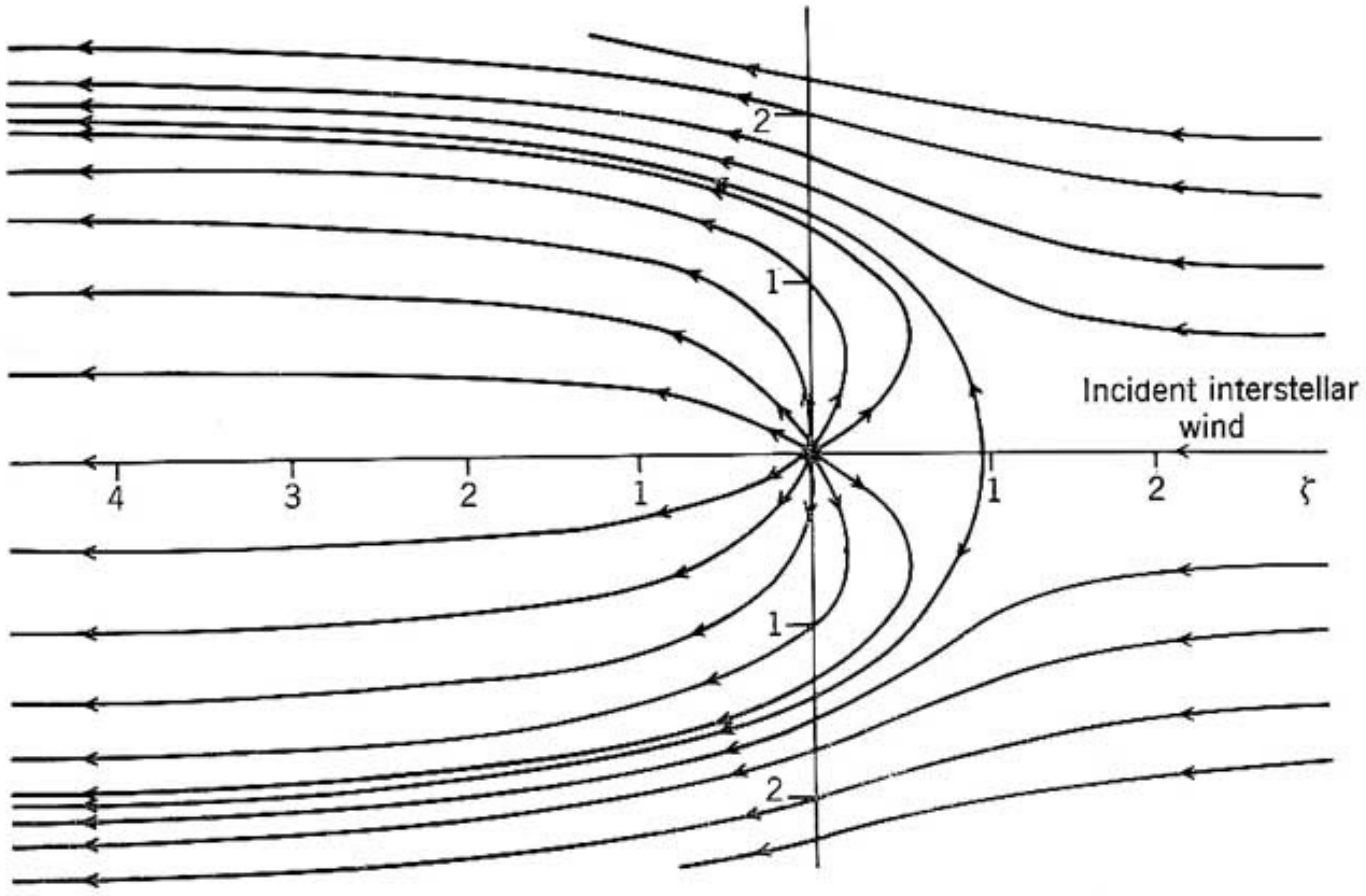
In one popular idea, the shape is determined partly by the magnetic field on the inside – the Parker spiral

“Croissant” shaped?
(***Opher et al., 2015, 2017***)





Parker's view of the heliosphere in 1961 – from an analytic formulation using "potential" flow.



- The solar wind flows *supersonically* and nearly *radially* from the Sun to the termination shock
- beyond the termination shock, the flow is subsonic and follows stream lines that deflect back towards the tail of the heliosphere.
- The tail is likely VERY long!

The Parker spiral field was derived in Parker's famous solar-wind paper

DYNAMICS OF THE INTERPLANETARY GAS AND MAGNETIC FIELDS*

E. N. Parker
Enrico Fermi Institute for Nuclear Studies, University of Chicago
Received January 2, 1958

ABSTRACT

We consider the dynamical consequences of Biermann's suggestion that gas is often streaming outward in all directions from the sun with velocities of the order of 500–1500 km/sec. These velocities of 500 km/sec and more and the interplanetary densities of 500 ions/cm³ (10^{14} gm/sec mass loss from the sun) follow from the hydrodynamic equations for a 3×10^6 °K solar corona. It is suggested that the outward-streaming gas draws out the lines of force of the solar magnetic fields so that near the sun the field is very nearly in a radial direction. Plasma instabilities are expected to result in the thick shell of disordered field (10^{-5} gauss) inclosing the inner solar system, whose presence has already been inferred from cosmic-ray observations.

I. INTRODUCTION

Biermann (1951, 1952, 1957*a*) has pointed out that the observed motions of comet tails would seem to require gas streaming outward from the sun. He suggests that gas is often flowing radially outward in all directions from the sun with velocities ranging
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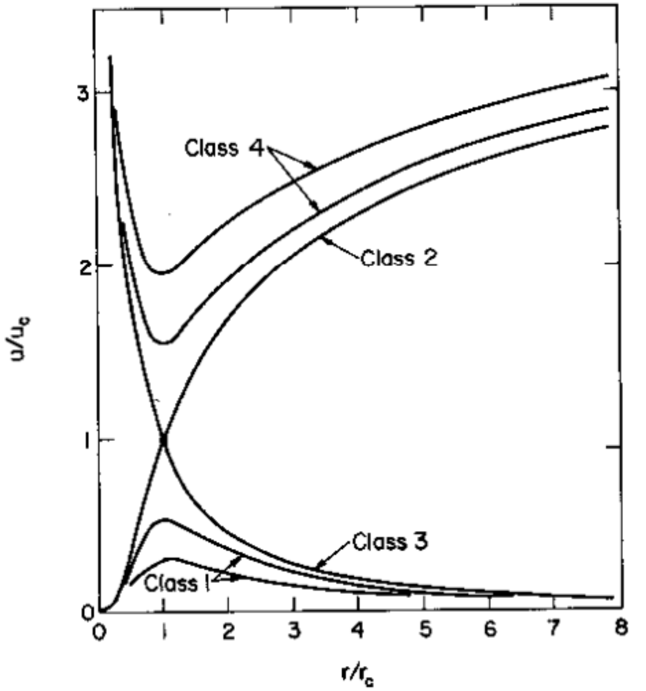
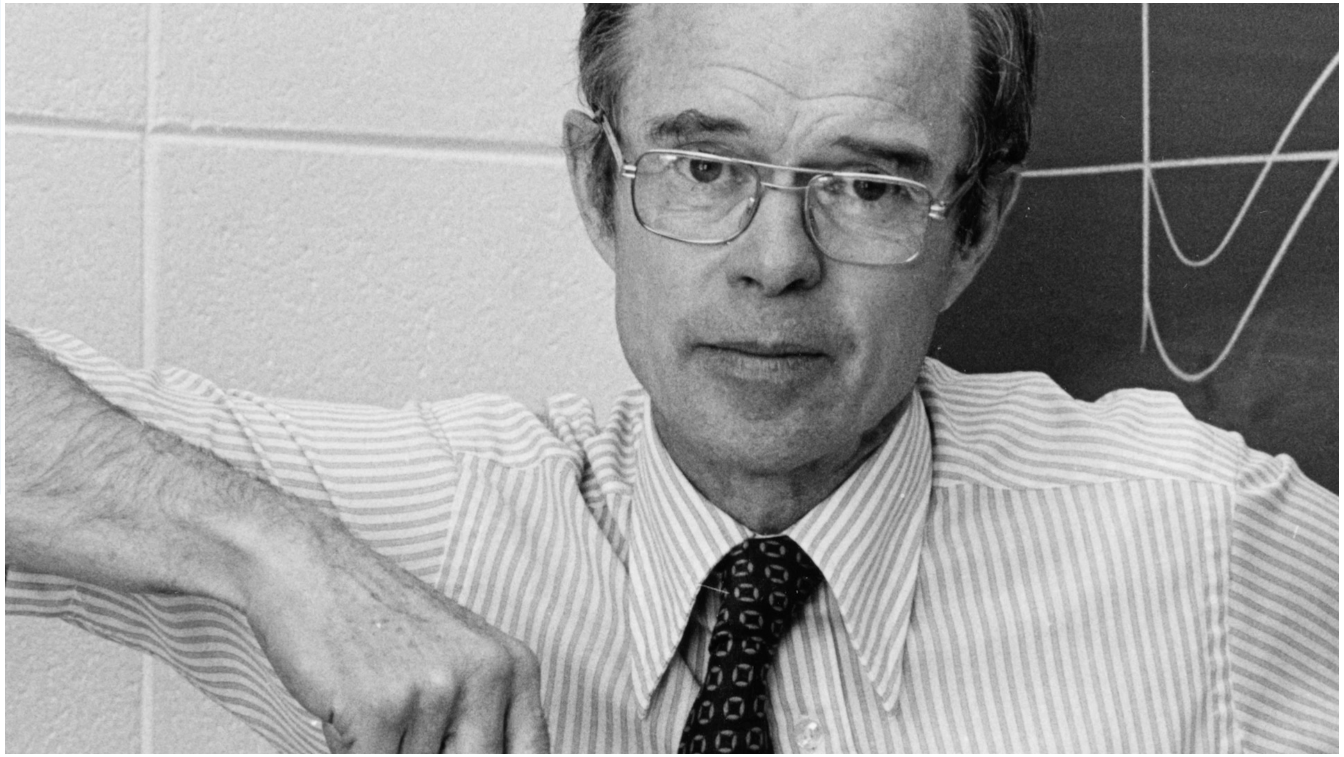


