Stellar dynamos

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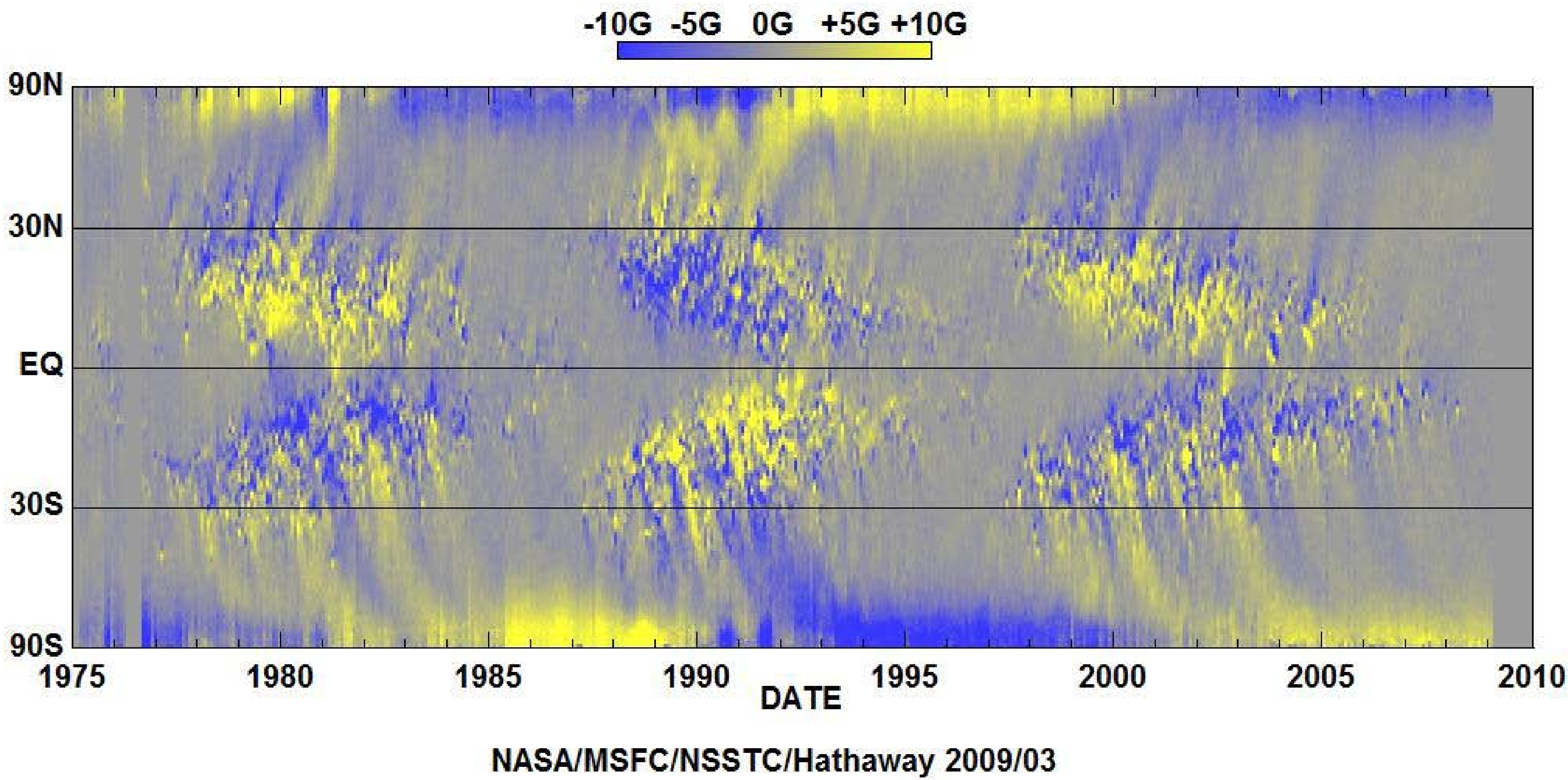
Magnetohydrodynamics (ch. I.3)
Simulations of solar/stellar dynamos (ch. III.5, +)
Mean-field electrodynamics (ch. I.3, III.6)

From MHD to simpler dynamos (ch. I.3, III.6)
Solar and stellar dynamo models (ch. III.2, III.6)
Introducing the lab

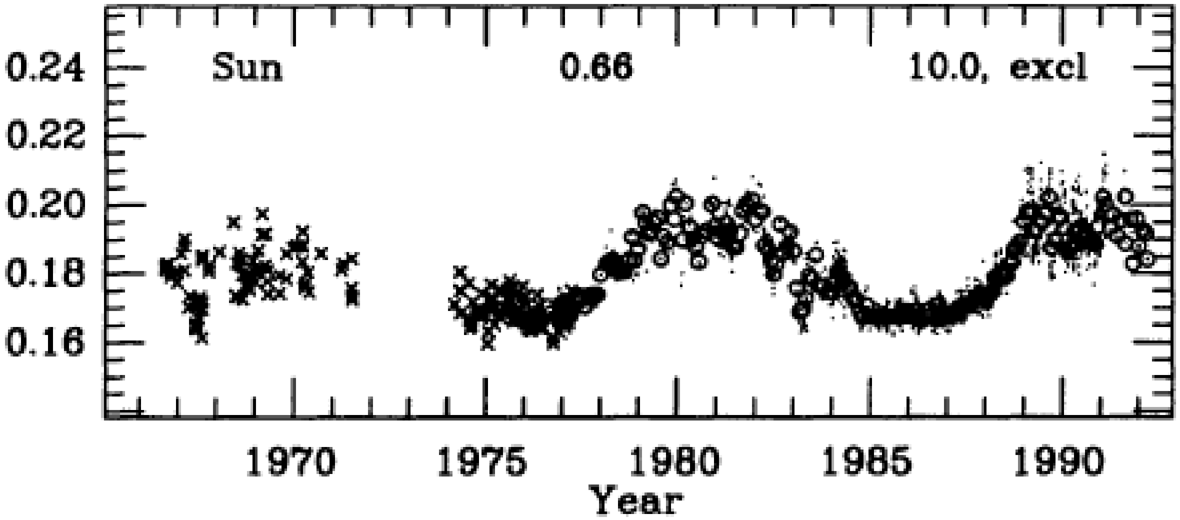
The solar magnetic cycle



I.2.2

The Sun as a star

Emission in the cores of the H and K spectral lines of Ca offers a good **proxy** of magnetic activity…

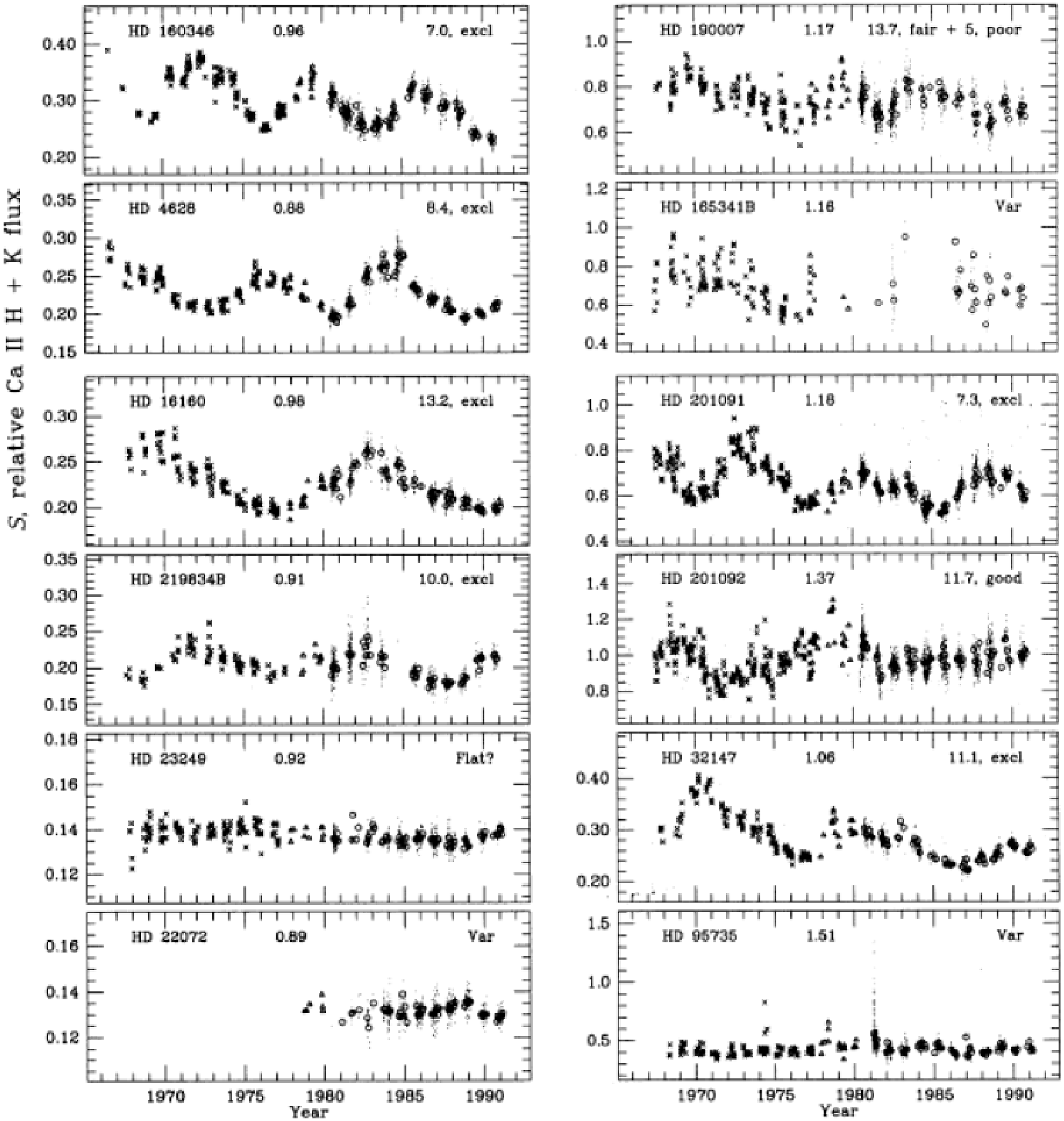


S. Baliunas *et al.* 1995, ApJ, **438**, 269.

…but, the amplitude of Ca H+K emission is difficult to relate **quantitatively** to measures of « magnetic cycle amplitude »; the cycle period, in contrast, is (probably) unambiguous.

I.2.3

…and stars as Suns…



S. Baliunas *et al.* 1995, ApJ, **438**, 269.

I.2.3

Magnetohydrodynamics
[Section I.3.2]

Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\varepsilon_0} , \qquad \text{[Gauss' Law]}$$

$$\nabla \cdot \mathbf{B} = 0 , \qquad \text{[Anonymous]}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} , \qquad \text{[Faraday's Law]}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} , \qquad \text{[Ampere/Maxwell's Law]}$$

I.3.2

From Maxwell to MHD (1)

Step 1: Drop displacement current to revert to the original form of Ampère's Law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Step 2: Write down Ohm's Law in a frame co-moving with the fluid:

$$\mathbf{J}' = \sigma \mathbf{E}'$$

Step 3: Non-relativistic transformation back to the laboratory frame of reference:

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

I.3.2

From Maxwell to MHD (2)

Step 4: Combine with Ampère's Law to express the electric field as:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \qquad \mathbf{E} = -\mathbf{u} \times \mathbf{B} + \frac{1}{\mu_0 \sigma}(\nabla \times \mathbf{B})$$

Step 5: Substitute into Faraday's Law to get the justly famous **magnetohydrodynamical induction equation:**

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B})}$$

where we defined the **magnetic diffusivity** as:

$$\eta = \frac{1}{\mu_0 \sigma} \qquad [\mathrm{m}^2 \mathrm{s}^{-1}]$$

I.3.2

Scaling analysis (1)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Suppose we can estimate, *a priori*, a typical time scale and a typical length scale over which both the flow and magnetic field vary appreciably; by replacing differential operators in the MHD equation by the inverse of these scales, we get:

$$\frac{\mathbf{B}}{\tau} = \frac{u_0 \mathbf{B}}{\ell} + \frac{\eta \mathbf{B}}{\ell^2}$$

The ratio of the two terms on the RHS defines a dimensionless quantity known as the **magnetic Reynolds number**:

$$\boxed{\mathrm{R}_m = \frac{u_0 \ell}{\eta}}$$

I.3.2.3

Astrophysical parameter regimes

Table 1.2
Properties of some astrophysical objects and flows

System/flow	L [km]	σ [Ω^{-1}m^{-1}]	η [m^{2}s^{-1}]	τ [yr]	u [km/s]	R_m
Solar interior	10^6	10^4	100	10^9	0.1	10^9
Solar atmosphere	10^3	10^3	1000	10^2	1	10^6
Solar corona	10^5	10^6	1	10^8	10	10^{12}
Solar wind (1 AU)	10^5	10^4	100	10^8	300	10^{11}
Molecular cloud	10^{14}	10^2	10^4	10^{17}	100	10^{18}
Interstellar medium	10^{16}	10^3	1000	10^{22}	100	10^{21}
Sphere of copper	10^{-3}	10^8	10^{-1}	10^{-7}	—	—

Magnetic diffusion would appear entirely negligible in most astrophysical systems; but beware…

Scaling analysis (2)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

$$\tau = \ell / u_0$$

$$\tau = \frac{\ell^2}{\eta}$$

Advection timescale

Rm >> 1

« Ideal MHD »

Diffusion timescale

Rm << 1

10^8-10^{10}yr for sun!

I.3.2.3

The MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \ ,$$

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \frac{1}{\mu_0 \rho}(\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{1}{\rho}\nabla \cdot \boldsymbol{\tau} \ ,$$

$$\frac{\mathrm{D}e}{\mathrm{D}t} + (\gamma - 1)e\nabla \cdot \mathbf{u} = \frac{1}{\rho}\Big[\nabla \cdot \Big((\chi + \chi_r)\nabla T\Big) + \phi_\nu + \phi_B\Big] \ ,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \ .$$

I.3.2.1

Dynamo problems

The kinematic dynamo problem:

HARD

TURBULENCE

« To find a flow **u** that can lead to field amplification when substituted in the MHD equation »

The self-excited dynamo problem:

MUCH HARDER

TURBULENCE

« To find a flow **u** that can lead to field amplification when substituted in the MHD equation, while being dynamically consistent with the fluid equations including the Lorentz force »

The solar/stellar dynamo problem(s):

HARDEST

????????

« To find a flow **u** that leads to a magnetic field amplification and evolution in agreement with observational inferences for the Sun and stars »

I.3.2.3

Simulations of solar/stellar dynamos
[Chapter III.5, + extra]

Selected milestones

Gilman 1983: Boussinesq MHD simulation, producing large-scale magnetic fields with polarity reversals on yearly timescale; but non-solar large-scale organization.

Glatzmaier 1984, 1985: Anelastic model including stratification, large-scale fields with polarity reversals within a factor 2 of solar period; tendency for poleward migration of the large-scale magnetic field.

Brun et al. 2004: Strongly turbulent MHD simulation, producing copious small-scale magnetic field but no large-scale magnetic component.

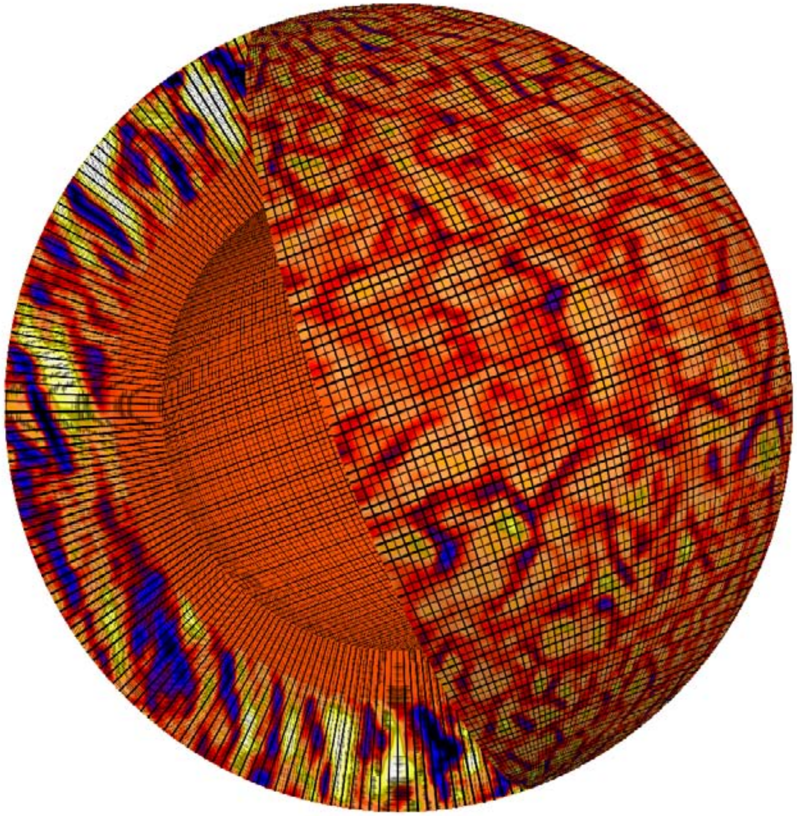
Browning et al. 2006: Demonstrate the importance of an underlying, convectively stable fluid layer below the convection zone in producing a large-scale magnetic component in the turbulent regime.

Brown et al. 2009, 2010: Obtain irregular polarity reversals of thin, intense toroidal field structure in a turbulent simulation rotating at 5X solar.

Ghizaru et al. 2010: Obtain regular polarity reversals of large-scale magnetic component on decadal timescales, showing many solar-like characteristics.

Nelson et al. 2012, 2013: Autonomous generation of buoyantly rising flux-ropes structures showing sunspot-like emergence patterns.

Simulation framework



Simulate anelastic convection in thick, **rotating** and unstably **stratified** fluid shell of electrically conducting fluid, overlaying a stably stratified fluid shell.

Recent such simulations manage to reach Re, Rm ~10^2-10^3, at best; a long way from the solar/stellar parameter regime.

Throughout the bulk of the convecting layers, **convection is influenced by rotation**, leading to alignment of convective cells parallel to the rotation axis.

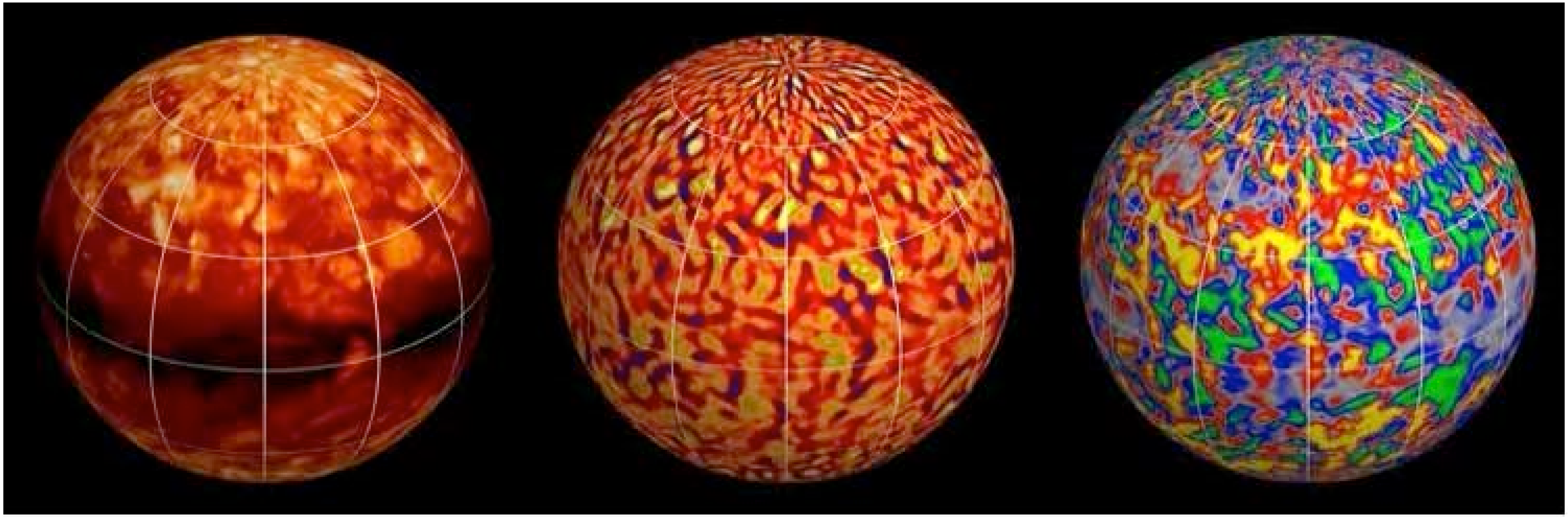
Stratification leads to **downward pumping of the magnetic field** throughout the convecting layers.

