Exploring the Sun and its effects on the Earth's atmosphere and physical environment...

HIGH ALTITUDE OBSERVATORY

Planetary Dynamos

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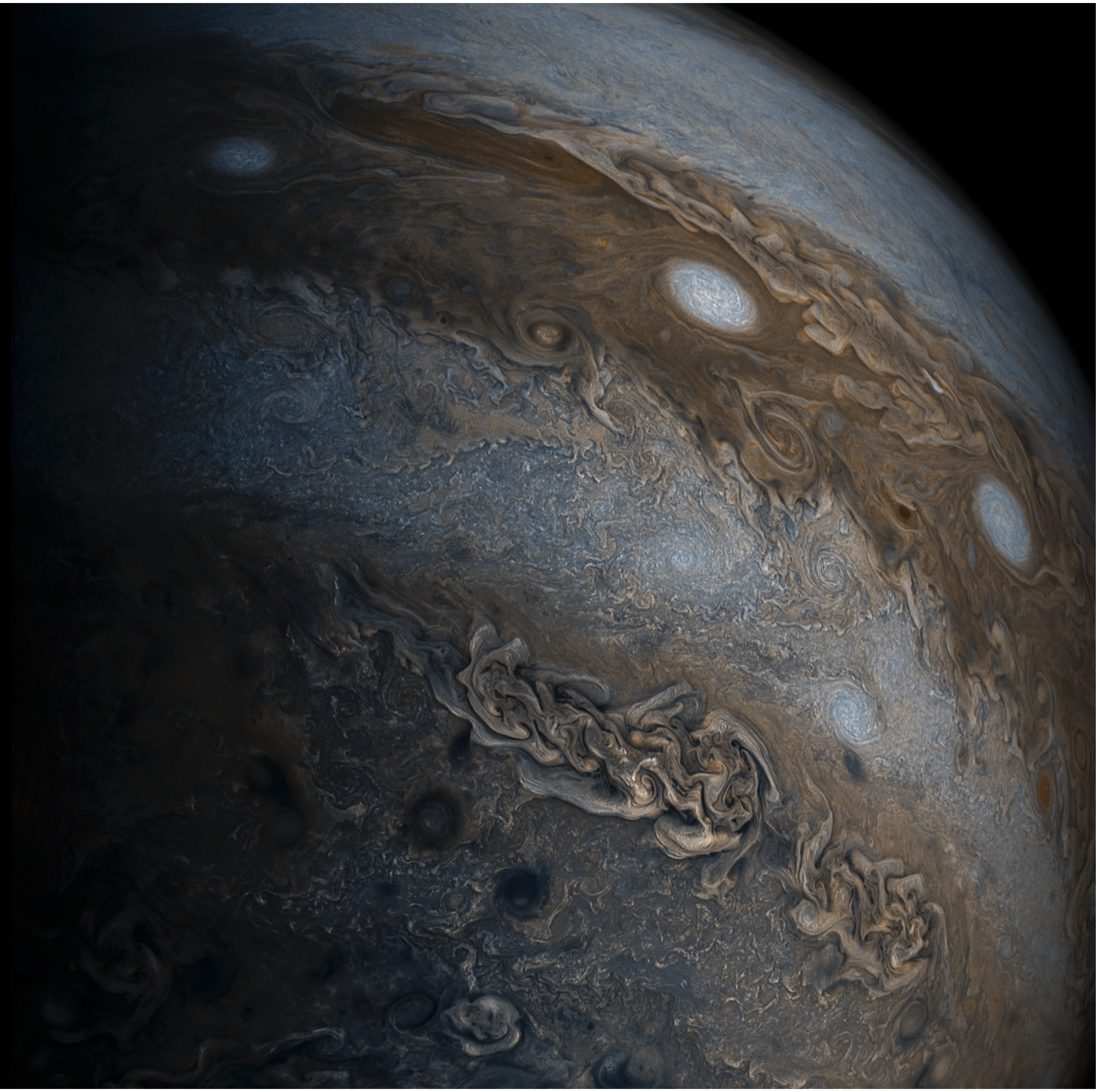
NASA Heliophysics Summer School
Boulder, Colorado

July, 2017



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NCAR

Outline

I) What is a Dynamo?

- Magnetic field creation vs dissipation
- Conditions for a planetary dynamo

II) A whirlwind tour of the Solar System

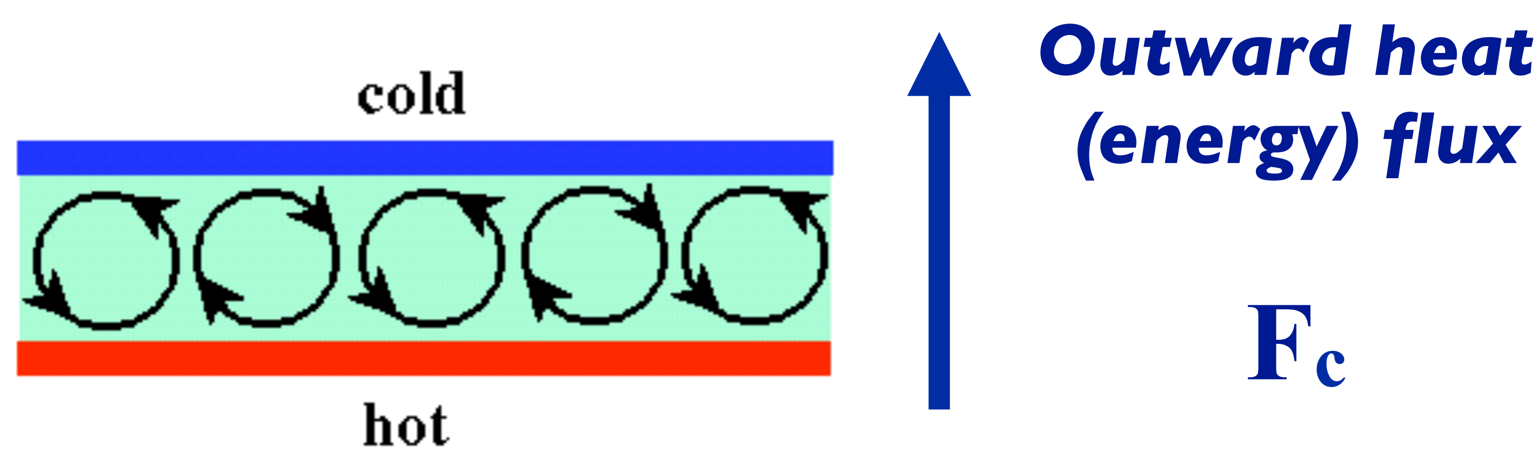
- Observed magnetic fields
- Internal structure

III) Convection in Rotating Spheres

- Dynamical balances
- Columns and waves

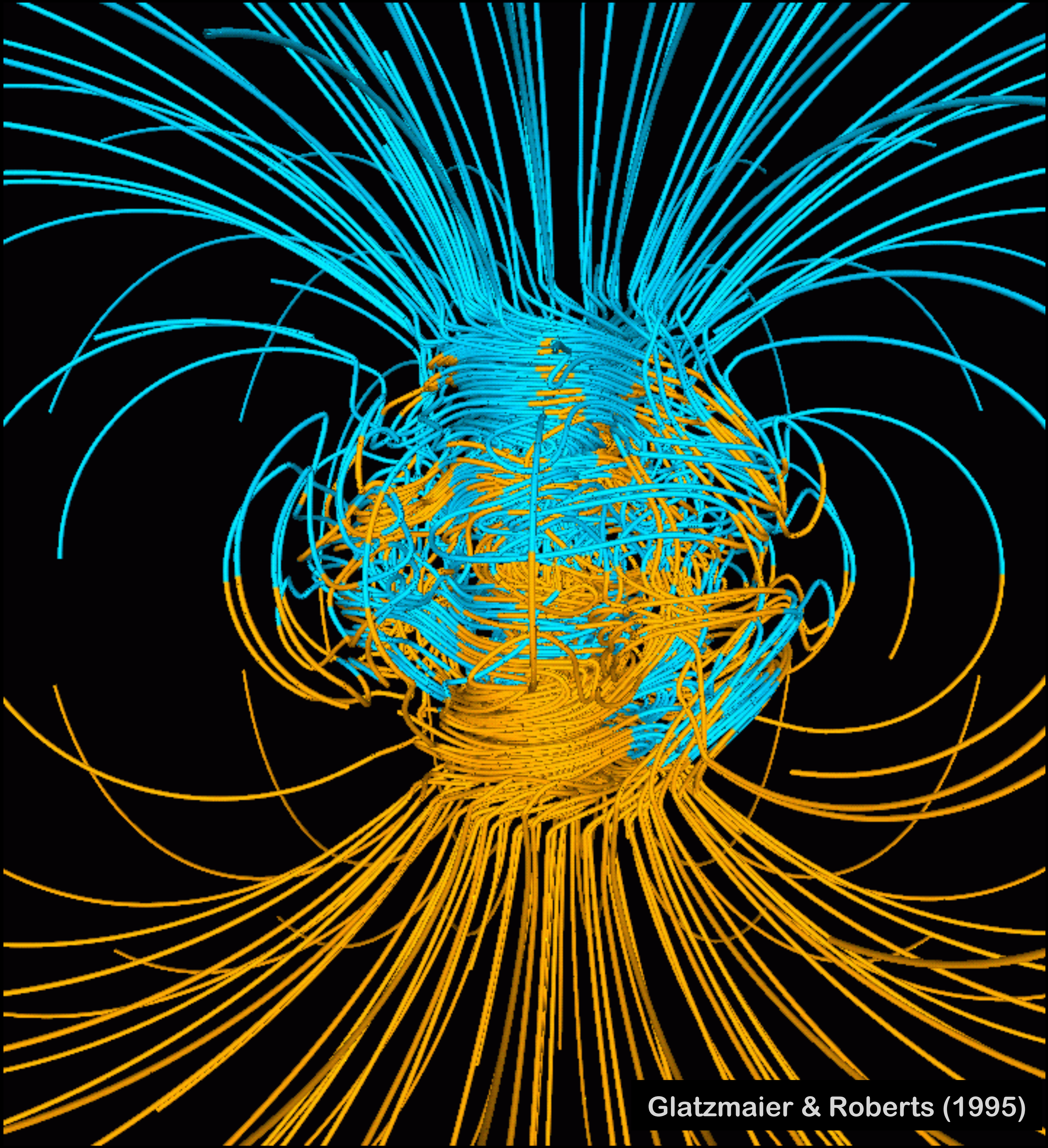
IV) Numerical Models

- General trends
- Case Studies (Earth, Jupiter…)



What is a (hydromagnetic) Dynamo?

An object (such as a star or a planet or a lab experiment) that converts the **kinetic energy** ***of*** **fluid motions** ***into*** **magnetic energy**



MHD Magnetic Induction equation

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})}$$

Comes from Maxwell's equations (Faraday's Law and Ampere's Law)

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}$$ ***(Assumes v << c)***

And Ohm's Law

$$\tilde{\mathbf{J}} = \sigma \tilde{\mathbf{E}}$$

electrical conductivity

Magnetic diffusivity

$$\eta = \frac{c^2}{4\pi\sigma}$$

Creation and destruction of magnetic fields

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})}$$

Source of Magnetic Energy

Sink of Magnetic Energy

How would you demonstrate this?

$$E_m = \frac{B^2}{8\pi}$$

(Hint: have a sheet handy with lots of vector identities!)

Creation and destruction of magnetic fields

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})}$$

Source of Magnetic Energy

Sink of Magnetic Energy

$$\frac{\partial E_m}{\partial t} = -\nabla \cdot \mathbf{F}_P - \frac{\mathbf{v}}{c} \cdot (\mathbf{J} \times \mathbf{B}) - \Phi_o$$

Poynting Flux

$$\mathbf{F}_P = \mathbf{E} \times \mathbf{B} = \left[\frac{\eta}{c} \mathbf{J} - \frac{1}{4\pi} (\mathbf{v} \times \mathbf{B}) \right] \times \mathbf{B}$$

Ohmic Heating

$$\Phi_o = \frac{4\pi\eta}{c^2} J^2$$

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy
~ U B / D

Sink of Magnetic Energy
~ η B / D^2

$$\mathrm{Rm} = \frac{UD}{\eta}$$

If Rm >> 1 the source term is much bigger than the sink term

....Or is it???

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy
~ U B / D

Sink of Magnetic Energy
~ η B / δ^2

δ can get so small that the two terms are comparable

It's not obvious which term will "win" - it depends on the subtleties of the flow, including geometry & boundary conditions

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy
$\sim U\ B\ /\ D$

Sink of Magnetic Energy
$\sim \eta\ B\ /\ \delta^2$

What is a Dynamo? (A corollary)

A dynamo must sustain the magnetic energy (through the conversion of kinetic energy) against Ohmic dissipation

The need for a Dynamo

If v = 0 and η = constant then the induction equation becomes

$$\frac{\partial \mathbf{B}}{\partial t} = -\eta \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{B} = \eta \nabla^2 \mathbf{B}$$

The field will diffuse away (dissipation of magnetic energy) on a time scale of

$$\tau_d \approx \frac{D^2}{\eta}$$

A more careful calculation for a planet gives

$$\tau_d \approx \frac{R^2}{\pi^2 \eta}$$

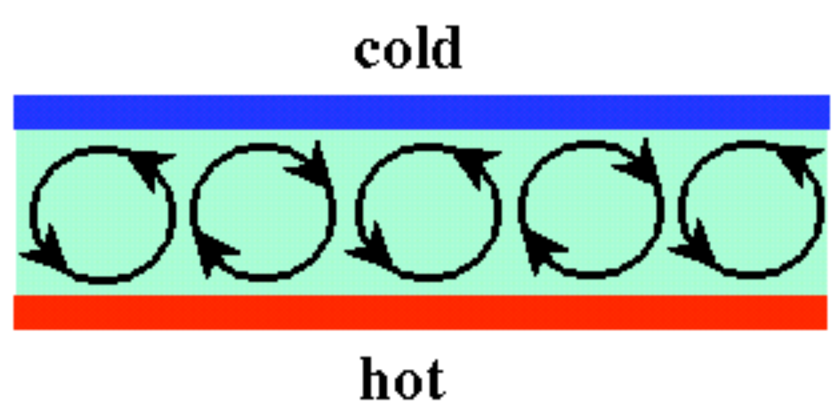
Earth: τ_d ~ 80,000 yrs

Jupiter: τ_d ~ 30 million yrs

Planetary fields must be maintained by a dynamo or they would have decayed by now!

Conditions for a Planetary (or Stellar) Dynamo

- **Absolutely necessary**
  - **An electrically conducting fluid**
    - Stars: plasma
    - Terrestrial planets: molten metal (mostly iron)
    - Jovian planets: metallic hydrogen (maybe molecular H)
    - Ice Giants: water/methane/ammonia mixture
    - Icy moons: salty water
  - **Fluid motions**
    - Usually generated by buoyancy (convection)
  - **Rm >> 1**
    - Too much ohmic diffusion will kill a dynamo



Conditions for a Planetary (or Stellar) Dynamo

- **Not strictly necessary but it (usually) helps**
  - **Rotation**
    - **Good**: helps to build strong, large-scale fields (promotes magnetic self-organization)
    - **Bad**: can suppress convection (though this is usually not a problem for planets)
  - **Turbulence** (low viscosity / Re >> 1)
    - **Good**: Chaotic fluid trajectories good at amplifying magnetic fields (chaotic stretching)
    - **Bad**: can increase ohmic dissipation

Earth

Dynamo!

Field strength
~ 0.4 G

Dipolarity
~ 0.61

Tilt
~ 10°

Archetype of a terrestrial planet!

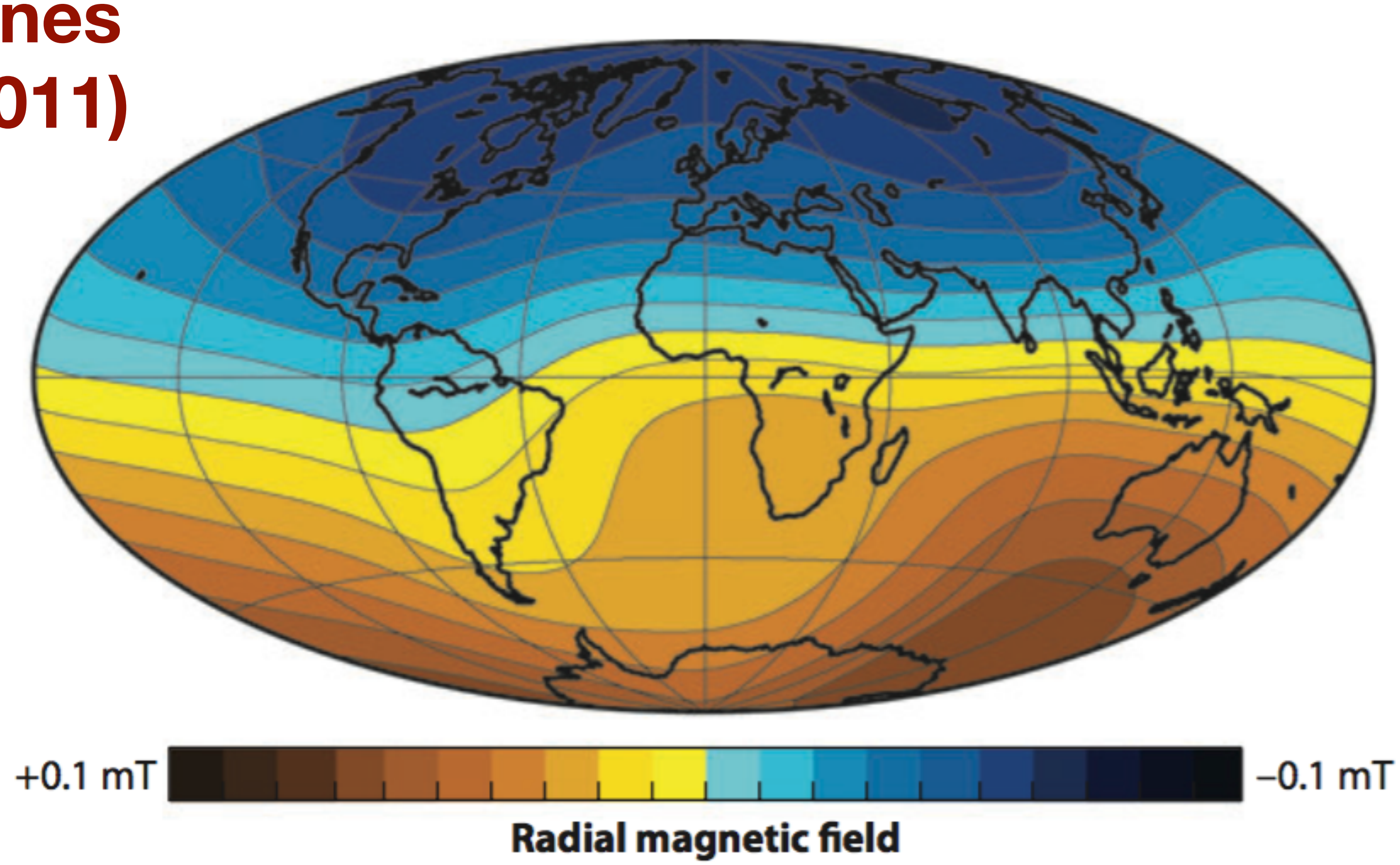
Earth

Magnetometer used by Alexander von Humboldt in his Latin America expedition of 1799-1804

Direct measurements of Earth's magnetic field date back to the early 1500's, with a boost in the early 1800's with the Magnetic Crusade led by Sabine in England and Gauss and Weber in Germany

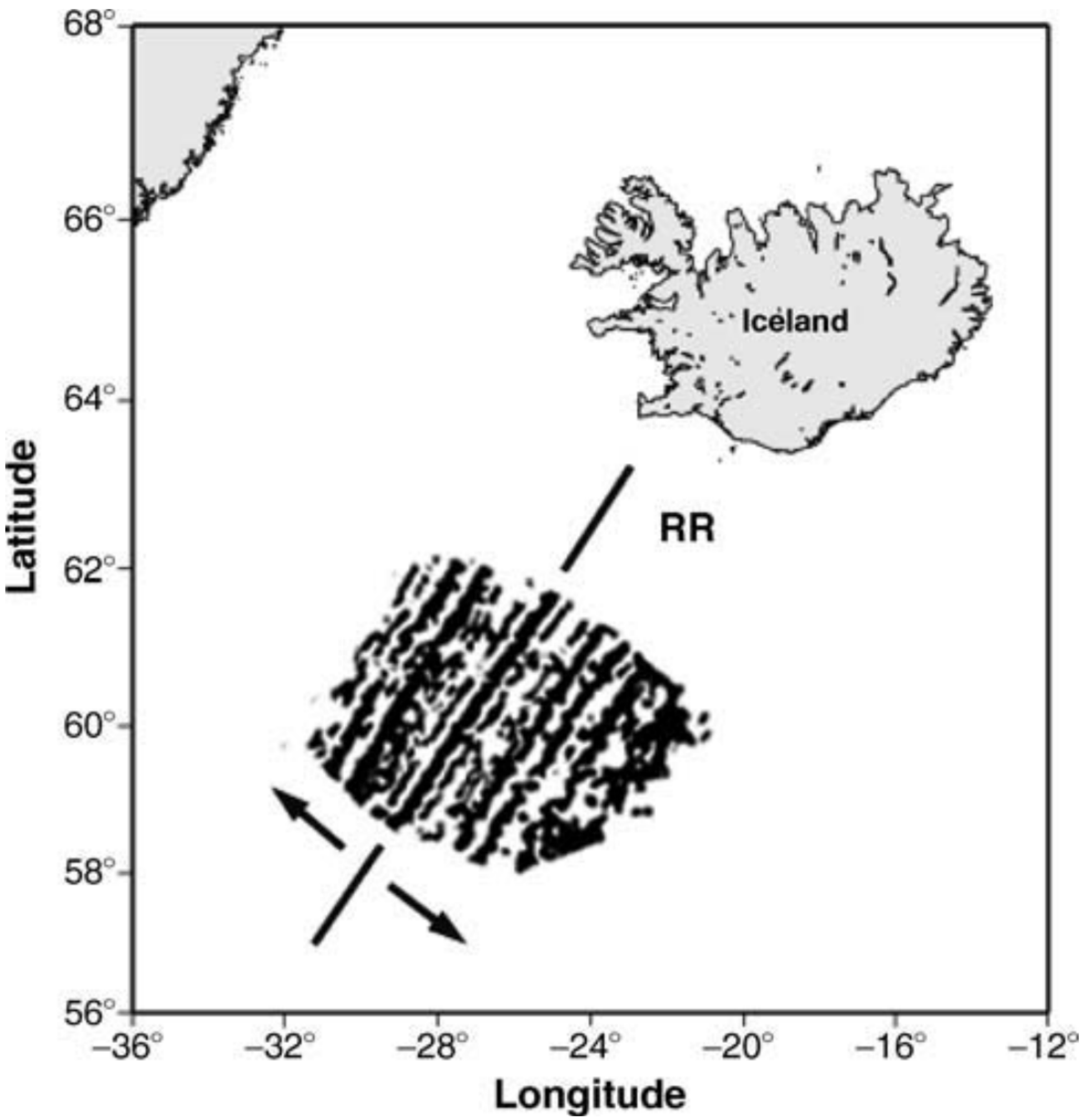
Today we also have satellite measurements

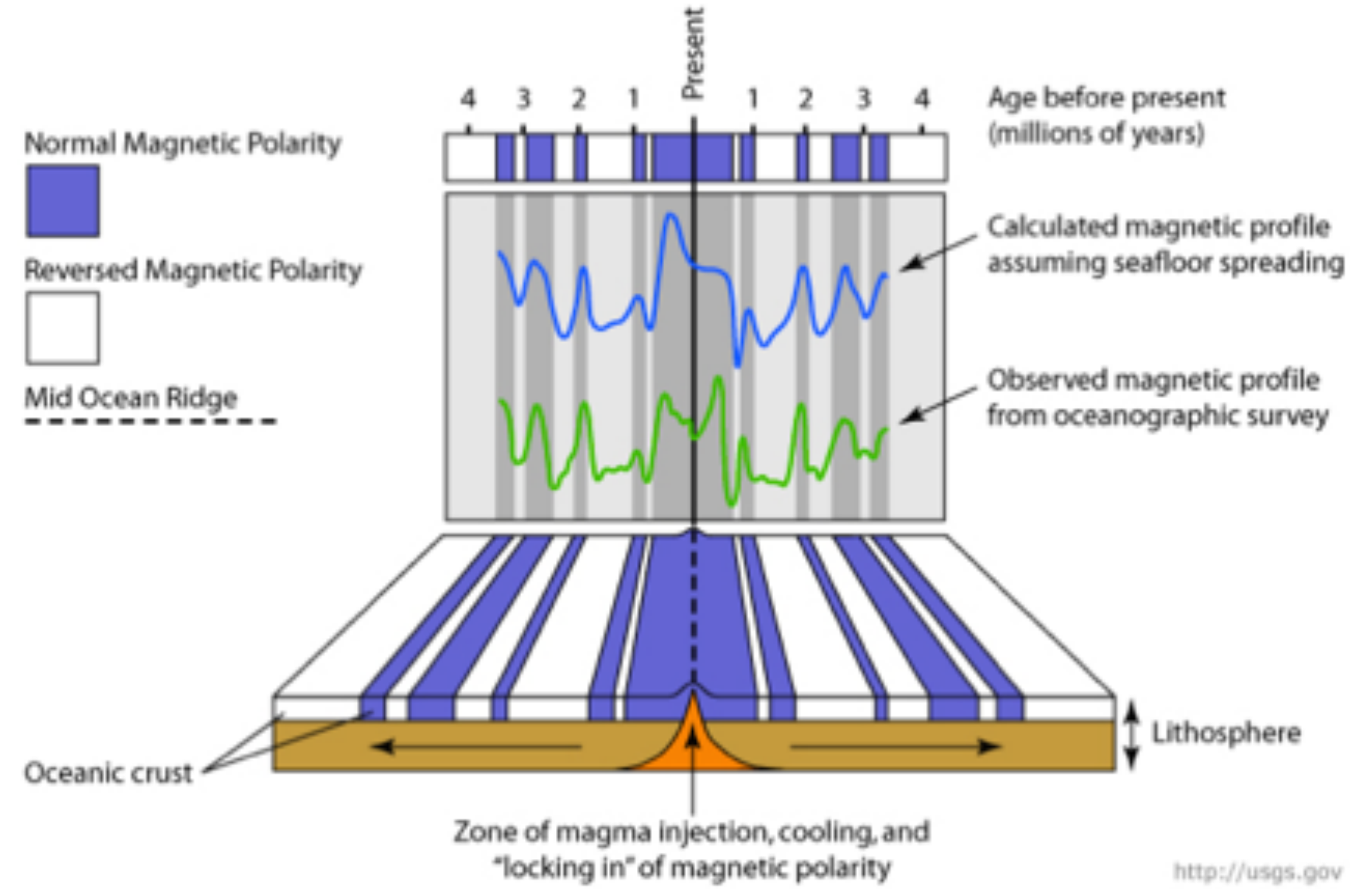
Jones (2011)