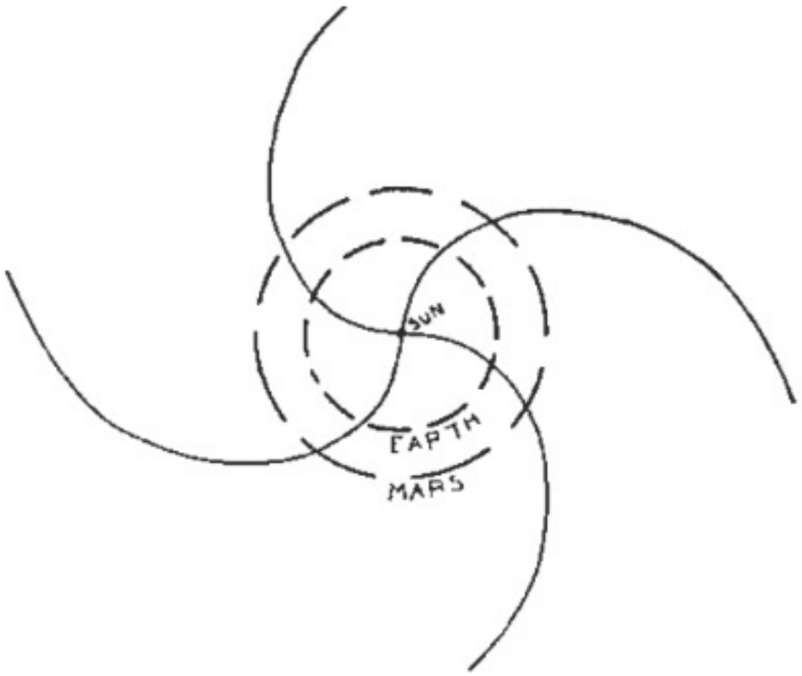
Figure 5.2: *The four classes of Parker outflow solutions for the solar wind.*



$$\left(\frac{U}{U_c}\right)^2 - ln\left(\frac{U}{U_c}\right)^2 = 4\,ln\left(\frac{r}{r_c}\right) + 4\frac{r}{r_c} + C$$

Parker's equation for the solar wind speed

Parker's drawing of magnetic field lines from his derivation. This is Figure 6 of his paper



The derived magnetic field vector (Eq. 26 from his paper)

$$B_r(r, \theta, \phi) = B(\theta, \phi_0)\left(\frac{b}{r}\right)^2,$$

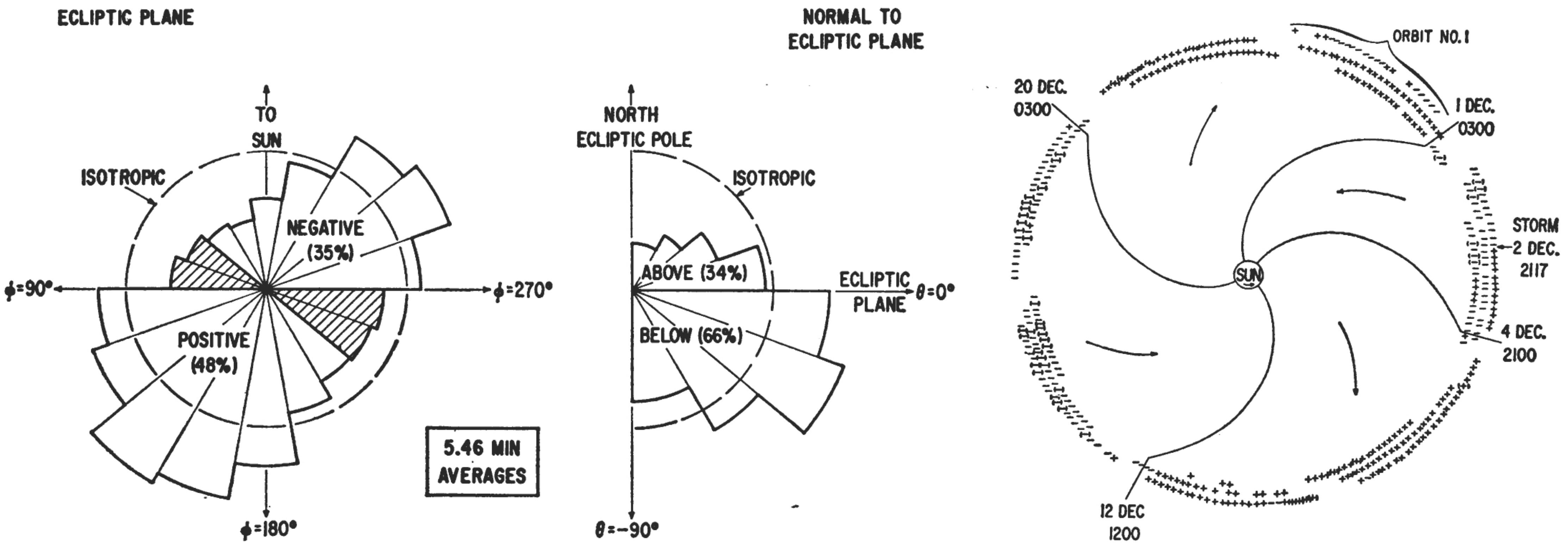
$$B_\theta(r, \theta, \phi) = 0\ ,$$

$$B_\phi(r, \theta, \phi) = B(\theta, \phi_0)\left(\frac{\omega}{v_m}\right)(r-b)\left(\frac{b}{r}\right)^2 \sin\ \theta\,,$$

The assumed solar-wind velocity in the frame co-rotating with the Sun (Eq. 24 from his paper)

$$v_r = v_m\ , \qquad v_\theta = 0\ , \qquad v_\phi = \omega\,(r-b)\sin\ \theta\ ,$$

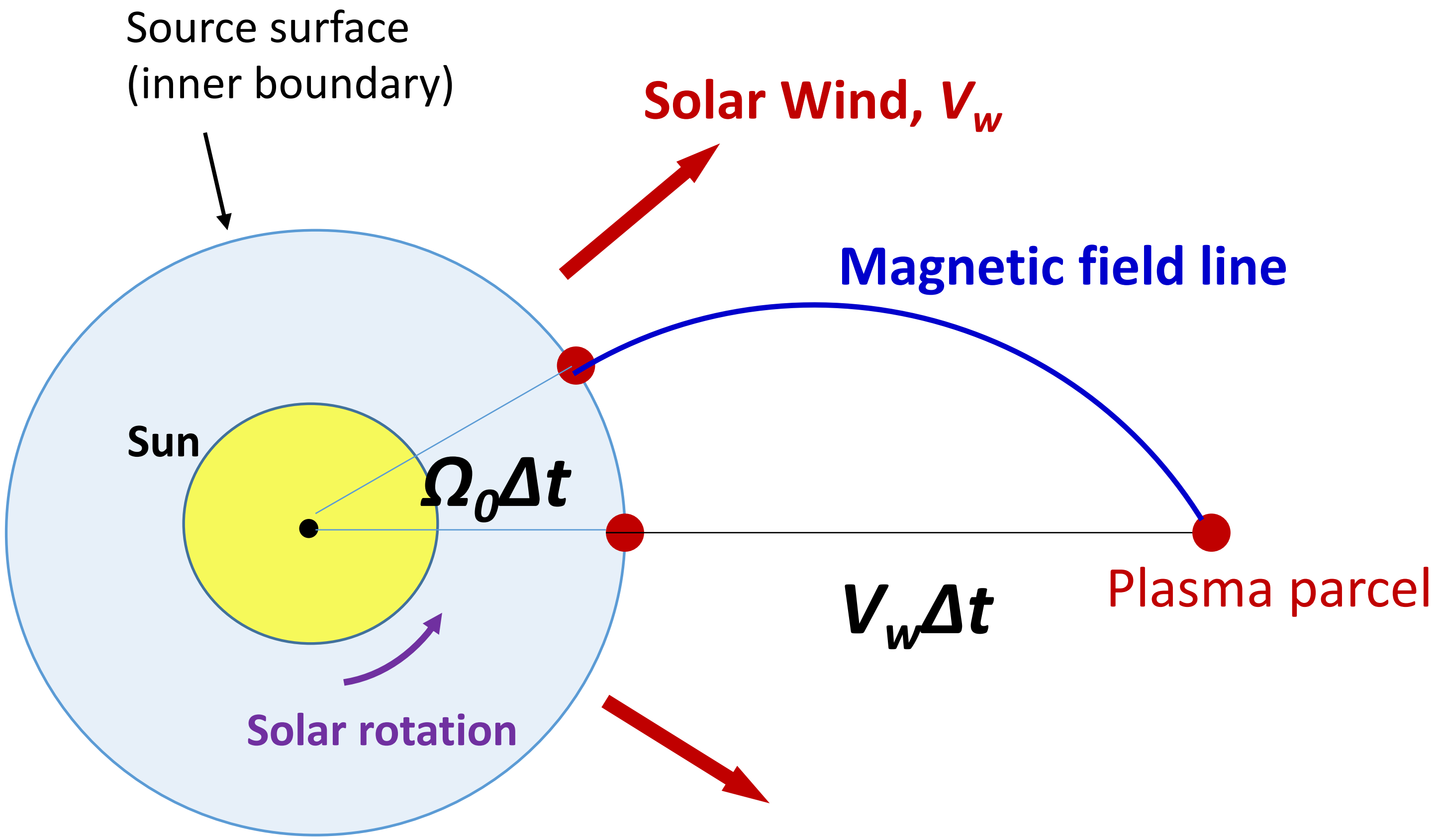
First spacecraft observations of the Parker spiral field



Ness & Wilcox, Science, 1965

Conceptual picture of the Parker spiral

- Open magnetic flux at the inner boundary is nearly radial
- The field is frozen into the plasma.
- As a plasma parcel moves outward with the solar wind, V_w, its "foot-point" is rooted at the Sun which rotates with the frequency Ω_0.
- The resulting path of the field line in space is that of an Archimedean spiral.



Quantitative derivation

- The derivation of the Parker spiral is an example of a “kinematic model”: we assume a flow velocity vector and solve the magnetic induction equation to get the magnetic field.
- Consider the following flow velocity vector:

$$\vec{U} = \begin{cases} R_0\Omega_0 \sin\theta\hat{\phi} & r = R_0 \\ V_w\hat{r} & r > R_0 \end{cases}$$

- The magnetic field is obtained by solving the magnetic induction equation, below, using this assumed flow speed

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times(\vec{U}\times\vec{B})$$

R_0 is the radius of the inner boundary.