III.5.4

MHD simulation of global dynamos

[Ghizaru et al. 2010, ApJL, 715, L133]

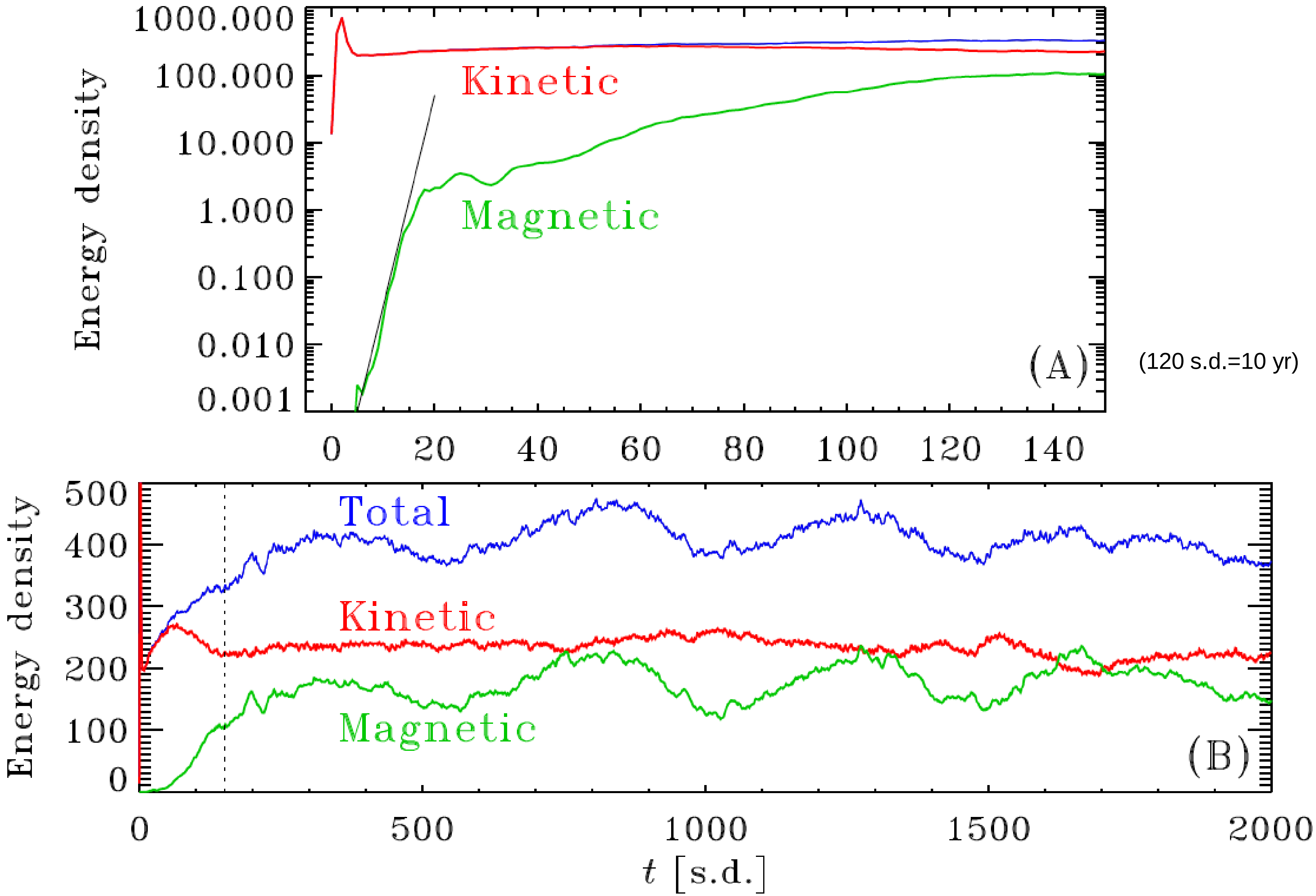
Temperature perturbation Radial flow component Radial magnetic field component



http://www.astro.umontreal.ca/~paulchar/grps > Que faisons nous > Simulations MHD

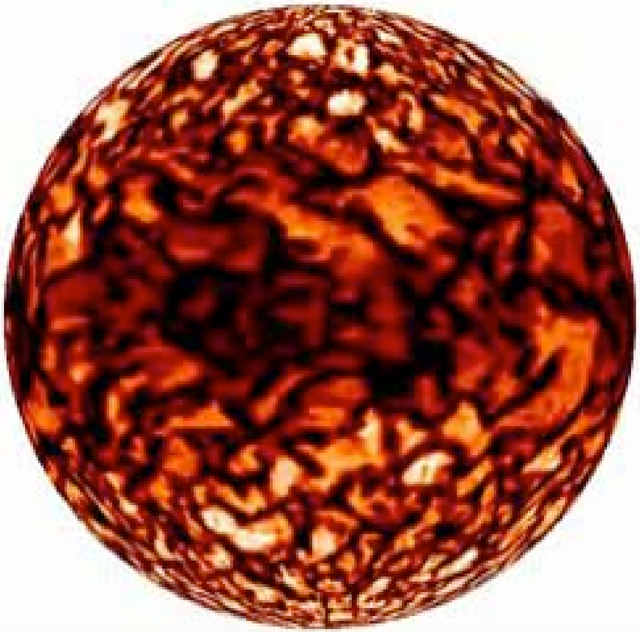
Electromagnetic induction by internal flows is the engine powering the solar magnetic cycle. The challenge is to produce a magnetic field well-structured on spatial and temporal scales much larger/longer than those associated with convection itself. This is the **magnetic self-organisation problem**.

Kinetic and magnetic energies

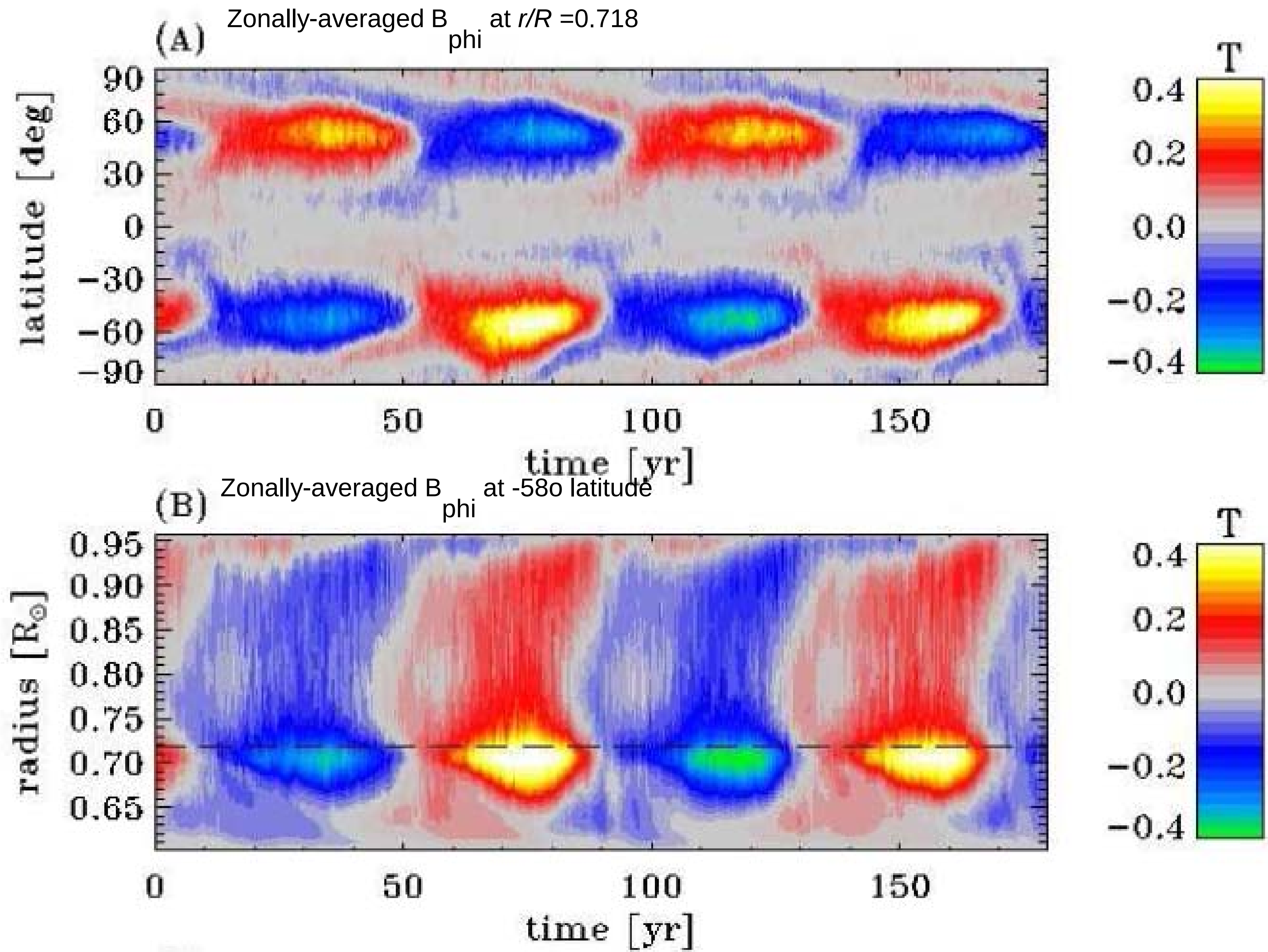


(120 s.d.=10 yr)

Simulated magnetic cycles (1)



Large-scale organisation of the magnetic field takes place primarily at and immediately below the base of the convecting fluid layers

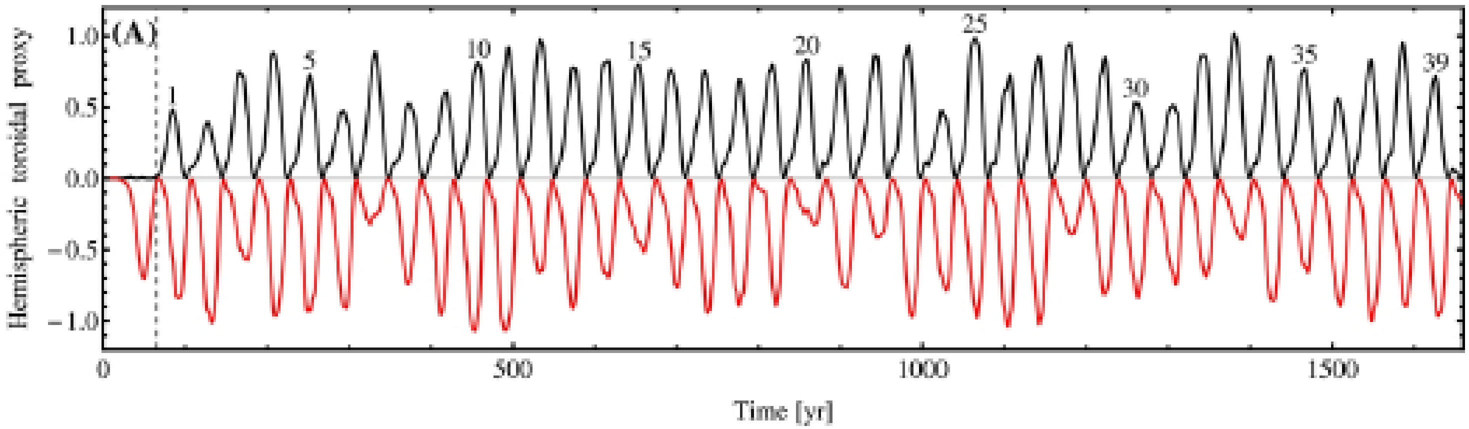


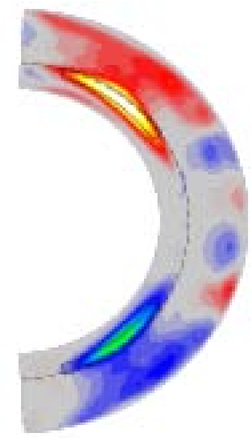
(A)
Zonally-averaged B phi at r/R =0.718
latitude [deg]
90
60
30
0
−30
−60
−90
0
50
100
150
time [yr]
T
0.4
0.2
0.0
−0.2
−0.4
(B)
Zonally-averaged B phi at -58o latitude
radius [R⊙]
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0
50
100
150
time [yr]
T
0.4
0.2
0.0
−0.2
−0.4

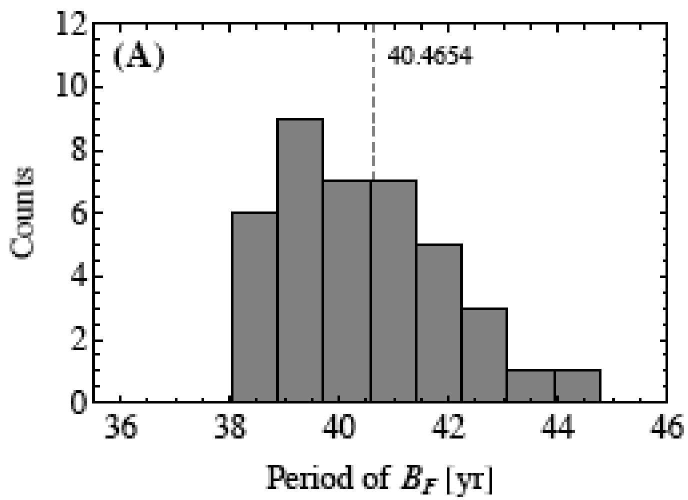
Characteristics of simulated cycles (1)

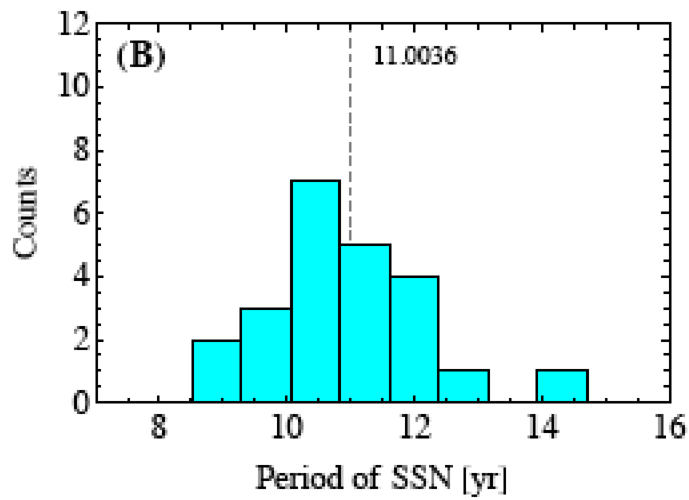
[Passos & Charbonneau 2014, A&A, in press]

Define a SSN proxy, measure cycle characteristics (period, amplitude…) and compare to observational record.





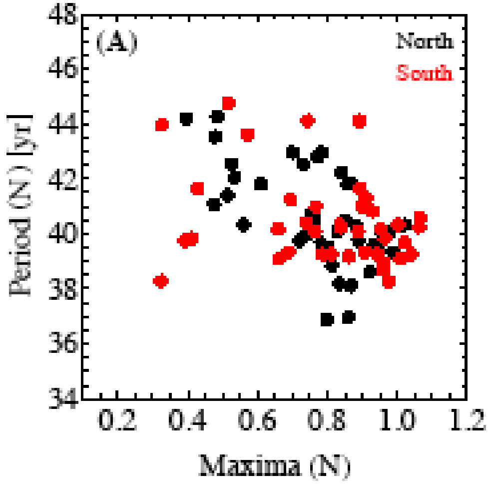


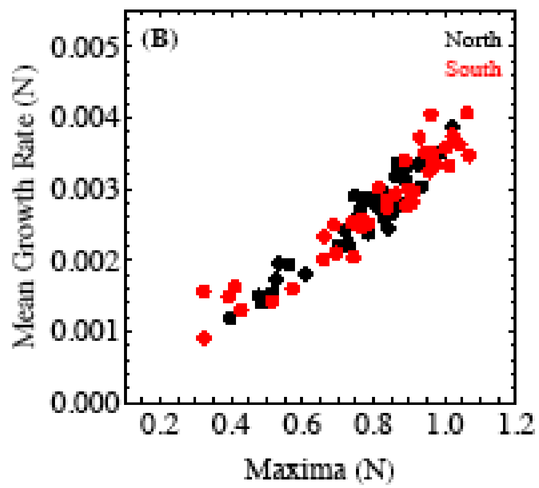


Characteristics of simulated cycles (2)

[Passos & Charbonneau 2014, A&A, in press]

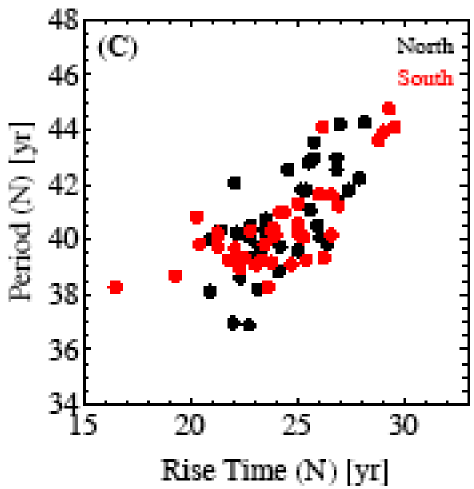
r =
-0.395/-0.147
[-0.552/-0.320]

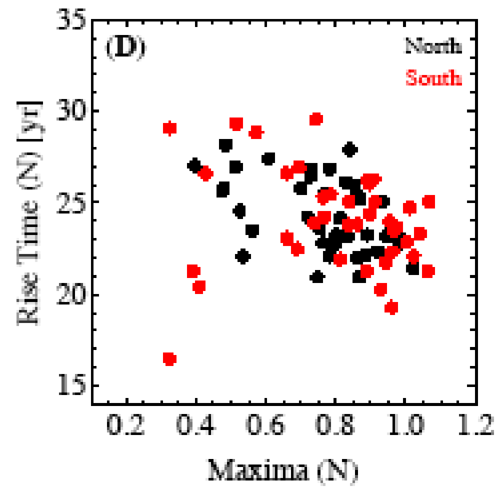




r =
0.957/0.947
[0.763/0.841]

r =
0.688/0.738
[0.322/0.451]





r =
-0.465/-0.143
[0.185/-0.117]

Successes and problems

KiloGauss-strength large-scale magnetic fields, antisymmetric about equator and undergoing regular polarity reversals on decadal timescales.

Cycle period four times too long, and strong fields concentrated at mid-latitudes, rather than low latitudes.

Internal magnetic field dominated by toroidal component and strongly concentrated immediately beneath core-envelope interface.