Longer time history can be inferred from measurements of magnetic signatures in crustal rocks

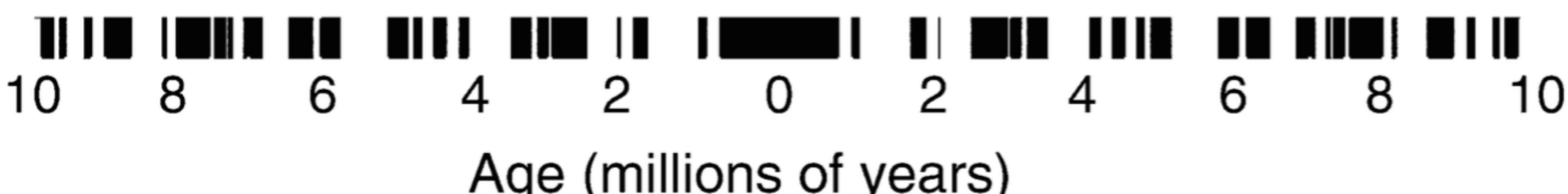
Heirtzler et al (1960's)



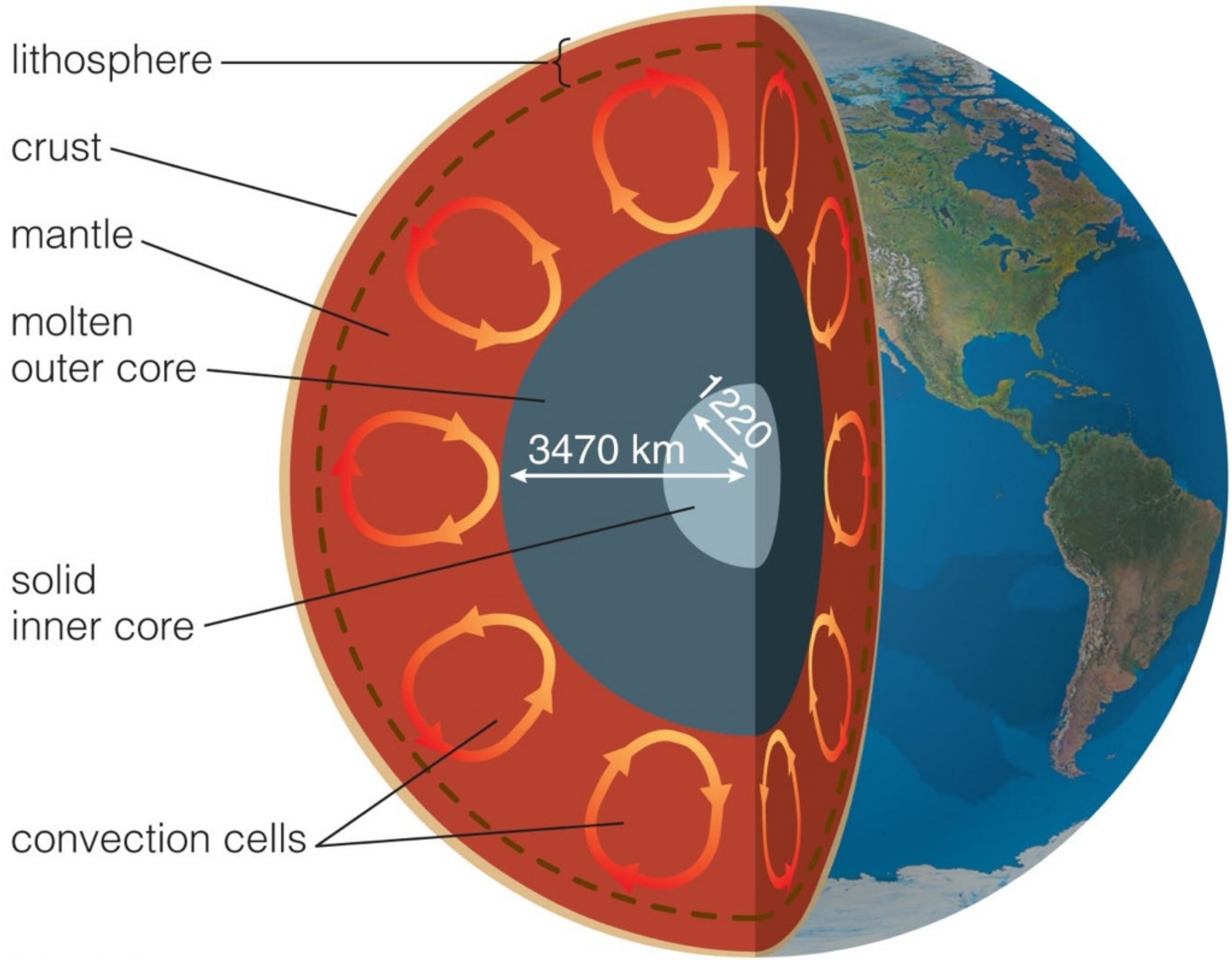


Magnetic poles flip every ~ 200,00 years on average, but randomly

Irregular reversals!



Earth



Mantle convection responsible for plate tectonics but not the geodynamo

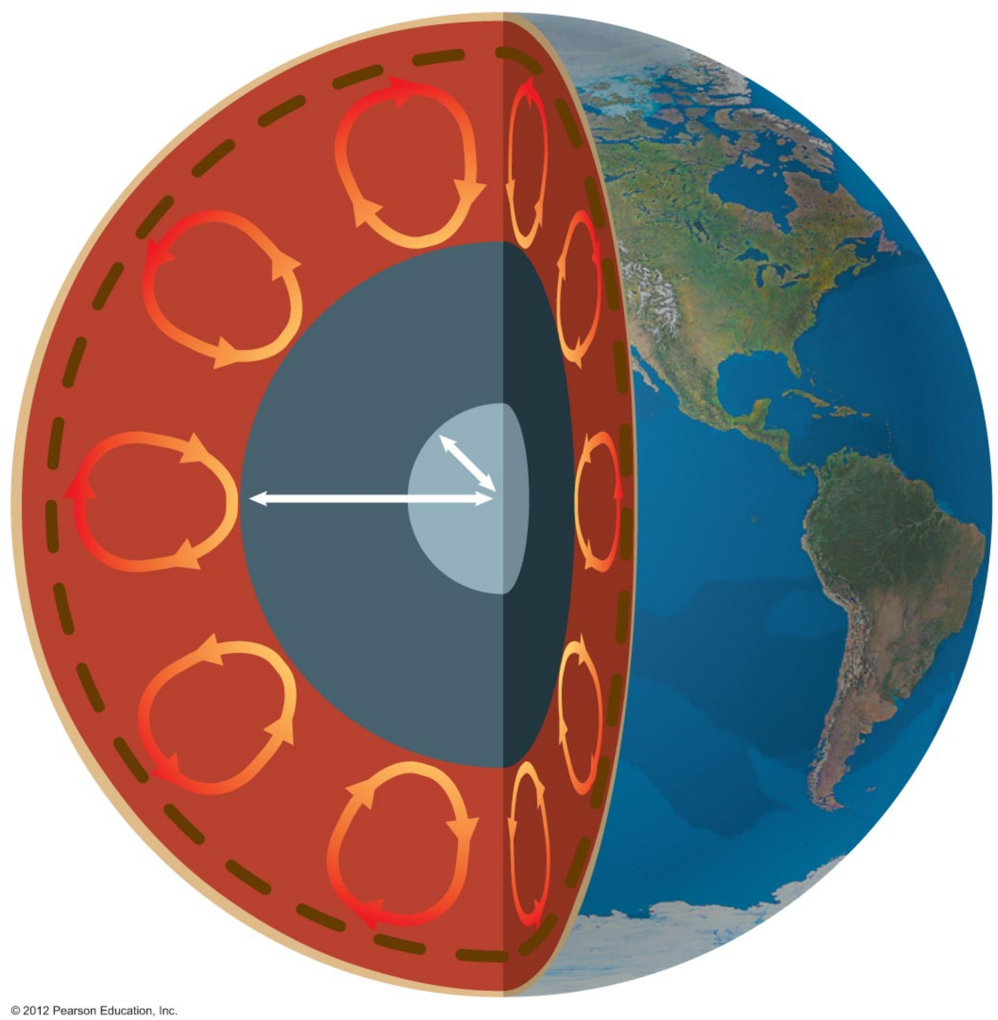
Why?

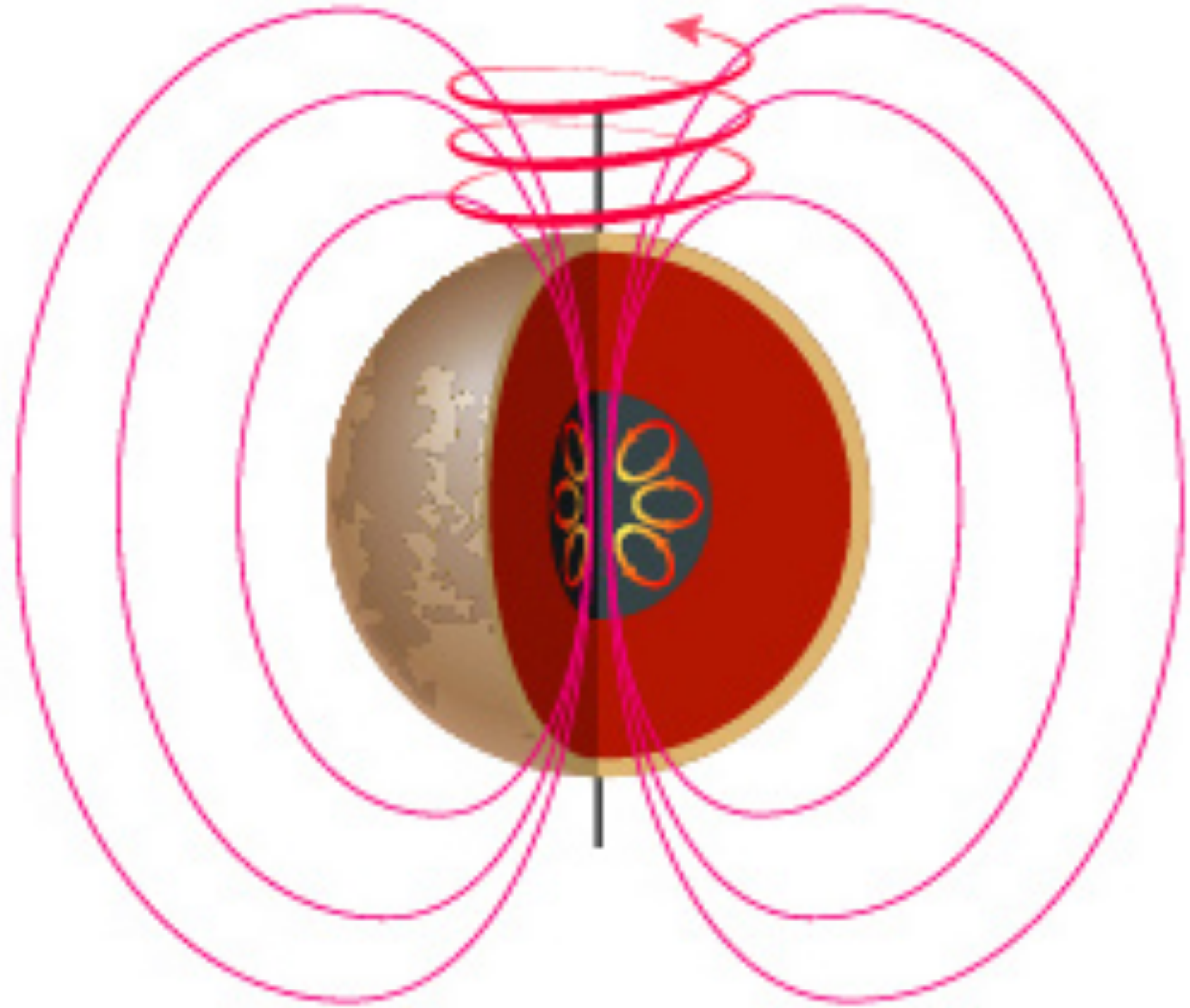
Earth

Mantle
non-conducting, slow

Overturning time

~100 million years





Outer Core
conducting, *fast*

Overturning time

~500 years

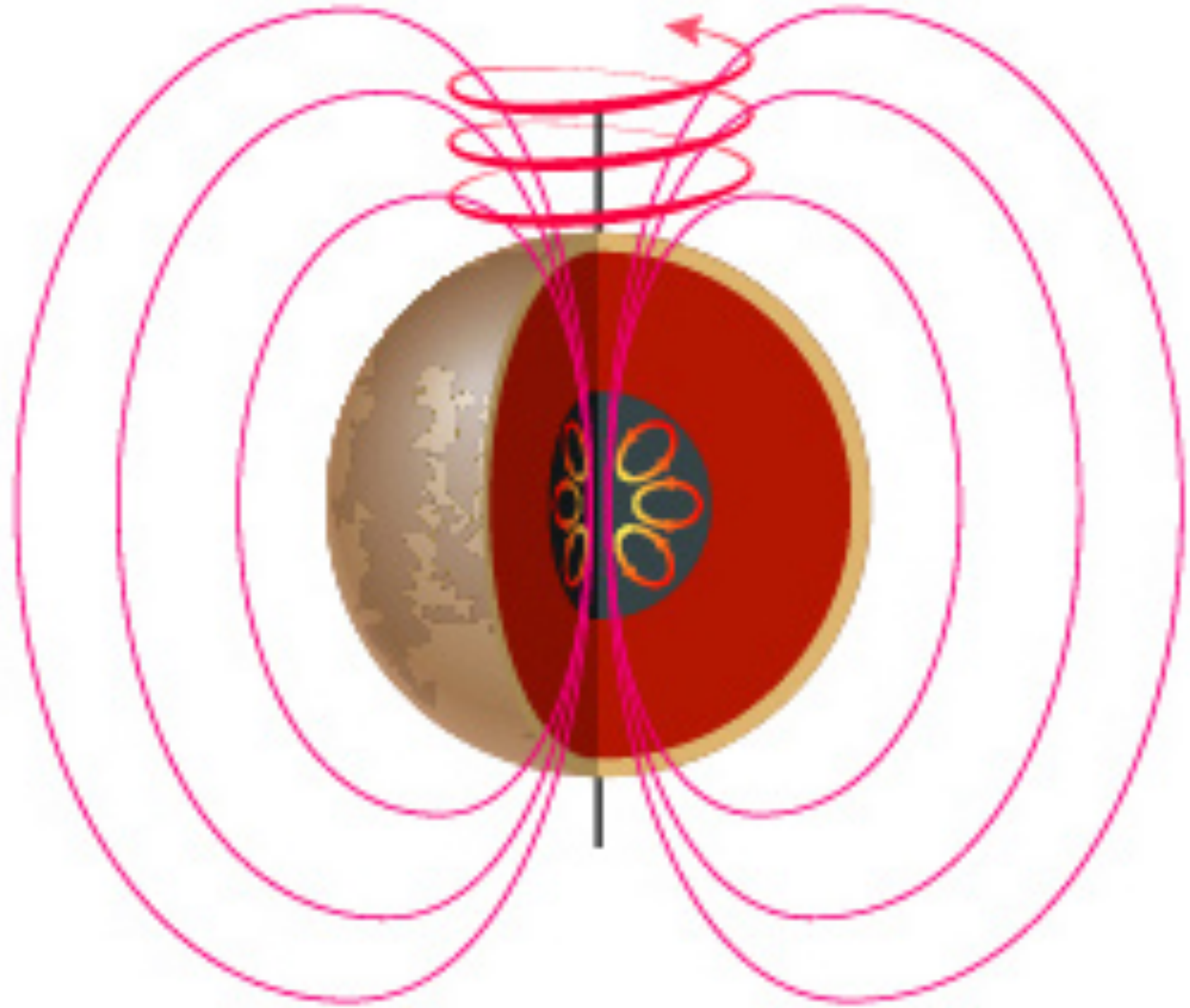
Earth

Rotational influence quantified by

Rossby number

$$\mathrm{Ro} = \frac{U}{2\Omega D} = \frac{1}{4\pi}\,\frac{P_{rot}}{\tau_c}$$

$$\mathrm{Ro} \sim 4 \times 10^{-7}$$

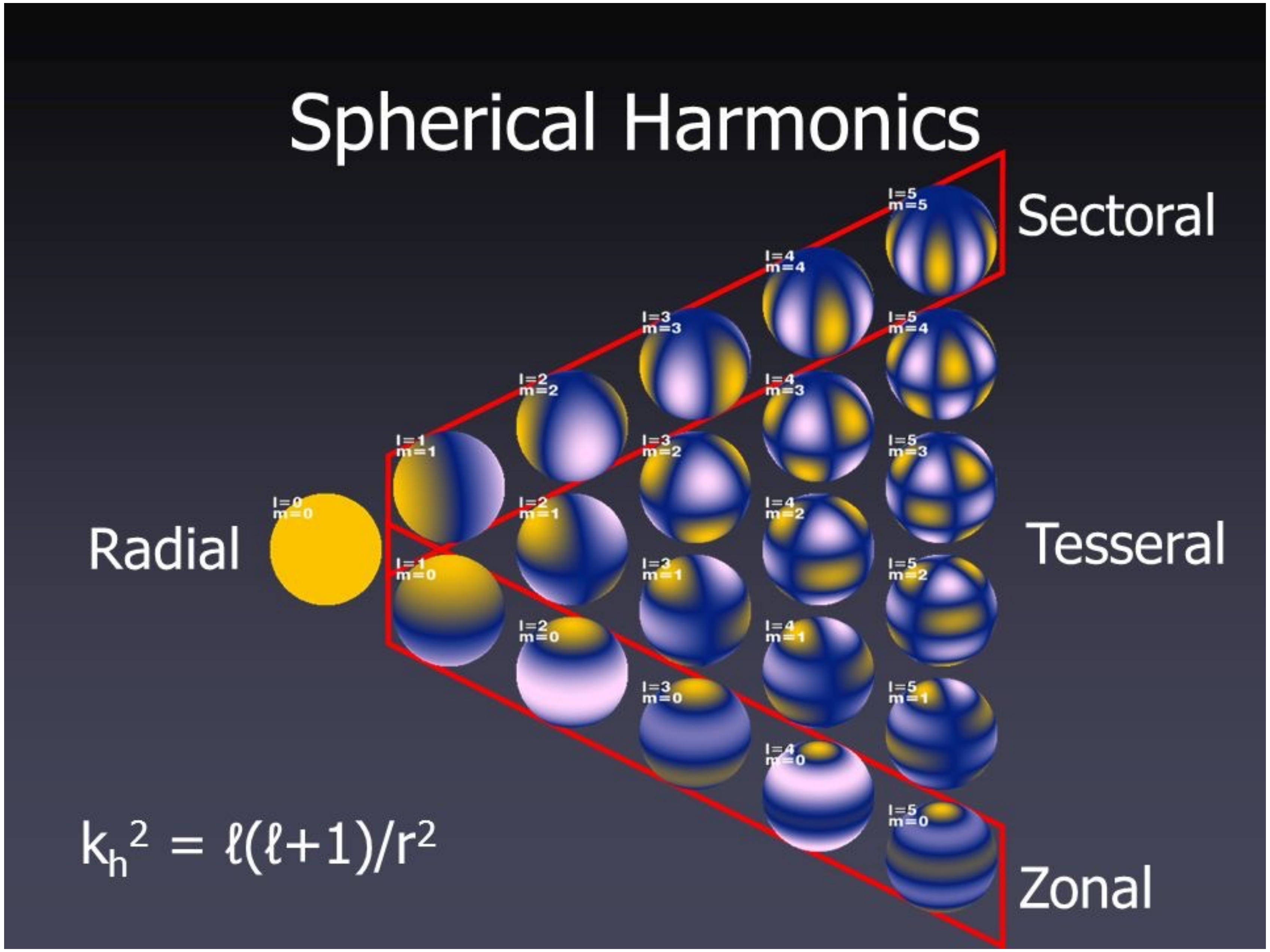


Outer Core
conducting, fast

Overturning time
~500 years

Earth

Spherical Harmonic expansion of the surface field allows for a backward extrapolation to the core-mantle boundary (CMB)



Assuming no currents in the non-conducting mantle & crust

$$B_r \propto r^{-(\ell+2)}$$

R. Townshend (Wisconsin)

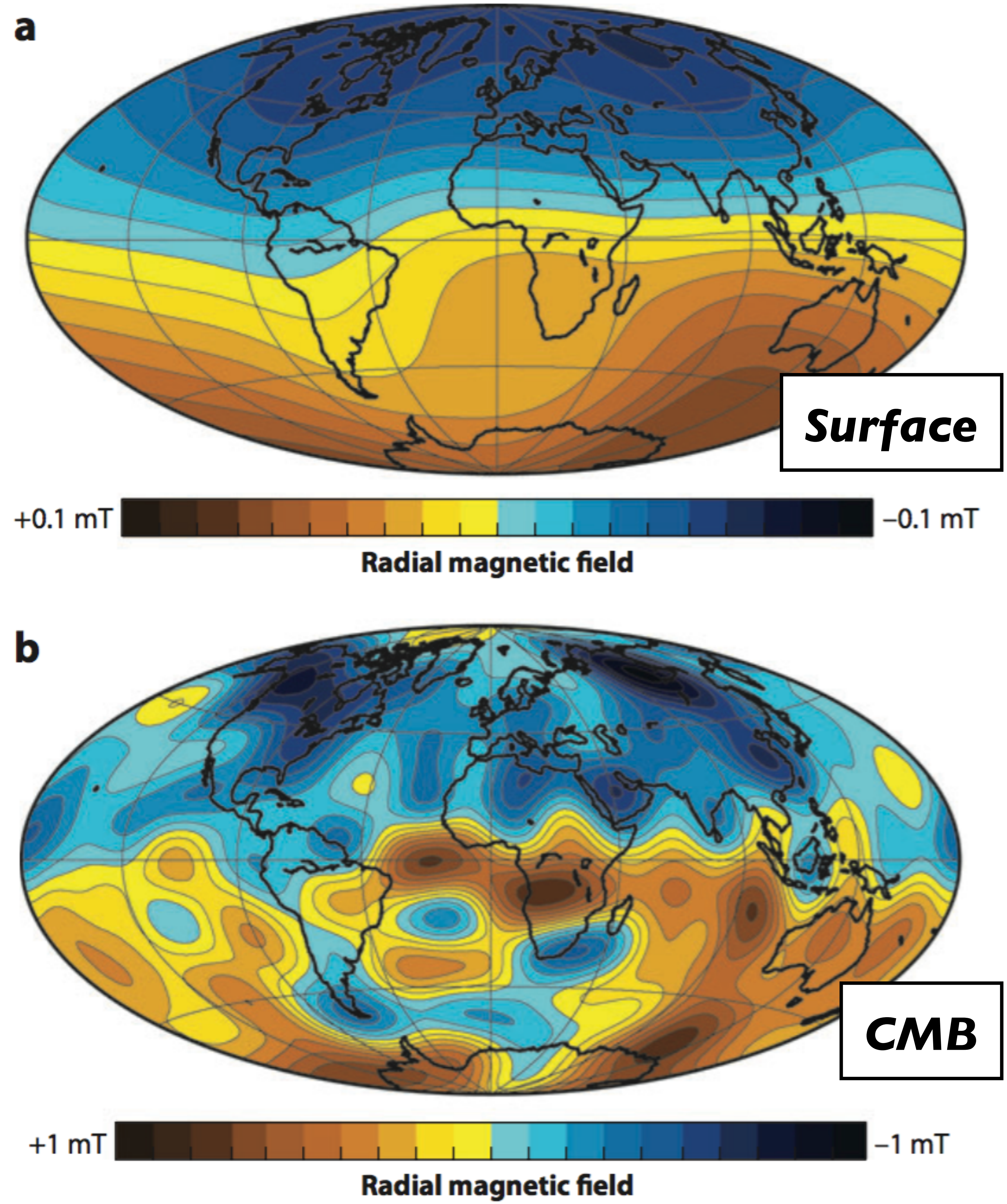
Jones (2011)