Ω_0 is the solar rotation frequency

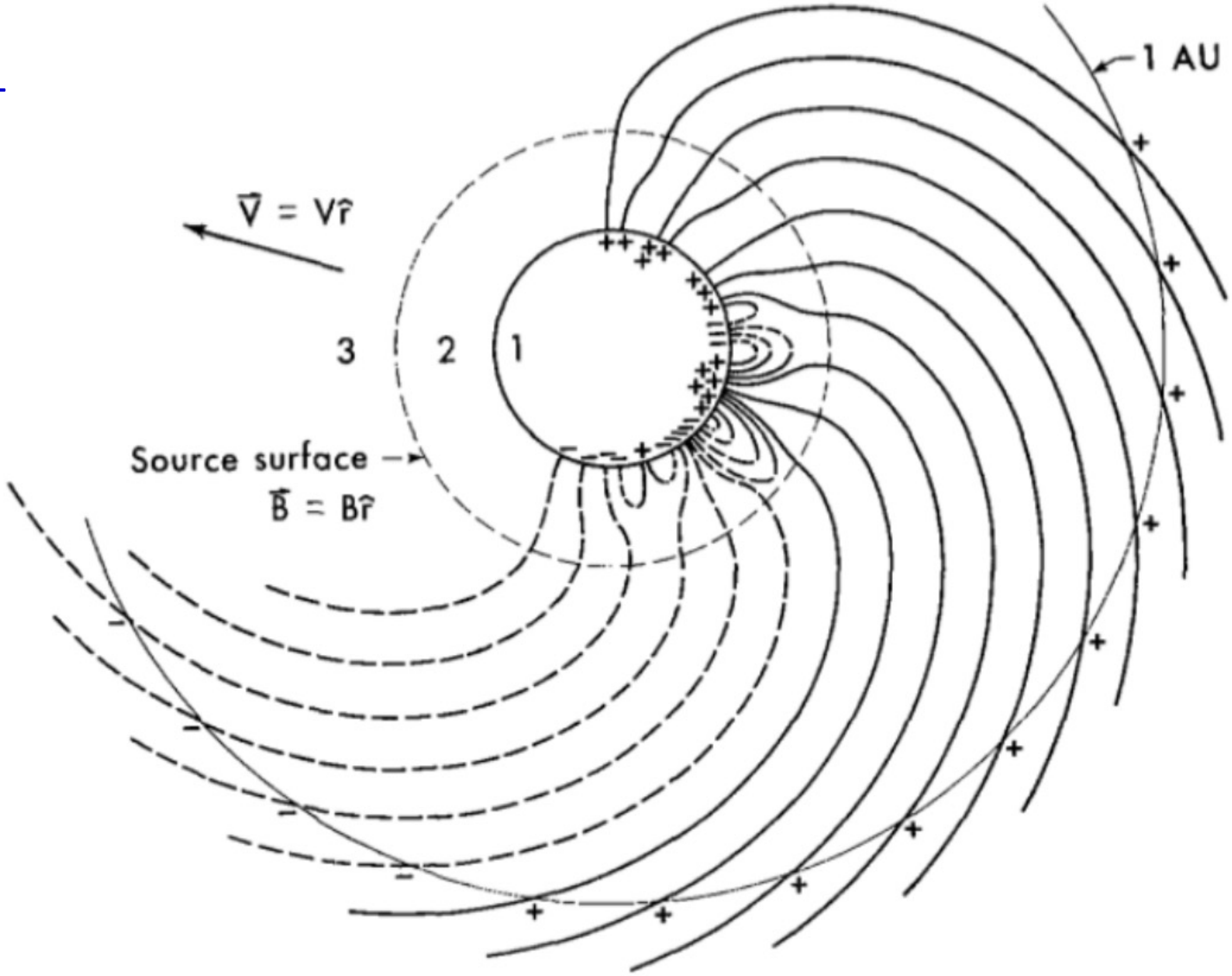
r is heliocentric distance , θ is polar angle, and ϕ is the azimuth angle

V_w is the solar wind speed, which is assumed radial an constant in this example. (Parker did not assume this)

$R_0\Omega_0 \approx 2$ km/s $V_w \approx 400$ km/s

The inner boundary – where does the coronal field end and the interplanetary field start?

- The "source surface" is used in so-called "potential-field source surface models" of the coronal magnetic field.
- This is where the magnetic field is nearly radial from (or inward towards) the Sun
- This can be taken as the inner boundary in the Parker spiral derivation
- The actual value of R_0 in the derivation is only important with regards to specifying the strength of the magnetic field at the inner boundary



Quantitative derivation (continued)

- Assuming that the magnetic field is in steady state, the magnetic induction equation requires $\nabla \times (\boldsymbol{U} \times \boldsymbol{B})=0$.

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$$r^2 B_r = constant = R_0^2 B_0$$

$$r B_\theta = constant = 0$$

(There is no θ component of the field at $r=R_0$)

$$r B_\phi = constant = R_0 B_\phi(R_0)$$

Quantitative derivation (continued)

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- It is tempting to set the last constant to zero as well, since we have assumed the field is radial at the source surface. But … we have only said that it is "nearly" radial at the source surface! It turns out that it cannot be exactly radial in this particular example. Here is why: consider the electric field (***E***=-***U***x***B***/c) at and just barely above the source surface

$$\vec{E}(r = R_0) = -\frac{1}{c} B_r V_\phi \hat{\theta} = -\frac{1}{c} B_0 R_0 \Omega_0 \sin\theta \, \hat{\theta}$$

$$\vec{E}(r = R_0 + \epsilon) = \frac{1}{c} B_\phi(R_0) V_r \hat{\theta} = \frac{1}{c} B_\phi(R_0) V_w \hat{\theta}$$

These MUST be the same! Thus, equating them gives:

$$B_\phi(R_0) = -B_0 \frac{R_0 \Omega_0 \sin\theta}{V_w}$$

The final form of the Parker Spiral (Model #1)

- Thus, the final form for the magnetic field in this case is given by:

$$B_r = B_0(\theta)\left(\frac{R_0}{r}\right)^2$$

$$B_\theta = 0$$

$$B_\phi = -B_0(\theta)\frac{R_0}{r}\frac{R_0\Omega_0 \sin\theta}{V_w}$$

B_0 is a signed quantity, and depends on θ: The radial component of the field changes sign across neutral line at the Sun, which leads to a heliospheric current sheet in the solar wind.

This dependence on θ does not effect the analysis that we have presented.

- This form differs slightly from that derived by Parker.
- However, this form is the correct solution to the magnetic induction equation for our assumed flow speed, is internally consistent, and is consistent with the MHD equations in general.

However …

- The previous form does not give a purely radial magnetic field at the inner boundary. There is a small azimuthal component.
- In order to have a purely radial field at the inner boundary, we must have an azimuthal component to the solar wind speed for $r>R_0$.
- Thus, let us consider instead the following form for the plasma velocity:

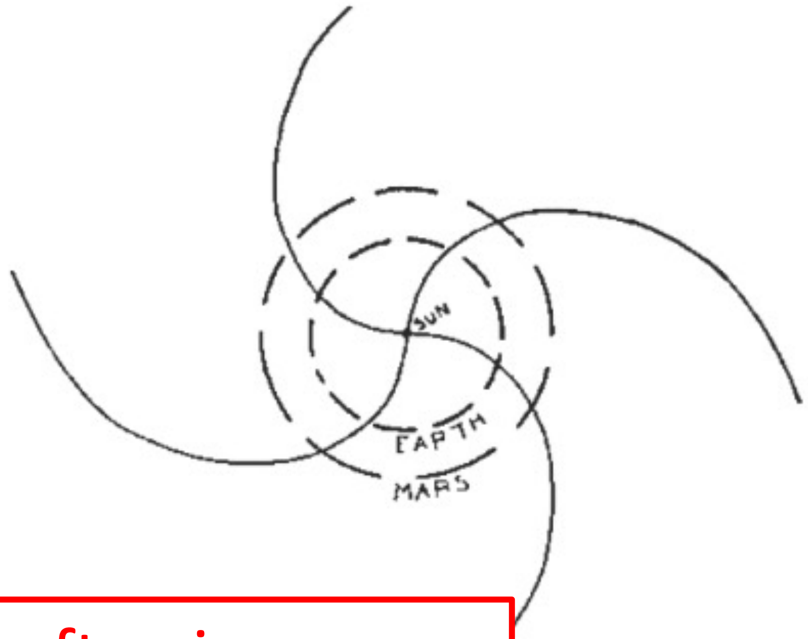
$$\vec{U} = \begin{cases} R_0 \Omega_0 \sin\theta \hat{\phi} & r = R_0 \\ V_w \hat{r} + V_\phi \hat{\phi} & r > R_0 \end{cases}$$

- In order for the electric field to be continuous across the inner boundary, the azimuthal flow speed must be continuous. Thus, we require:

$$V_\phi (R_0 + \epsilon) = R_0 \Omega_0 \sin\theta$$

- But how does V_ϕ depend on r?
- Parker assumed it doesn’t and took it to be constant. But, this is NOT consistent with the conservation of angular momentum.

Parker's drawing of magnetic field lines from his derivation. This is Figure 6 of his paper



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$$B_r(r, \theta, \phi) = B(\theta, \phi_0)$$

$$B_\theta(r, \theta, \phi) = 0 ,$$

$$B_\phi(r, \theta, \phi) = B(\theta, \phi_0)$$

An important assumption which often is forgotten in the literature and in text books. This assumption is not consistent with the conservation of angular momentum

The assumed solar-wind velocity in the frame co-rotating with the Sun (Eq. 24 from his paper)

$$v_r = v_m , \qquad v_\theta = 0 , \qquad v_\phi = \omega (r - b) \sin \theta ,$$

Consider the ϕ-component of the MHD momentum equation:

- The (ideal) MHD momentum equation is given by:

$$\frac{\partial \vec{U}}{\partial t} + \vec{U} \cdot \nabla \vec{U} = -\nabla P + \frac{1}{c} \vec{J} \times \vec{B} - \rho \vec{g} = -\nabla P + \frac{1}{4\pi} \left(\nabla \times \vec{B} \right) \times \vec{B} - \rho \vec{g}$$

- Taking the ϕ component, assuming steady state, ignoring magnetic stresses, and assuming there is no ϕ dependence of the thermal pressure or gravity, we obtain (after using the spherical coordinate representation of the second term on the left)

$$\frac{\partial U_\phi}{\partial r} = -\frac{U_\phi}{r}$$

- U_φ = constant is not a solution to this. Thus, Parker's assumption does not satisfy conservation of angular momentum.
- The solution gives the correct form of the azimuthal solar wind velocity (note that we use V notation instead of U to remain consistent with our previous derivation)

$$V_\phi = \left(\frac{R_0}{r} \right) R_0 \Omega_0 \sin \theta$$

Quantitative derivation #2

- For this case, the kinematic model uses the following flow velocity vector:

$$\vec{U} = \begin{cases} R_0\Omega_0 \sin\theta\hat{\phi} & r = R_0 \\ V_w\hat{r} + \left({R_0}/{r}\right) R_0\Omega_0 \sin\theta\,\hat{\phi} & r > R_0 \end{cases}$$

- The magnetic field is obtained by solving the magnetic induction equation, using this assumed flow speed.
- The *r* and θ components of the field are the same as in the previous model.
- The ϕ component is determined from:

$$-rV_wB_\phi + rV_\phi B_r = constant = R_0V_\phi(R_0)B_0 = R_0^2\Omega_0B_0 \sin\theta$$

$$\Rightarrow \quad -rV_wB_\phi + r\left[\frac{R_0^2\Omega_0 \sin\theta}{r}\right]\left[B_0\left(\frac{R_0}{r}\right)^2\right] = R_0^2\Omega_0B_0 \sin\theta$$

The final form of the Parker Spiral (Model #2)

- Thus, the final form for the magnetic field in this case is given by:

$$B_r = B_0(\theta)\left(\frac{R_0}{r}\right)^2$$ ← **This follows from ∇•B=0**

$$B_\theta = 0$$

$$B_\phi = -B_0(\theta)\frac{R_0}{r}\frac{R_0\Omega_0 \sin\theta}{V_w}\left[1-\left(\frac{R_0}{r}\right)^2\right]$$