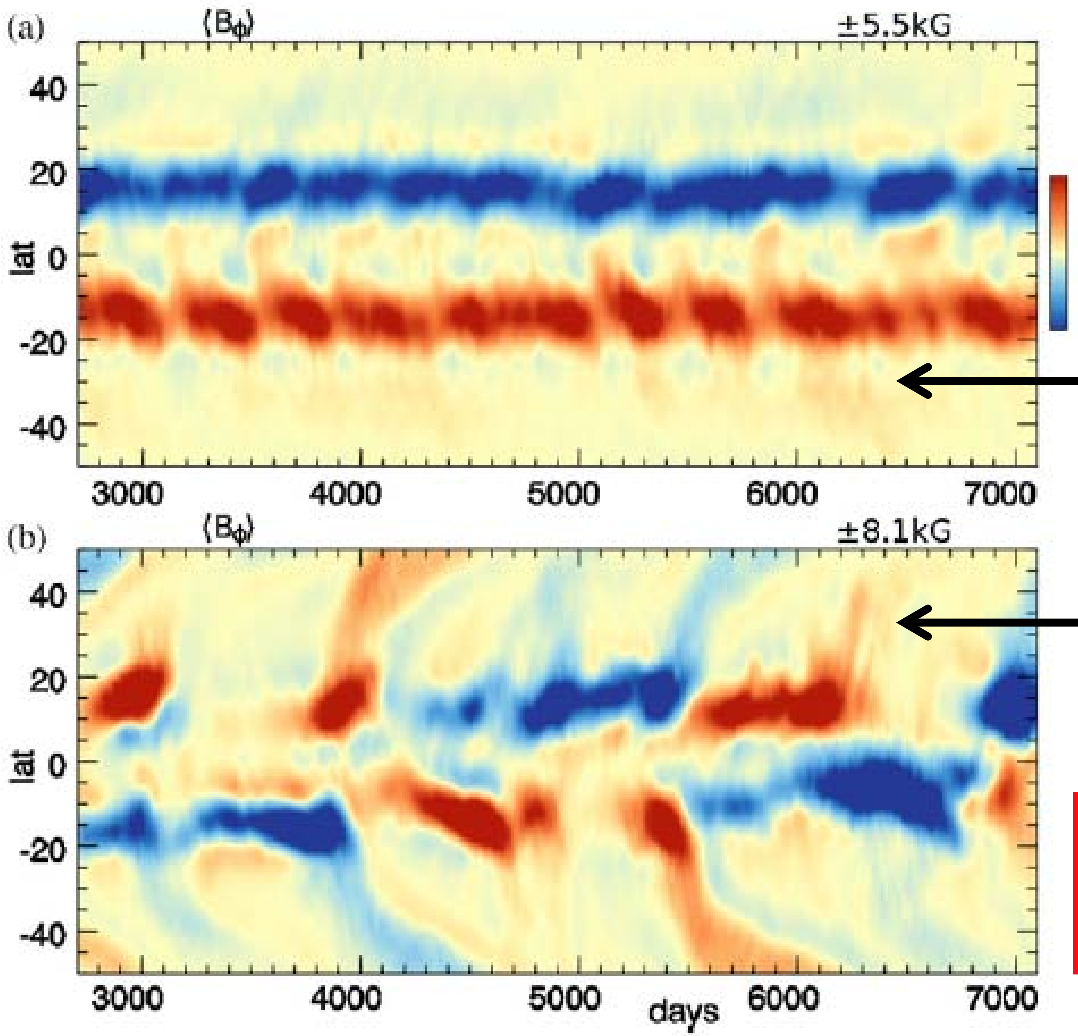
Well-defined dipole moment, well-aligned with rotation axis, but oscillating in phase with internal toroidal component.

Reasonably solar-like internal differential rotation, and solar-like cyclic torsional oscillations (correct amplitude and phasing).

On long timescales, tendency for hemispheric decoupling, and/or transitions to non-axisymmetric oscillatory modes.

Stellar cycles (1)

B.P. Brown et al. 2011, *Astrophys. J.*, **XXX**, YYY



ASH LES: at solar rotation rate and luminosity, no large-scale field produced;

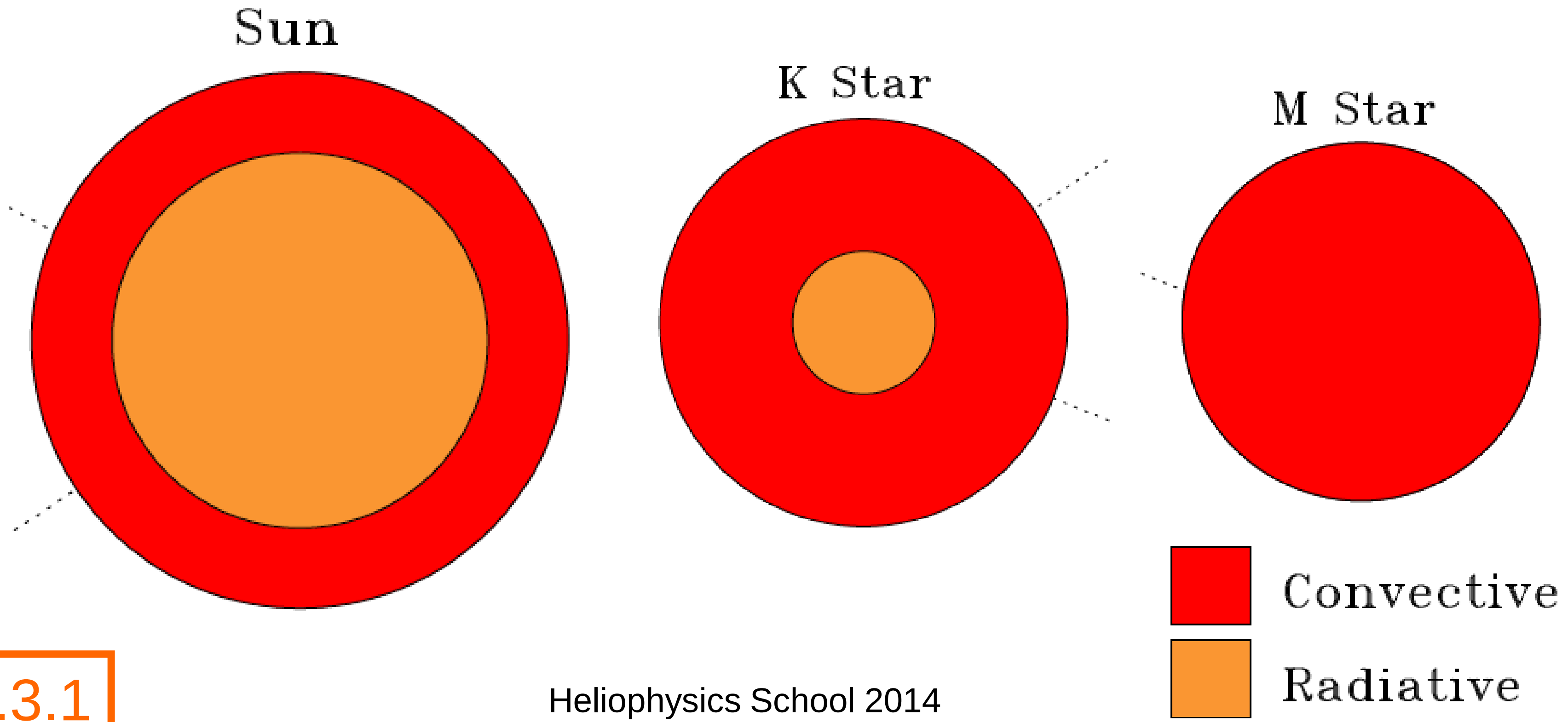
At 3X solar rotation, steady axisymmetric large-scale field is produced;

At 5X solar rotation, irregular polarity reversals occur.

Convection + Rotation breed magnetism !!

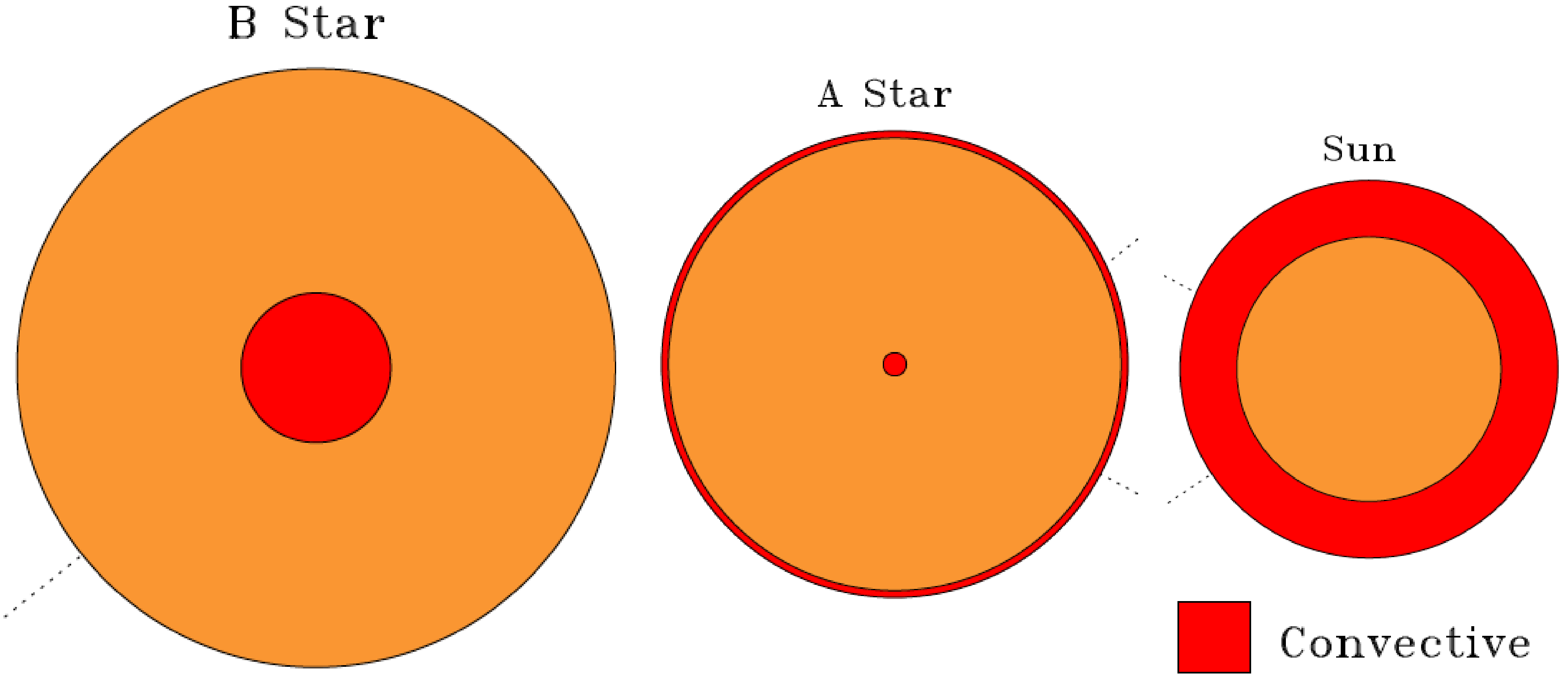
Convection in main-sequence star (1)

In « cool » stars, the convective envelope deepens as the surface temperature/mass decreases; stars are fully convective around M5



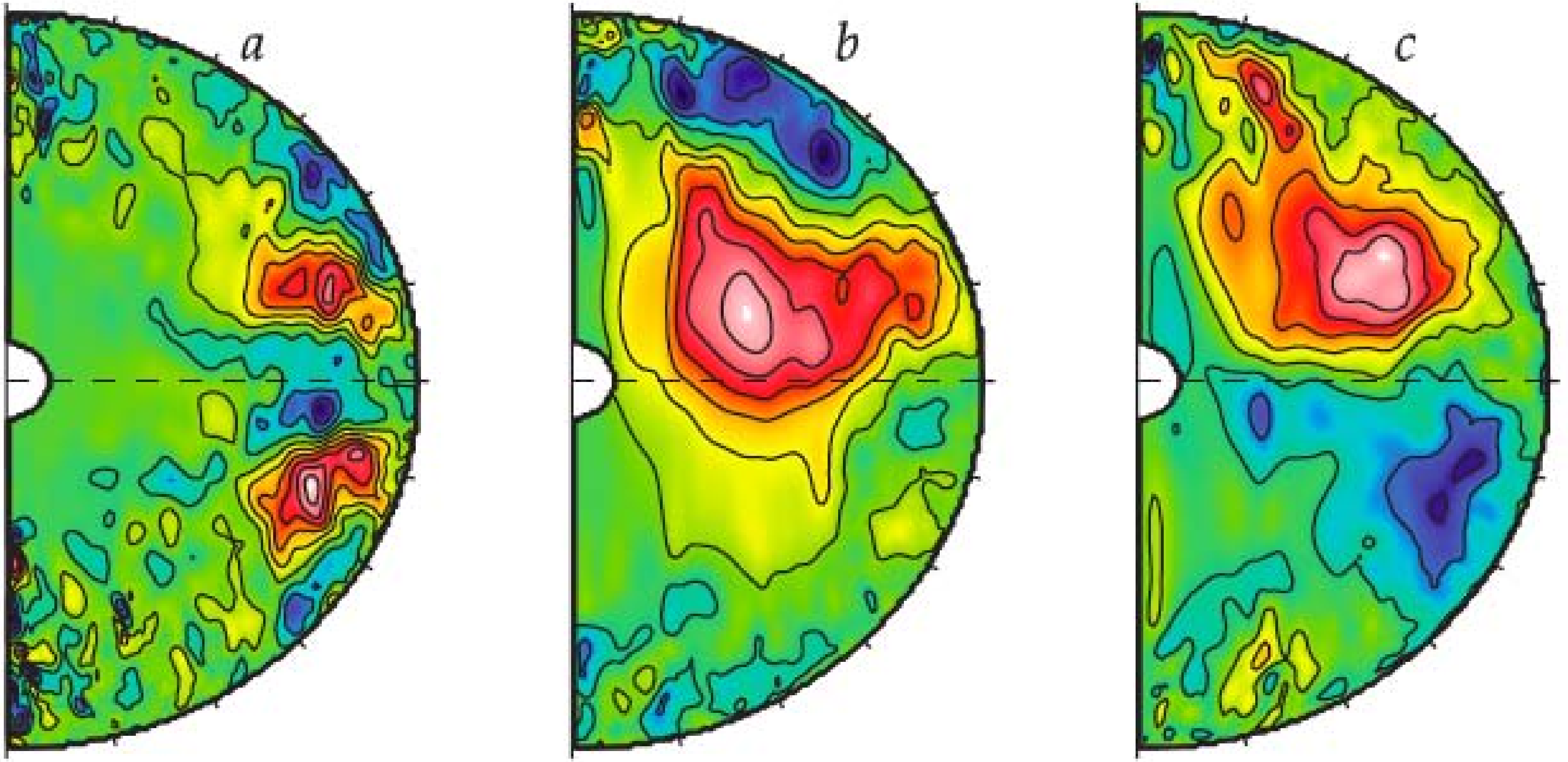
Convection in main-sequence stars (2)

In « hot » stars, the convective envelope disappears, but a convective core builds up as mass/effective temperature increases



3D MHD in fully convective stars (1)

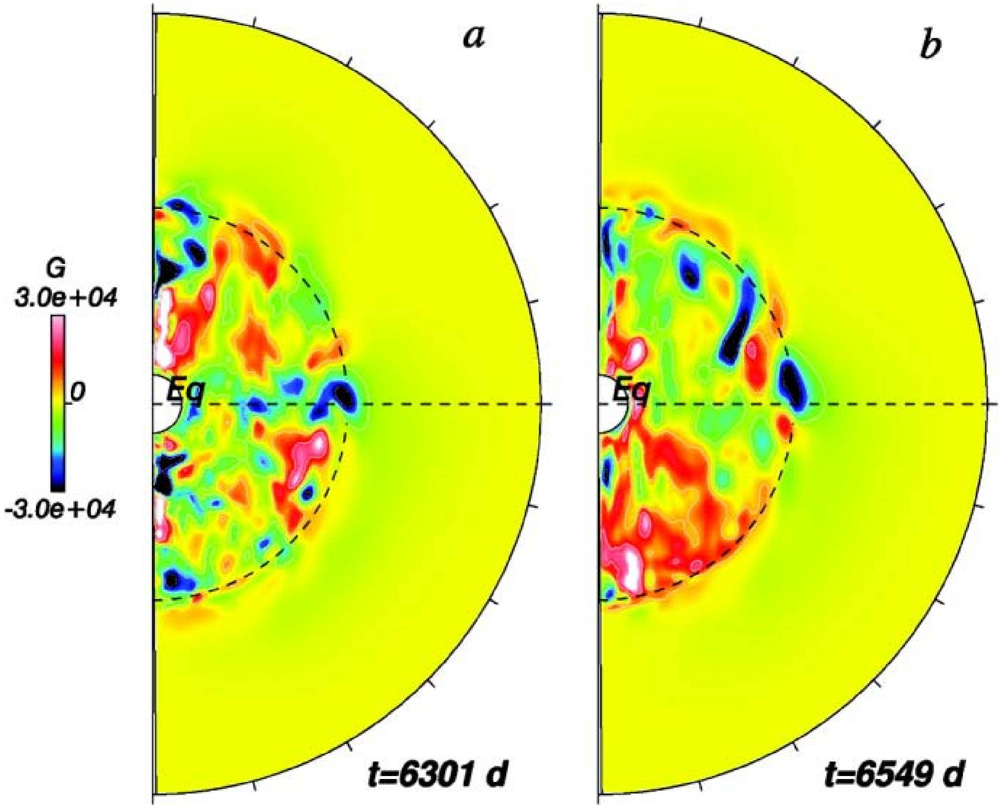
Browning 2008, *Astrophys. J.*, **676**, 2362



[Azimuthally-averaged toroidal magnetic component, in meridional plane]

3D MHD core dynamo action (2)

Brun, Browning & Toomre, 2005, *Astrophys. J.*, **629**, 461



[Azimuthally-averaged toroidal magnetic component, in meridional plane]

The magnetic self-organization conundrum

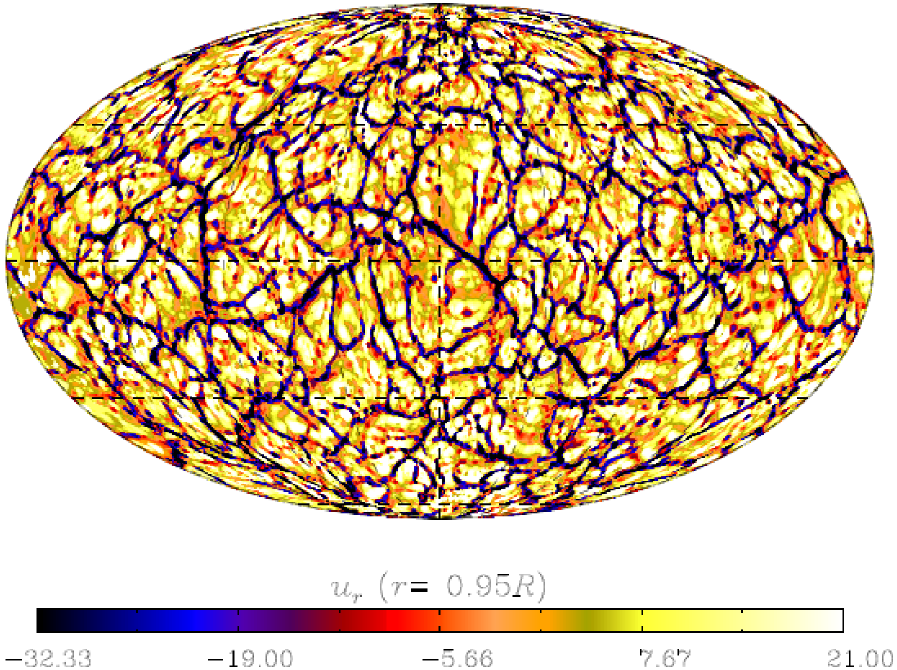
How can turbulent convection, a flow with a length scale <<R and coherence time of ~month, generate a magnetic component with scale ~R varying on a timescale of ~decade ??