Earth

$$B_r \propto r^{-(\ell+2)}$$

Dipole dominates at large distances from the dynamo region $\sim r^{-3}$

Time evolution of surface field can be used to infer flows at the CMB



Earth

- **Energy sources for convective motions**
  - **Outward heat transport by conduction**
    - Cooling of the core over time
    - Proportional to the heat capacity
  - **Latent heat**
    - Associated with the freezing (phase change) of iron onto the solid core
  - **Gravitational Differentiation**
    - Redistribution of light and heavy elements, releasing gravitational potential energy
  - **Radioactive Heating**
    - Energy released by the decay of heavy elements

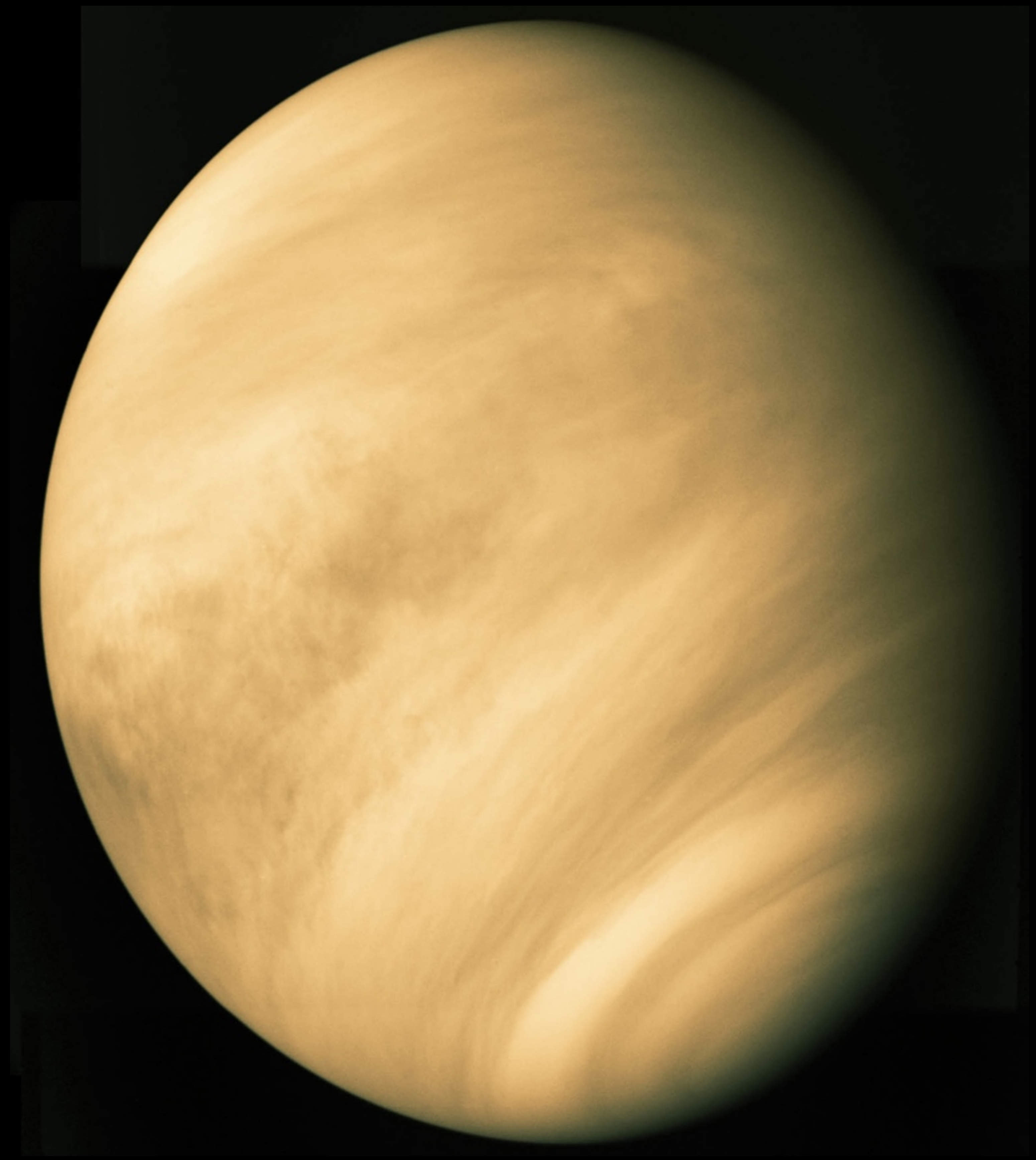
Venus

No Dynamo

No field detected

Why?

Core may be liquid and conducting, but it may not be convecting (rigid top may inhibit cooling)



Also - slow rotation

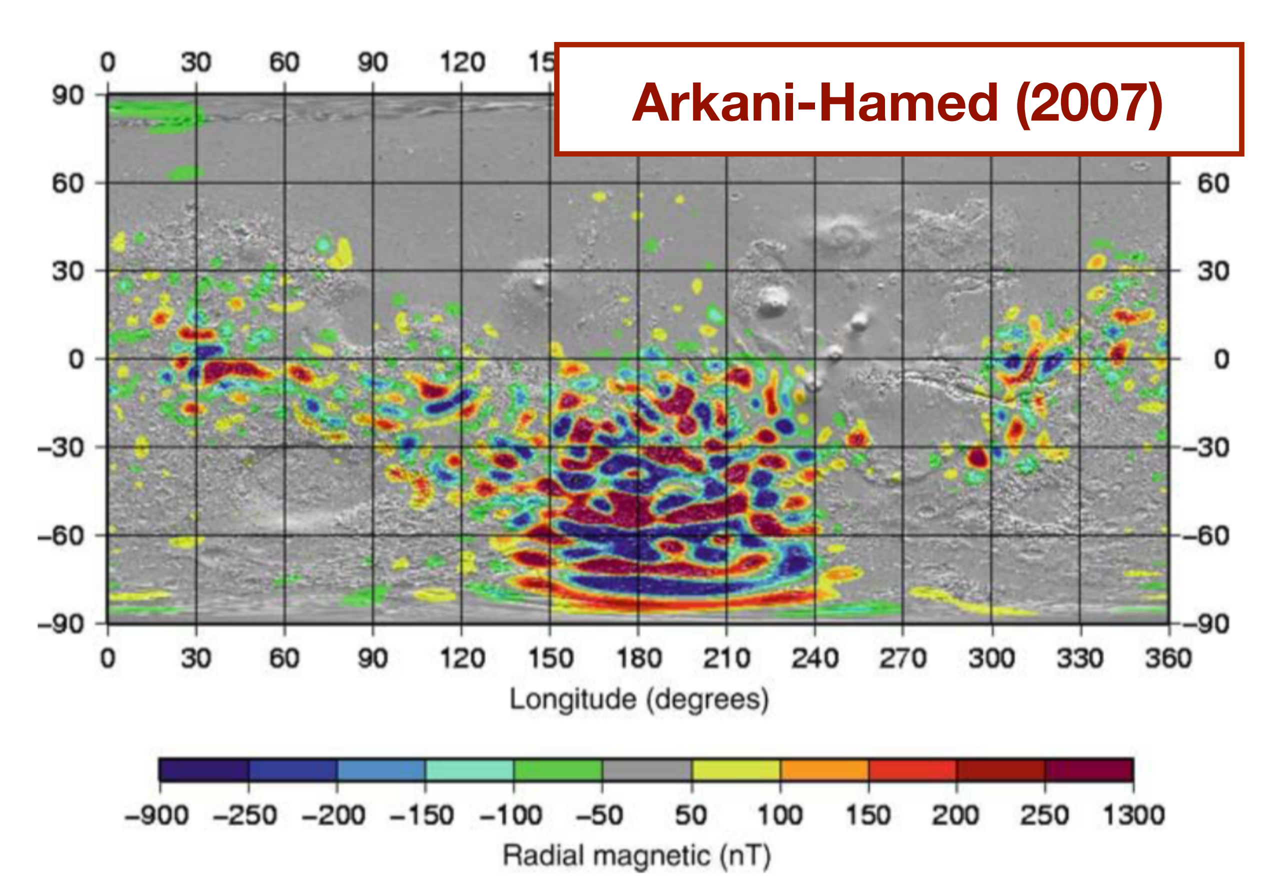
Mars

No Dynamo

Fields patchy, reaching ~ 0.01 G in spots but no dipole

Why?

It had a dynamo in the past (remnant crustal magnetism) but it cooled off fast, freezing out its molten core



Mercury

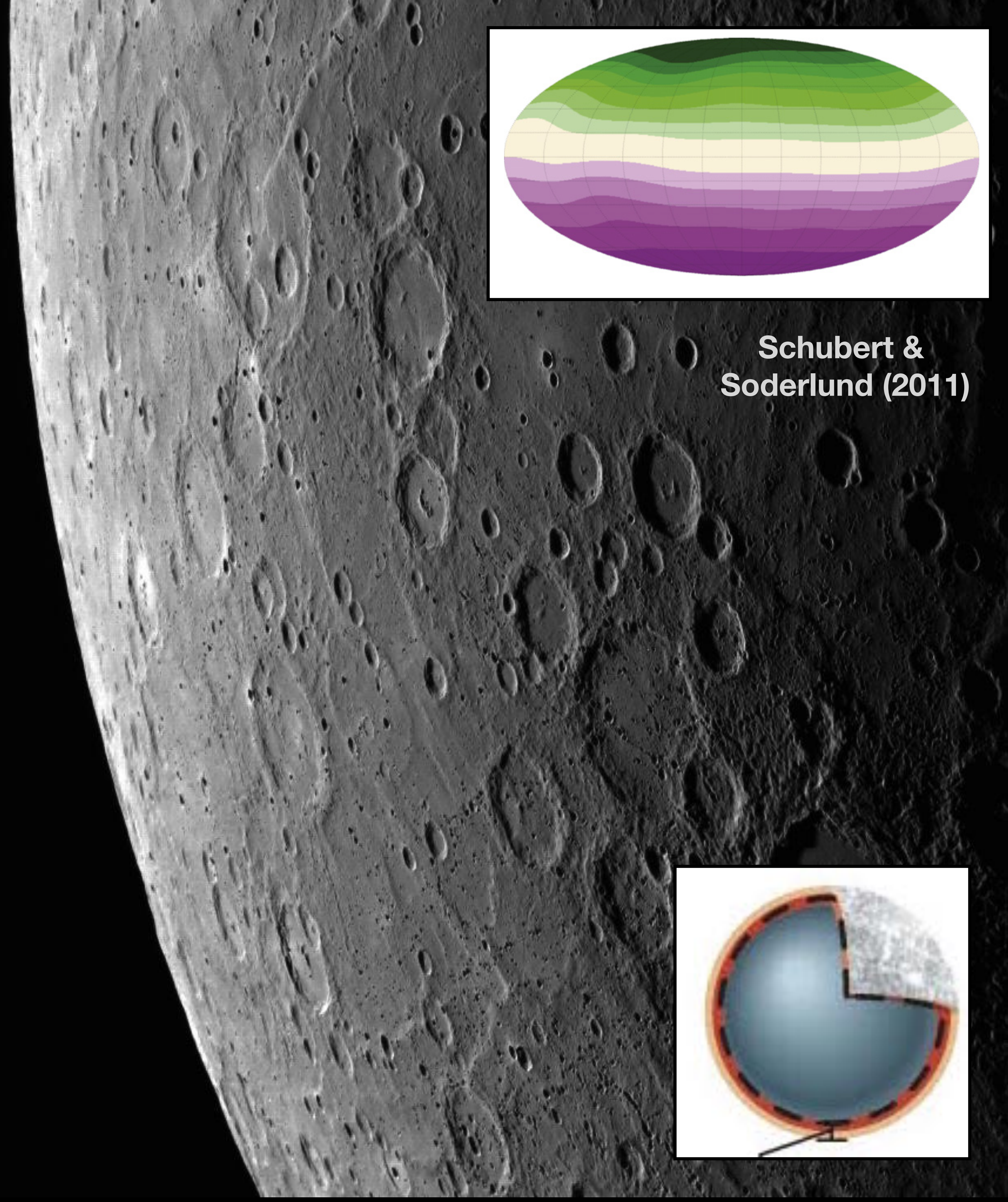
Dynamo!

Field strength
~ 0.003 G

Dipolarity
~ 0.71 G

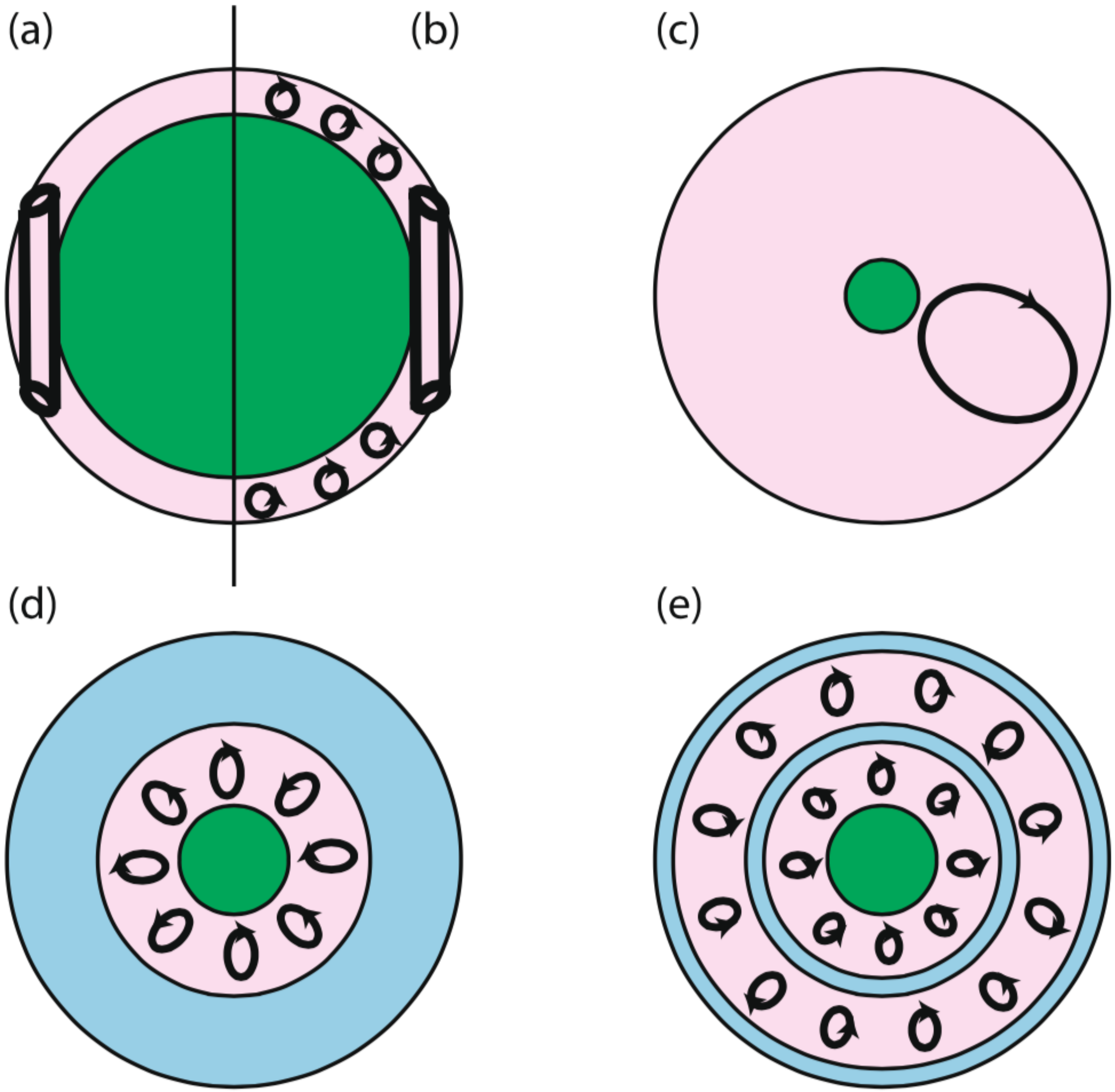
Tilt ~ 3°

Huge iron core relative to size of planet that is still partially molten



Schubert & Soderlund (2011)

But we're still not really sure what's going on!



Stanley & Glatzmaier (2009)

Ganymede!

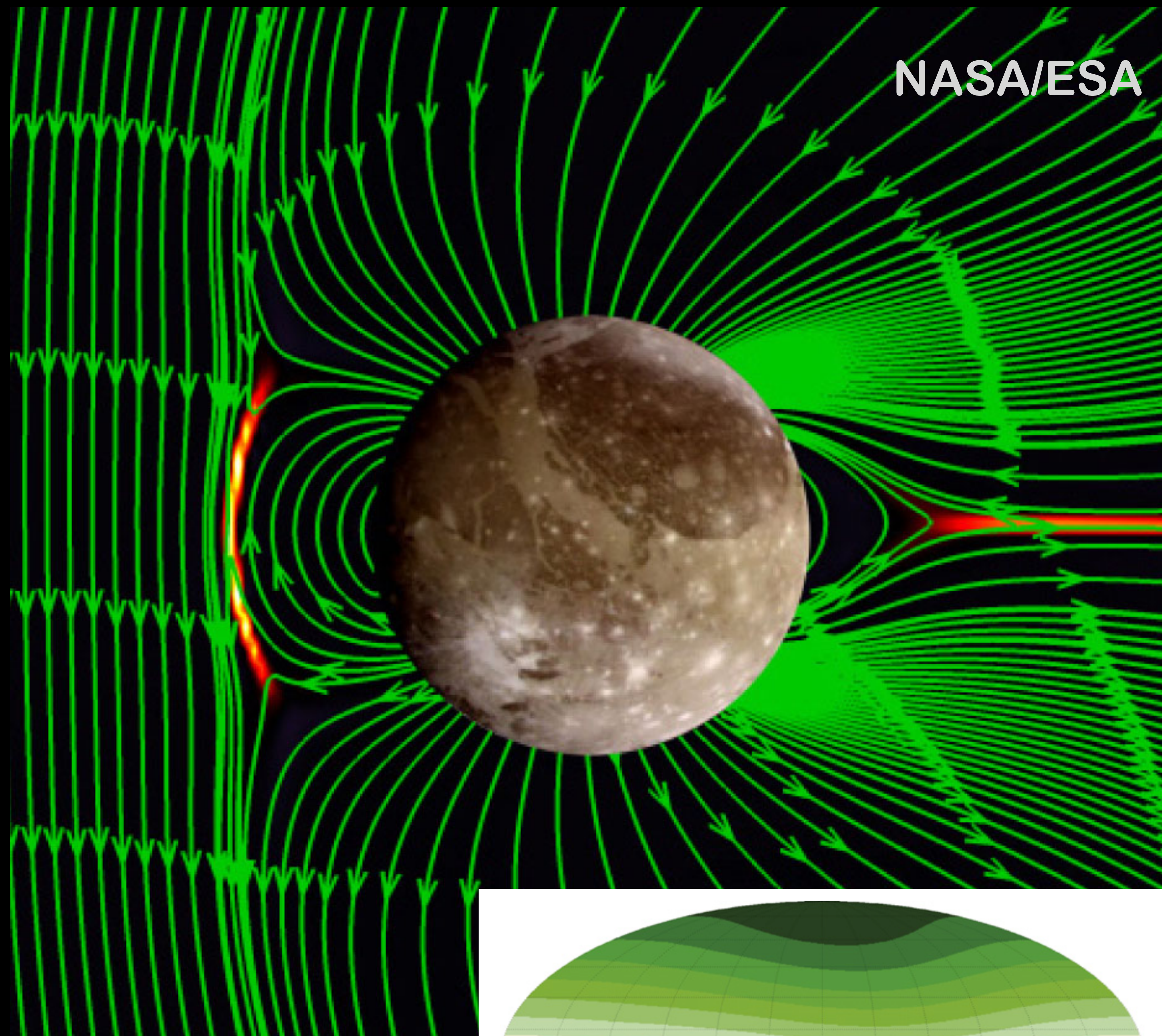
Dynamo!

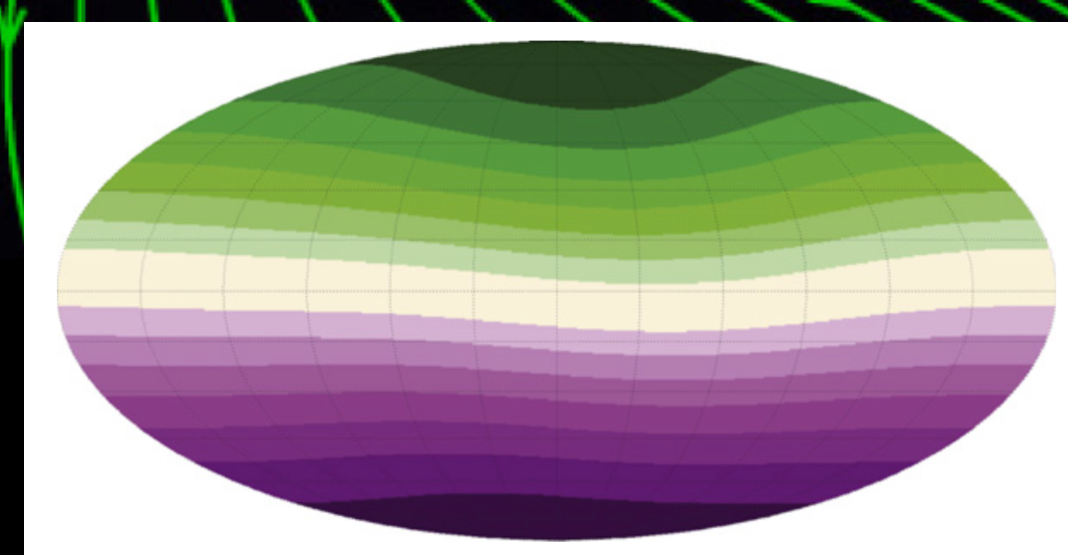
Field strength
~ 0.01 G

Dipolarity
~ 0.95 G

Tilt
~ 4°

Other icy satellites have induced magnetic fields from passing through the magnetospheres of their planets





Schubert & Soderlund (2011)

Jupiter

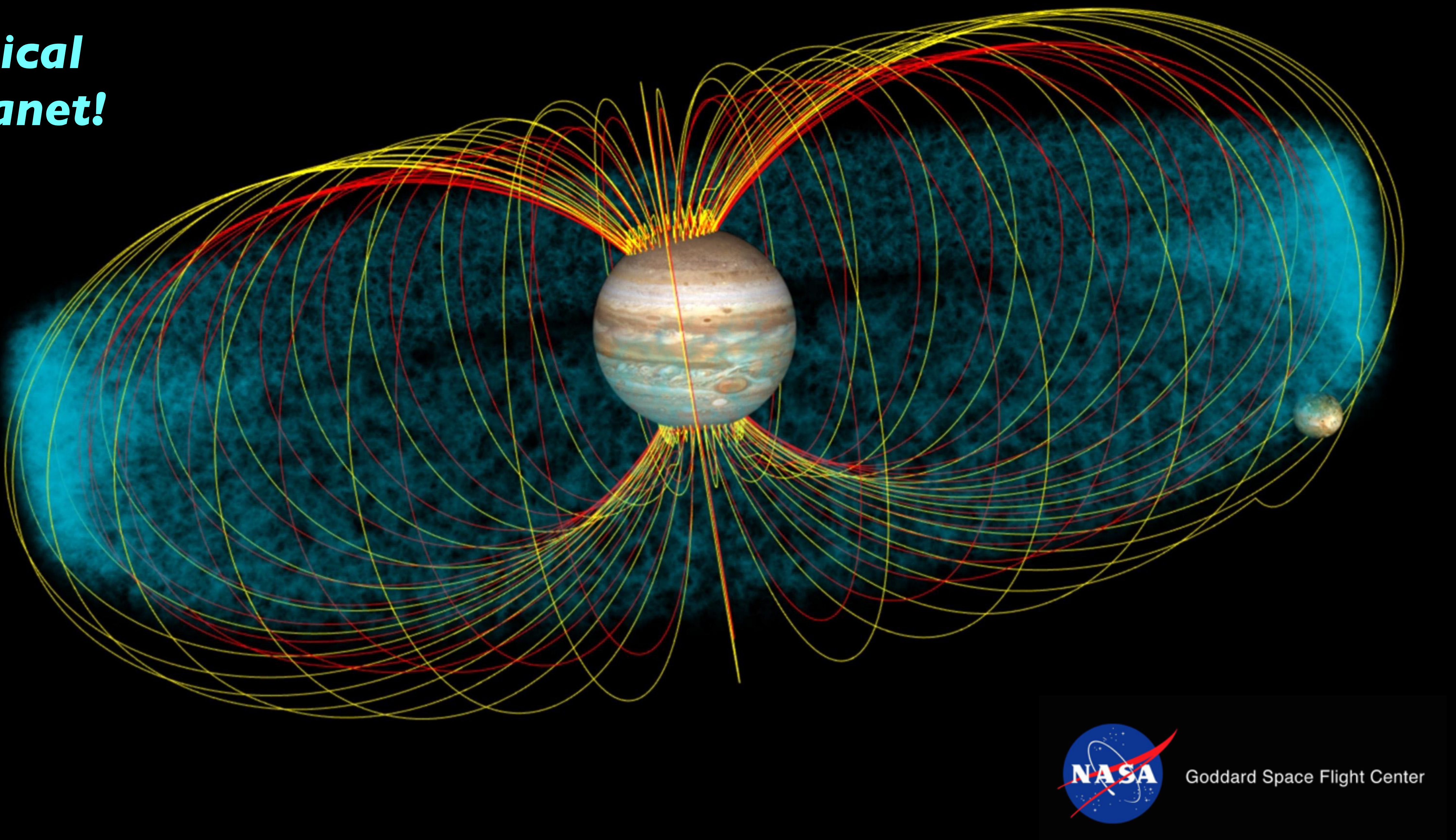
Big Whopping Dynamo!