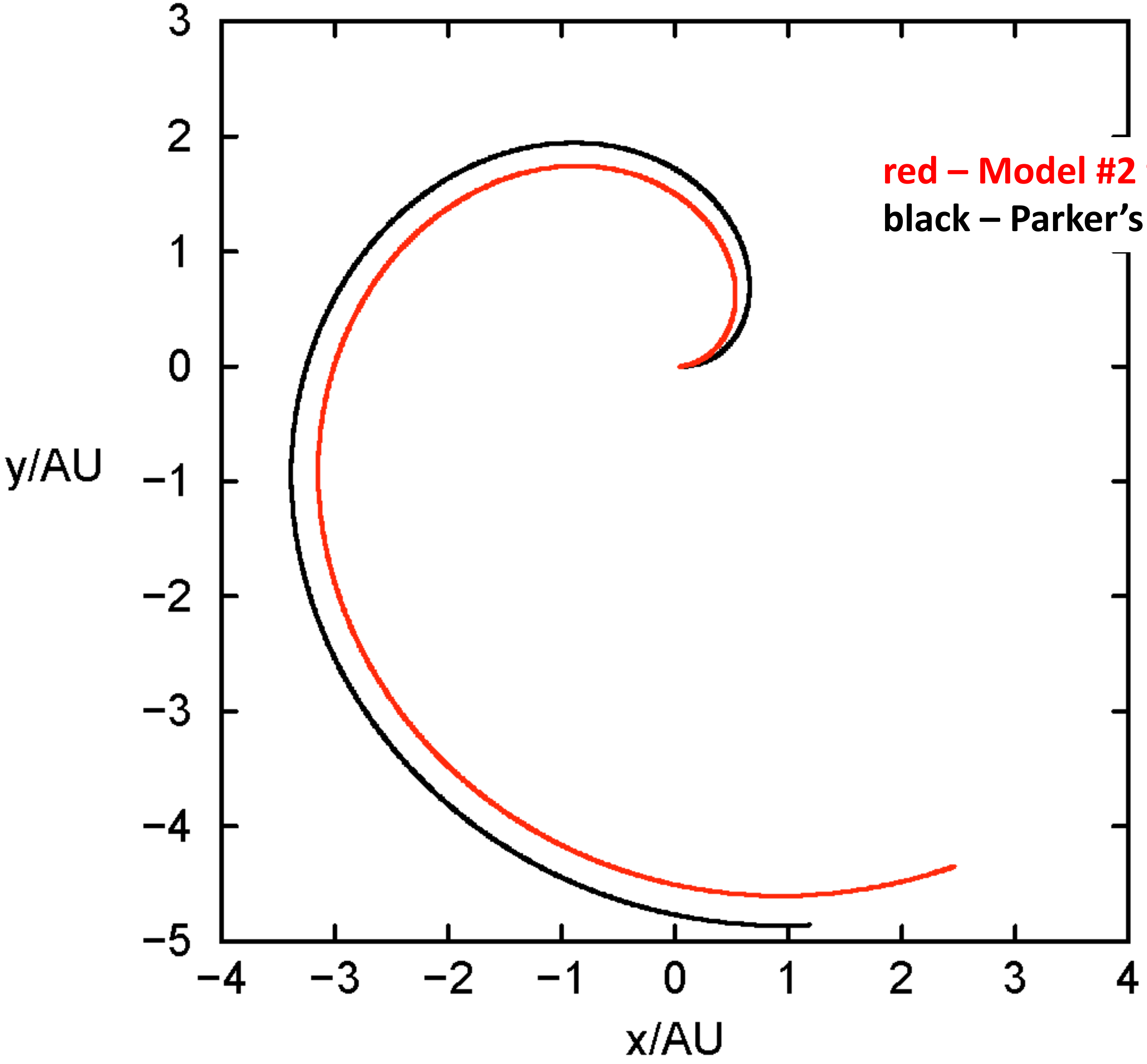
- This form has a radial magnetic field at the inner boundary **and** conserves angular momentum.
- Compare this with Parker's result.

$$B_r(r,\theta,\phi) = B(\theta,\phi_0)\left(\frac{b}{r}\right)^2,$$

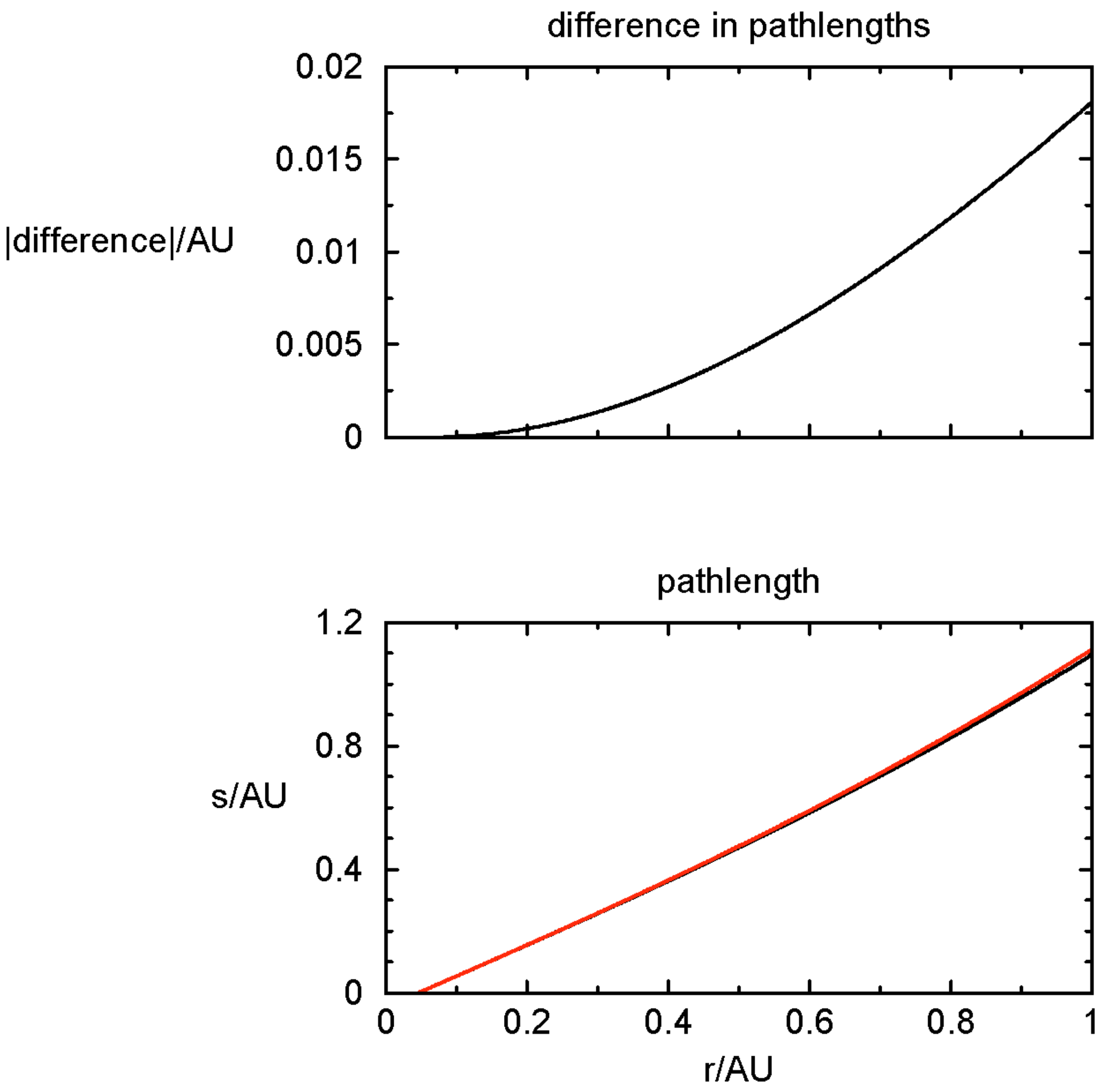
$$B_\theta(r,\theta,\phi) = 0,$$

$$B_\phi(r,\theta,\phi) = B(\theta,\phi_0)\left(\frac{\omega}{v_m}\right)(r-b)\left(\frac{b}{r}\right)^2 \sin\theta,$$

Field lines starting at the same point (at $\boldsymbol{r} = \boldsymbol{R}_0$)