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B})}$$

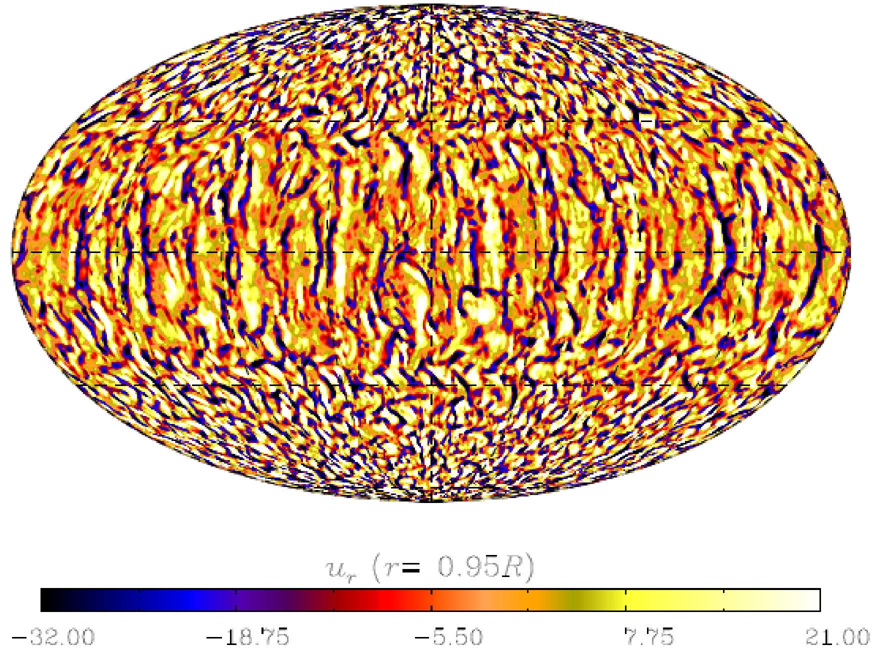
Mechanism/Processes favoring organization on large spatial scales: 1. rotation (cyclonicity); 2. differential rotation (scale ~R); and 3. turbulent inverse cascades.

Rotation and differential rotation (1)

No rotation



Rotation at solar rate

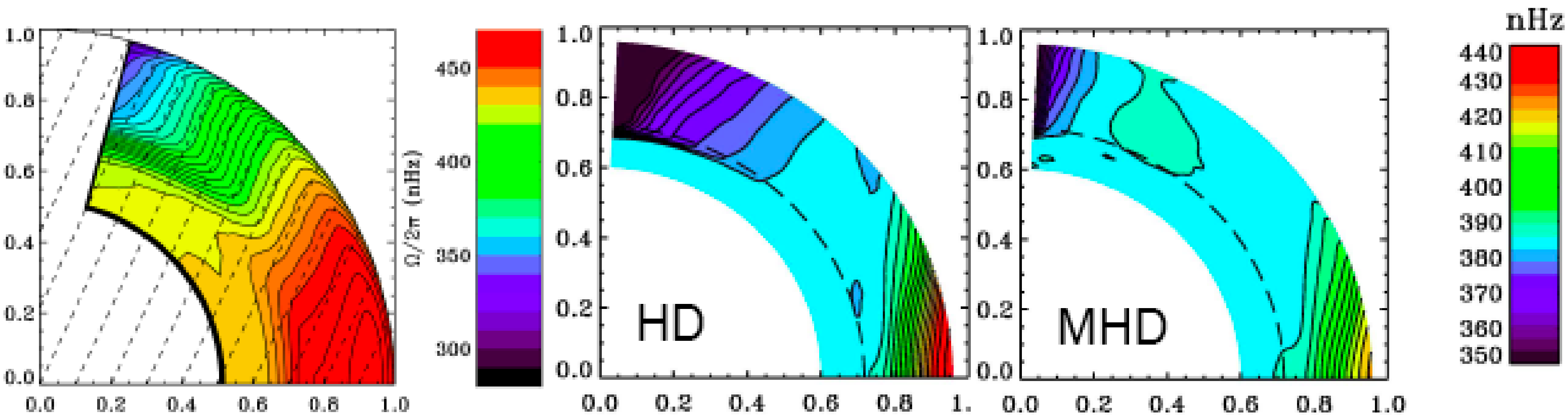


Vertical (radial) flow velocity, in Mollweide projection
[from Guerrero et al. 2013, *Astrophys. J.*, **XXX**, YYY]

III.5.2, III.5.5

Rotation and differential rotation (2)

Helioseismology HD simulation MHD simulation



Angular velocity profiles, in meridional quadrant

Differential rotation in the Sun and solar-type stars is powered by turbulent Reynolds stresses, arising from rotationally-induced anisotropy in turbulent transport of momentum and heat

III.5.5

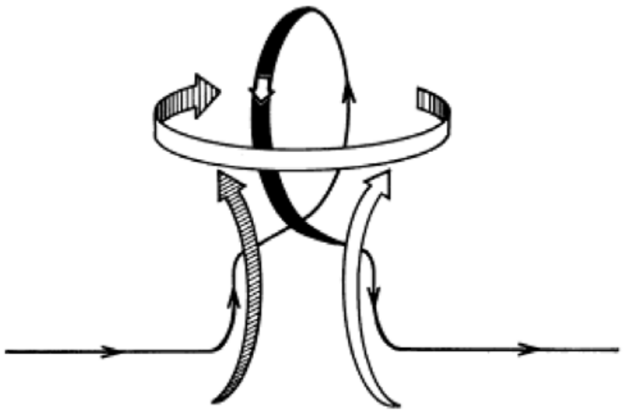
Mean-field electrodynamics and dynamo models

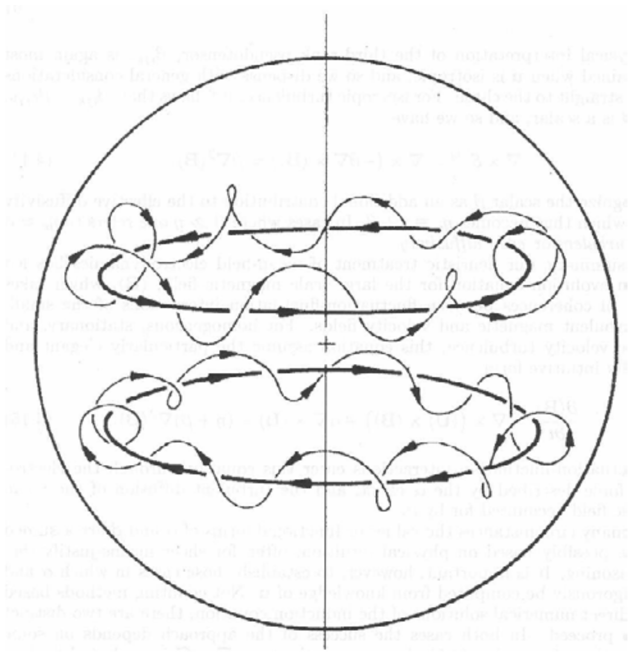
[see also Rempel chapter, vol. 1]

The basic idea

[Parker, E.N., ApJ, **122**, 293 (1955)]

Cyclonic convective updraft/downdrafts acting on a pre-existing toroidal magnetic field will twist the fieldlines into poloidal planes (in the high R*m* regime)





The collective effect of many such events is the production of an electrical current flowing parallel to the background toroidal magnetic field; such a current system contributes to the production of a poloidal magnetic component

The turbulent EMF (1)

Separate flow and magnetic field into large-scale, « laminar » component, and a small-scale, « turbulent » component:

$$\mathbf{U} = \langle \mathbf{U} \rangle + \mathbf{u} \qquad \mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}$$

Assume now that a good separation of scales exists between these two components, so that

$$\langle \mathbf{u} \rangle = \langle \mathbf{b} \rangle = 0.$$

Substitute into MHD induction equation and apply averaging operator:

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times \left(\langle \mathbf{U} \rangle \times \langle \mathbf{B} \rangle \right) + \nabla \times \boldsymbol{\mathcal{E}} + \eta \nabla^2 \langle \mathbf{B} \rangle$$

with : $\boldsymbol{\mathcal{E}} = \langle \mathbf{u} \times \mathbf{b} \rangle$ TURBULENT ELECTROMOTIVE FORCE !

I.3.4

The turbulent EMF (2)

Now, the whole point of the mean-field approach is NOT to have to deal explicitly with the small scales; since the PDE for **b** is linear, with the term $\nabla\times\left(\mathbf{u}\times\langle\mathbf{B}\rangle\right)$ acting as a source; therefore there must exit a linear relationship between **b** and **B**, and also between **B** and $\langle\mathbf{u}\times\mathbf{b}\rangle$ We develop the mean emf as

$$\mathcal{E}_i = \alpha_{ij}\langle B\rangle_j + \beta_{ijk}\partial_k\langle B\rangle_j + \gamma_{ijkl}\partial_j\partial_k\langle B\rangle_l + \cdots,$$

I.3.4

Where the various tensorial coefficients can be a function of $\langle\mathbf{U}\rangle$ of the statistical properties of **u**, on the magnetic diffusivity, but NOT of $\langle\mathbf{B}\rangle$

Specifying these closure relationships is the crux of the mean-field approach

I.3.4

The alpha-effect (1)

Consider the first term in our EMF development:

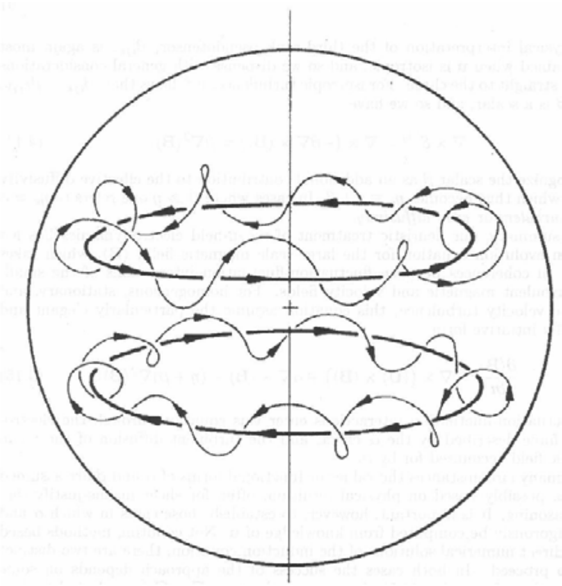
$$\mathcal{E}_i^{(1)} = \alpha_{ij} \langle B \rangle_j$$

If **u** is an isotropic random field, there can be no preferred direction in space, and the alpha-tensor must also be isotropic:

$$\alpha_{ij} = \alpha \delta_{ij}$$

This leads to:

$$\boldsymbol{\mathcal{E}}^{(1)} = \alpha \langle \mathbf{B} \rangle$$



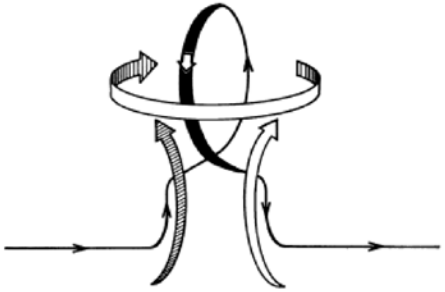
The mean turbulent EMF is parallel to the mean magnetic field!
This is called the « **alpha-effect** »

I.3.4.3

The alpha-effect (2)

Computing the alpha-tensor requires a knowledge of the statistical properties of the turbulent flow, more precisely the cross-correlation between velocity components; under the assumption that **b** << **B**, if the turbulence is only mildly anisotropic and inhomogeneous, the so-called Second-Order Correlation Approximation leads to

$$\alpha = -\frac{1}{3}\tau_c\langle \mathbf{u}\cdot(\nabla\times\mathbf{u})\rangle \ , \qquad [\mathrm{m\,s^{-1}}]$$



where τ_c is the correlation time for the turbulence.

The alpha-effect is proportional to the fluid helicity!

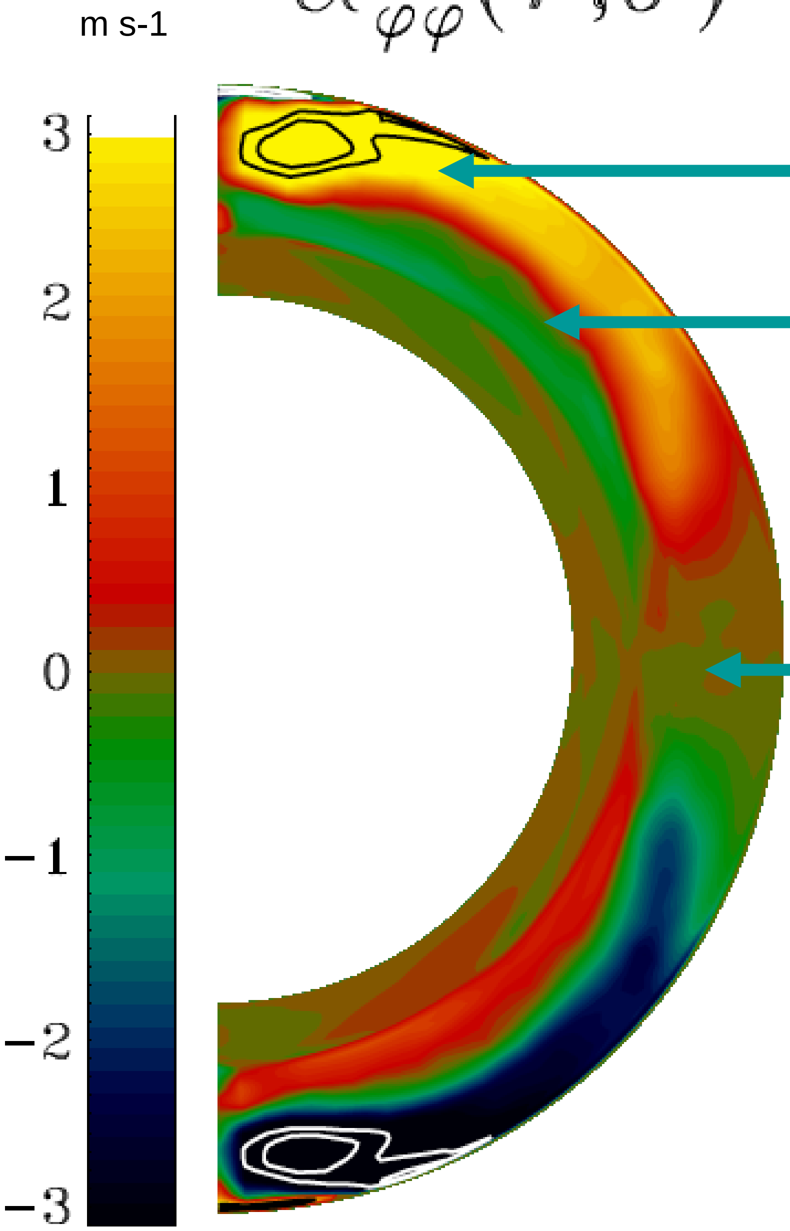
If the mild-anisotropy is provided by rotation, and the inhomogeneity by stratification, then we have

$$\alpha = -\frac{1}{3}\tau_c^2 u^2 \mathbf{\Omega}\cdot\nabla\ln(\rho u)$$

I.3.4.3

The alpha-effect (3)

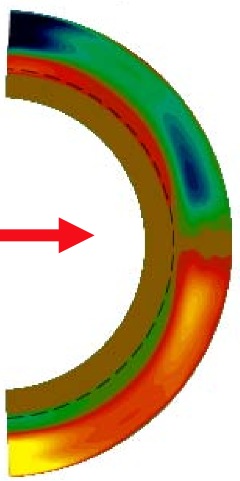
$\alpha_{\varphi\varphi}(r,\theta)$



Positive in N-hemisphere, peaking at high latitudes; sign change near the base of convecting fluid layer.

Antisymmetric about equatorial plane.

Roughly proportional to the negative of kinetic helicity of turbulent flow component !!



Turbulent pumping (1)

Turbulent pumping is a basic physical effect arising in inhomogeneous, anisotropic turbulence; mathematically, it shows up as the antisymmetric part of the alpha-tensor relating the turbulent EMF to the mean magnetic field:

$$\mathcal{E}_i(t,r,\theta) = \alpha_{ij}(r,\theta)\langle B_j\rangle(t,r,\theta)$$

Extracting the symmetric part of the tensor yields:

$$\mathcal{E}_i = \alpha_{(ij)}\langle B_j\rangle + \Big(\gamma \times \langle \boldsymbol{B}\rangle\Big)_i ,$$

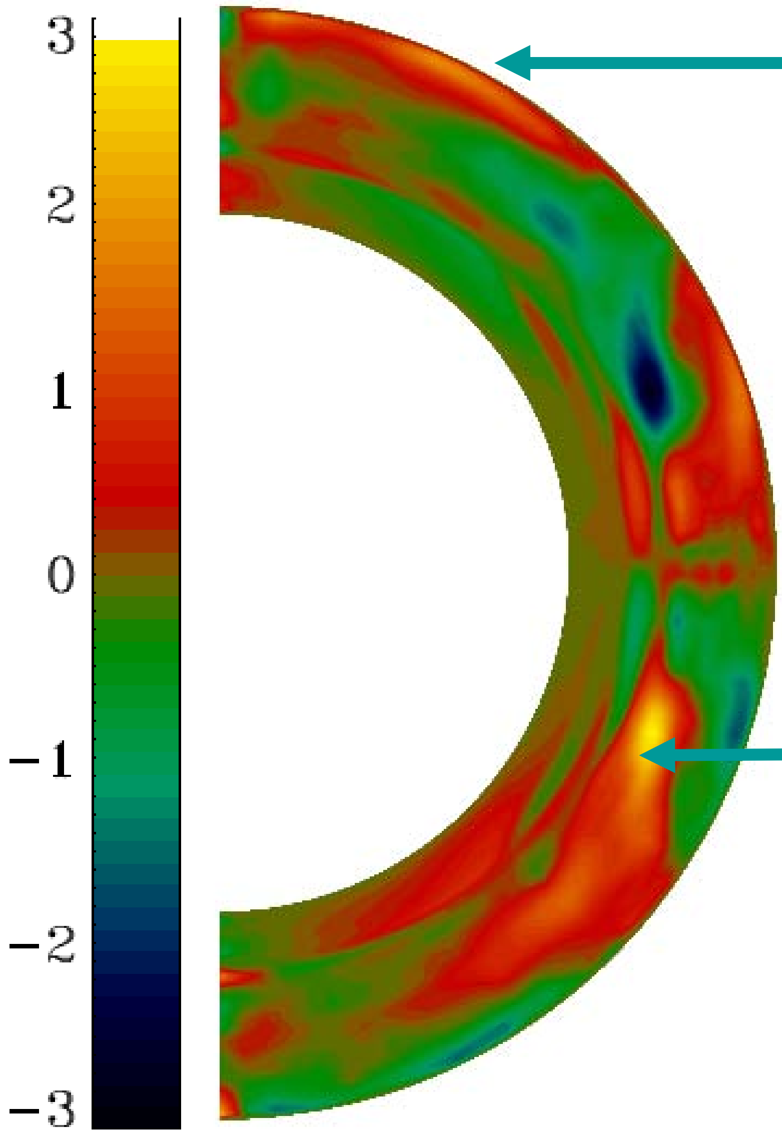
where

$$\gamma_i = -\frac{1}{2}\epsilon_{ijk}\alpha_{jk}$$

acts as a velocity in the mean-field dynamo equations. For mildly anisotropic, inhomogeneous turbulence:

I.3.4.3

Turbulent pumping (2)



$$\gamma_\theta(r,\theta)$$

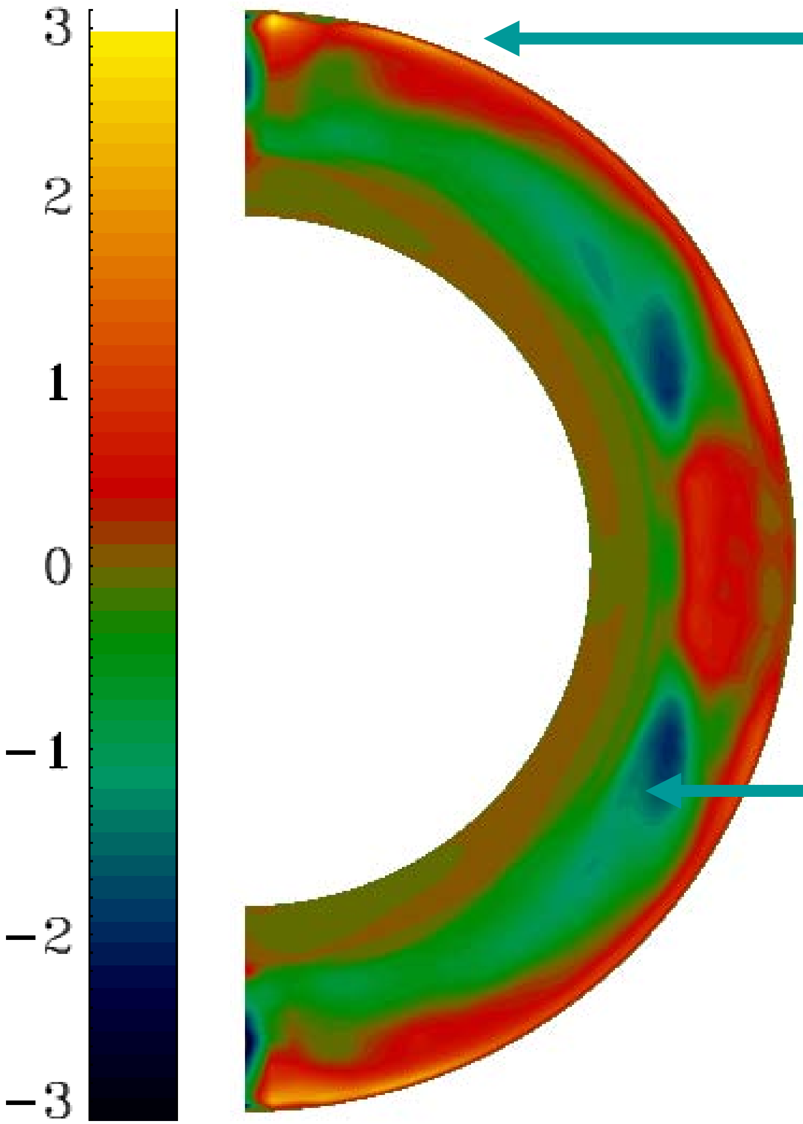
Poleward transport of surface magnetic field by turbulent pumping; speed in range 1-3 m s-1 above +/- 45º latitudes

Equatorward transport of deep magnetic field by turbulent pumping between +/- 15 and 60º latitudes; speed 1-2 m s-1

Turbulent pumping (3)

m s-1

$$\gamma_r(r,\theta)$$



Upward transport of magnetic field by turbulent pumping in subsurface layers; speed exceeding 1 m s-1 above +/- 60o latitudes

Downward transport of magnetic field by turbulent pumping in bulk of deep convection zone; speed exceeding 1 m s-1 between +/- 15 and 60o latitude

Turbulent diffusivity

Turn now to the second term in our EMF development:

$$\mathcal{E}_i^{(2)} = \beta_{ijk}\partial_k\langle B\rangle_j$$

In cases where **u** is isotropic, we have $\beta_{ijk} = \beta\epsilon_{ijk}$, and thus:

$$\nabla\times\mathcal{E}^{(2)} = \nabla\times\left(-\beta\nabla\times\langle\mathbf{B}\rangle\right) = \beta\nabla^2\langle\mathbf{B}\rangle.$$

The mathematical form of this expression suggests that β can be interpreted as a **turbulent diffusivity** of the large-scale field. for homogeneous, isotropic turbulence with correlation time τ_c it can be shown that

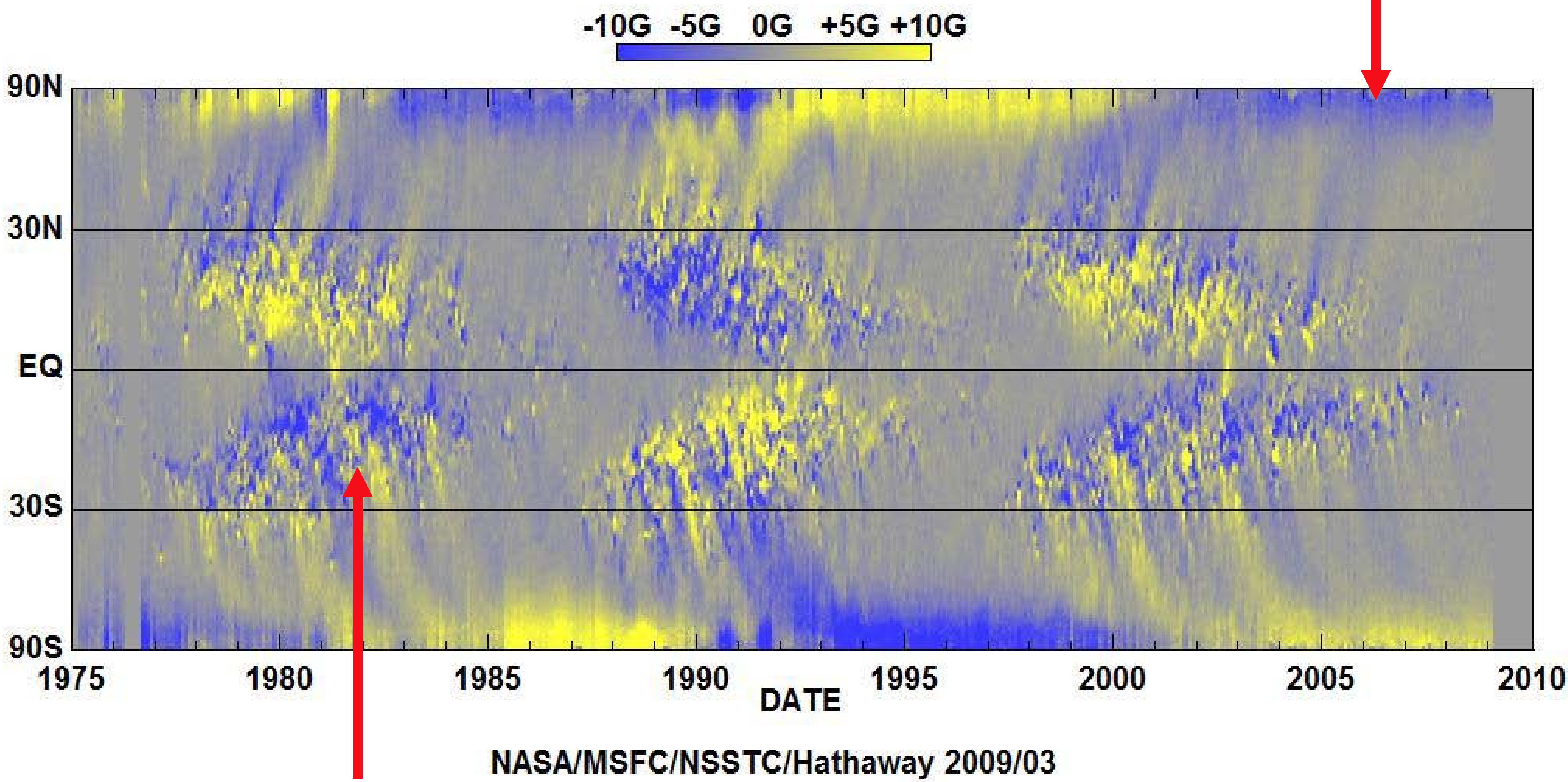
$$\beta = \frac{1}{3}\tau_c\langle\mathbf{u}^2\rangle\ , \qquad [\mathrm{m}^2\mathrm{s}^{-1}]$$

This result is expected to hold also in mildly anisotropic, mildly inhomogeneous turbulence. In general, $\beta \gg \eta$

I.3.4.3

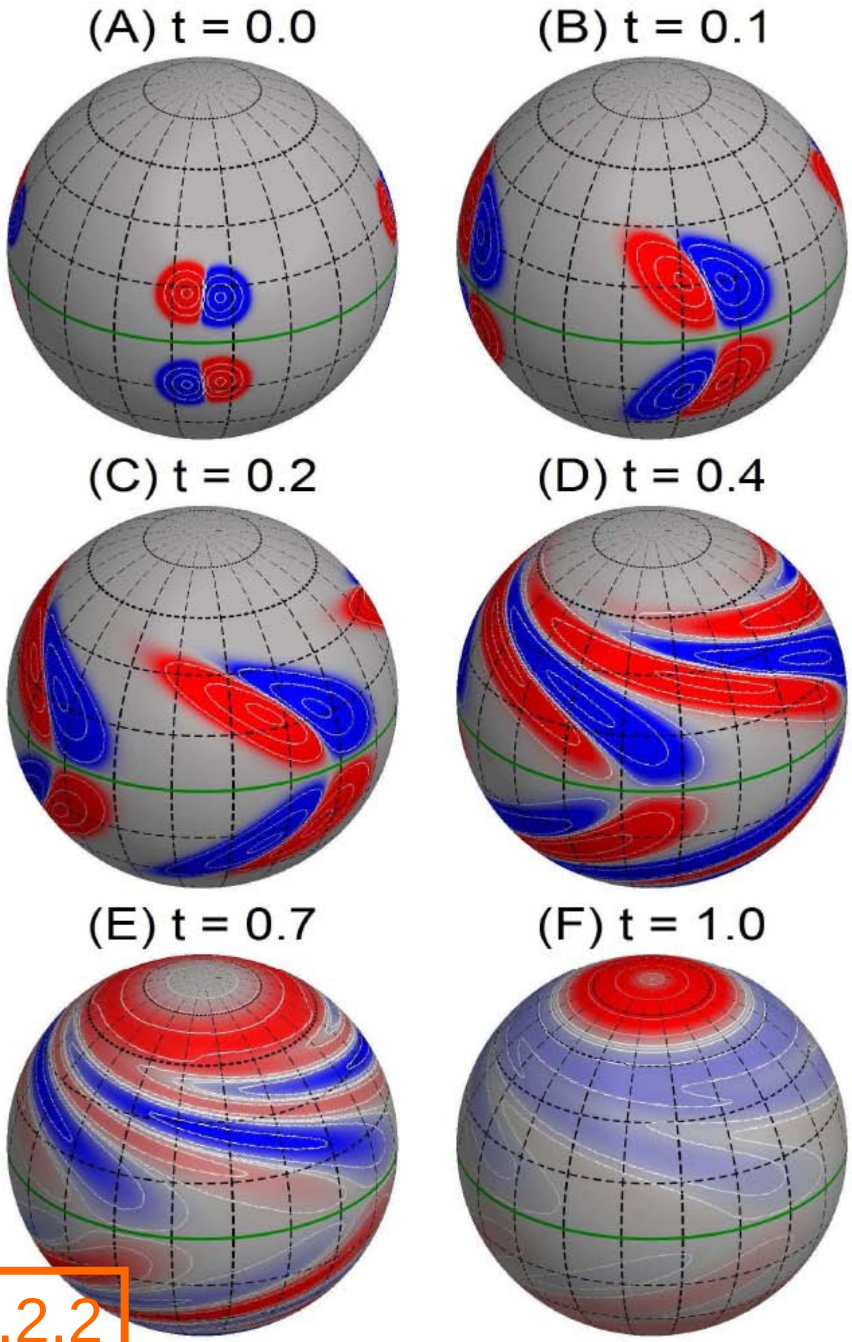
Active region decay (1)

Peak polar cap flux: ~10^{14} Wb



Toroidal flux emerging in active regions in one cycle: ~10^{17} Wb

III.2.2.1



Sunspot pairs are the photospheric manifestation of an emerging, formerly toroidal magnetic flux rope generated in the deep interior ;

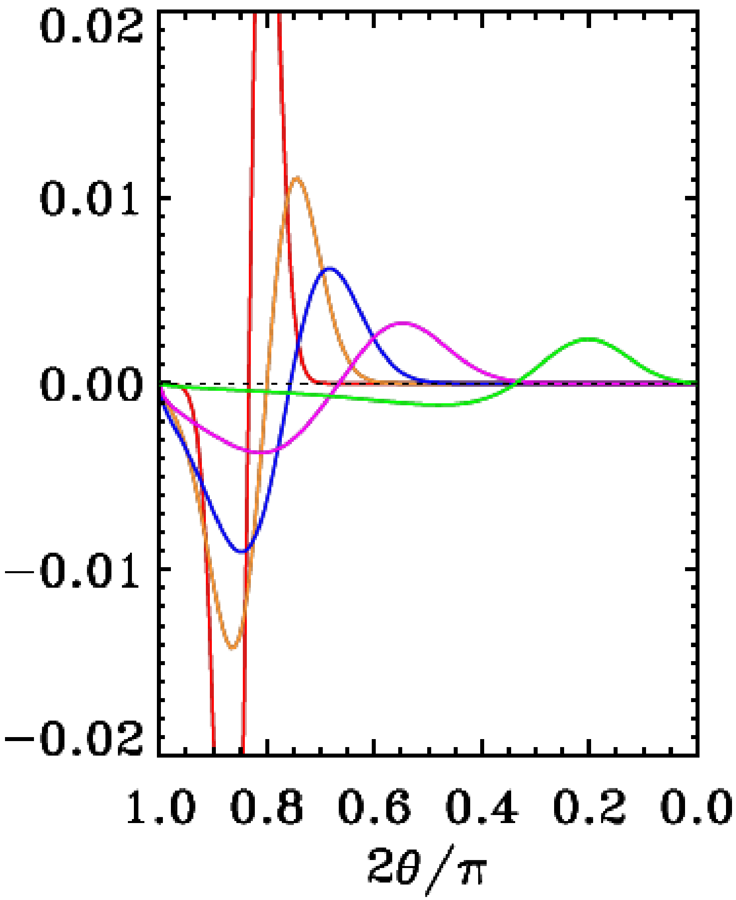
after surface decay and transport by diffusion, differential rotation, and the surface meridional flow…

…an axisymmetric dipole moment is produced; this Babcock-Leighton mechanism produces a poloidal field from a toroidal component.

III.6.2.2

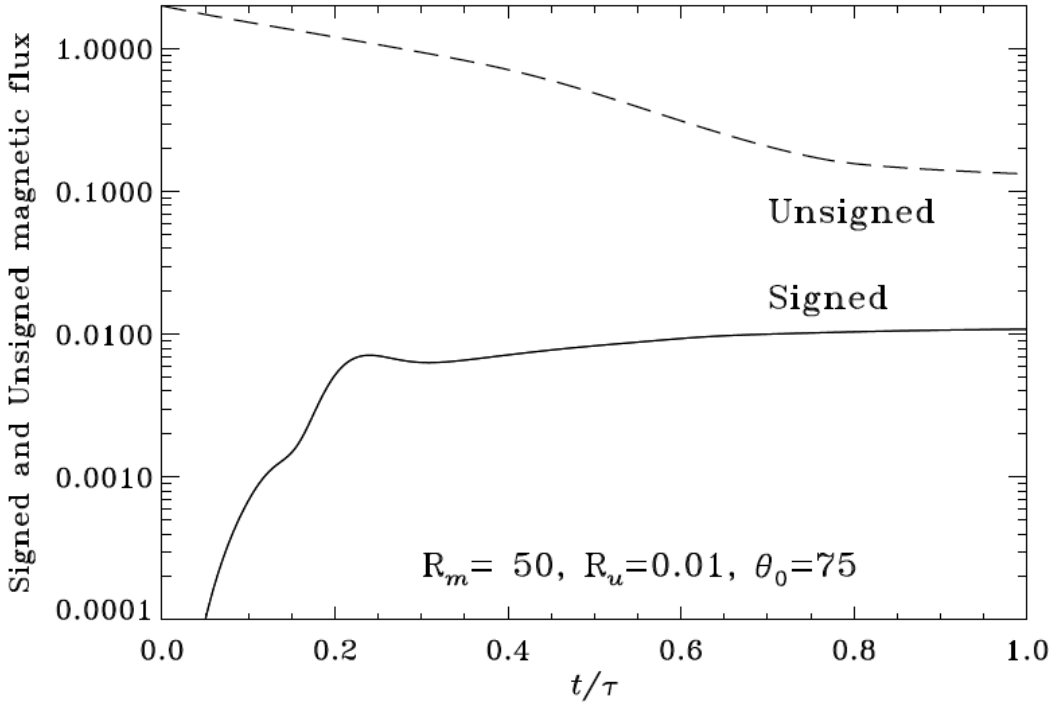
Active region decay (3)

Zonal means



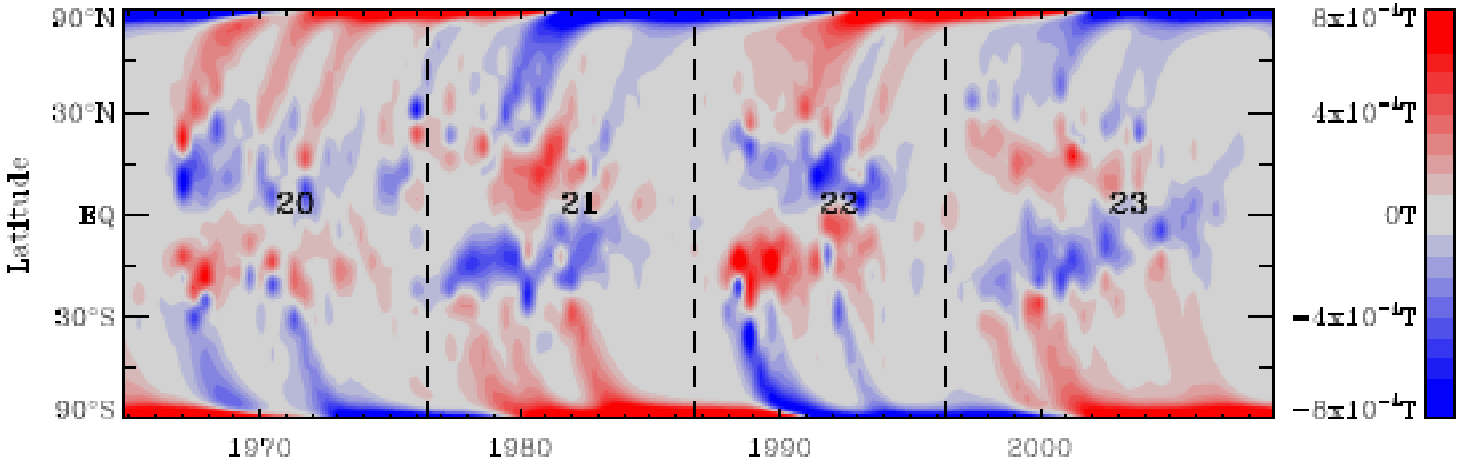
$$\langle B_r \rangle = -\int_0^{2\pi}\int_{-1}^{+1} B_r \mathrm{d}\mu \mathrm{d}\phi$$

$$\Phi = |\langle B_r \rangle| \ , \qquad F = \langle | B_r | \rangle$$

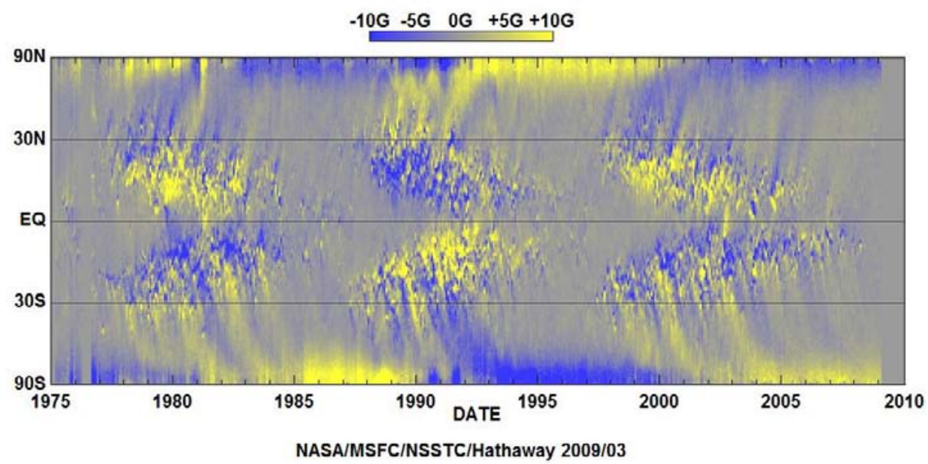


Active region decay (3)