Field strength ~ 7 G

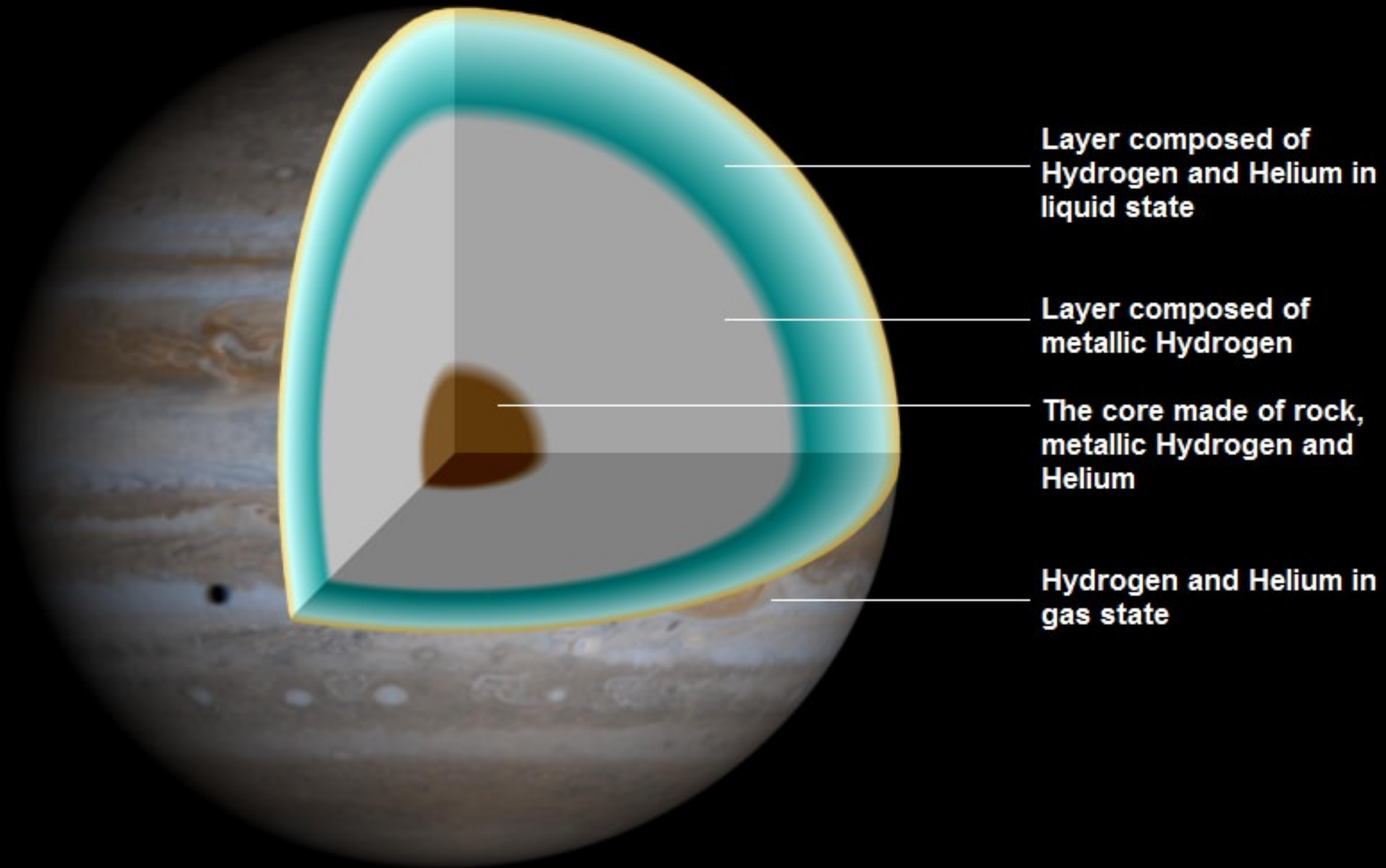
Dipolarity ~ 0.61

Tilt ~ 10°

Archetypical Jovian planet!



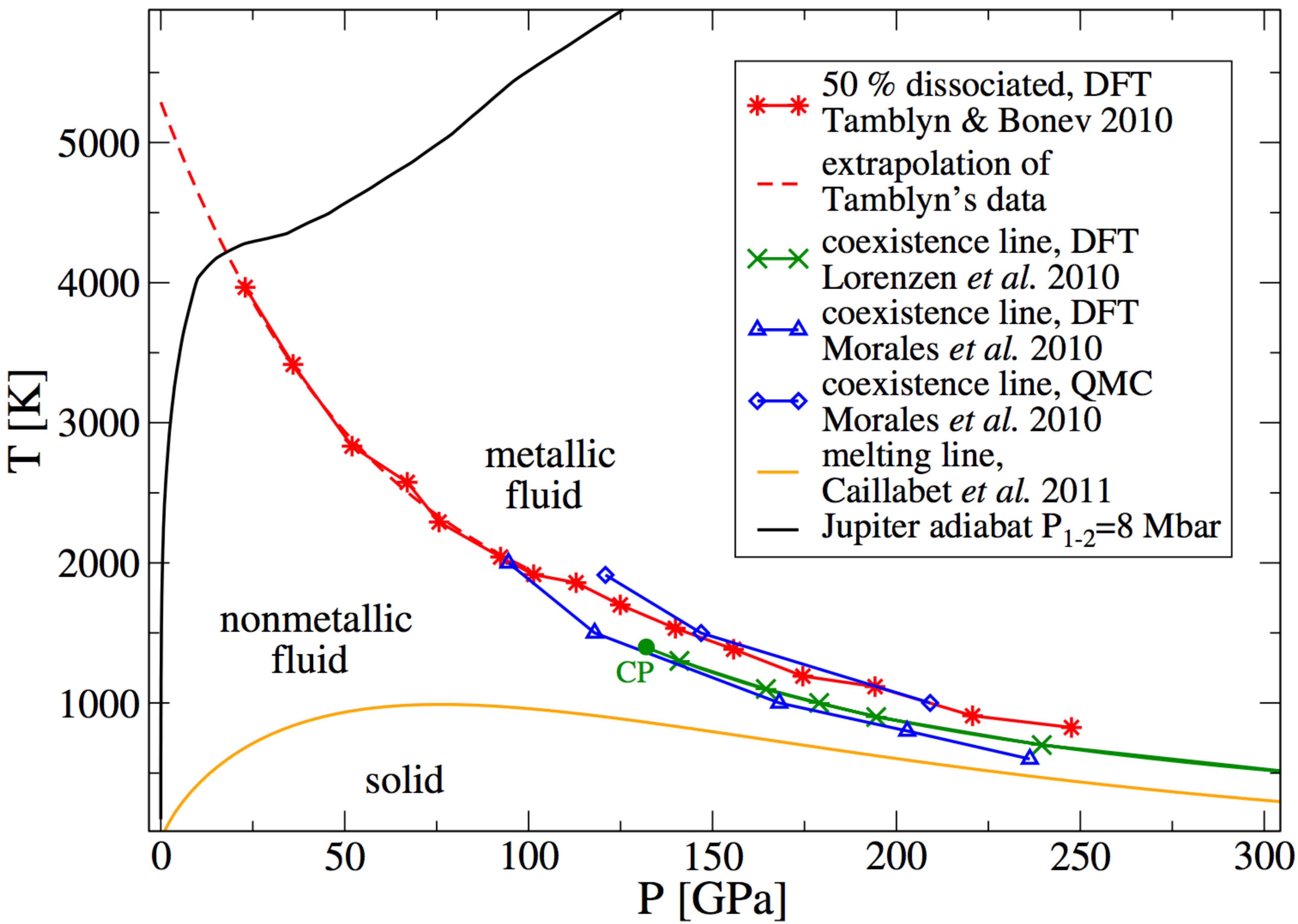
Jupiter



Layer composed of Hydrogen and Helium in liquid state
Layer composed of metallic Hydrogen
The core made of rock, metallic Hydrogen and Helium
Hydrogen and Helium in gas state

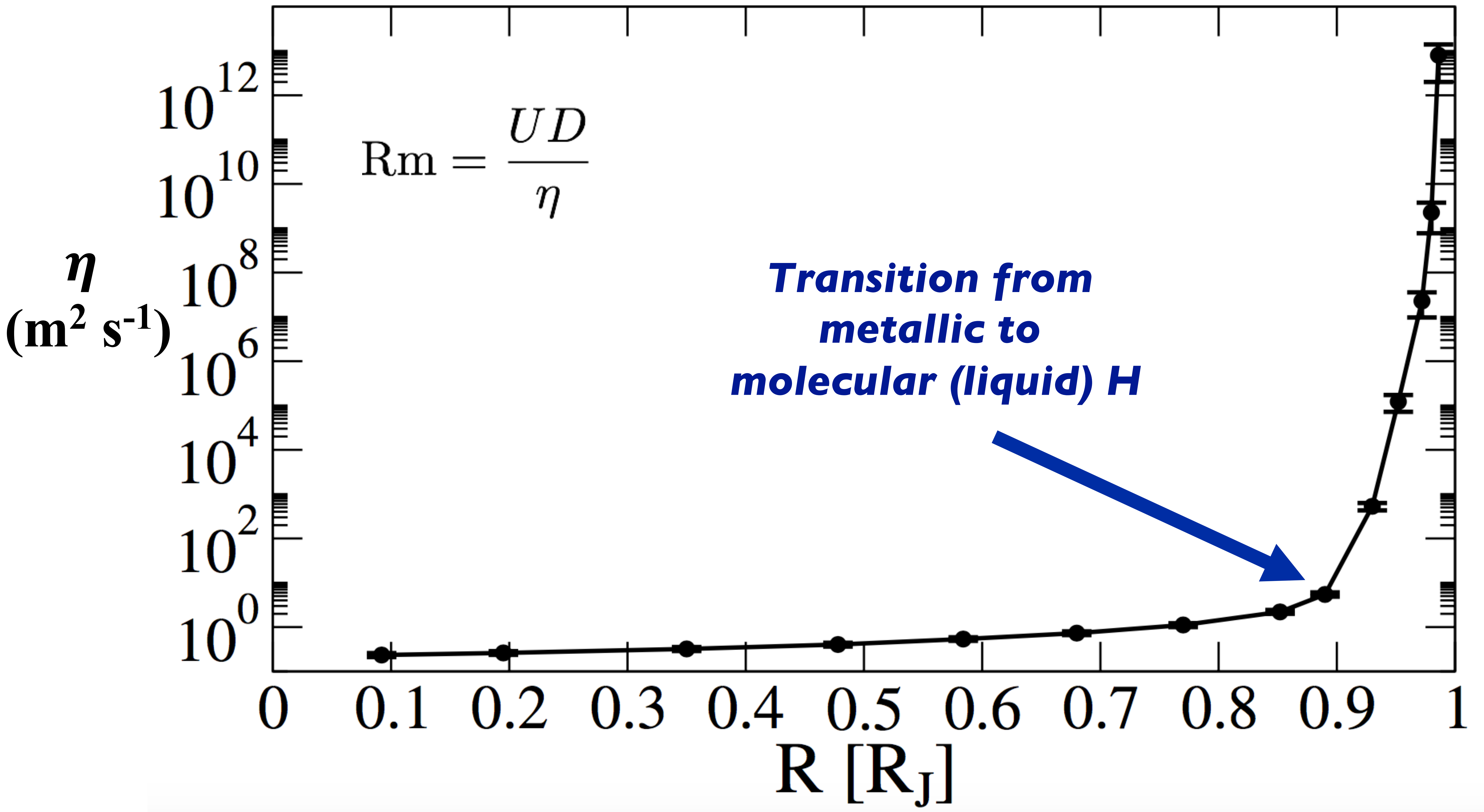
Jupiter: Internal Structure

French et al (2012)

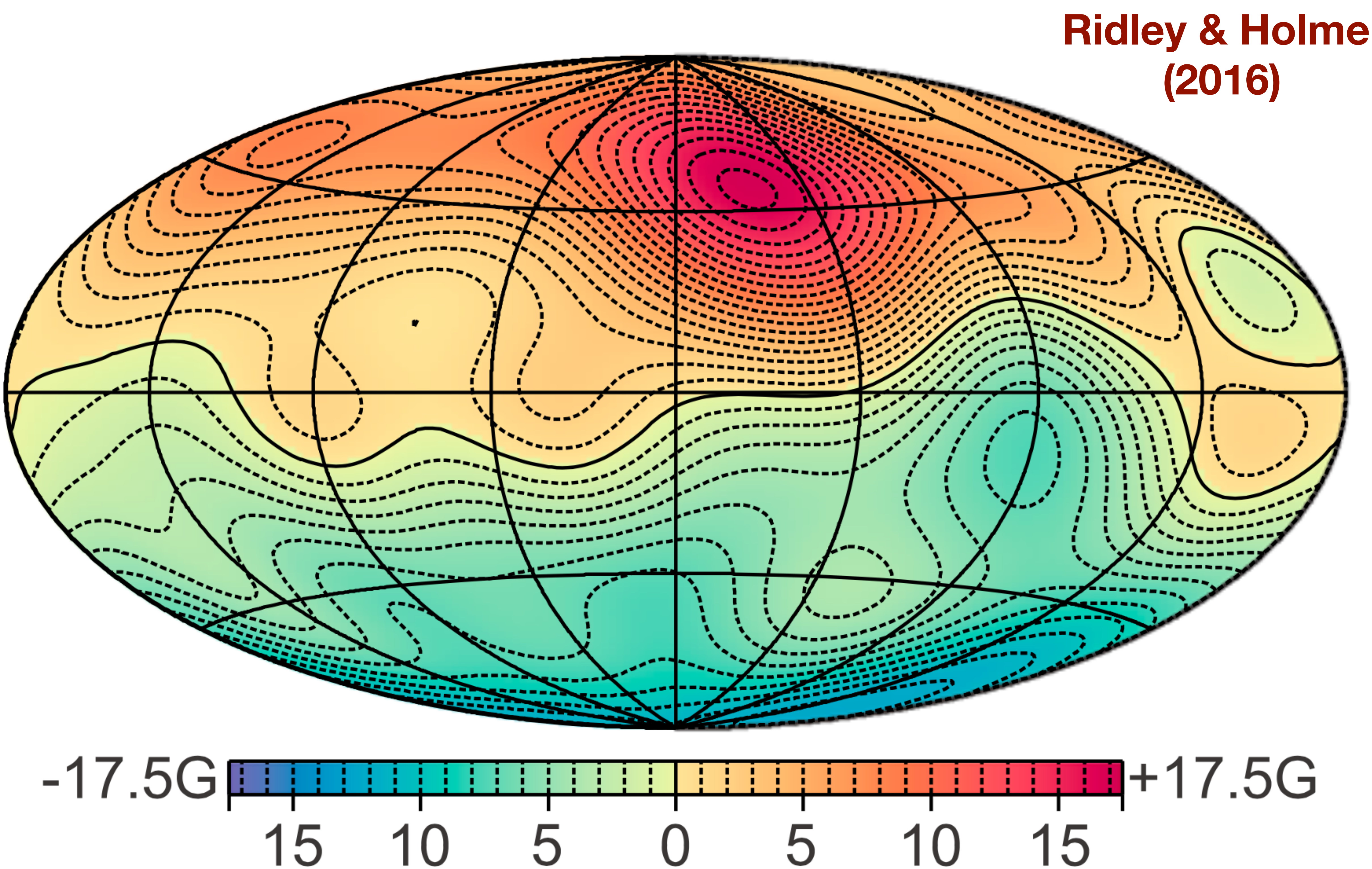


Jupiter: Internal Structure

French et al. (2012)

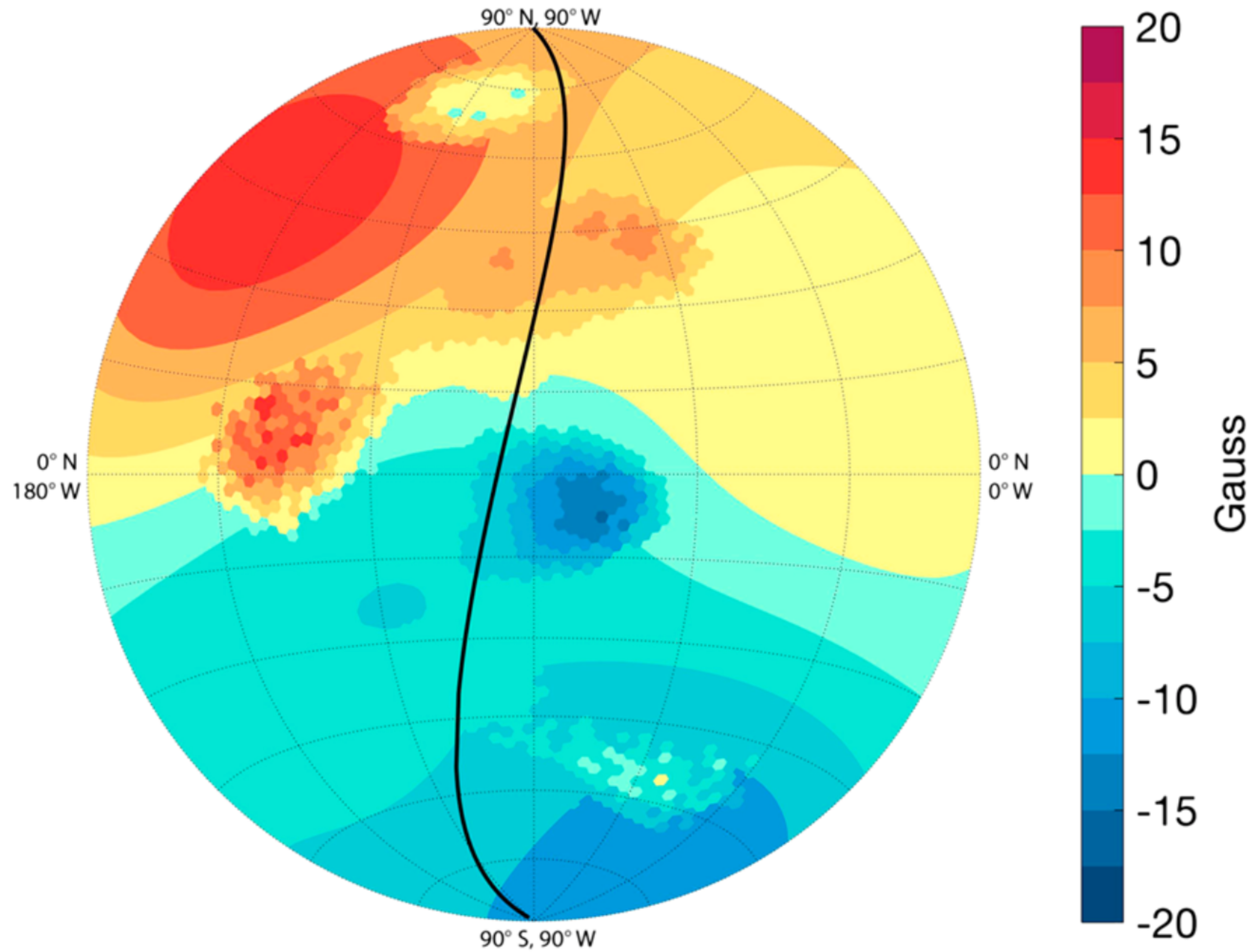


Jupiter: Magnetic Field (Pre-Juno)



Initial results from Juno

Stronger and more patchy than expected (higher-order multipoles)



This suggests that dynamo action might exist closer to the surface than previously thought

$$B_r \propto r^{-(\ell+2)}$$

Moore et al (2017)

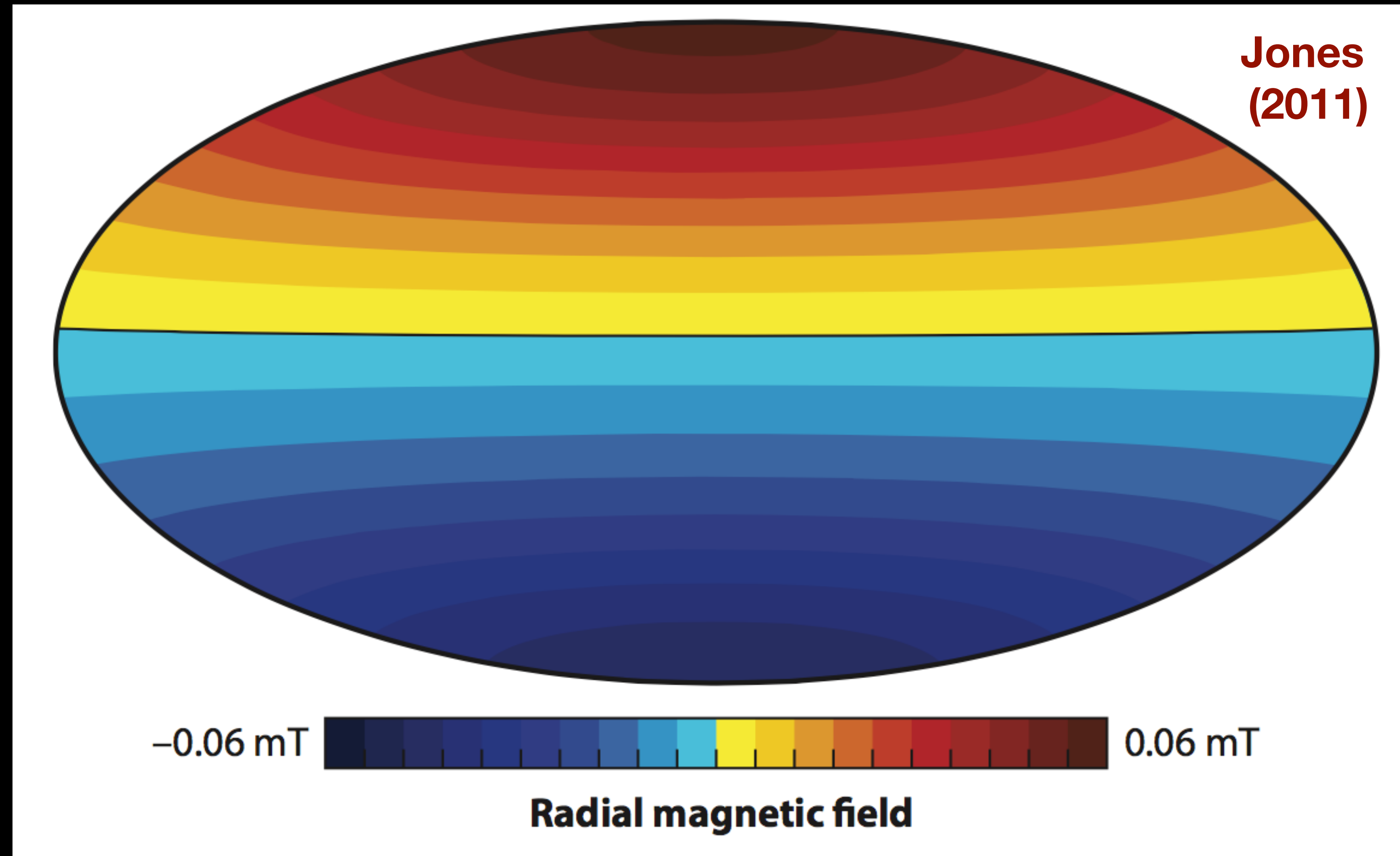
Saturn

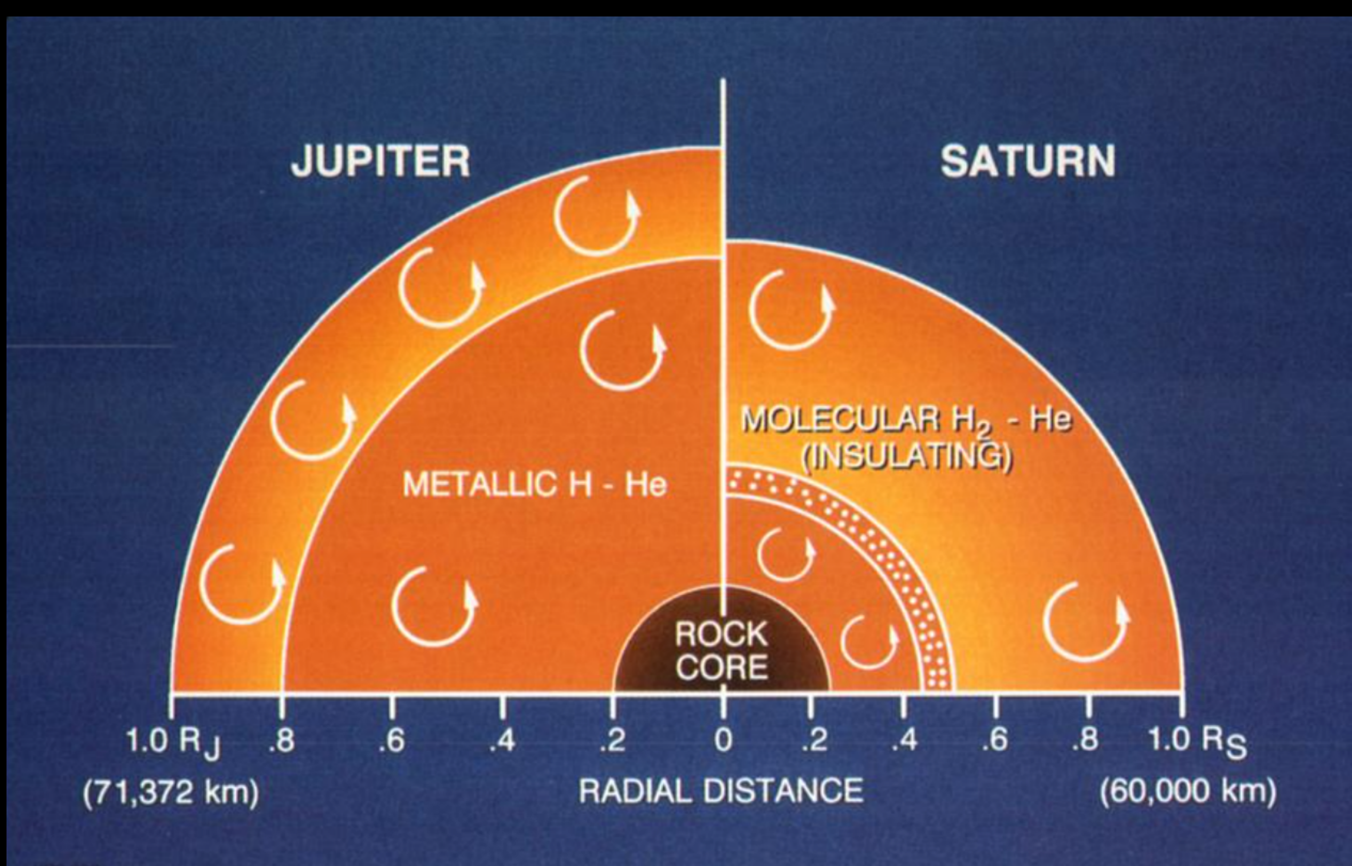
Dynamo!