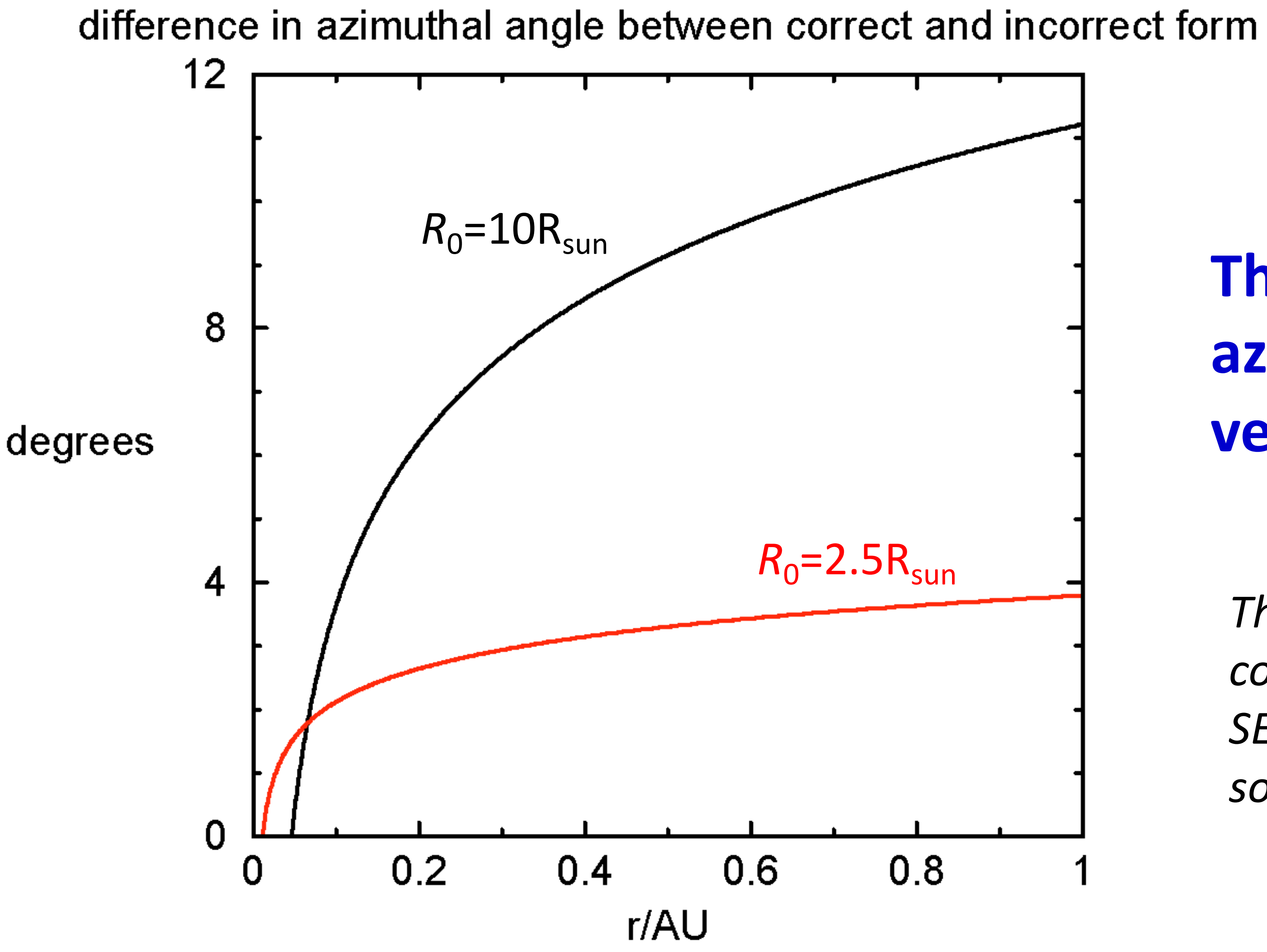


red – Model #2 form (angular momentum conserved)
black – Parker's form



The difference in path-length is fairly small

red – model #2 (angular momentum conserved)
black – Parker's form



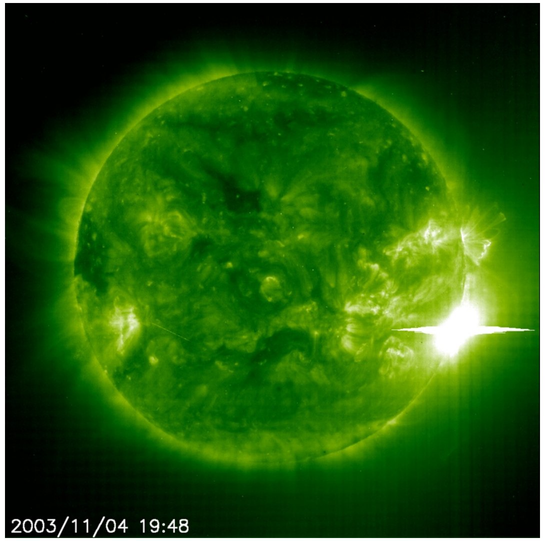
The difference in azimuth angle is very significant

This has important consequences for relating SEPs seen at 1AU with their source at the Sun

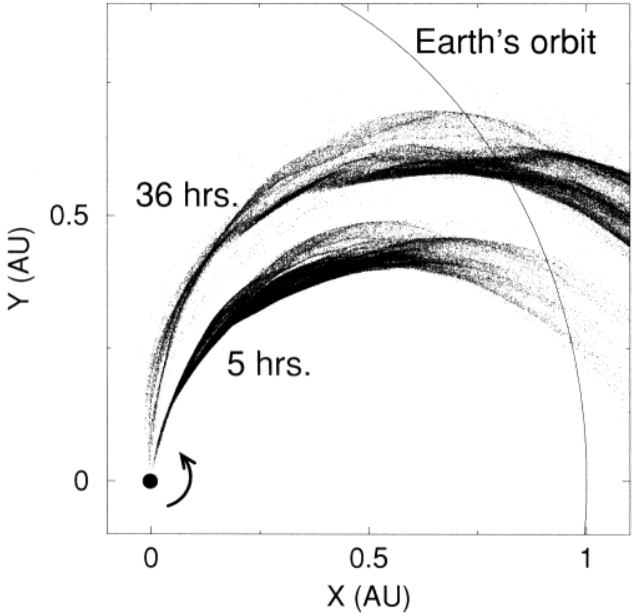
Impulsive Solar-Energetic Particle Events

- Impulsive SEP events seen at 1AU are often characterized by clear velocity dispersion signatures.
  - resembles the "Nike®-swoosh" when plotted as a time vs. MeV/nuc. scatter plot
- That they are impulsive implies that whatever accelerates them does so on a time scale shorter than the transport time scale.
- Ideal for studying charged-particle transport in the interplanetary magnetic field between the Sun and Earth

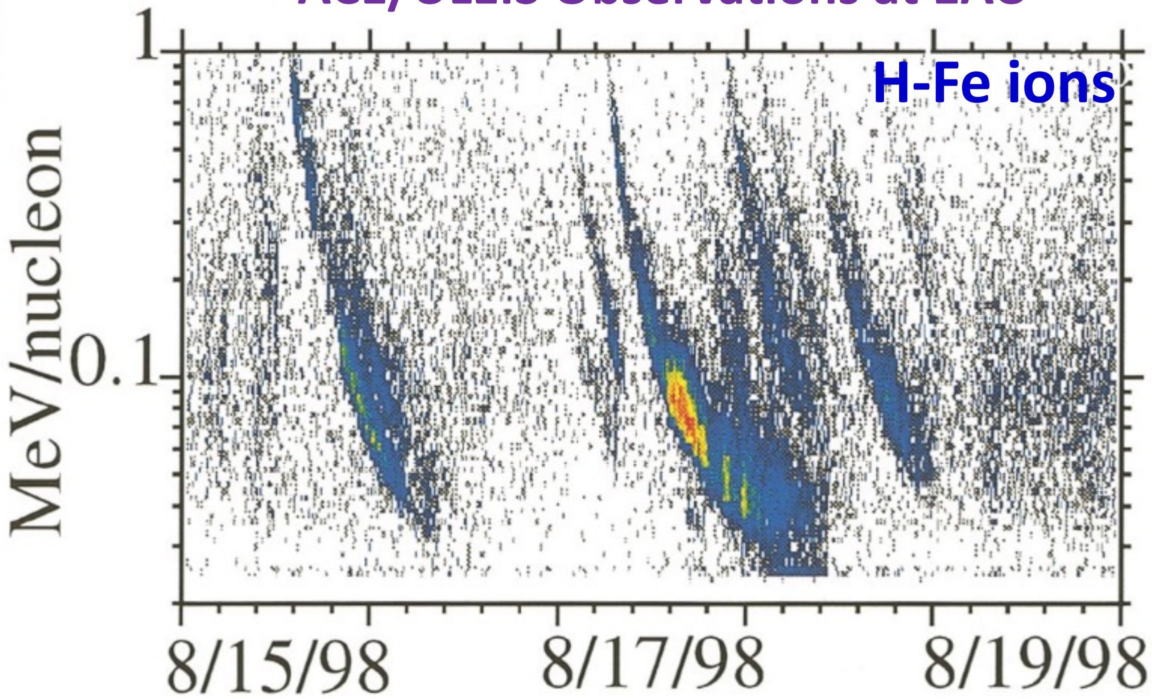
Flare at Sun



Transport in IMF



ACE/ULEIS Observations at 1AU



Mason et al., 1999

Including magnetic stresses

- Recall the (ideal) MHD momentum equation:

$$\frac{\partial \vec{U}}{\partial t} + \vec{U} \cdot \nabla \vec{U} = -\nabla P + \frac{1}{4\pi}(\nabla \times \vec{B}) \times \vec{B} - \rho \vec{g}$$

- The second term on the right can also be included.
- The resulting derivation is not repeated here, but leads to yet another form of the field and solar-wind velocity. Given below:

$$V_\phi = r\Omega_0 \sin\theta \left(\frac{\frac{L}{r^2 \Omega_0 \sin\theta} M_A^2 - 1}{M_A^2 - 1} \right)$$

where

$$M_A = V_w / V_A$$

V_A is the Alfven speed, which depends on r

$$L = R_A^2 \Omega_0 \sin\theta$$

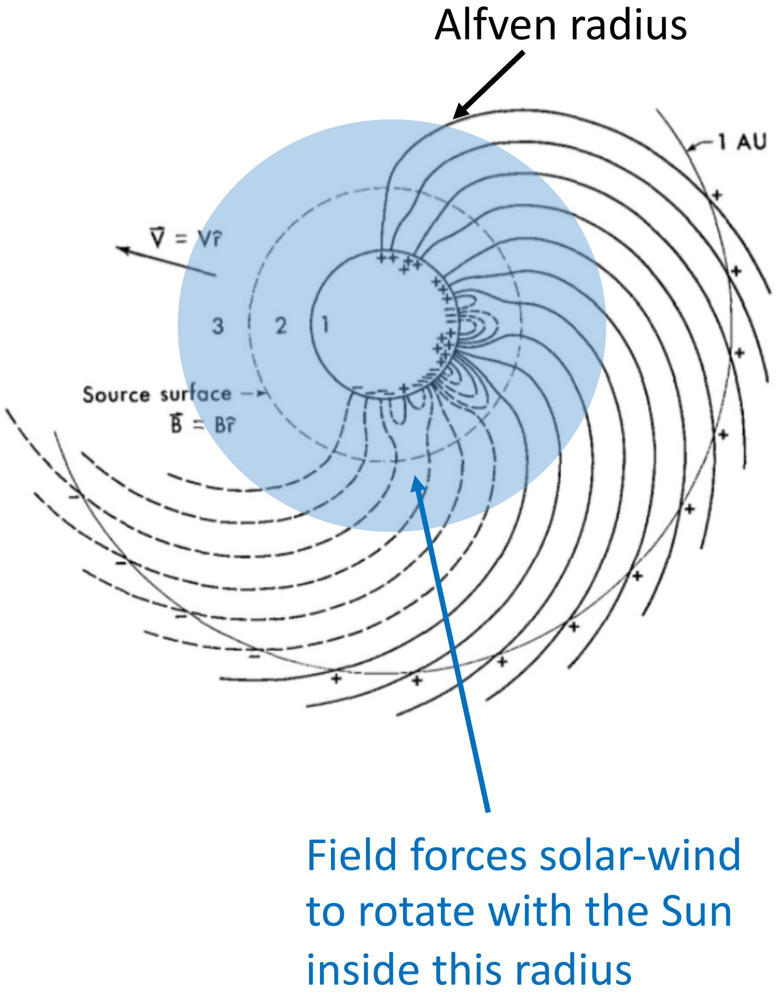
This is a constant

$$B_\phi = \frac{V_\phi}{V_w} B_r - \frac{R_0^2 \Omega_0 B_0 \sin\theta}{r V_w}$$

R_A is the Alfven radius, and is the location where M_A=1

The Alfven radius, angular momentum conservation, and the "spin-down" rate of the Sun (e.g. "magnetic breaking")

- When magnetic stresses are included, the magnetic field forces the solar wind to rotate rigidly with the Sun out to a location of R_A. R_A is about 10-15 solar radii (note: Parker Solar Probe will move inside this radius!)
- Beyond R_A, the solar wind no longer rotates rigidly with the Sun, and the total angular momentum is lost with the solar wind.
- While the loss of solar mass due to solar wind is only a small fraction of the solar mass, the loss of angular momentum is significant enough that the solar rotation rate slows with time.
- "stellar spin down" is an important problem in astrophysics!
- Some key references:



Weber and Davis, Astrophysical Journal, 148, 217, 1967
Priest, E. J., "Magnetohydrodynamics of the Sun", Cambridge Univ. Press
Boyd, T.J.M and J. J. Sanderson, "The Physics of Plasmas", Cambridge University Press
Foukal, P. V., "Solar Astrophysics", Wiley

DIAGRAM NOT TO SCALE!