[Simulation of surface magnetic flux evolution by A. Lemerle]



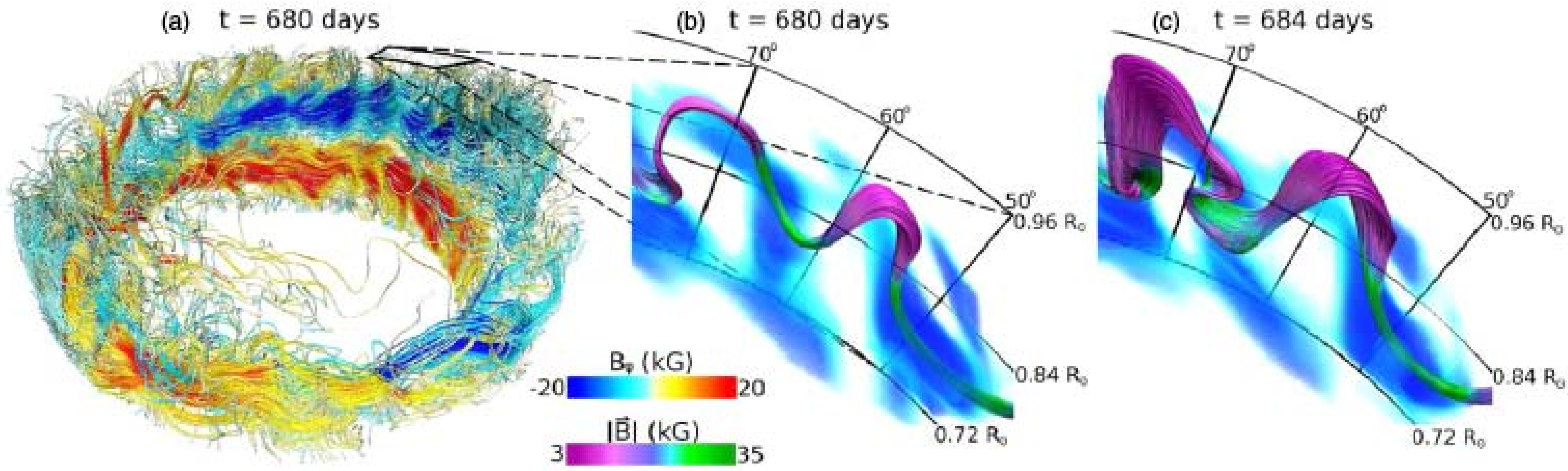
The Babcock-Leighton mechanism is definitely seen operating at the solar photosphere! But, does it really feed back into the dynamo loop ?



III.2.2.1.4

Formation of magnetic flux strands (1)

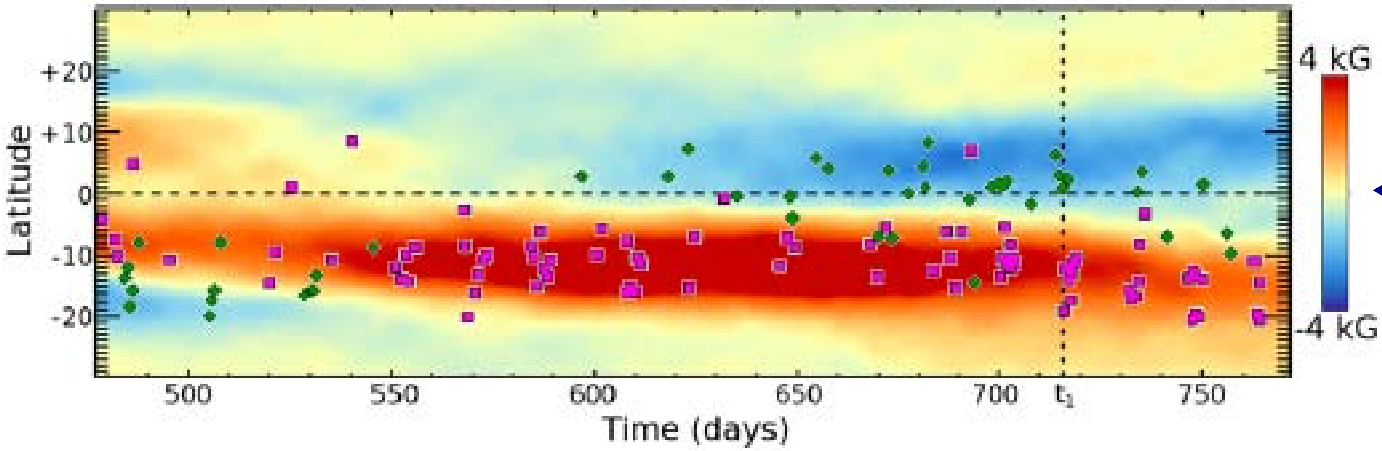
[Nelson *et al.* 2013, *Astrophys*. J., **762**: 73]



Recent, very high resolution 3D MHD simulations of solar convection Have achieved the formation of flux-rope-like super-equipartition-strength « magnetic strands » characterized by a significant density deficit in their core; ripped from the parent large-scale structure by turbulent entrainement, subsequent buoyant rise ensues.

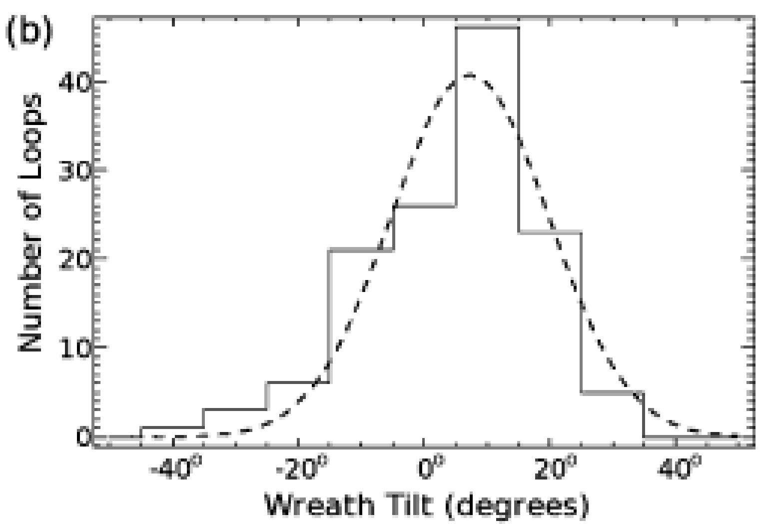
Formation of magnetic flux strands (2)

[Nelson et al. 2014, *Solar Phys*., **289**, 441]



The strands « remember » their origin !

The strands develop a pattern of East-West tilt similar to that inferred obervationally for the sun



From MHD to simpler dynamos

[Sections I.3.4, I.3.5, III.6.1-3]

The MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \ ,$$

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \frac{1}{\mu_0 \rho}(\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{1}{\rho}\nabla \cdot \boldsymbol{\tau} \ ,$$

$$\frac{\mathrm{D}e}{\mathrm{D}t} + (\gamma - 1)e\nabla \cdot \mathbf{u} = \frac{1}{\rho}\left[\nabla \cdot \Big((\chi + \chi_r)\nabla T\Big) + \phi_\nu + \phi_B\right] \ ,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \ .$$

Model setup

Solve MHD induction equation in spherical polar coordinates for large-scale (~R), axisymmetric magnetic field in a sphere of electrically conducting fluid:

$$\mathbf{B}(r,\theta,t) = \nabla \times (A(r,\theta,t)\hat{\mathbf{e}}_\phi) + B(r,\theta,t)\hat{\mathbf{e}}_\phi$$

Evolving under the influence of a steady, axisymmetric large-scale flow:

$$\mathbf{u}(r,\theta) = \frac{1}{\rho}\nabla \times (\Psi(r,\theta)\hat{\mathbf{e}}_\phi) + \varpi\Omega(r,\theta)\hat{\mathbf{e}}_\phi$$

Match solutions to potential field in $r > R$.

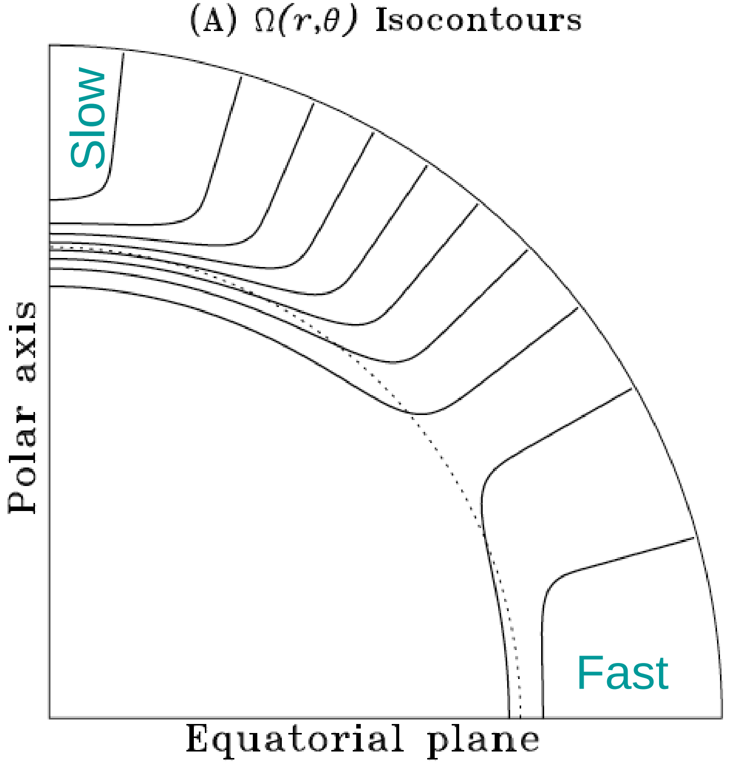
Kinematic axisymmetric dynamo

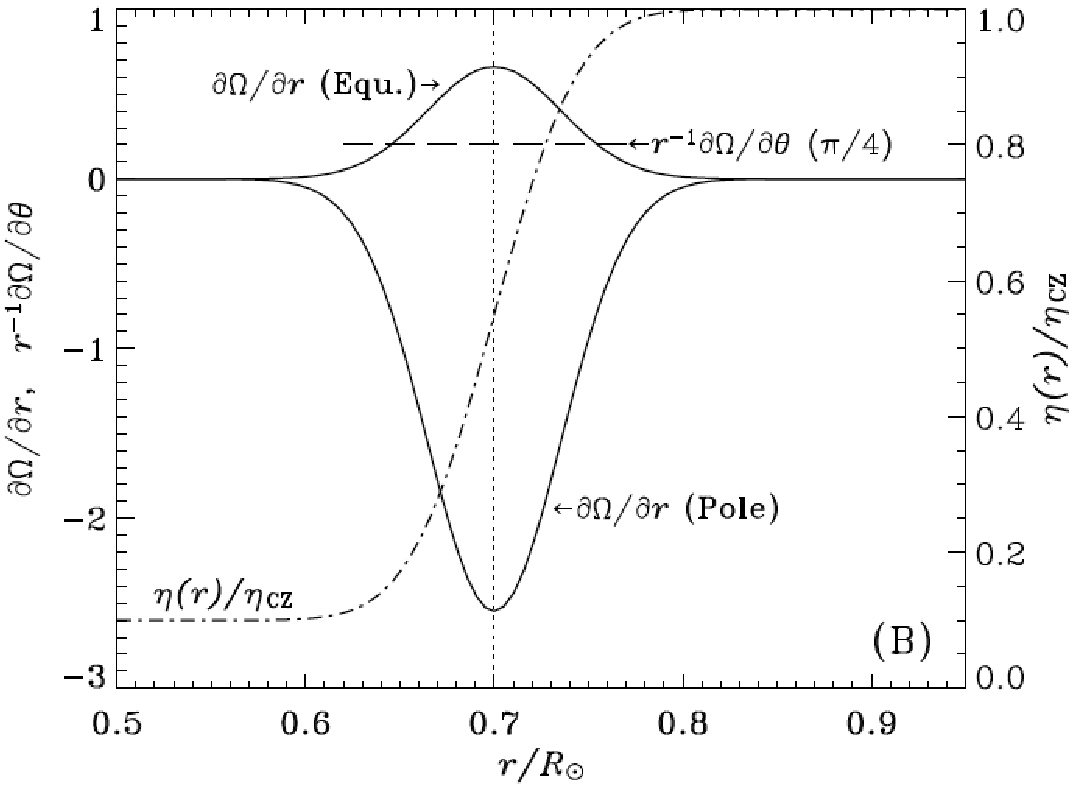
$$\frac{\partial A}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)A}_{\text{resistive decay}} - \underbrace{\frac{\mathbf{u}_p}{\varpi}\cdot\nabla(\varpi A)}_{\text{advection}} \qquad [+\ \text{Source}]\ ,$$

$$\frac{\partial B}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)B}_{\text{resistive decay}} + \underbrace{\frac{1}{\varpi}\frac{\partial(\varpi B)}{\partial r}\frac{\partial \eta}{\partial r}}_{\text{diamagnetic transport}} - \underbrace{\varpi\mathbf{u}_p\cdot\nabla\left(\frac{B}{\varpi}\right)}_{\text{advection}}$$

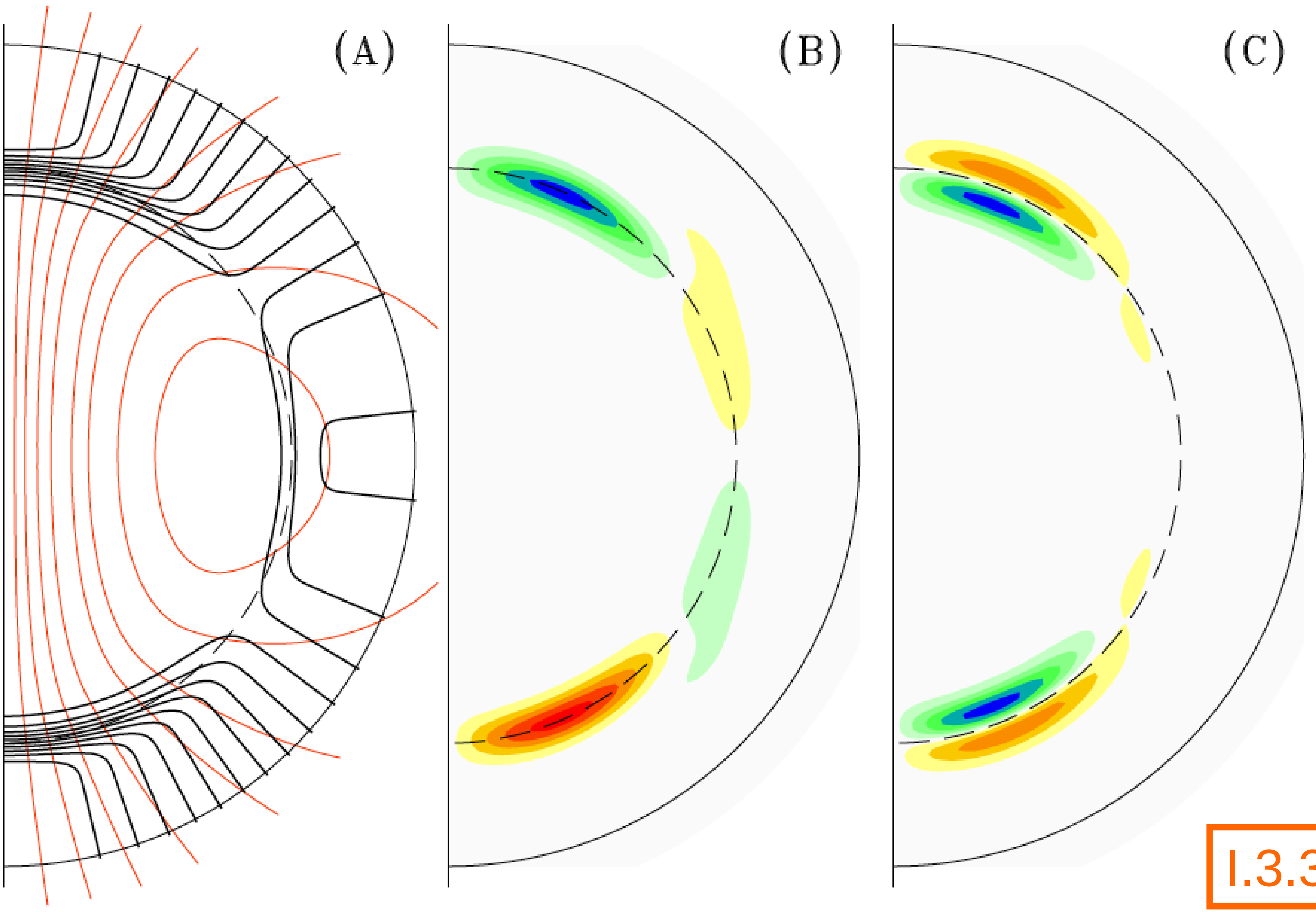
$$- \underbrace{B\nabla\cdot\mathbf{u}_p}_{\text{compression}} + \underbrace{\varpi(\nabla\times(A\hat{\mathbf{e}}_\phi))\cdot\nabla\Omega}_{\text{shearing}}\ .$$