Field strength
~ 0.6 G

Dipolarity
~ 0.85 G

Tilt
< 0.5°





Remarkably axisymmetric!

A surprise!

Why?

Connerney(1993)

Cowling's Theorem

Why is this a surprise?

Assume B is axisymmetric and consider the longitudinally-averaged MHD induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\langle \mathbf{v} \rangle \times \mathbf{B} - \langle \eta \rangle \nabla \times \mathbf{B} \right)$$

Express B as $\mathbf{B} = \nabla \times \left(A\hat{\phi} \right) + B\hat{\phi}$

Evolution eqn for A (after some manipulation)

$$\lambda = r \sin \theta$$

$$\frac{\partial}{\partial t}(\lambda A) = -\mathbf{v} \cdot \nabla (\lambda A) + \eta \lambda \left(\nabla^2 A - \lambda^{-2} \right) A$$

Multiply by λA and integrate over volume: if $\nabla \cdot \mathrm{v} = 0$ ***then the first term on the RHS is zero and the second term is negative***

Cowling's Theorem (cont.)

$$\frac{\partial}{\partial t}(\lambda A) = -\mathbf{v}\cdot\boldsymbol{\nabla}(\lambda A) + \eta\lambda\left(\nabla^2 A - \lambda^{-2}\right) A$$

A decays with time

If A decays with time, then B will decay with time too (Work it out!)

Even if $\nabla \cdot \mathrm{v} \neq 0$ you can show that a steady field ($\partial \mathrm{A}/\partial \mathrm{t} = 0$) cannot be maintained

Conclusion: it is not possible to sustain a steady axisymmetric B field against ohmic dissipation

Corollary: It is not possible for a dynamo to produce a steady axisymmetric field!!

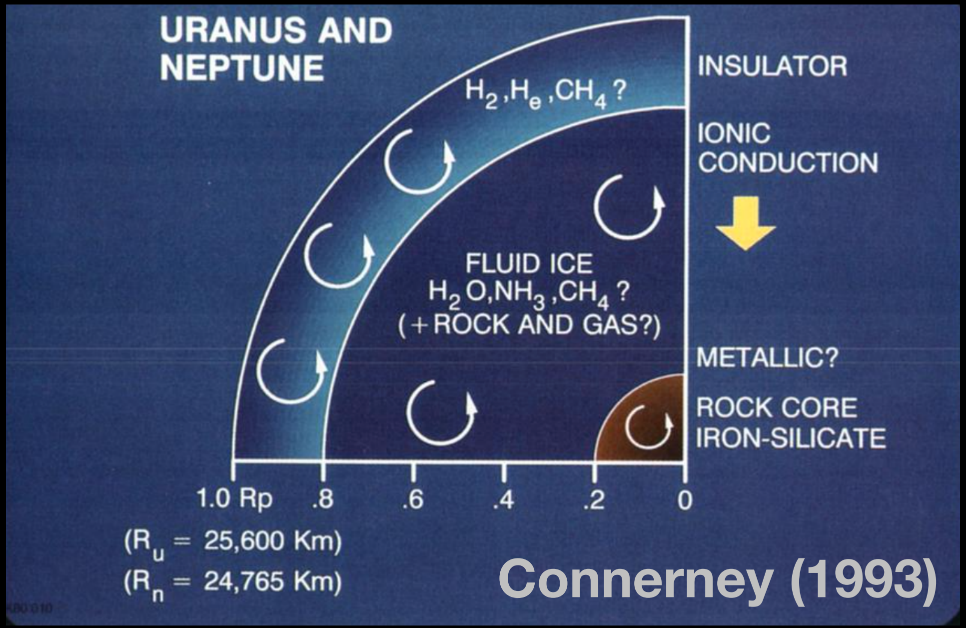
Uranus & Neptune

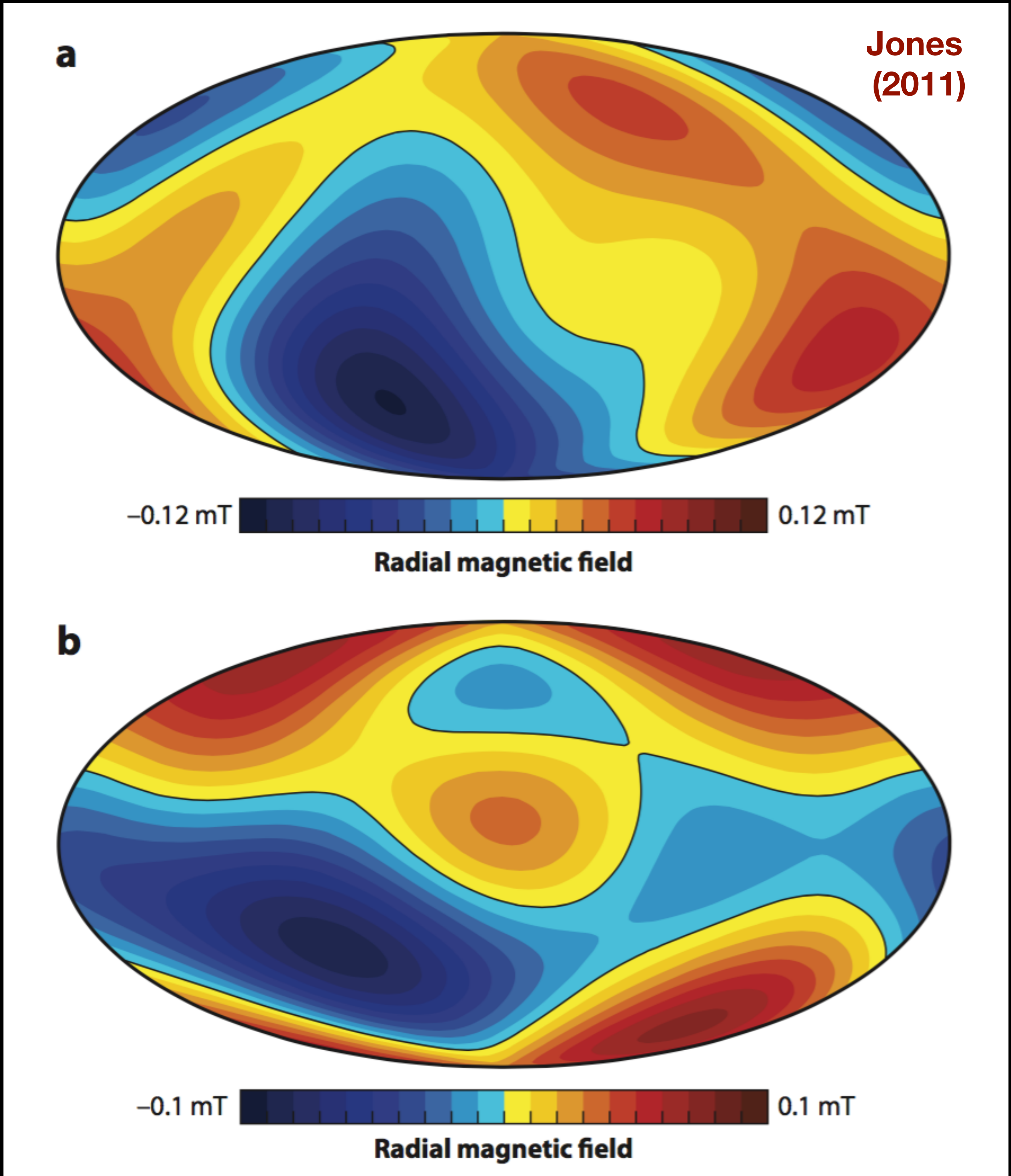
Dynamos!

Field strength ~ 0.3 G

Dipolarity ~ 0.42, 0.31

Tilt ~ 59°, 45°





Understanding the Dynamics

Conservation of momentum in MHD

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\left(\rho\mathbf{v}\cdot\boldsymbol{\nabla}\right)\mathbf{v} - 2\rho\left(\boldsymbol{\Omega}\times\mathbf{v}\right) - \boldsymbol{\nabla}P + \rho\mathbf{g} + c^{-1}\mathbf{J}\times\mathbf{B} - \boldsymbol{\nabla}\cdot\mathcal{D}$$

Convection established by buoyancy

But rotation exerts an overwhelming influence
Coriolis accelerations happen quickly (days) compared to convection and dynamo time scales (hundreds to thousands of years)

$$\mathrm{Ro} = \frac{U}{2\Omega D} << 1 \qquad \mathrm{Ek} = \frac{\nu}{2\Omega D^2} << 1$$

Dynamical Balances

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\left(\rho\mathbf{v}\cdot\nabla\right)\mathbf{v} - 2\rho\left(\mathbf{\Omega}\times\mathbf{v}\right) - \nabla P + \rho\mathbf{g} + c^{-1}\mathbf{J}\times\mathbf{B} - \nabla\cdot\mathcal{D}$$

Result: Flows evolve quasi-statically in so-called

Magnetostrophic (MAC) Balance

$$\boxed{c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\mathbf{\Omega}\times\mathbf{v}\right) + \nabla P - \rho\mathbf{g}}$$

Conservation of mass

hydrostatic background

Anelastic approximation (valid for small Ma)

$$\nabla\cdot\left(\hat{\rho}\mathbf{v}\right) = 0$$

Boussinesq approximation (valid for small Ma, H_ρ >> D)

$$\nabla\cdot\mathbf{v} = 0$$

Dynamical Balances

$$c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\boldsymbol{\Omega}\times\mathbf{v}\right) + \boldsymbol{\nabla} P - \rho\mathbf{g}$$

Now set B = 0 and assume that $\nabla\rho$ is mainly radial

Then the ϕ component of the curl gives (anelastic approximation):

$$\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\left(\rho\mathbf{v}\right) = \frac{\partial}{\partial z}\left(\rho\mathbf{v}\right) = 0$$

Taylor-Proudman Theorem

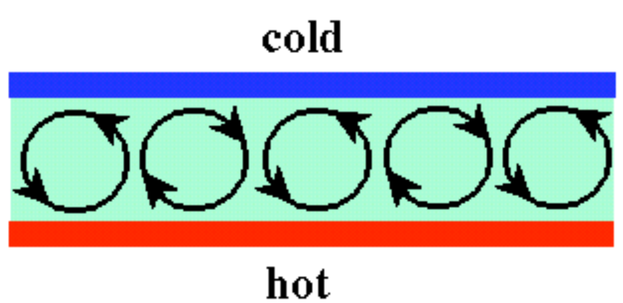
Boussinesq version:

$$\frac{\partial\mathbf{v}}{\partial z} = 0$$

Rapidly rotating flows tend to align with the rotation axis

Question

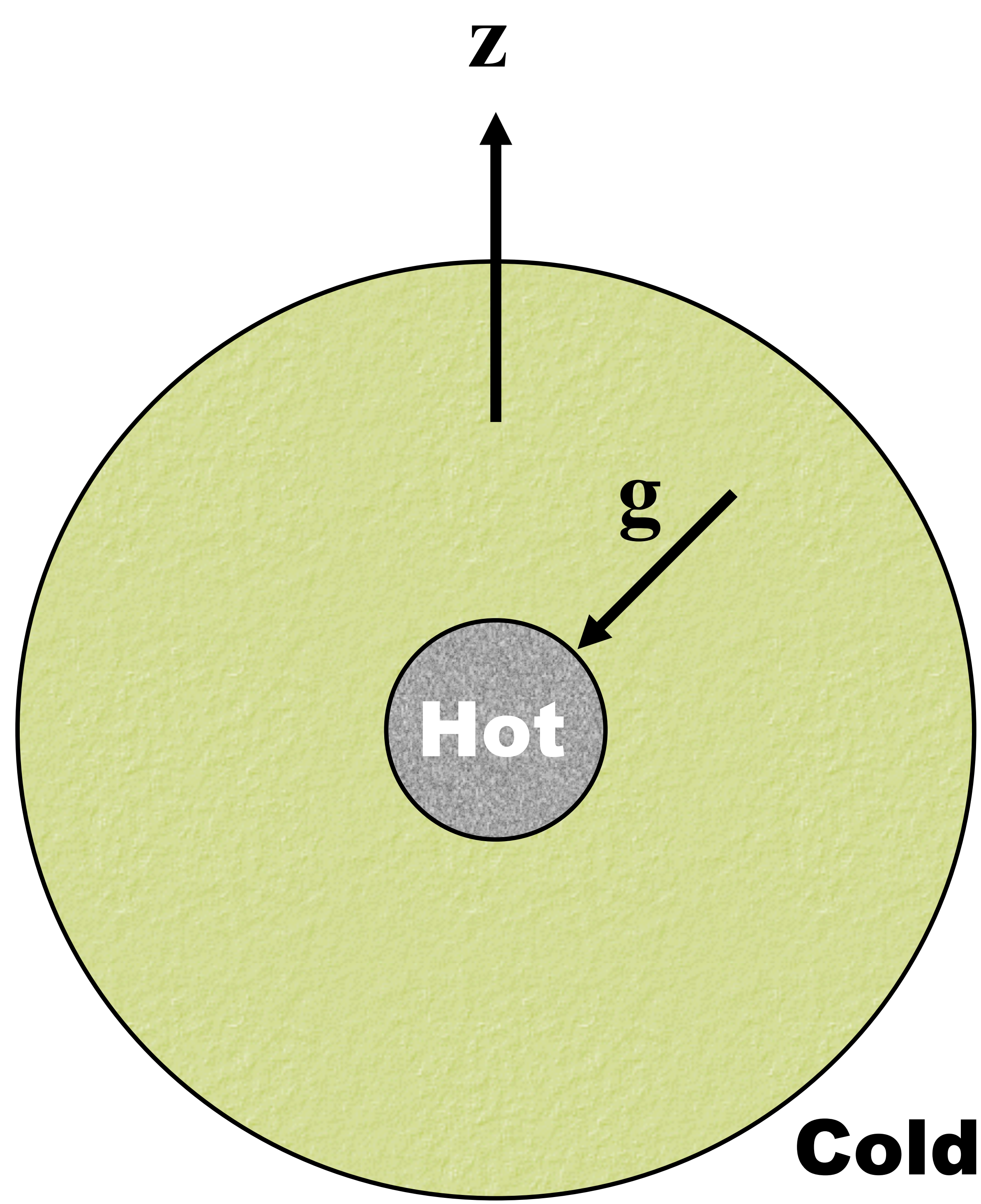
Work with a partner to draw what you think convective motions might look like in a rapidly-rotating spherical shell



How can you get the heat out while still satisfying the Taylor-Proudman theorem

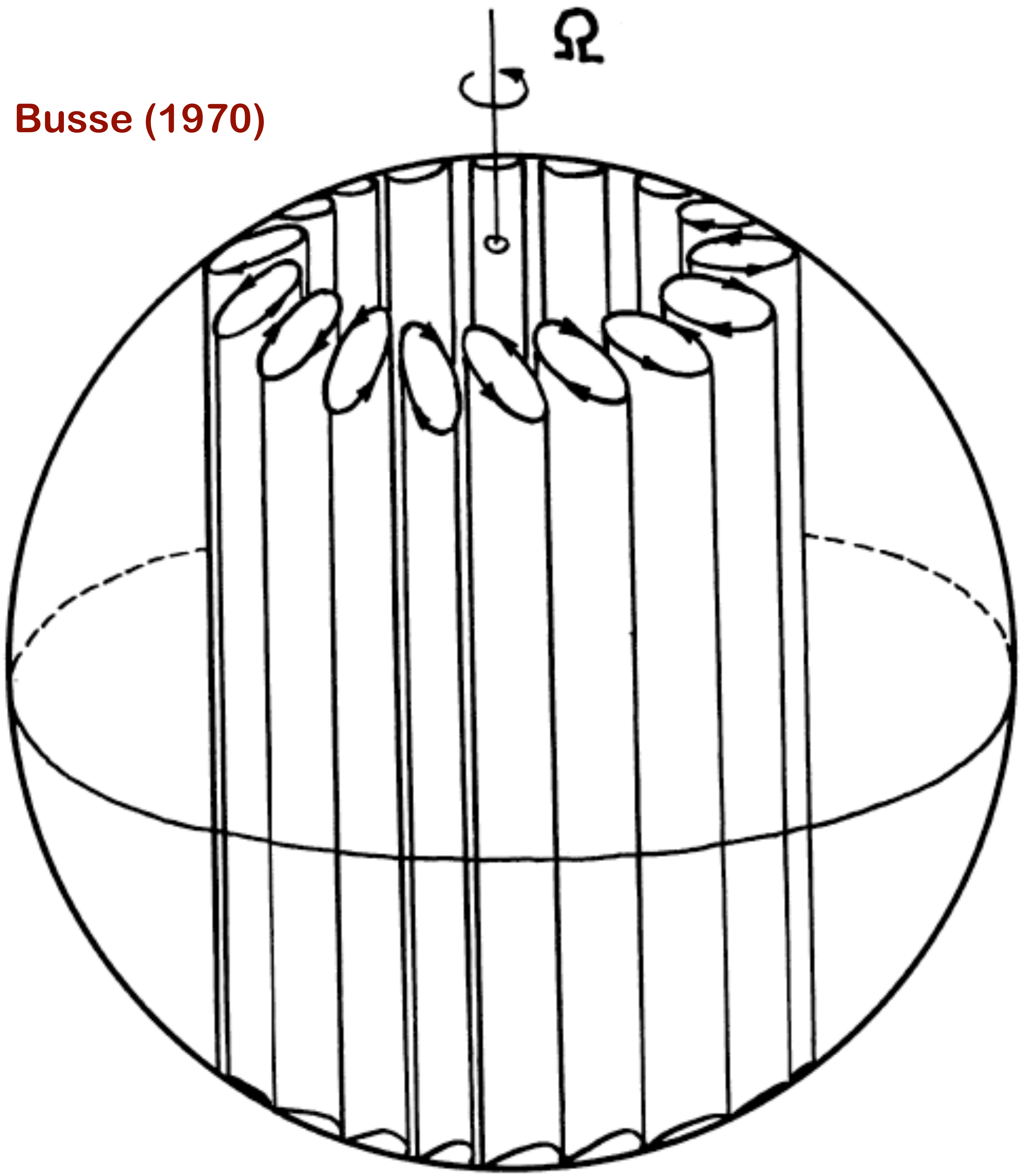
$$\frac{\partial \mathbf{v}}{\partial z} = 0$$

Can you satisfy it everywhere?



Linear Theory

Busse (1970)



The most unstable convective modes in a rapidly-rotating, weakly-stratified shell are