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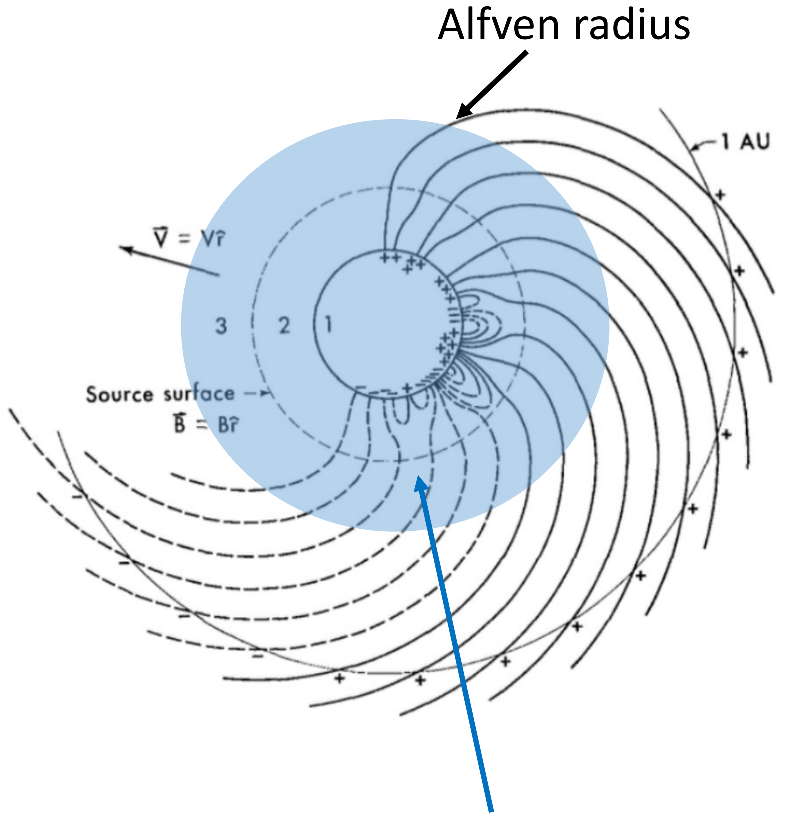

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Re-solved the complete SW problem including magnetic stresses

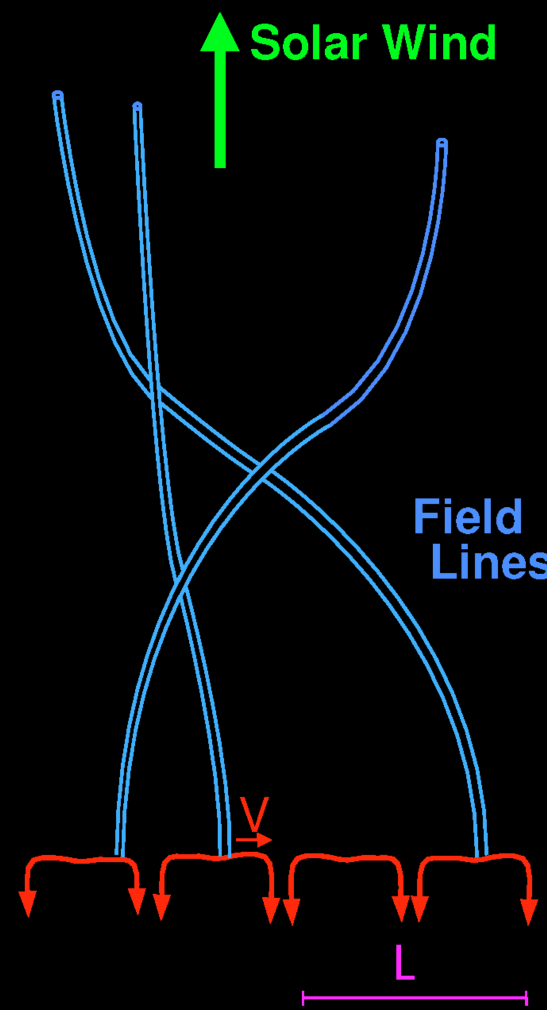
Another model, which includes fluctuations of the fluid velocity at the inner boundary.

In this case, the fluid speed has the form:

$$\vec{U} = \begin{cases} \delta U_\theta(\theta,\phi,t)\hat{\theta} + \left(R_0\Omega_0 \sin\theta + \delta U_\phi(\theta,\phi,t)\right)\hat{\phi} & r = R_0 \\ V_w \hat{r} & r > R_0 \end{cases}$$

the speeds δU_θ and δU_ϕ are "random" or turbulent speeds due to, for example, solar supergranulation

SUPERGRANULATION AND FIELD–LINE RANDOM WALK



From Jokipii & Parker, 1969

SOHO/MDI Doppler image showing transverse motions associated with supergranulation

Photospheric convection spectrum derived from observations by Hathaway et al. (2000)

304 D. H. HATHAWAY ET AL.

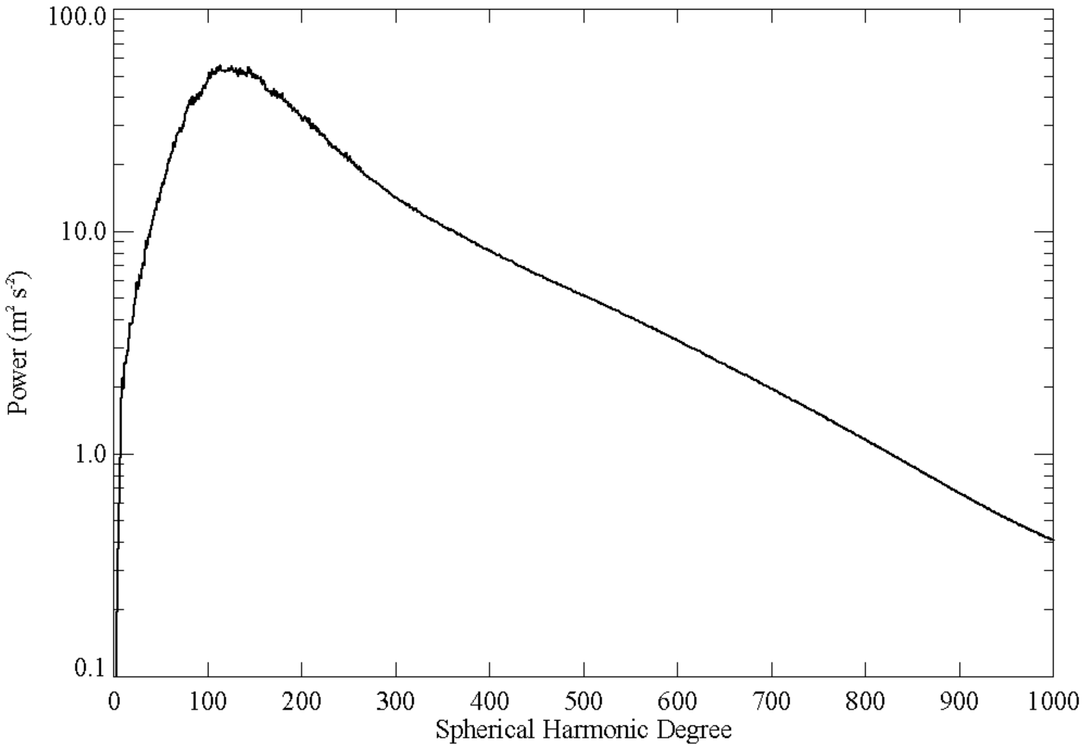


Figure 3. The average observed power spectrum for the cellular photospheric flows in the full-disk MDI data. The peak at $\ell \sim 120$ represents supergranules. There are no significant features to indicate that either mesogranules ($\ell \sim 600$) or giant cells ($\ell < 30$) are distinctly different from supergranules.

The form of the field with fluctuations at the inner boundary (Model #3)

- The form for the magnetic field in this case is given by:

$$B_r = B_0(\theta)\left(\frac{R_0}{r}\right)^2$$

$$B_\theta = -B_0(\theta)\frac{R_0}{r}\frac{\delta U_\theta(\theta,\phi,t_0)}{V_w}$$

$$B_\phi = -B_0(\theta)\frac{R_0}{r}\frac{R_0\Omega_0\sin\theta + \delta U_\phi(\theta,\phi,t_0)}{V_w}$$