Differential rotation





Shearing by axisymmetric differential rotation



I.3.3.7

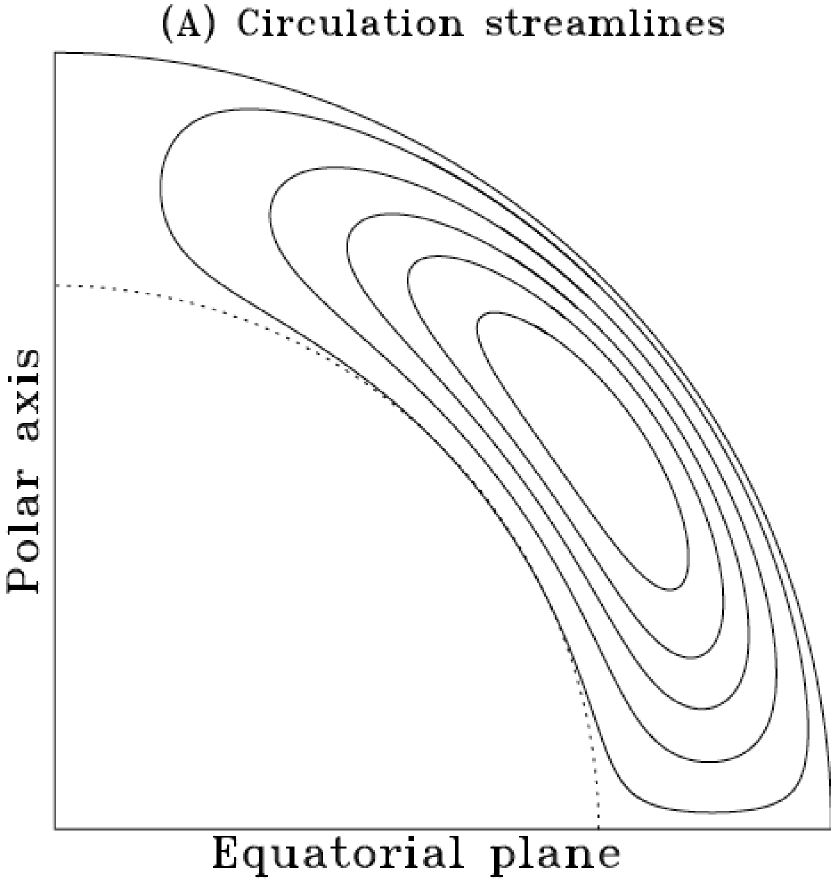
Kinematic axisymmetric dynamo

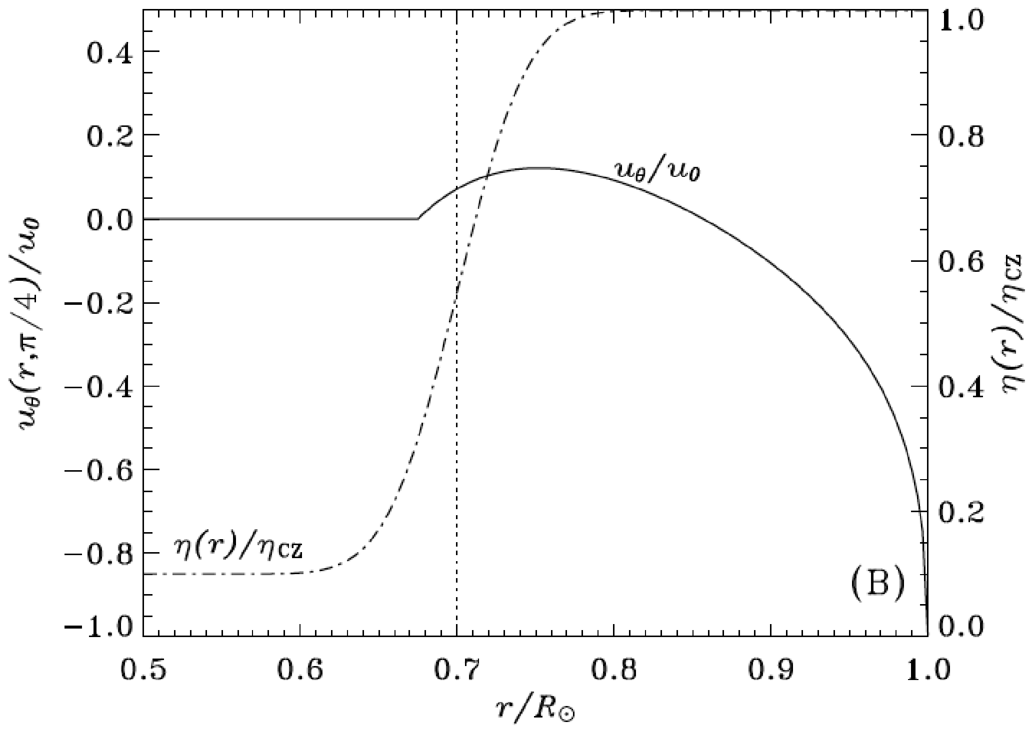
$$\frac{\partial A}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)A}_{\text{resistive decay}} - \underbrace{\frac{\mathbf{u}_p}{\varpi}\cdot\nabla(\varpi A)}_{\text{advection}} \qquad [+\ \text{Source}]\ ,$$

$$\frac{\partial B}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)B}_{\text{resistive decay}} + \underbrace{\frac{1}{\varpi}\frac{\partial(\varpi B)}{\partial r}\frac{\partial \eta}{\partial r}}_{\text{diamagnetic transport}} - \underbrace{\varpi\mathbf{u}_p\cdot\nabla\left(\frac{B}{\varpi}\right)}_{\text{advection}}$$

$$- \underbrace{B\nabla\cdot\mathbf{u}_p}_{\text{compression}} + \underbrace{\varpi(\nabla\times(A\hat{\mathbf{e}}_\phi))\cdot\nabla\Omega}_{\text{shearing}}\ .$$

Meridional circulation





Kinematic axisymmetric dynamos

$$\frac{\partial A}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)A}_{\text{resistive decay}} - \underbrace{\frac{\mathbf{u}_p}{\varpi}\cdot\nabla(\varpi A)}_{\text{advection}} \quad [+\ \text{Source}]\ ,$$

$$\frac{\partial B}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)B}_{\text{resistive decay}} + \underbrace{\frac{1}{\varpi}\frac{\partial(\varpi B)}{\partial r}\frac{\partial \eta}{\partial r}}_{\text{diamagnetic transport}} - \underbrace{\varpi\mathbf{u}_p\cdot\nabla\left(\frac{B}{\varpi}\right)}_{\text{advection}}$$

$$- \underbrace{B\nabla\cdot\mathbf{u}_p}_{\text{compression}} + \underbrace{\varpi(\nabla\times(A\hat{\mathbf{e}}_\phi))\cdot\nabla\Omega}_{\text{shearing}}\ .$$

Poloidal source terms

1. Turbulent alpha-effect
2. Active region decay (Babcock-Leighton mechanism)
3. Helical hydrodynamical instabilities
4. Magnetohydrodynamical instabilities
   (flux tubes, Spruit-Tayler)

III.6.2

Solar and Stellar dynamo models
[Sections III.2, III.6]

Models based on the turbulent alpha-effect

Scalings and dynamo numbers

Length scale: solar/stellar radius: R

Time scale: turbulent diffusion time: $\tau = R^2/\eta_0$

$$\frac{\partial A}{\partial t} = \eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)A - \frac{\mathrm{R}_m}{\varpi}\mathbf{u}_p\cdot\nabla(\varpi A) + C_\alpha \alpha B\,,$$

$$\frac{\partial B}{\partial t} = \eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)B + \frac{1}{\varpi}\frac{\mathrm{d}\eta}{\mathrm{d}r}\frac{\partial(\varpi B)}{\partial r}$$

$$-\mathrm{R}_m\varpi\nabla\cdot\left(\frac{B}{\varpi}\mathbf{u}_p\right) + C_\Omega\varpi(\nabla\times A)\cdot(\nabla\Omega) + C_\alpha\hat{\mathbf{e}}_\phi\cdot\nabla\times[\alpha\nabla\times(A\hat{\mathbf{e}}_\phi)]$$

Three dimensionless groupings have materialized:

$$C_\alpha = \frac{\alpha_0 R}{\eta_0}\,,$$

$$C_\Omega = \frac{\Omega_0 R^2}{\eta_0}\,,$$

$$\mathrm{R}_m = \frac{u_0 R}{\eta_0}\,,$$

III.6.2.1

The mean-field zoo

α^2 dynamo

The alpha-effect is the source of both poloidal and toroidal magnetic components; works without a large-scale flow! planetary dynamos are believed to be of this kind.

$\alpha\Omega$ dynamo

Rotational shear is the sole source of the toroidal component; the alpha-effect is the source of only the poloidal component. the solar dynamo is believed to be of this kind.

$\alpha^2\Omega$ dynamo

Both the alpha-effect and differential rotation shear contribute to toroidal field production; stellar dynamos could be of this kind if differential rotation is weak, and/or if dynamo action takes place in a very thin layer.

I.3.4.6

Linear alpha-Omega solutions (1)

Solve the axisymmetric kinematic mean-field alpha-Omega dynamo equation in a differentially rotating sphere of electrically conducting fluid, embedded in vacuum; in spherical polar coordinates:

$$\frac{\partial A}{\partial t} = \left(\nabla^2 - \frac{1}{\varpi^2}\right) A + C_\alpha B \ ,$$

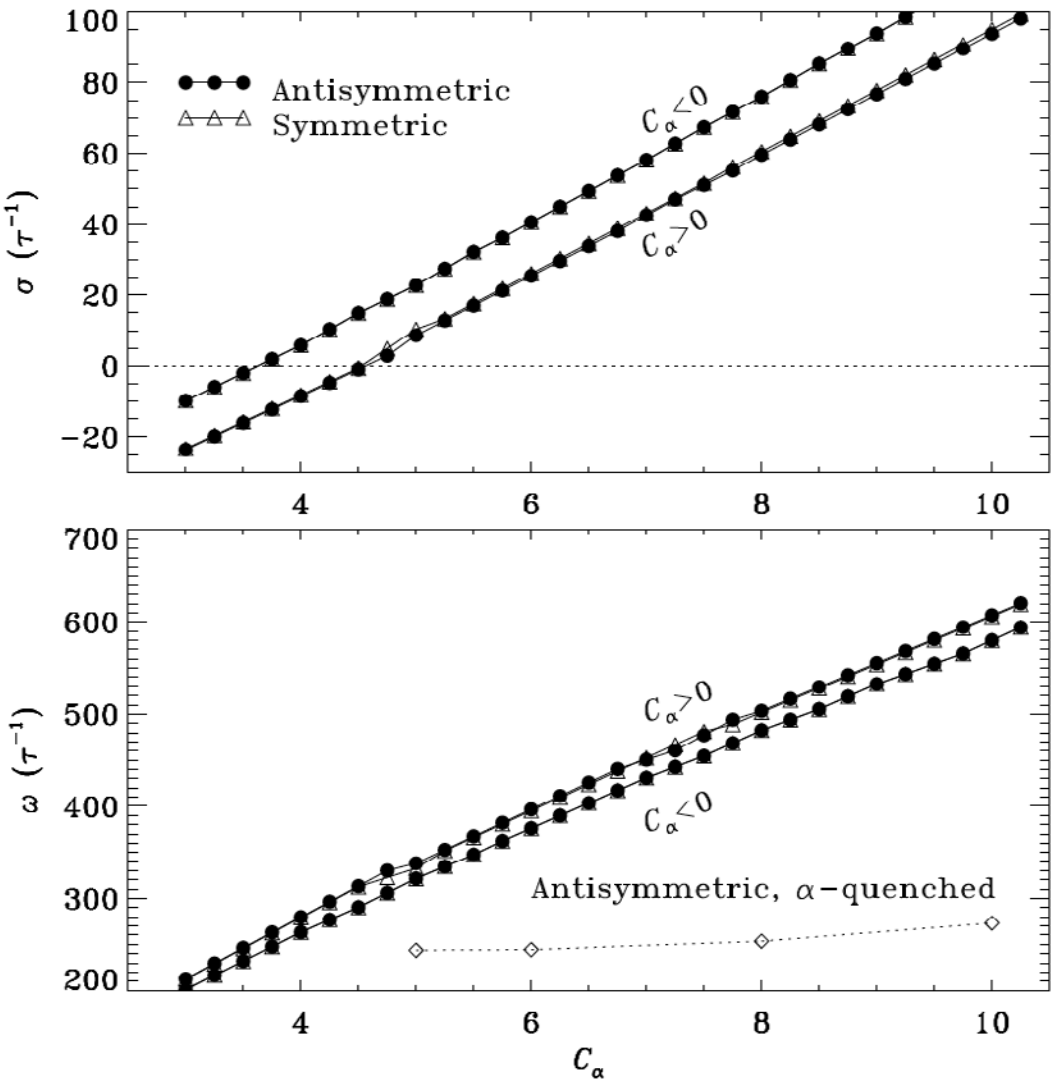
$$\frac{\partial B}{\partial t} = \left(\nabla^2 - \frac{1}{\varpi^2}\right) B + C_\Omega \varpi (\nabla \times A\hat{\mathrm{e}}_\phi) \cdot (\nabla\Omega) + \frac{1}{\varpi}\frac{\mathrm{d}\eta}{\mathrm{d}r}\frac{\partial(\varpi B)}{\partial r}$$

Choice of alpha: $\alpha(r,\theta) = f(r) g(\theta) \ ,$

$$g(\theta) = \cos\theta$$

$$f(r) = \frac{1}{4}\left[1 + \mathrm{erf}\left(\frac{r - r_c}{w}\right)\right]\left[1 - \mathrm{erf}\left(\frac{r - 0.8}{w}\right)\right]$$

Linear alpha-Omega solutions (2)



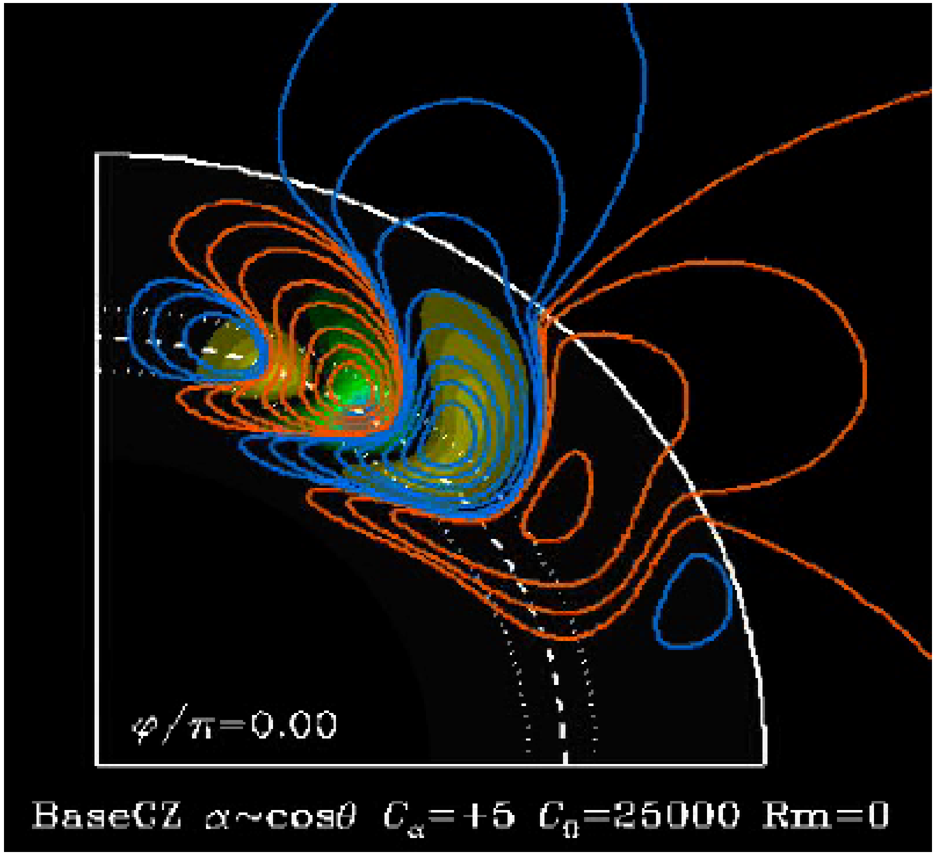
$$\begin{bmatrix} A(r,\theta,t) \\ B(r,\theta,t) \end{bmatrix} = \begin{bmatrix} a(r,\theta) \\ b(r,\theta) \end{bmatrix} e^{\lambda t}$$

$$\lambda = \sigma + i\omega$$

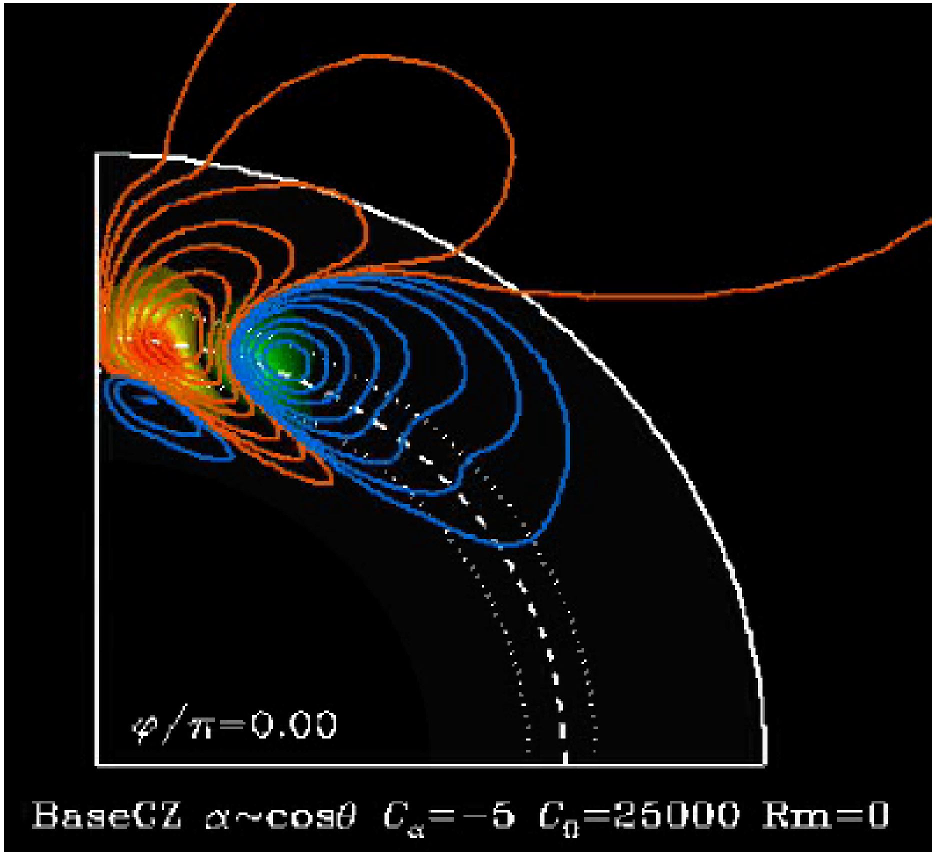
The growth rate, frequency, and eigenmode morphology are completely determined by the product of the two dynamo numbers

$$D \equiv C_\alpha \times C_\Omega = \frac{\alpha_0 \Omega_0 R^3}{\eta_0^2}$$

Linear alpha-Omega solutions (3)



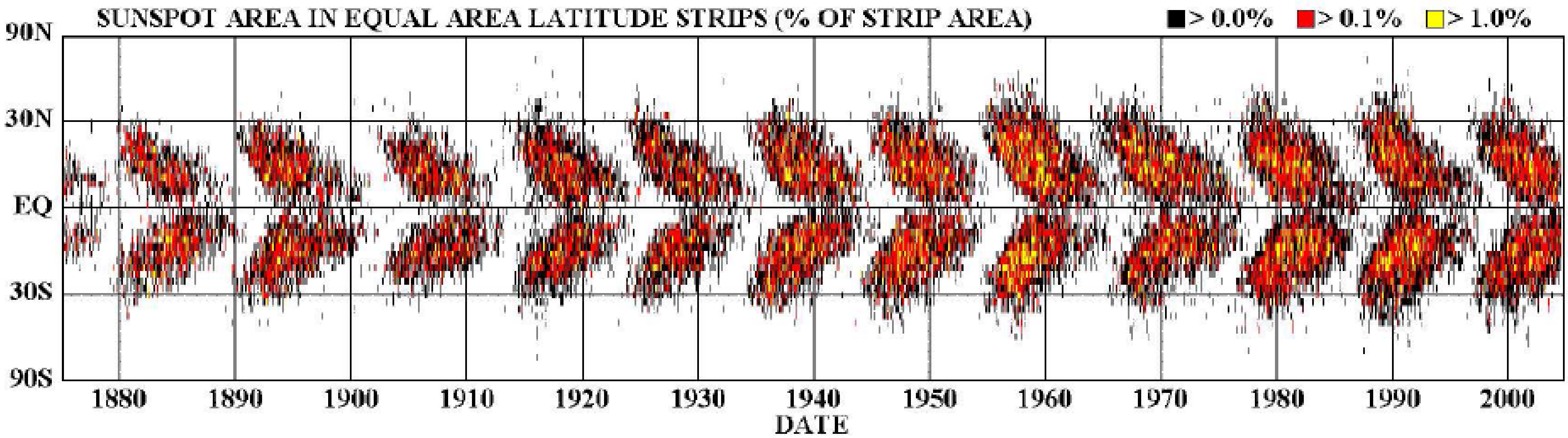
Positive alpha-effect



Negative alpha-effect

Linear alpha-Omega solutions (4)