Busse columns

aka

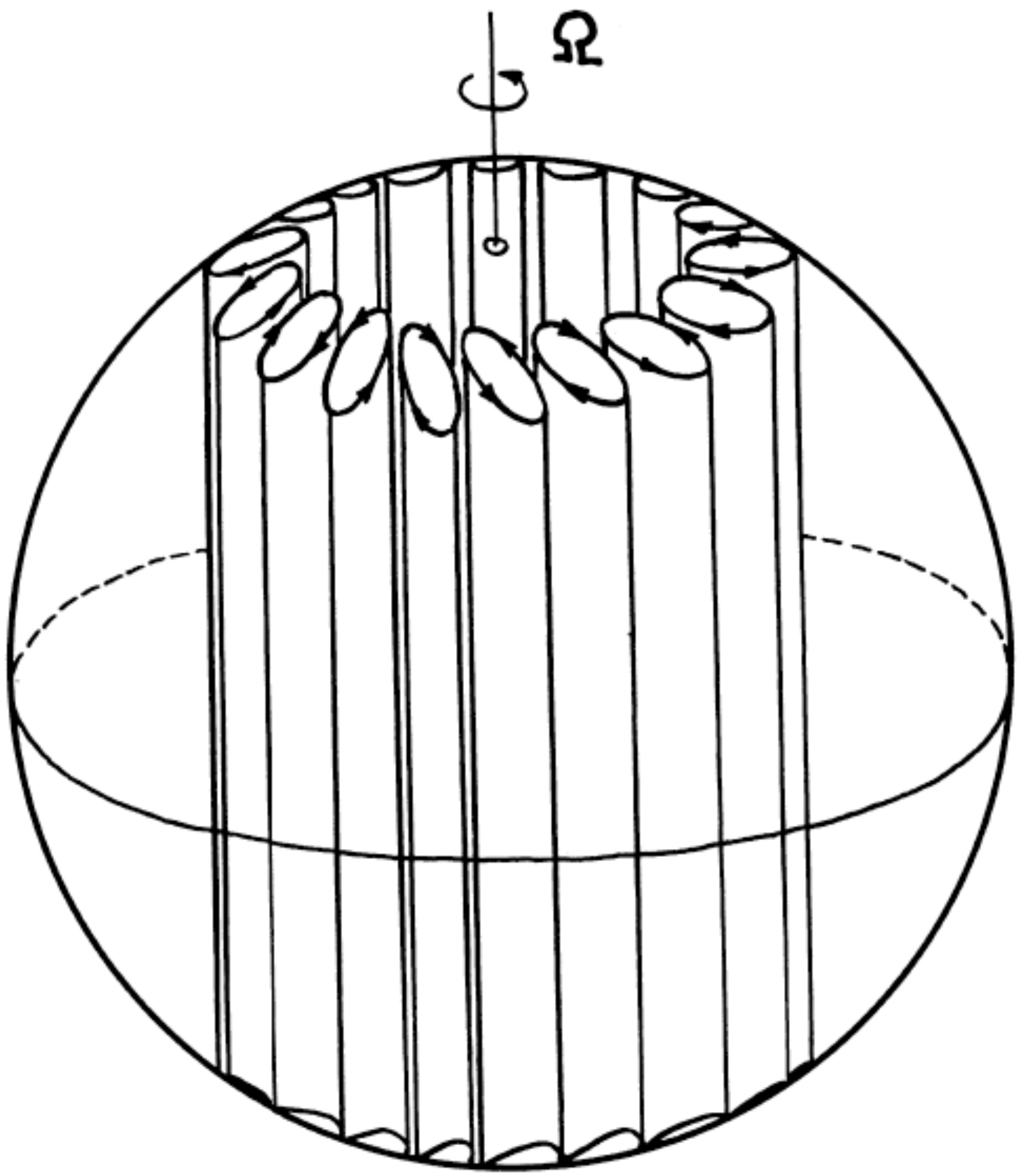
Banana Cells

The preferred longitudinal wavenumber (m) scales as

$$Ek^{-1/3}$$

Coriolis vs viscous diffusion

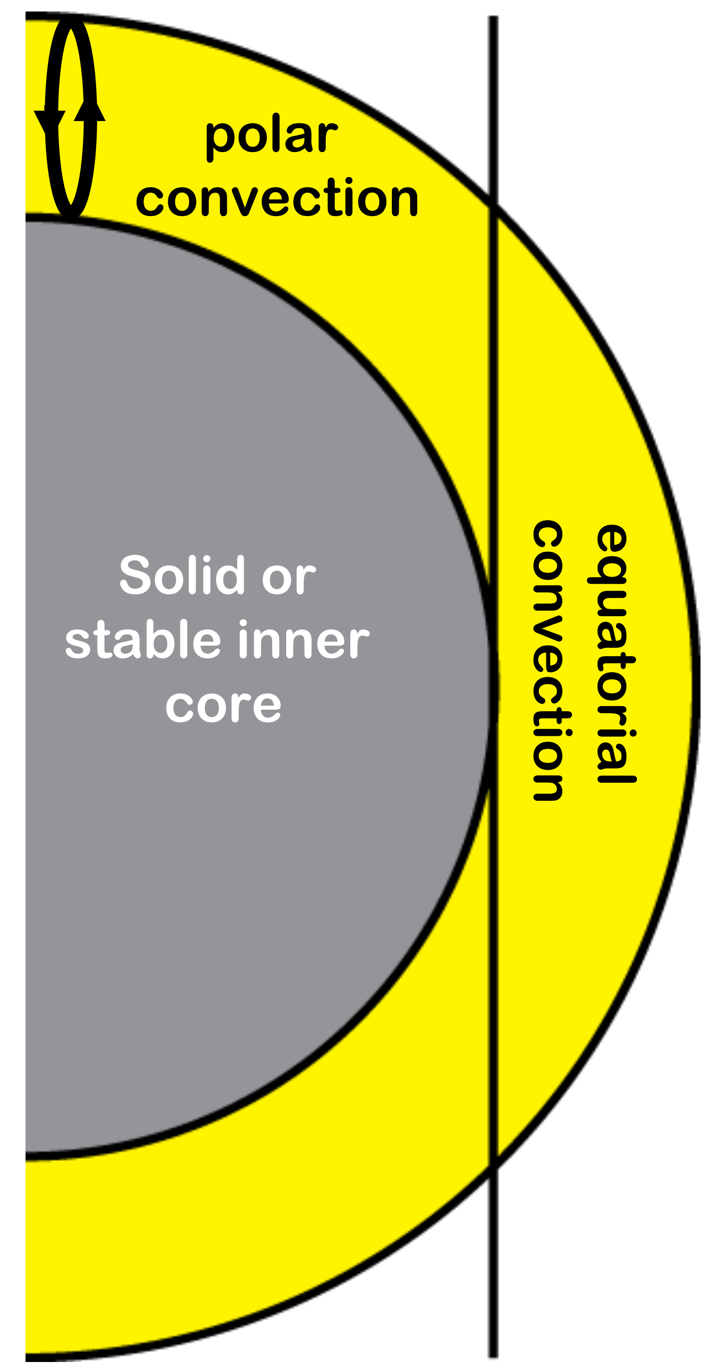
Linear Theory



The

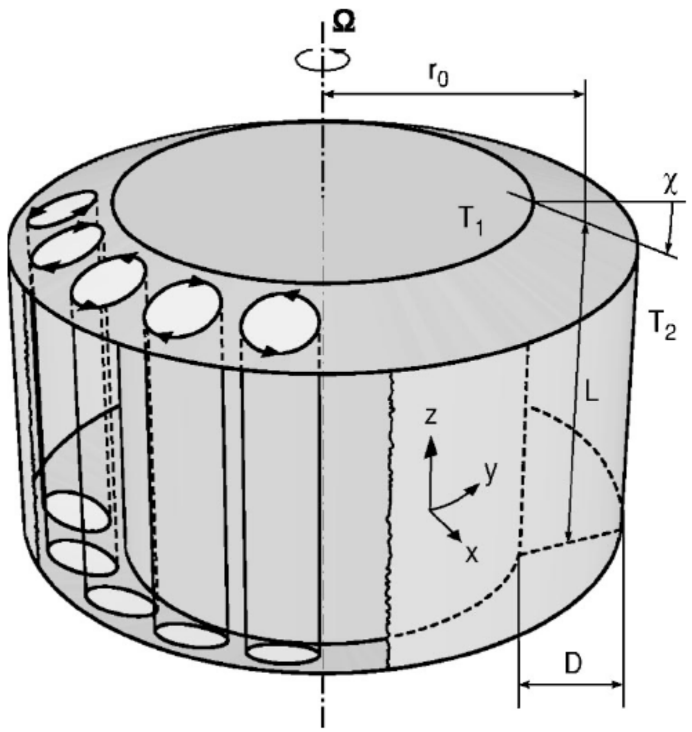
Tangent Cylinder

Delineates two distinct dynamical regimes

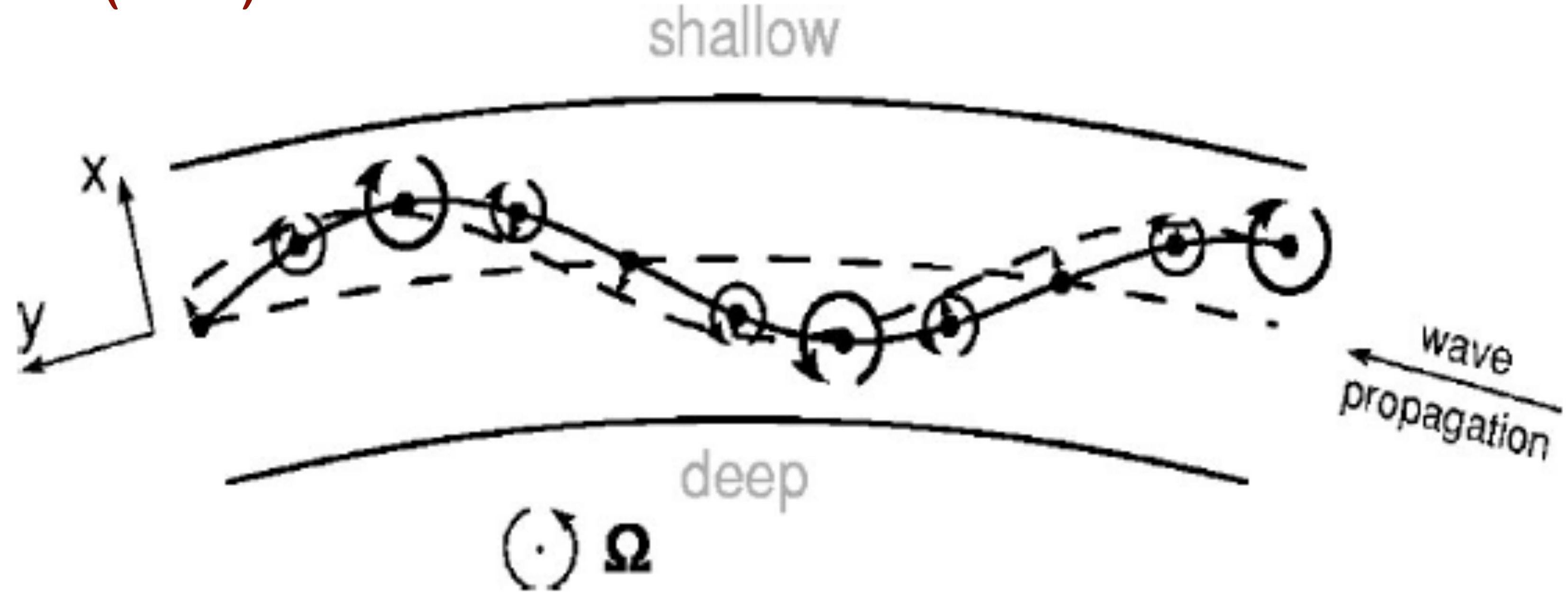


Implication of the Taylor-Proudman theorem

Linear Theory: Traveling Waves



Busse (2002)



Prograde propagation
(thermal Rossby waves)

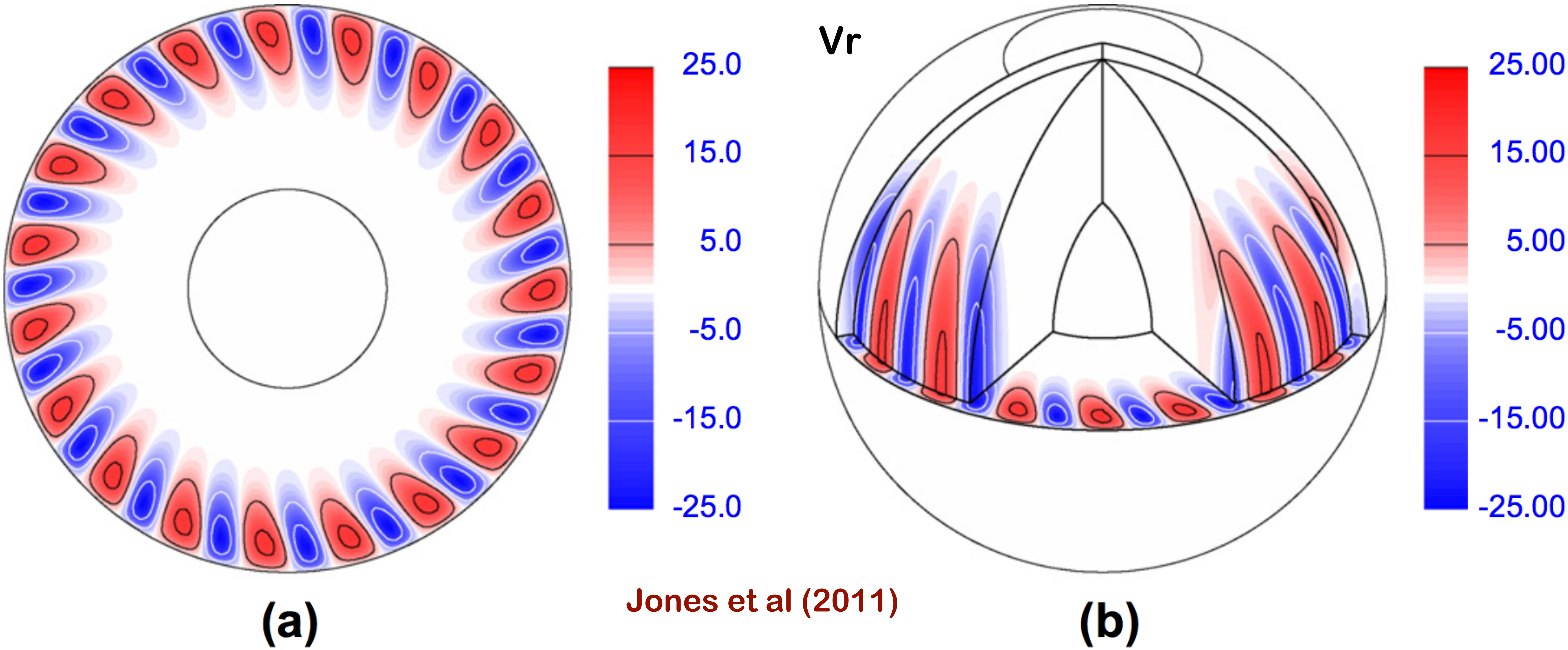
Induced by curvature of outer boundary and/or density stratification

$$\frac{\omega_z}{H} = \text{constant}$$

Simplest example: Boussinesq fluid, centrifugal gravity, local, linear perturbations, small boundary curvature (Busse 2002)

$$v_p = \frac{4\Omega}{L}\frac{\tan\chi}{(1+P_r)(k_y^2+k_x^2)}$$

Nonlinear Regimes require Numerical Models



Jones et al (2011)

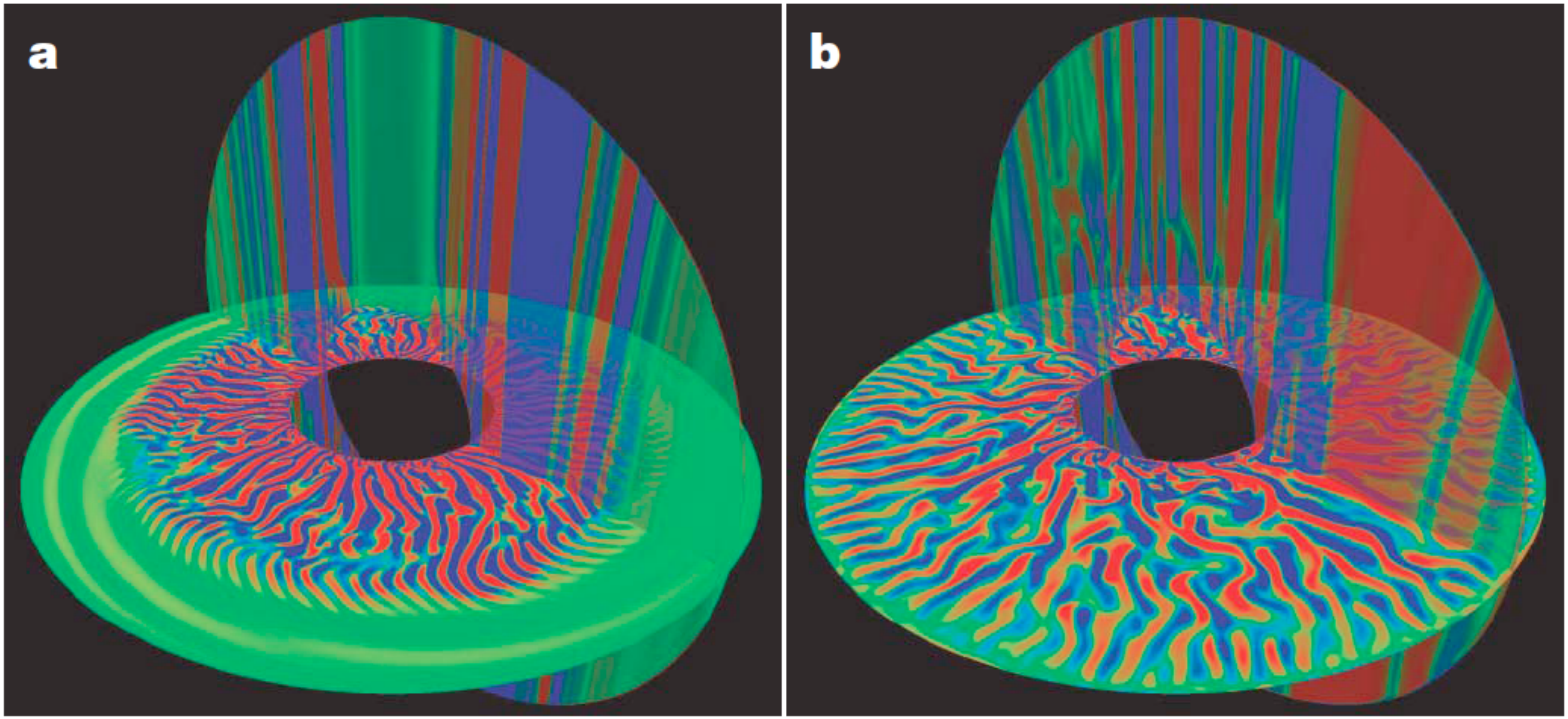
Solve the MHD equations in a rotating spherical shell
Anelastic or Boussinesq approximation
ρ, T, P, S are linear perturbations about a
hydrostatic, adiabatic background state

Convection simulations: heating from below, cooling from above

Axial alignment persists even in turbulent parameter regimes

Kageyama et al (2008)

Axial vorticity $\omega \cdot \Omega$



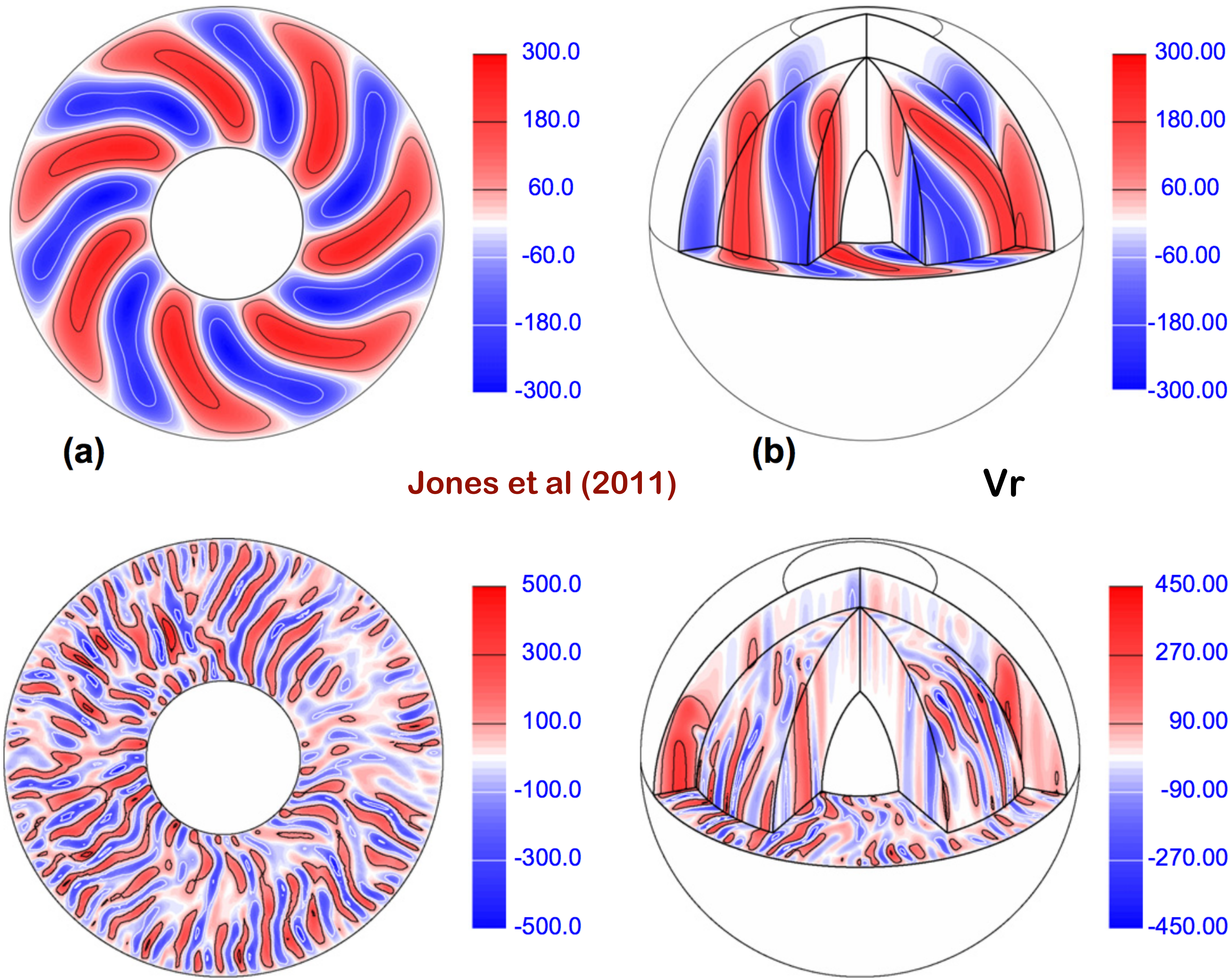
$\mathrm{Ek} = 2.3 \times 10^{-7}$ $\mathrm{Ek} = 2.6 \times 10^{-6}$

Busse columns give way to vortex sheets but the flow is still approximately 2D

$$\mathrm{Ek} = \frac{\nu}{2\Omega R^2}$$

...and in MHD

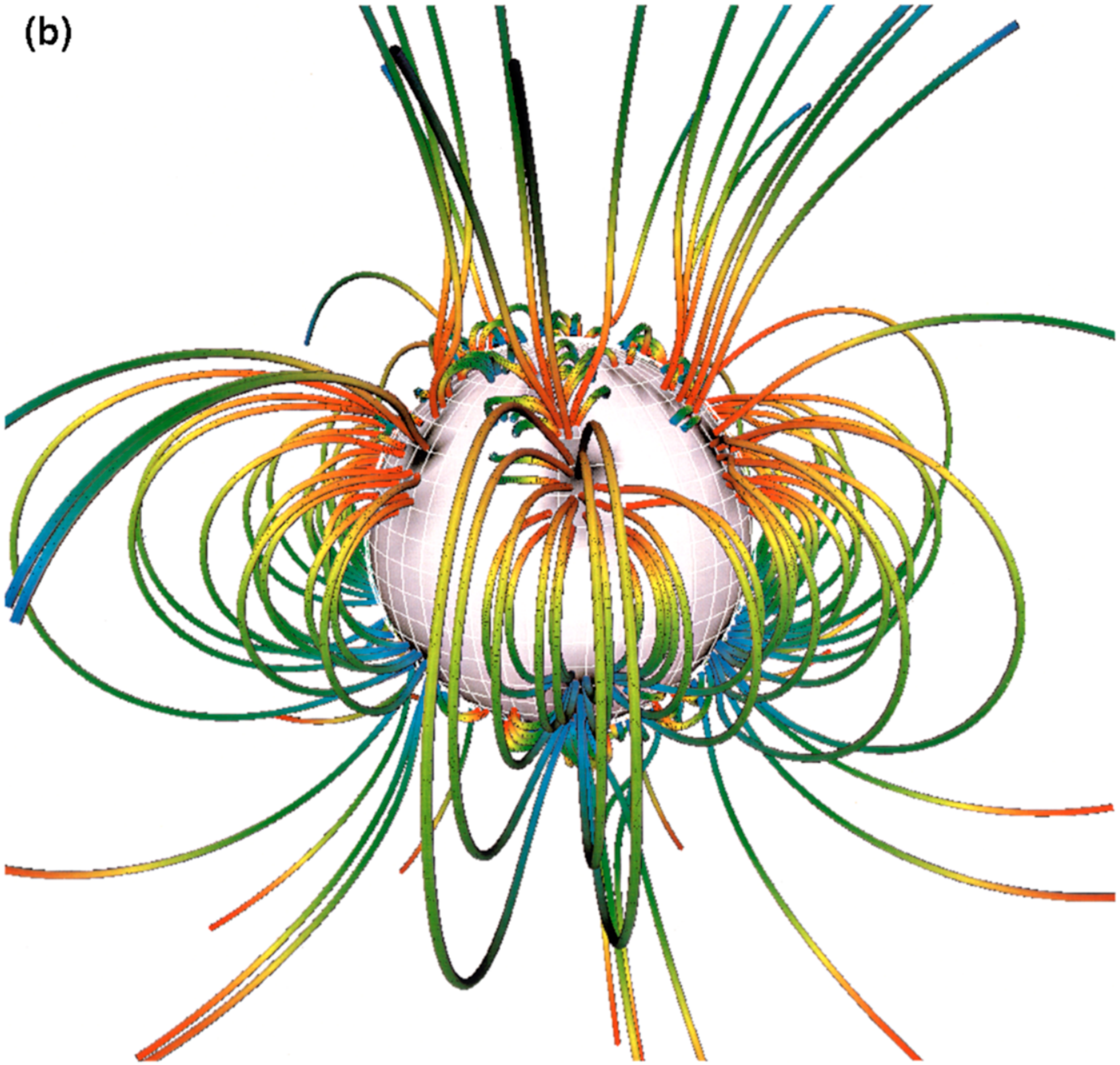
$$\mathrm{Ra} = \frac{GMD\Delta S}{\nu\kappa C_P} = \frac{\text{buoyancy driving}}{\text{dissipation}}$$



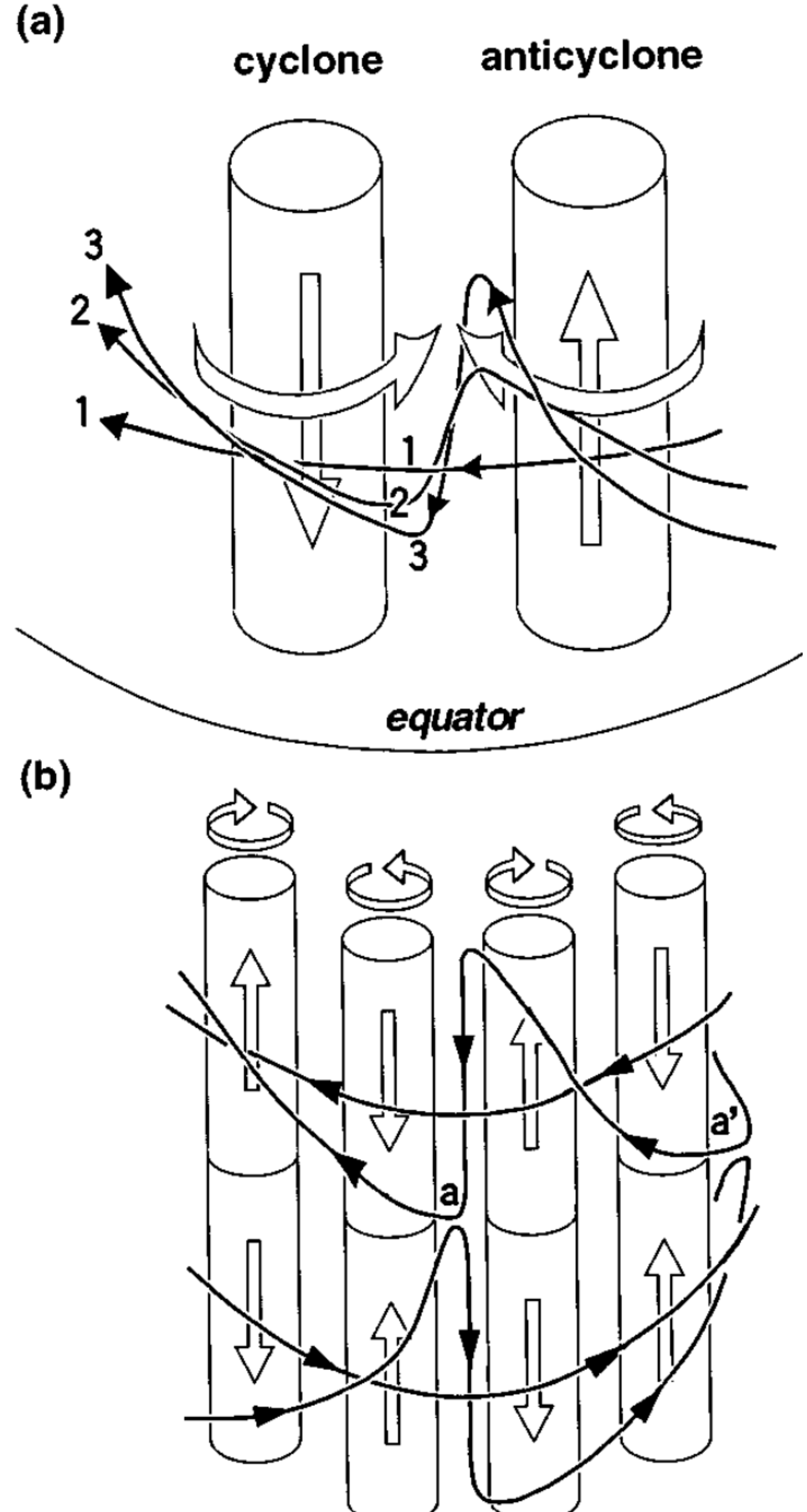
$\mathrm{Ra} = 8 \times 10^5$

$\mathrm{Ra} = 2.5 \times 10^7$

Busse columns are really good at making roughly dipolar fields



Kageyama & Sato (1997)