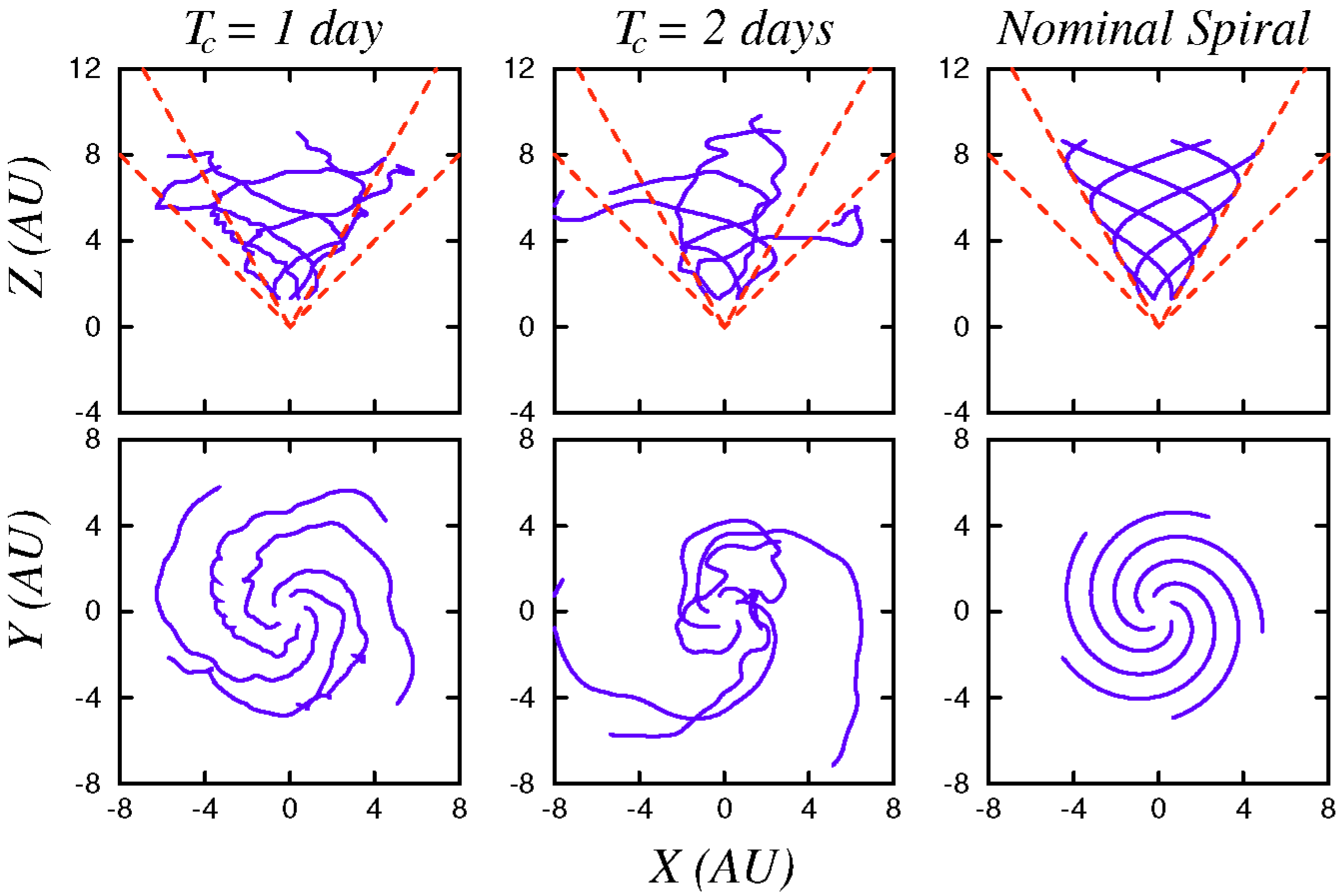
Where

$$t_0 = t - \frac{r - R_0}{V_w}$$

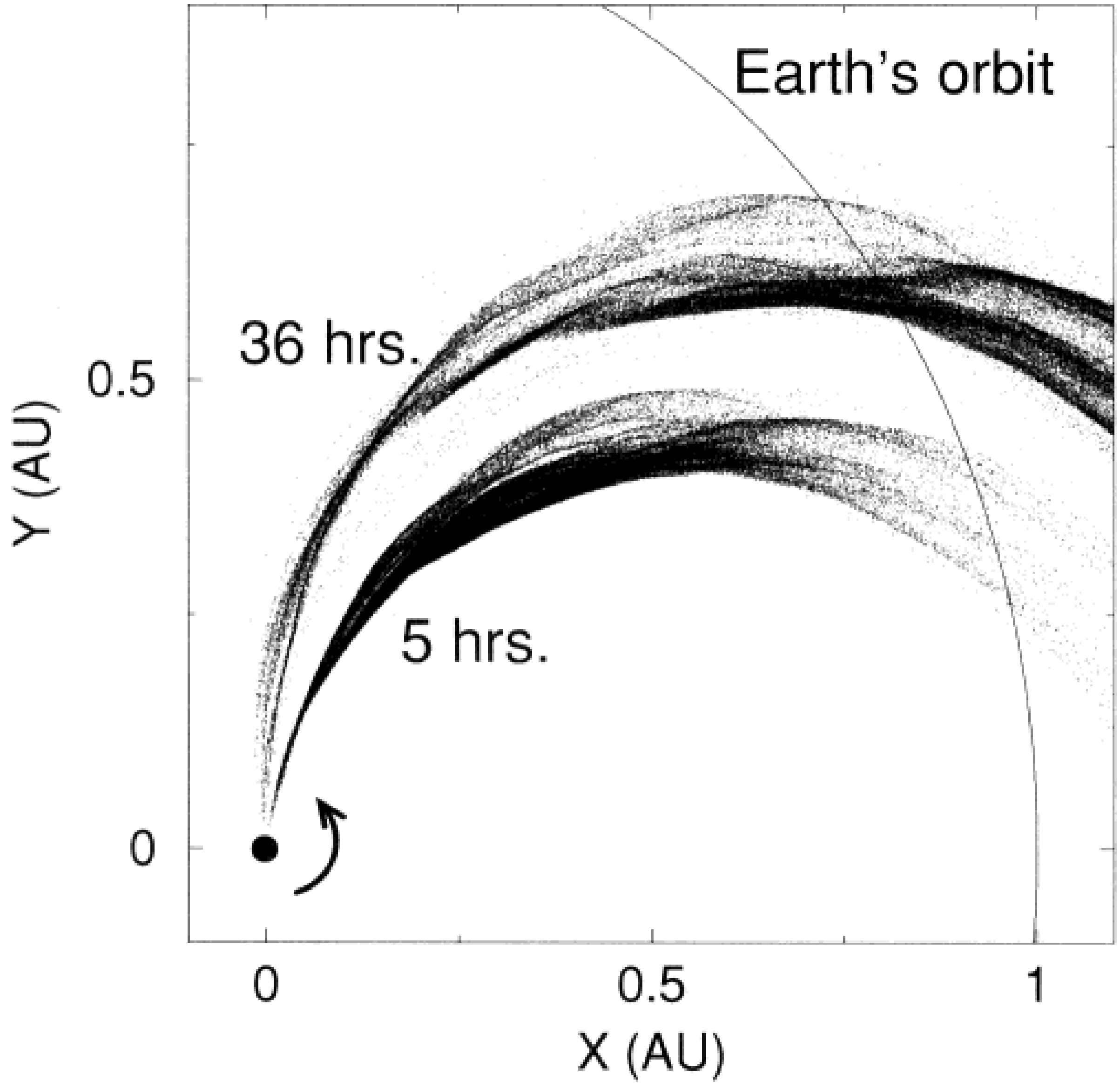
This assumes B_0 is uniform over the surface, but changes sign across the neutral line (leading to heliospheric current sheet)

This model is applicable only at large scales since it ignores magnetic stresses on the plasma

Model Interplanetary Field including Fluctuations Created by Supergranulation



T_c is the characteristic scale of supergranulation

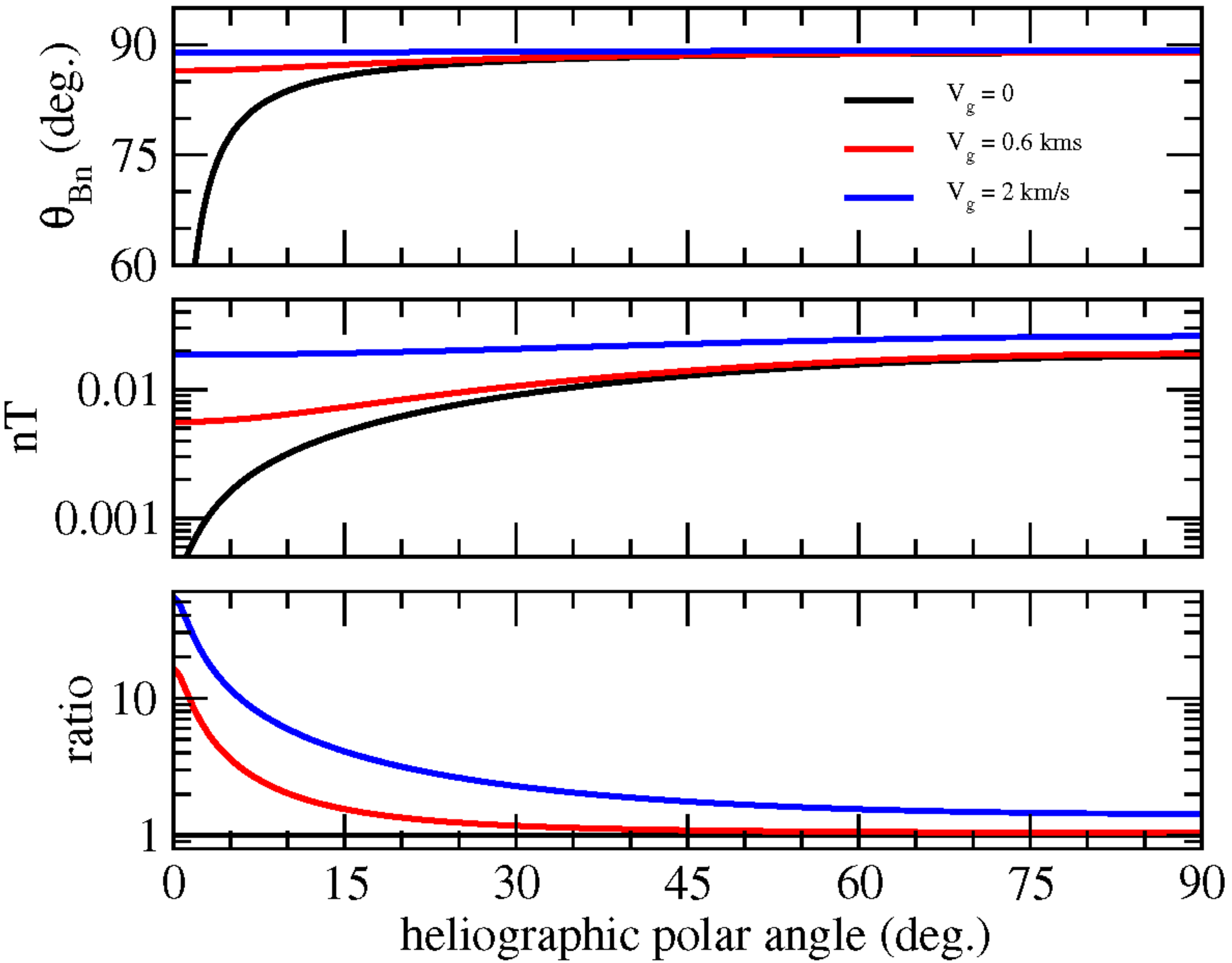


Giacalone, Jokipii, & Mazur, ApJ Lett. 532, 2000

- This field effects the trajectories of energetic particles from solar flares
- In this model, the trajectories of 8 keV/n to 20 MeV/n oxygen from an impulsive flare are determined by numerical integration of the equations of motion with the magnetic field discussed previously
- After ~1 day, ions were still present inside 1 AU and populated field lines spanning ~10º in longitude

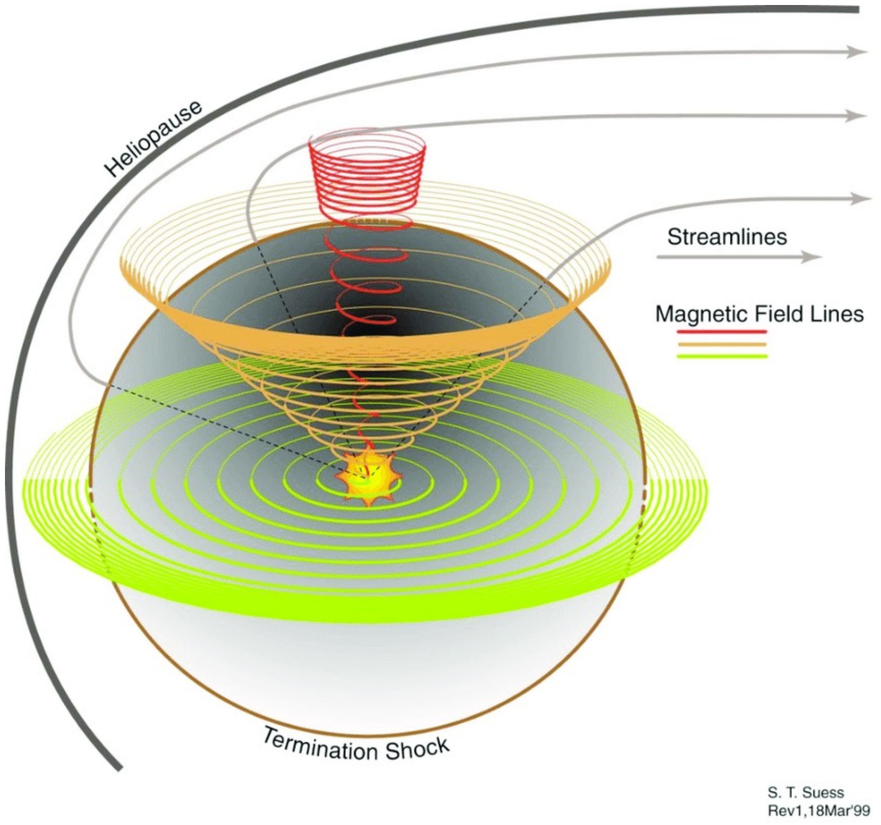
Another important consequence of this model: the nature of the polar heliospheric magnetic field