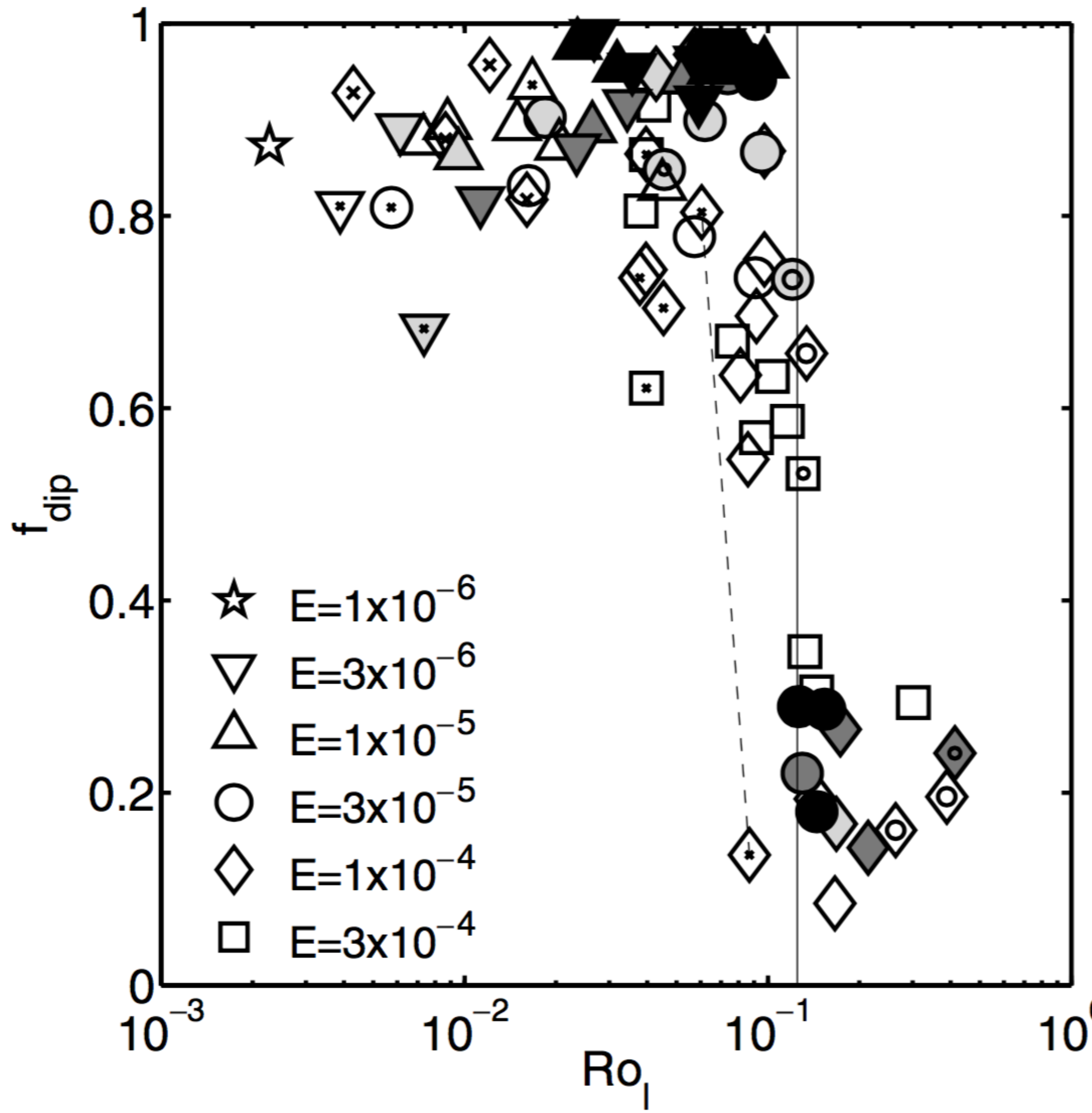


General trends

Complexity of magnetic field depends mainly on the rotational influence

Rapid rotators tend to be more dipolar

Christensen & Aubert (2006)



Question

$$c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\boldsymbol{\Omega}\times\mathbf{v}\right) + \boldsymbol{\nabla} P - \rho\mathbf{g}$$

Assuming MAC balance, compute the ratio of ME/KE
How does it scale with Ro?

$$\boldsymbol{\nabla}\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J}$$

$$\mathrm{ME} = \frac{B^2}{8\pi}$$

$$\mathrm{Ro} = \frac{U}{2\Omega D}$$

$$\mathrm{KE} = \frac{1}{2}\rho v^2$$

Question

$$c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\mathbf{\Omega}\times\mathbf{v}\right) + \mathbf{\nabla}P - \rho\mathbf{g}$$

Assuming MAC balance, compute the ratio of ME/KE
How does it scale with Ro?

$$\frac{ME}{KE} \sim \mathrm{Ro}^{-1}$$

>>1 *if* Ro << 1*!*

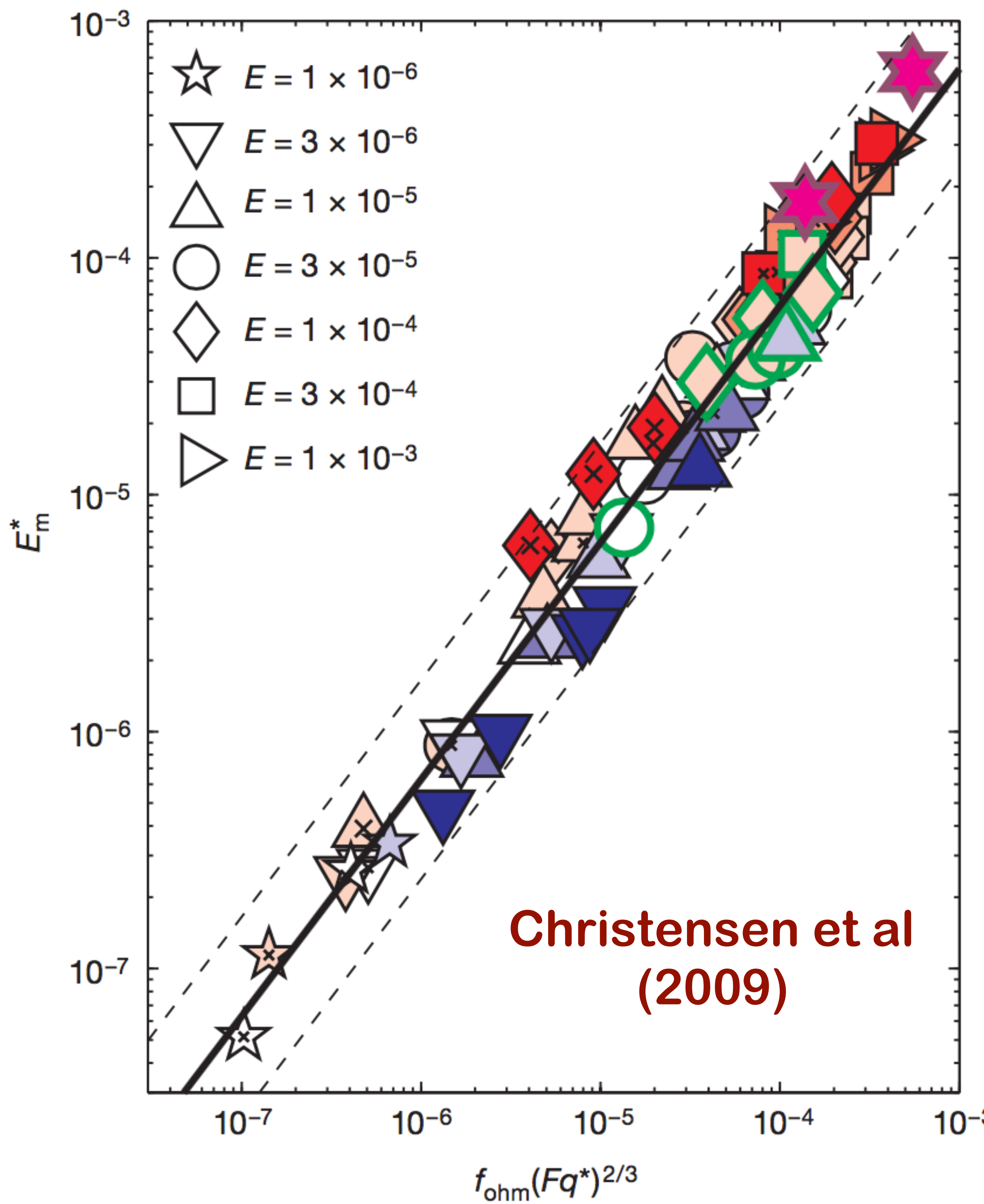
But how do KE and Ro (and thus, ME) depend on observable* global parameters like Ω and F_c?

****in principle***

General trends

Field strength scales with the heat flux through the shell (independent of Ω!)

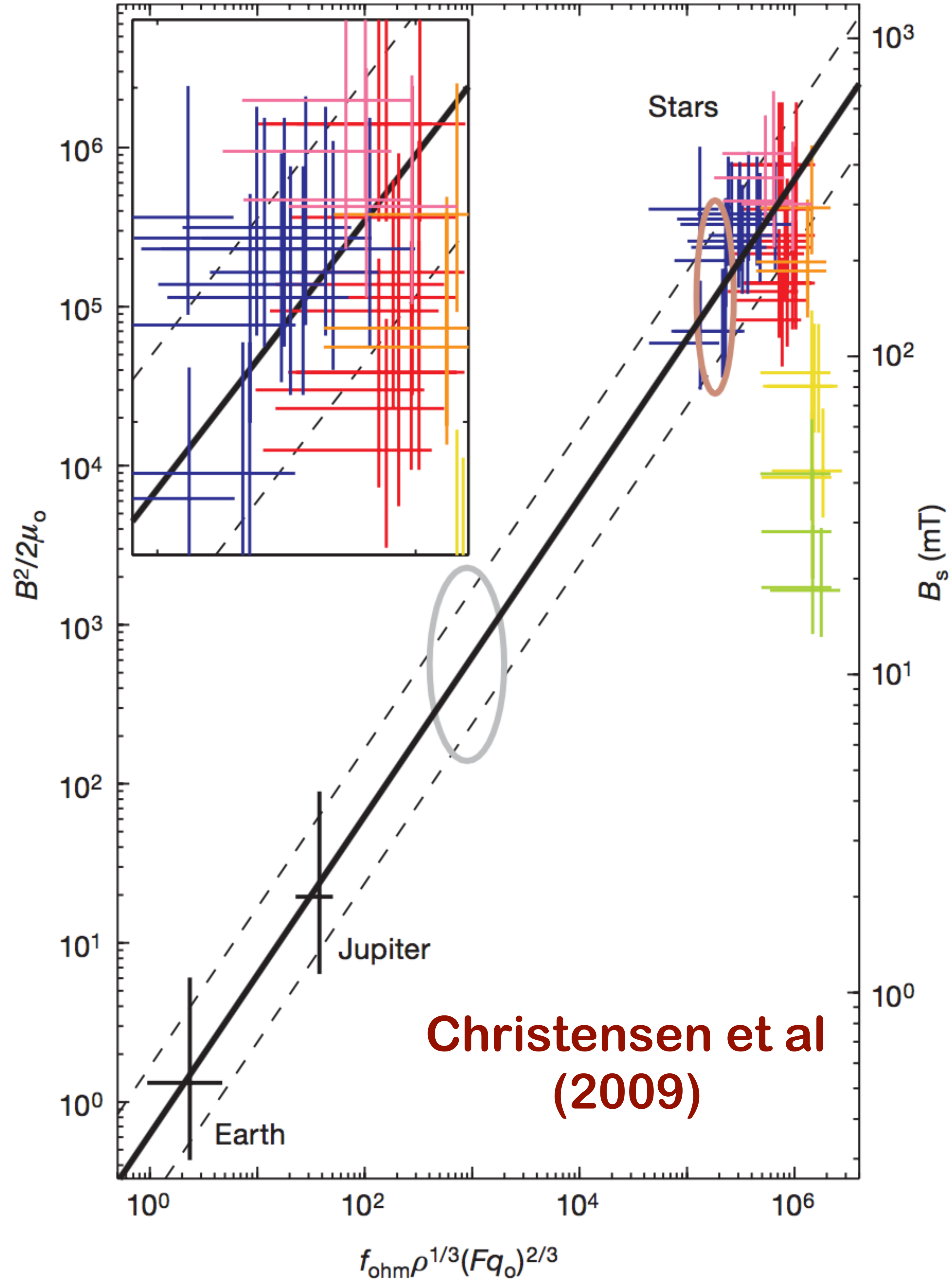
Rapid rotators seem to operate at maximum efficiency, tapping all the energy they can



General trends

Rapid rotators seem to operate at maximum efficiency, tapping all the energy they can

This may apply to rapidly-rotating stars as well as planets!



Numerical Models: The Challenge

$$\mathrm{Pm} = \frac{\nu}{\eta}$$

	Earth	Jupiter	Simulations
Ra	10^{31}	10^{37}	10^{6}-10^{7}
Ek	3×10^{-15}	10^{-9}	10^{-6} - 10^{-7}
Rm	300-1000	400-3×10^{4}	50-3000
Pm	5-6×10^{-7}	6×10^{-7}	0.1-0.01

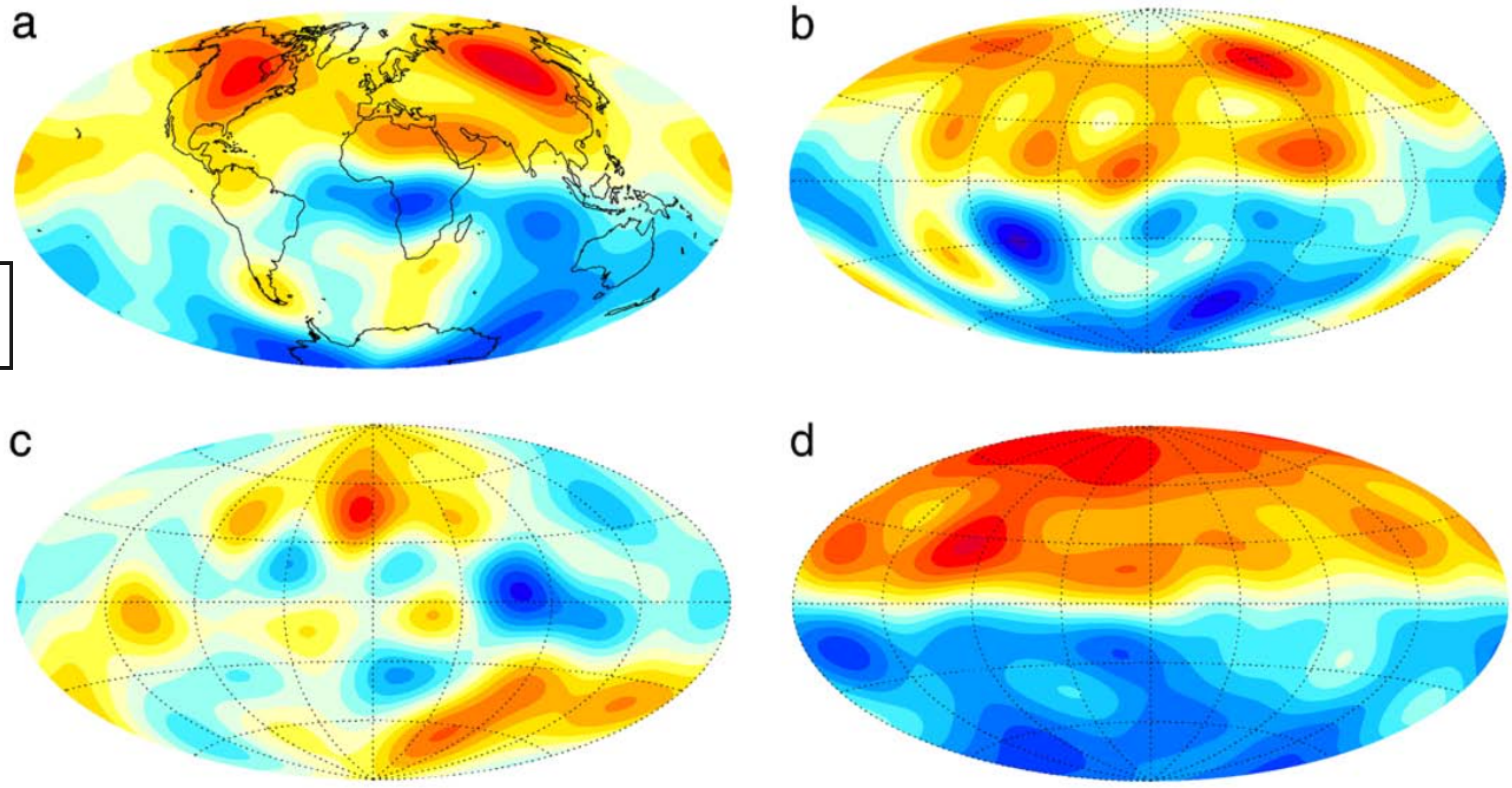
Numerical Models: The Hope

Realistic simulations might be possible if you can achieve the right dynamical balances (e.g. MAC balance)

- **The most important parameters to get right**
  (or as right as possible)
  - **Ro**
    - **Appropriate rotational influence on the convection**
  - **Rm**
    - **Reasonable estimate of the ohmic dissipation**
  - **Ek**
    - **At least get it small enough that viscosity isn't part of the force balance**

Example: The Geodynamo

Points of comparison: Field strength, morphology (spectrum, symmetry, etc), Reversal timescale



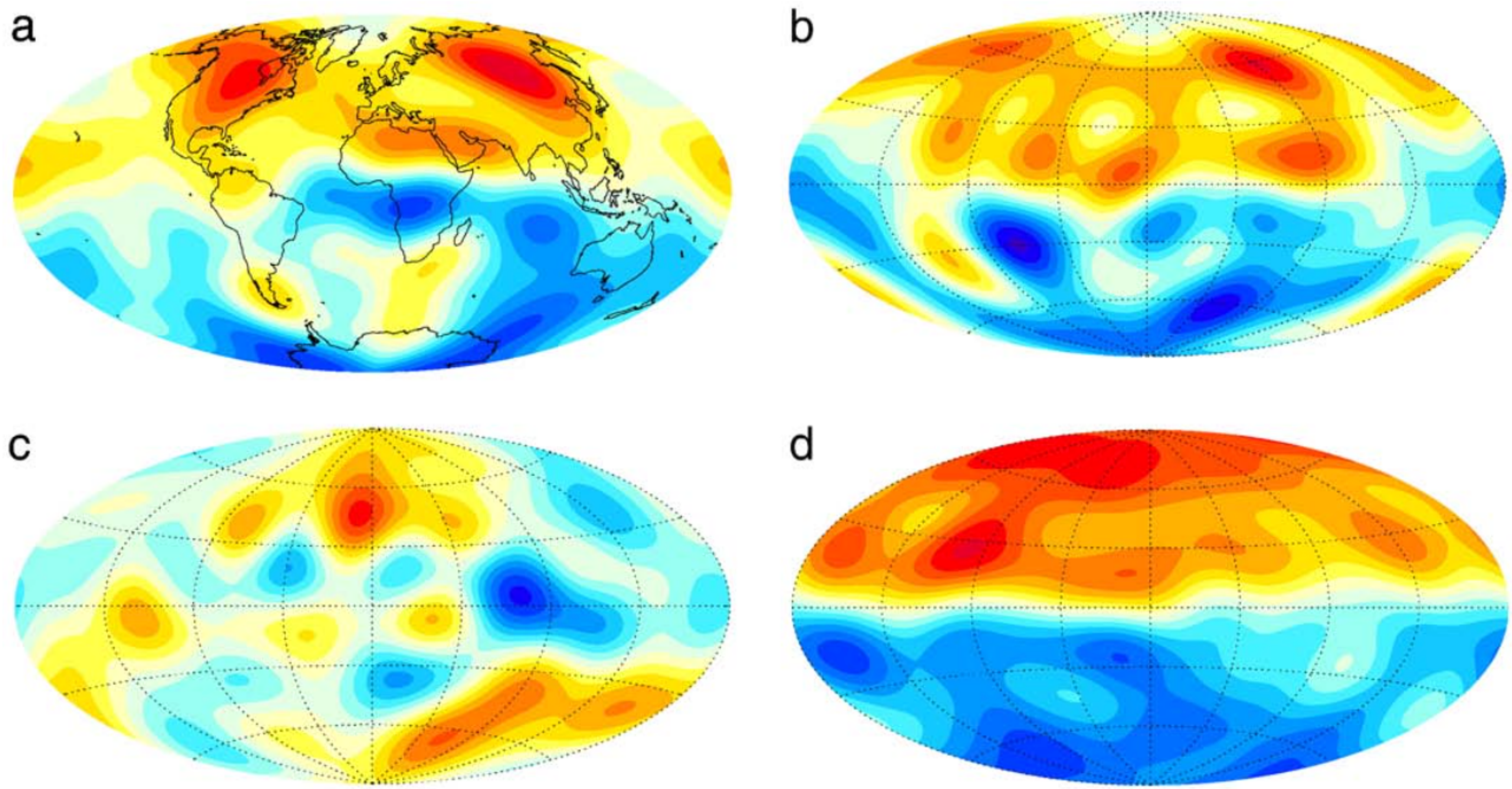
Inferred from observations

Christensen et al (2010)

Best matches are those with Ek < 10^{-4} and Rm "large enough"

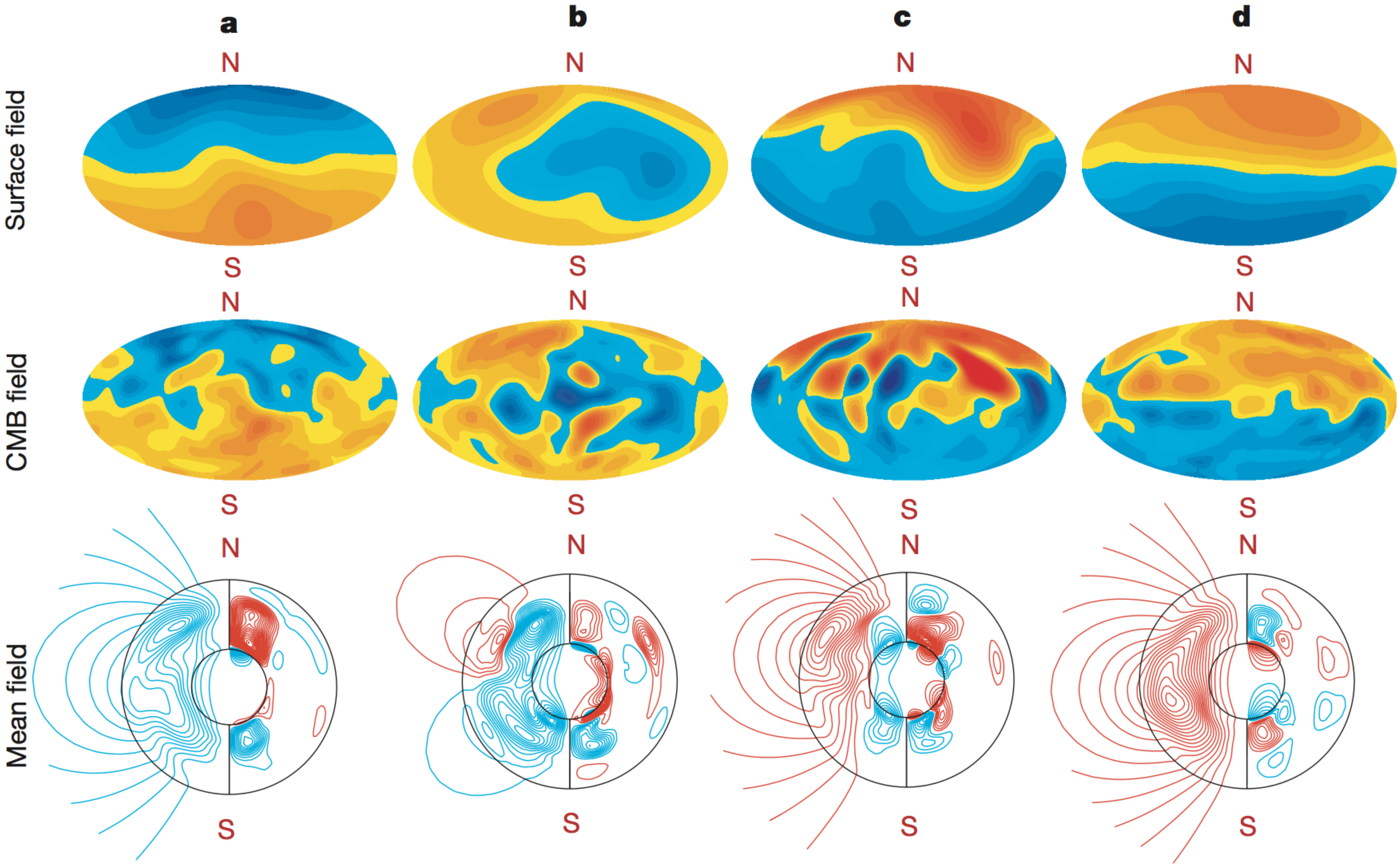
Example: The Geodynamo

Inferred from observations



But be careful! They could be right for the wrong reasons!
For example, both c and d have a higher Ra and lower Ek than b they <u>should</u> be more realistic, right?