- The transverse component of the magnetic field falls off only as 1/r
- Radial component as $1/r^2$
- This leads to a large difference between the simple Parker spiral field and that which includes large scale turbulence, especially at high (and low) heliographic latitudes
- This has implications for the basic structure of the heliosphere (and has not been implemented in global heliosphere models, to my knowledge)

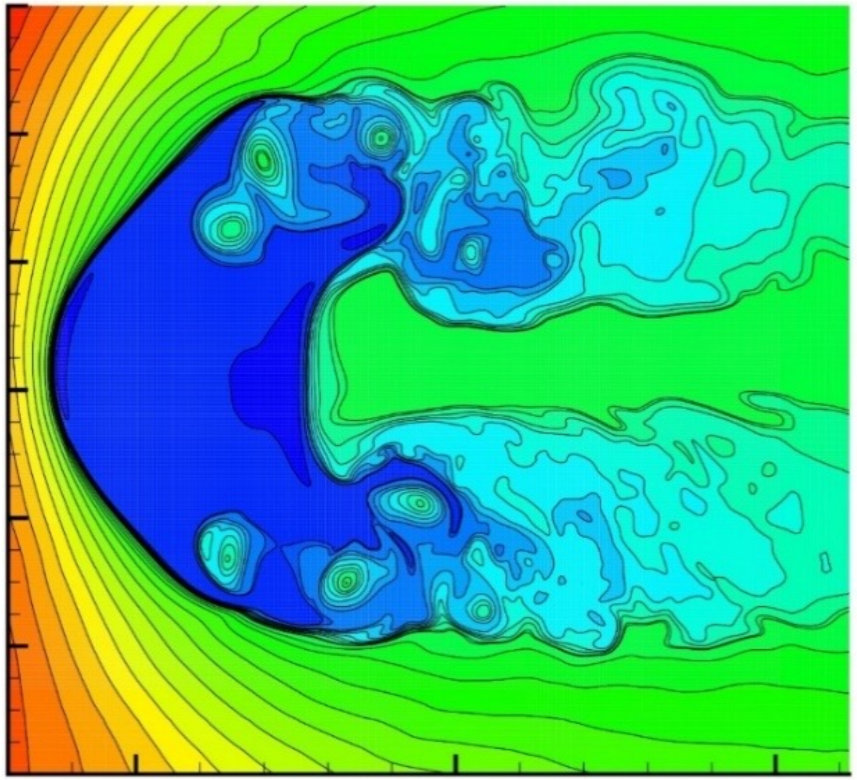


Effect on the shape of the heliosphere?

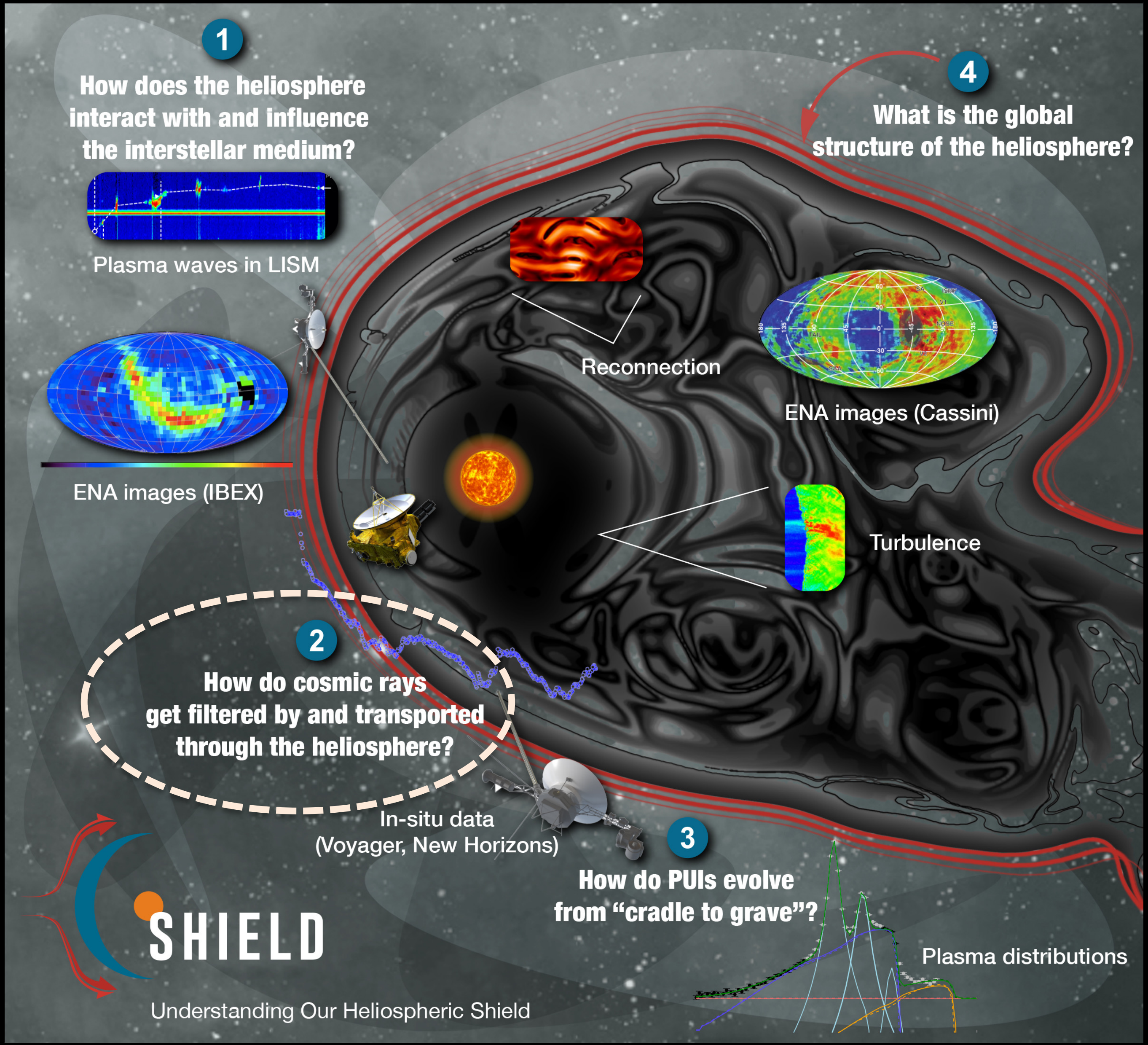
The magnetic field at high heliographic latitudes does not look like the picture below

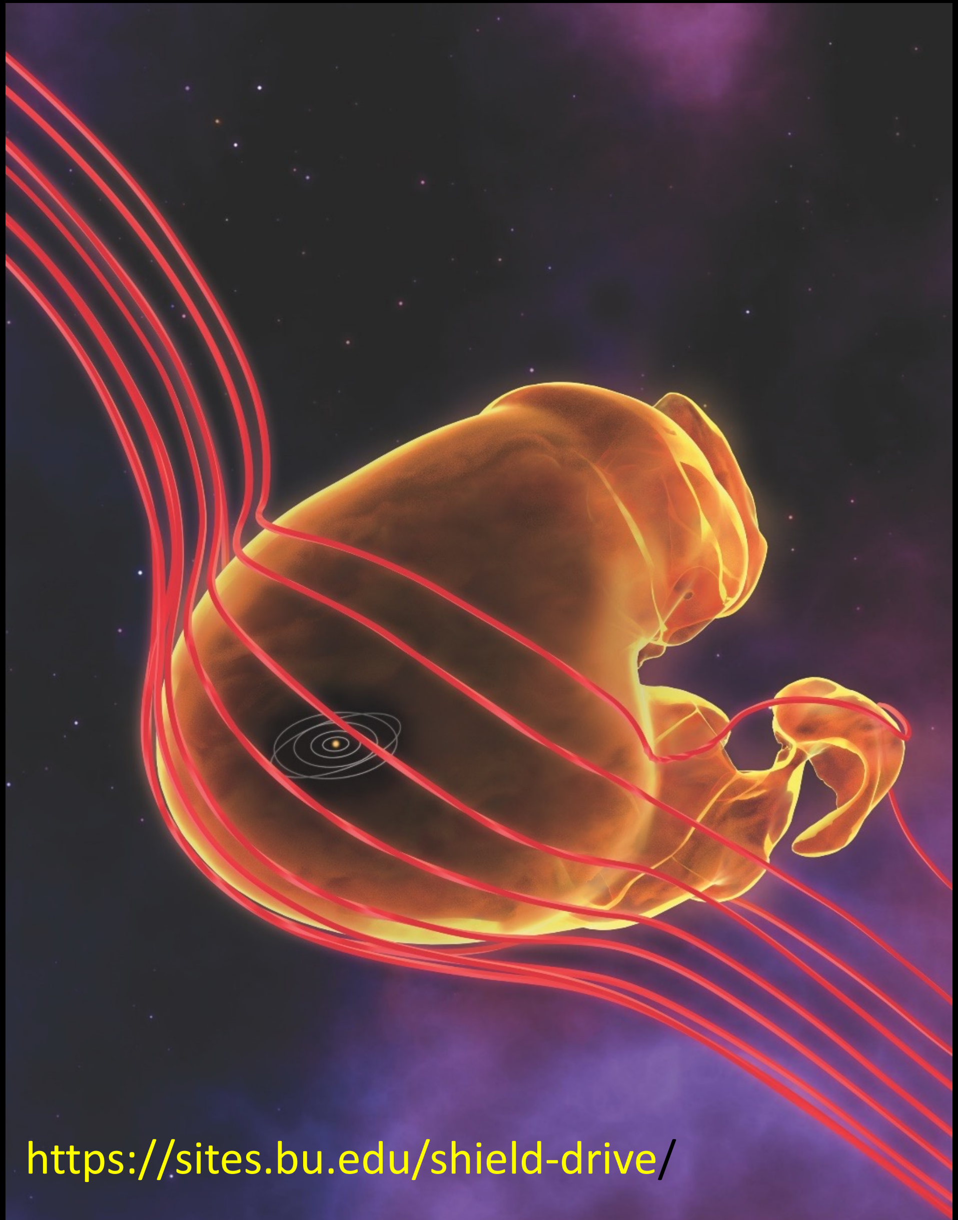


The "Croissant" shaped Heliosphere (***Opher et al., 2015, 2017***) assumes the field *does* look like that at the left. What would it look like using a more-realistic interplanetary field?



NASA Drive Center: Our Heliospheric Shield





https://sites.bu.edu/shield-drive/

Final comments

- Be mindful of your choice of "Parker spiral" magnetic field for whatever application you are interested
- Even in the "nominal Parker spiral", there are 2 correct versions, and one version that is incorrect in that it does not conserve angular momentum.
- Parker's derived field does not conserve angular momentum, yet there are several textbooks that still use this model without acknowledging this basic caveat.
- Understand the assumptions, and their effect on the results, that go into any particular model!