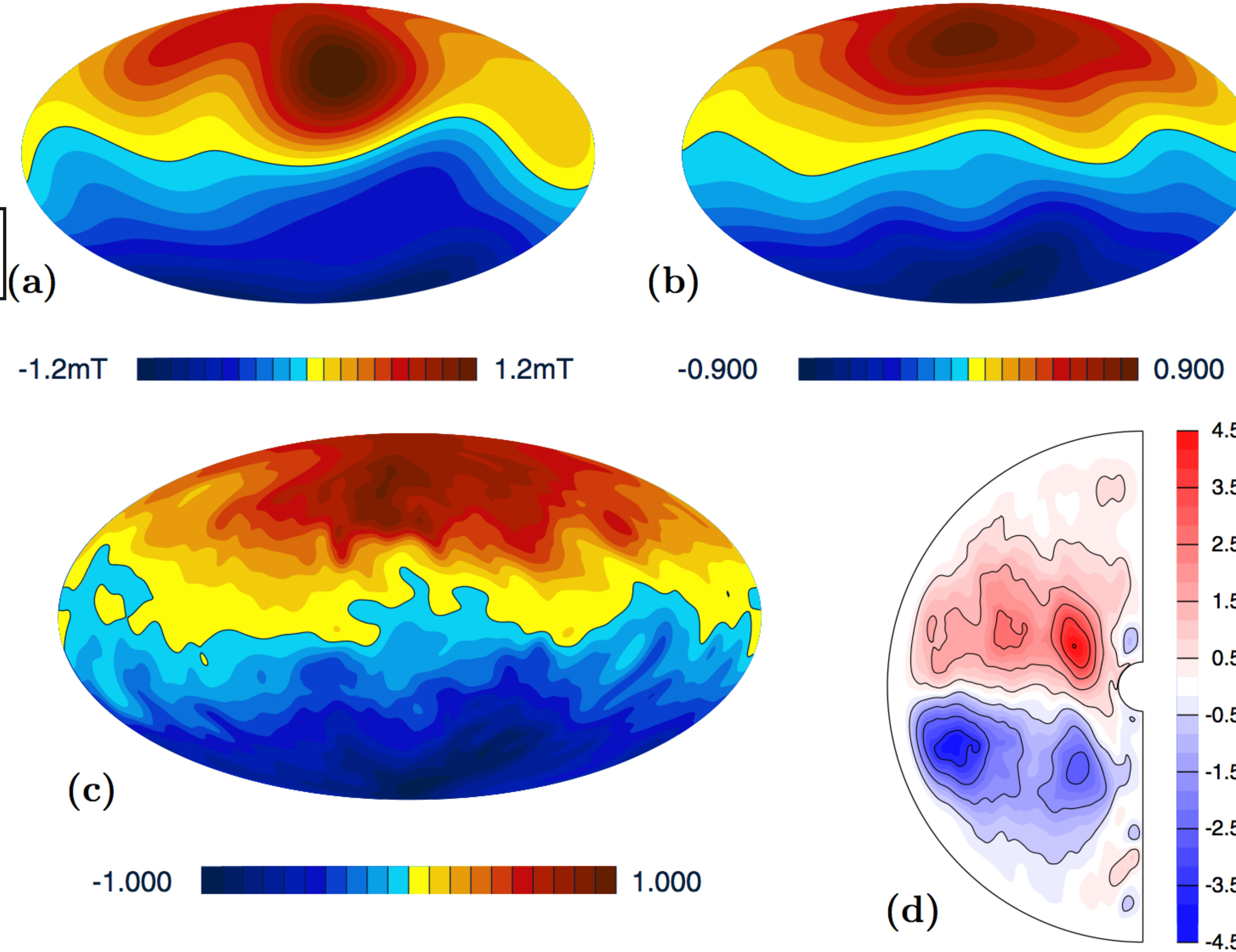
Example: The Geodynamo



Coupling to inner core needed to get the reversal time scale right (Glatzmaier & Roberts 1995; Glatzmaier et al 1999)

Example: Jupiter

Inferred from observations

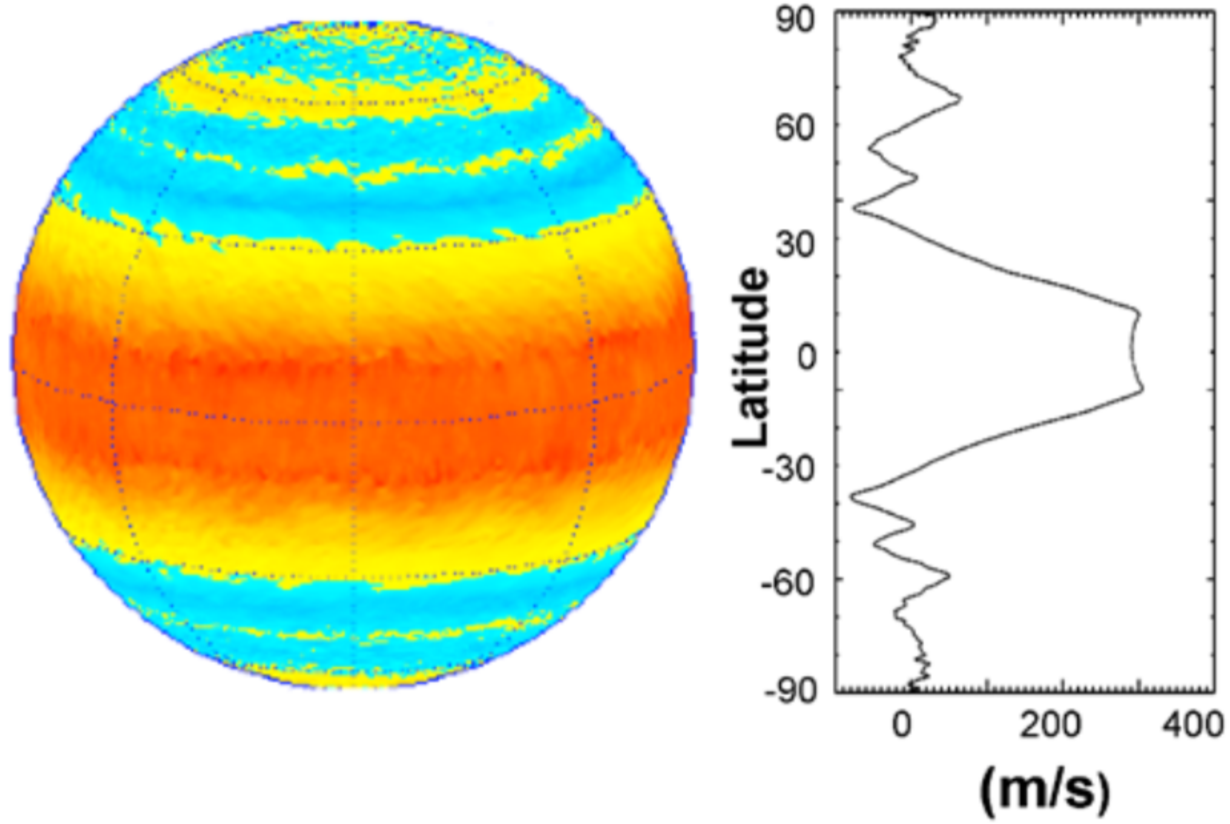


Jones (2014)

But "dipole solutions are not easy to find" for the "best" parameters

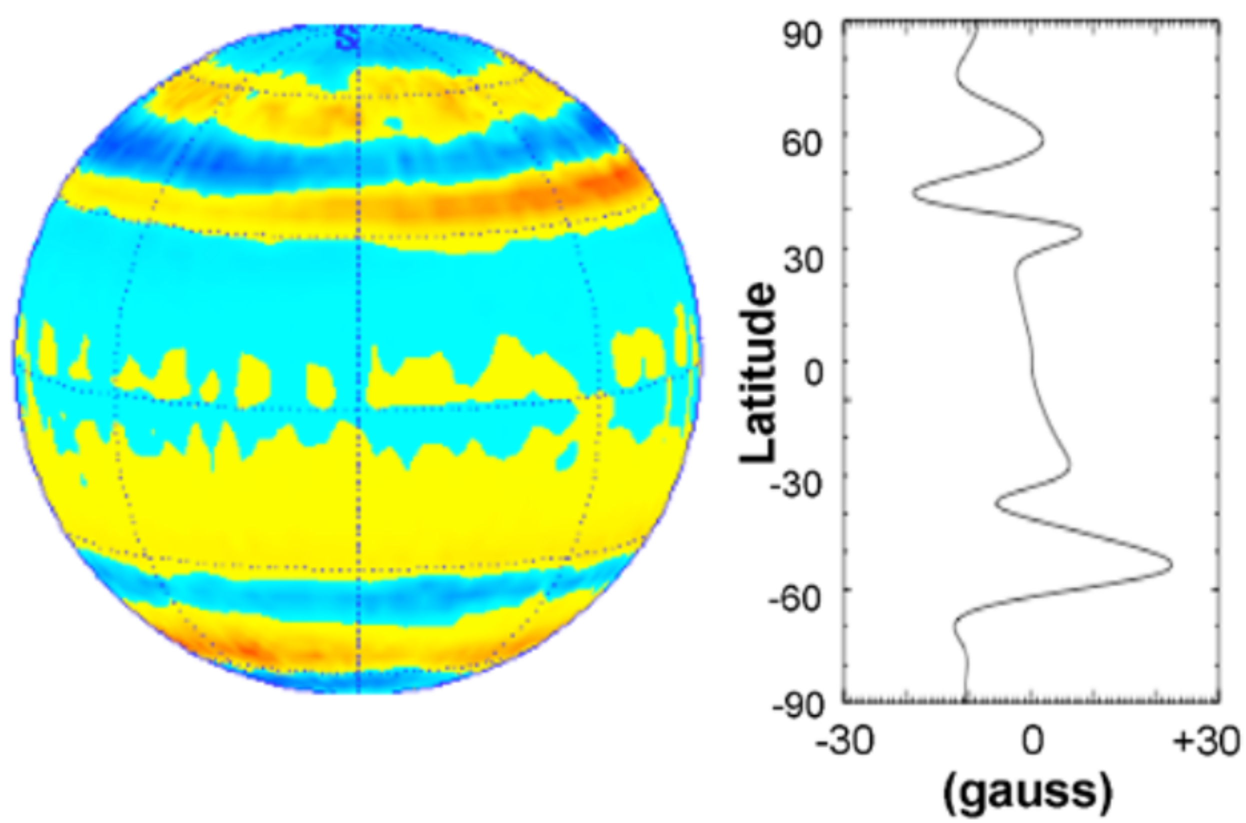
Another example of a Gas Giant dynamo highlighting banded zonal flows

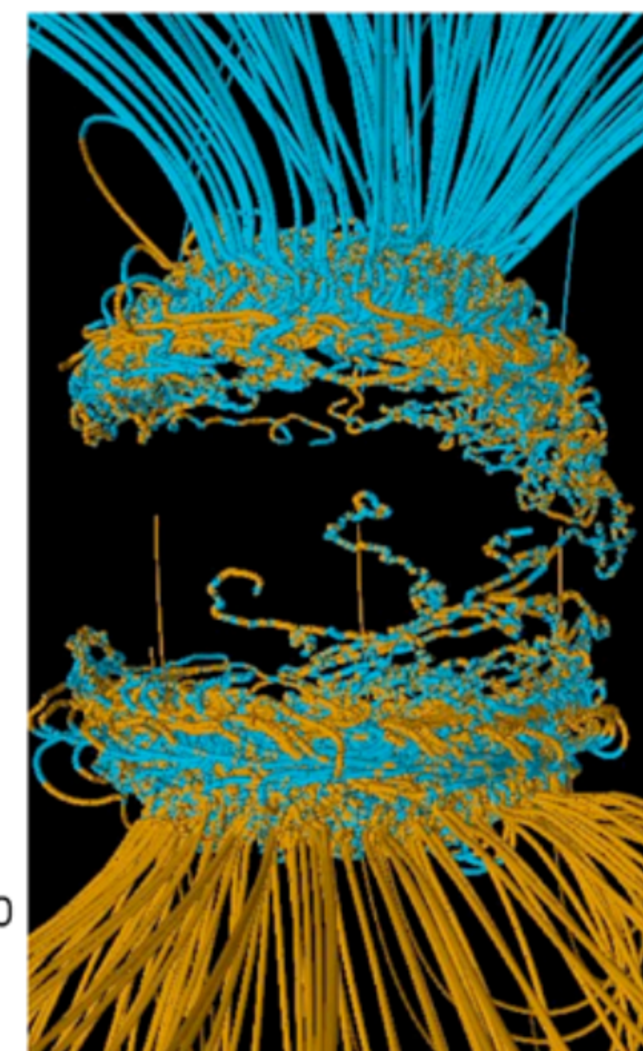
Surface longitudinal winds



Zonal winds in meridian plane

Surface radial magnetic field

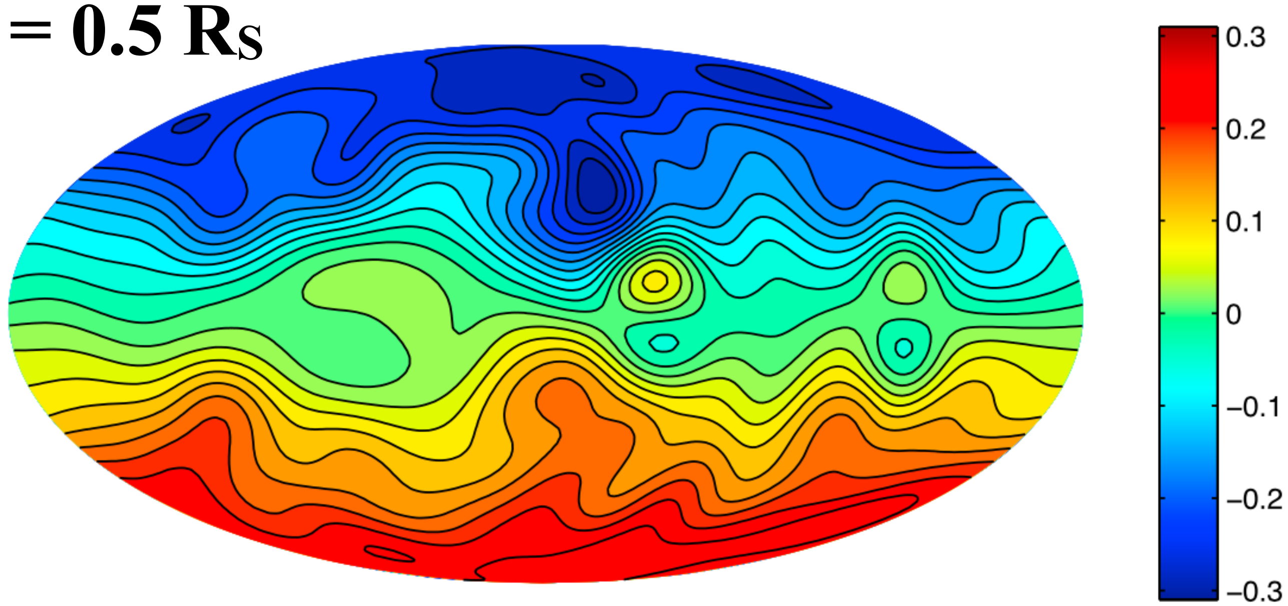




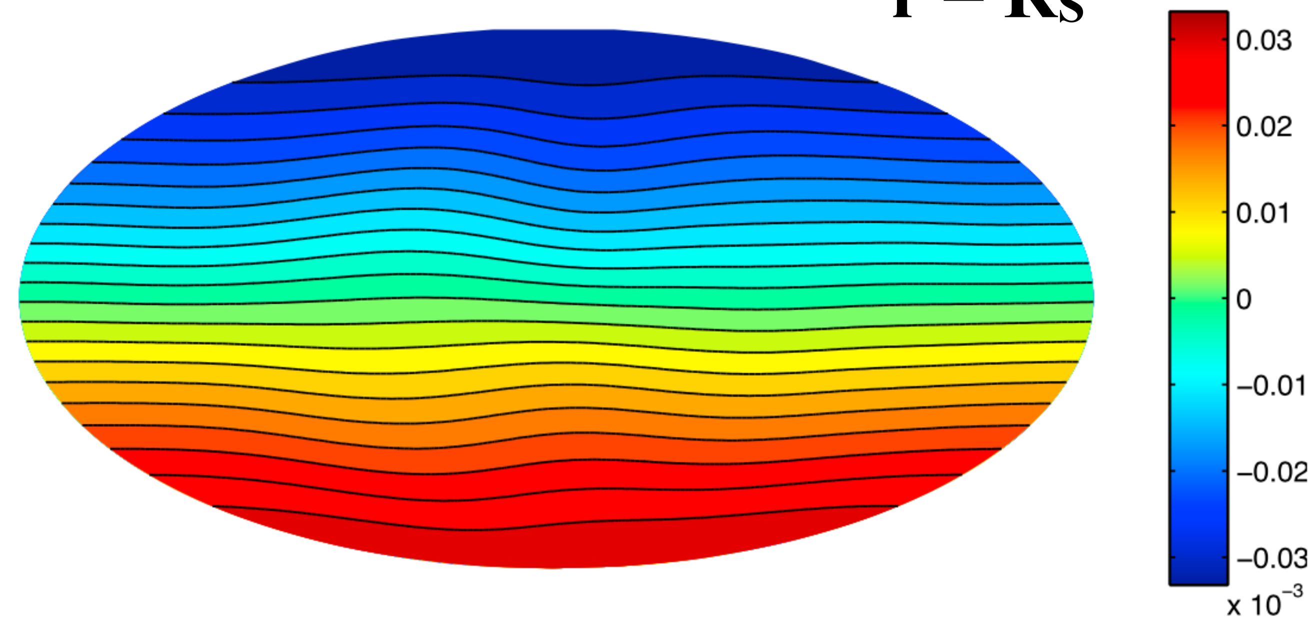
Stanley & Glatzmaier (2009)

So what's going on with Saturn?

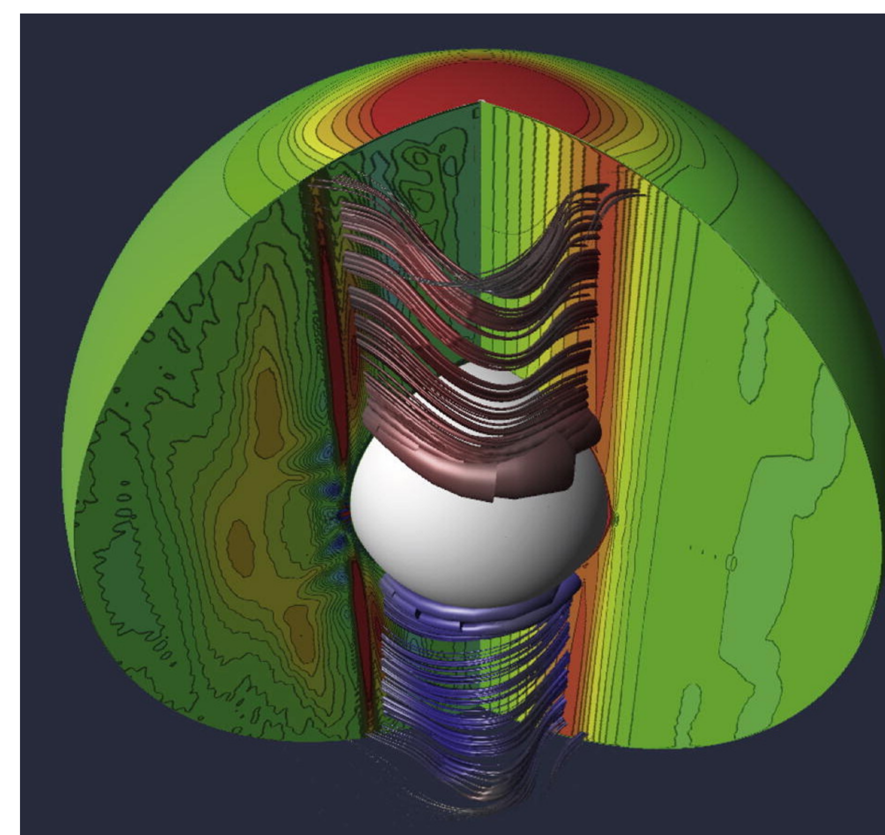
r = 0.5 R_S



r = R_S



Maybe the field is "axisymmeterized" by an overlying stable layer that has differential rotation but no convection (Stevenson 1982, Stanley 2010)

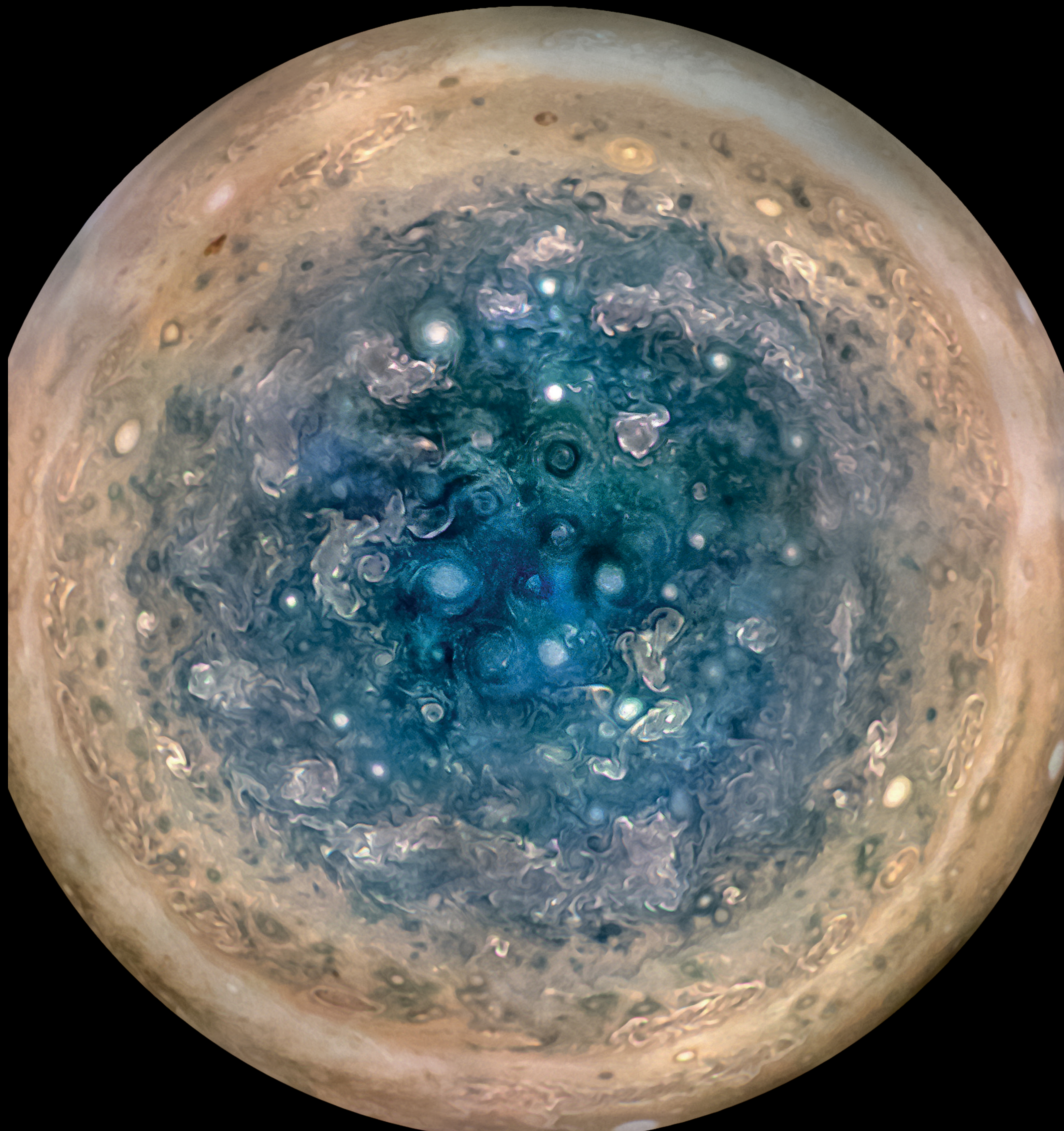


Or, maybe it's running a different type of dynamo, driven more by shear than buoyancy (Cao et al 2012)

Numerical Models: Summary

- **Lessons Learned**
  - ***Rapid Rotation has a profound influence on the dynamics***
  - ***Success attributed to correct dynamical balances and (when possible) realistic Rm***
- **Future challenges**
  - ***What happens at really low Ek (tiny ν)?***
  - ***Peculiarities of particular planets (Saturn, Mercury, Uranus, Neptune…)***
    - Boundary conditions (adjacent layers)
    - Rapid variations of η
    - Energy sources
    - Compositional convection
  - ***Moving to more realistic parameters doesn't always improve the fidelity of the model***
  - ***Exoplanets!***

Juno!



Featherstone & Heimpel 2017