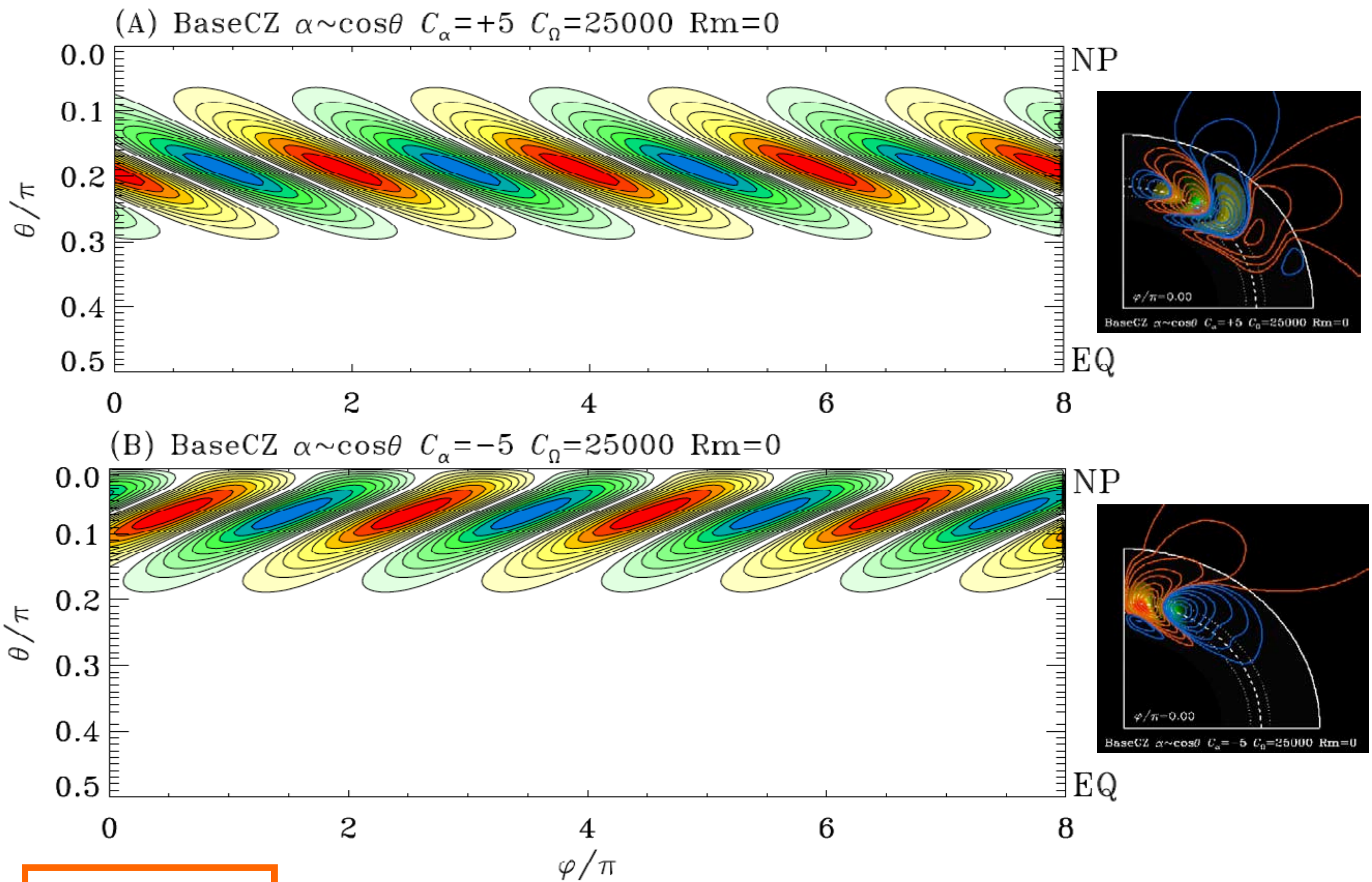
Time-latitude « butterfly » diagram



[http://solarscience.msfc.nasa.gov/images/bfly.gif]

Equivalent in axisymmetric numerical model: constant-*r* cut at *r/R*=0.7, versus latitude (vertical) and time (horizontal)

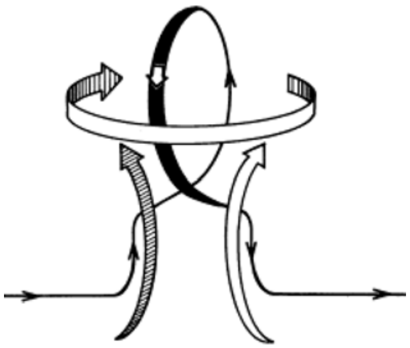
Linear alpha-Omega solutions (5)



III.6.2.1.1

Nonlinear models: alpha-quenching (1)

We expect that the Lorentz force should oppose the cyclonic motions giving rise to the alpha-effect;



We also expect this to become important when the magnetic energy becomes comparable to the kinetic energy of the turbulent fluid motions, i.e.:

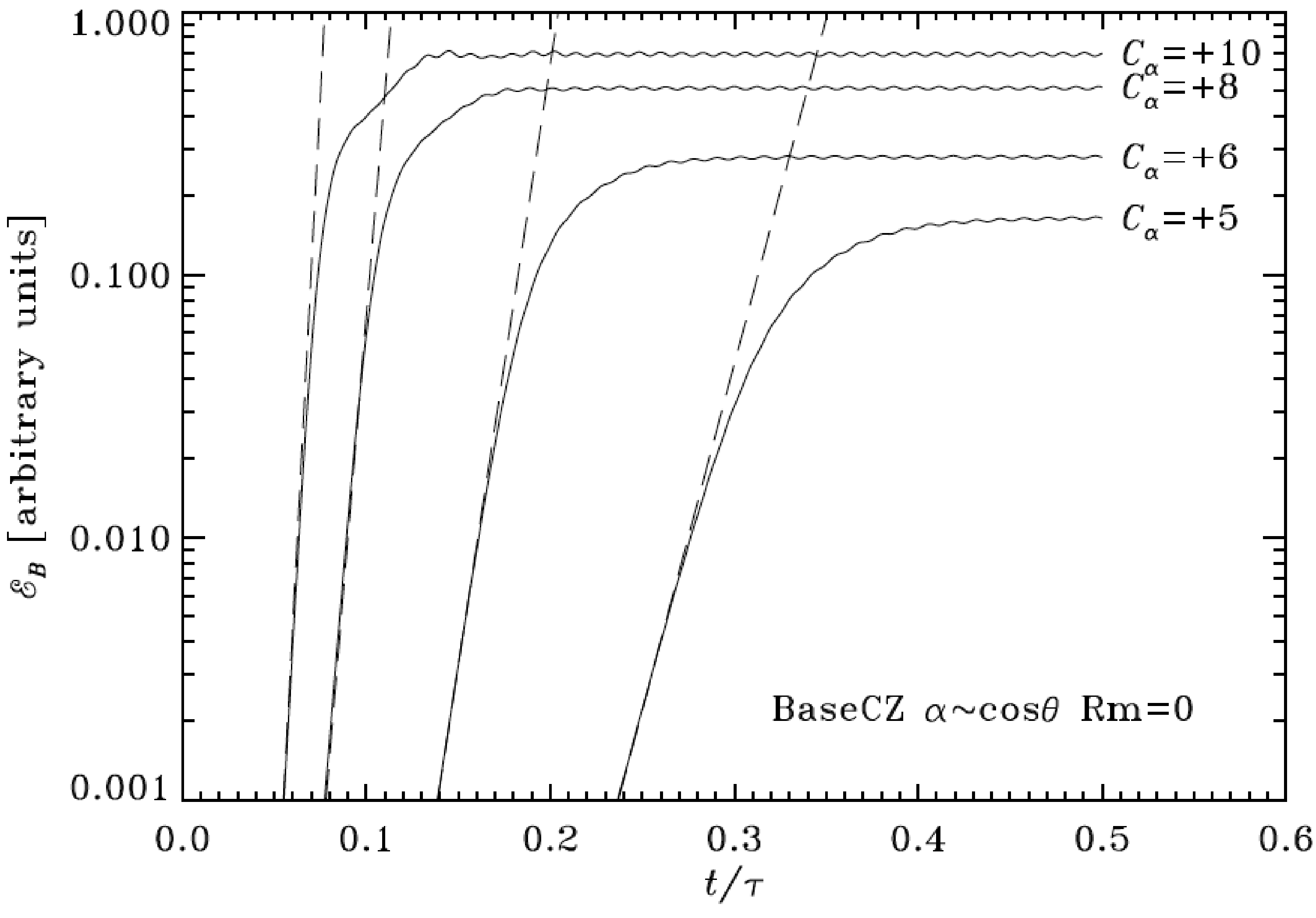
$$\frac{B_{\rm eq}^2}{2\mu_0} = \frac{\rho u_t^2}{2} \quad \rightarrow \quad B_{\rm eq} = u_t \sqrt{\mu_0 \rho}$$

This motivates the following *ad hoc* expression for « alpha-quenching »:

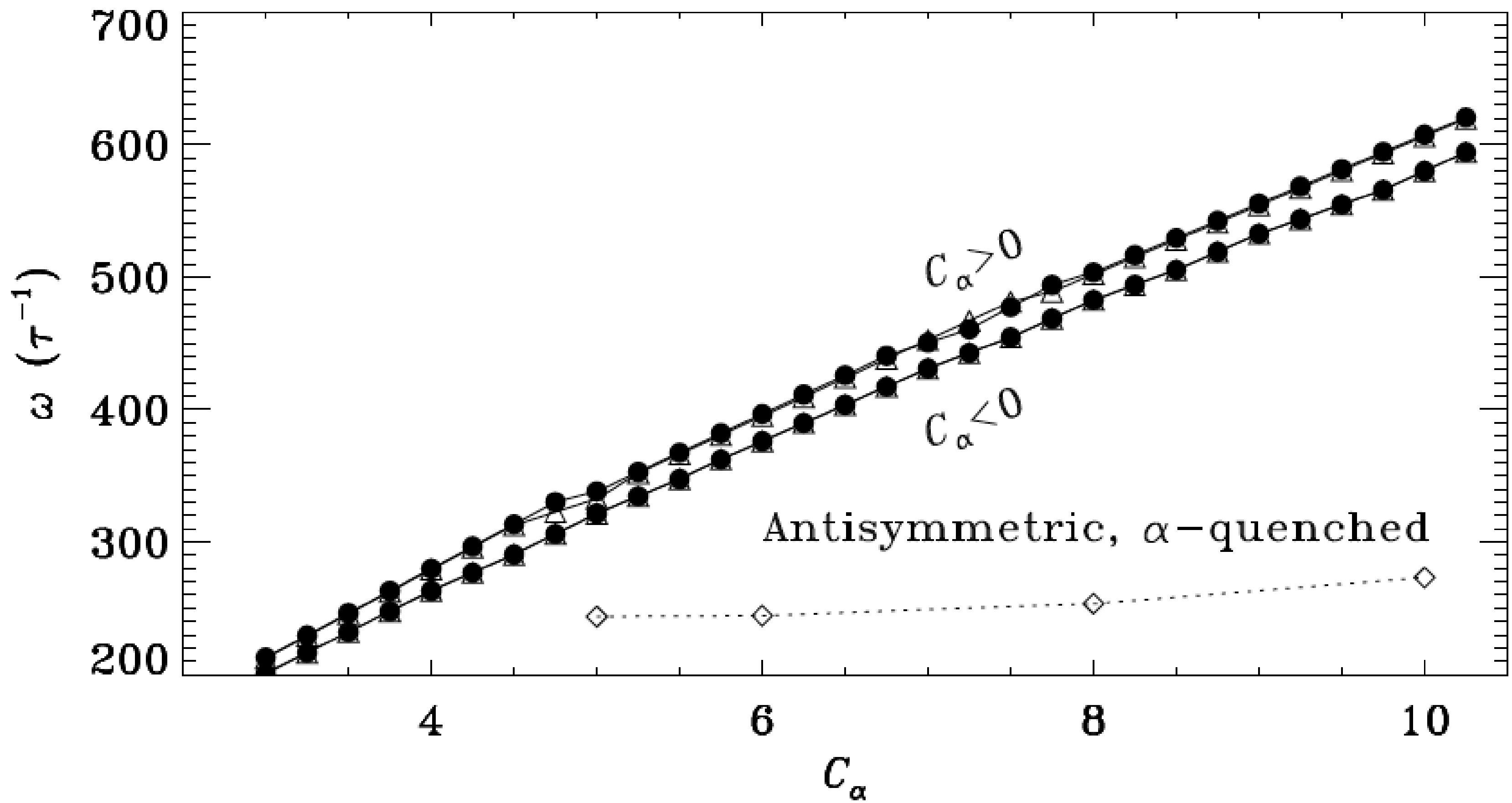
$$\alpha \rightarrow \alpha(B) = \frac{\alpha_0}{1 + (B/B_{\rm eq})^2}$$

III.6.2.1.2

Nonlinear models: alpha-quenching (2)



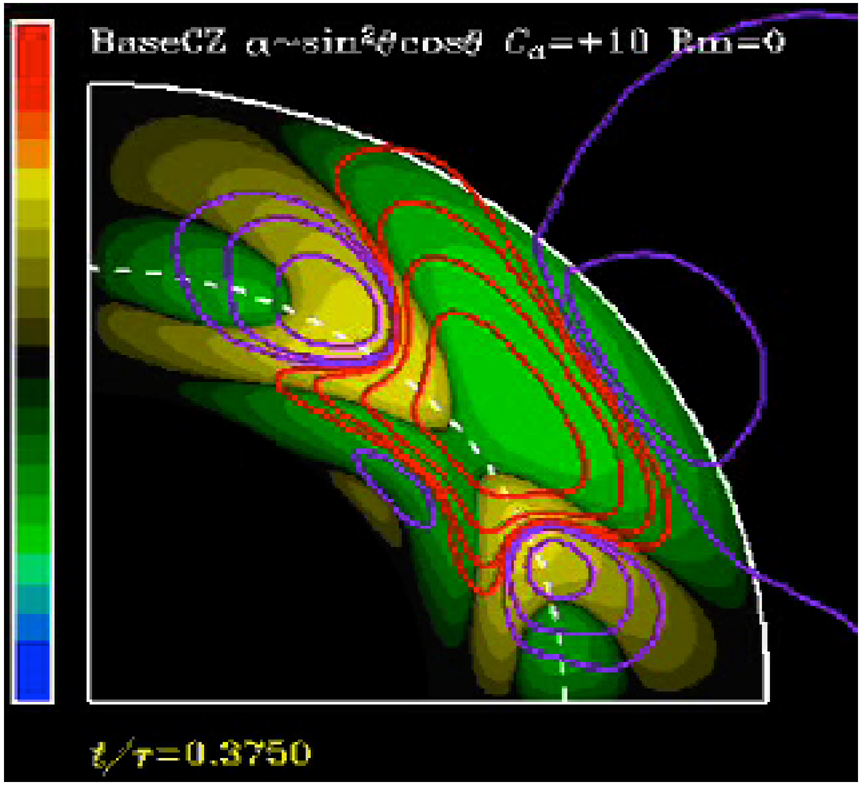
Nonlinear models: alpha-quenching (3)

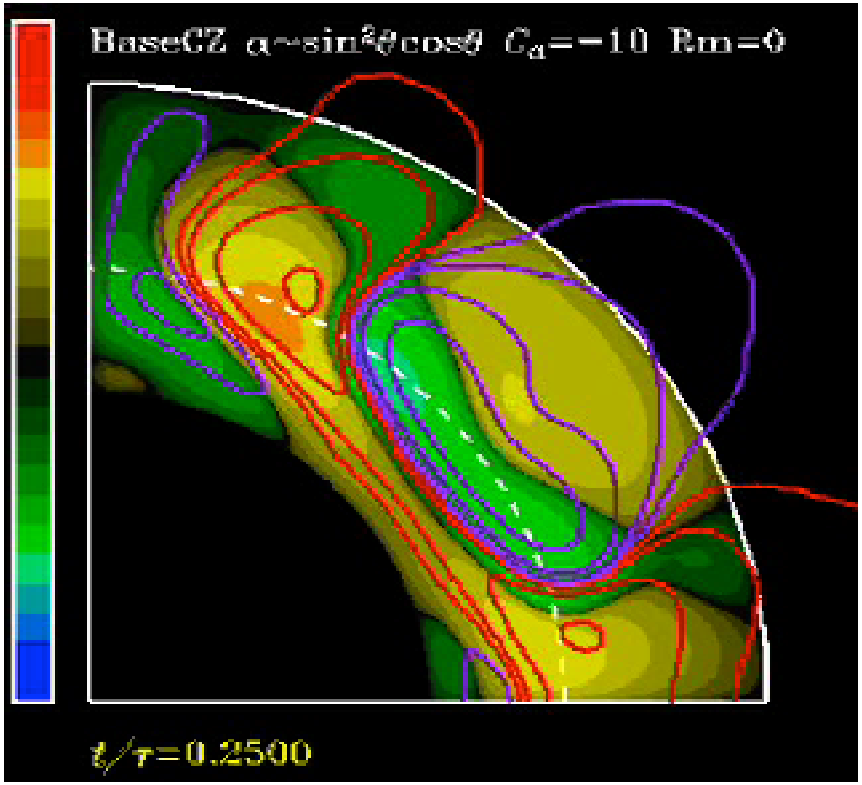


The magnetic diffusivity is the primary determinant of the cycle period

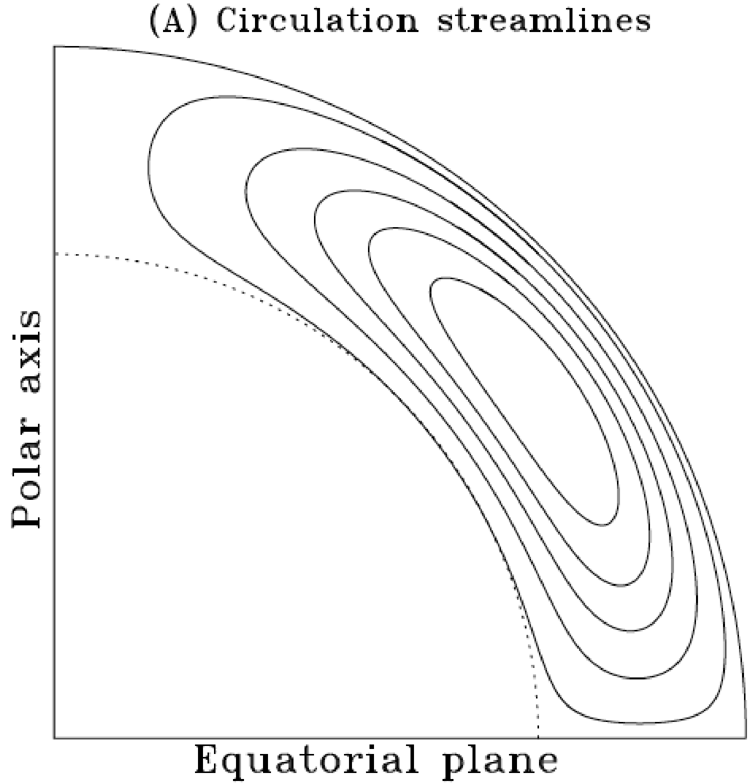
Nonlinear models: alpha-quenching (4)

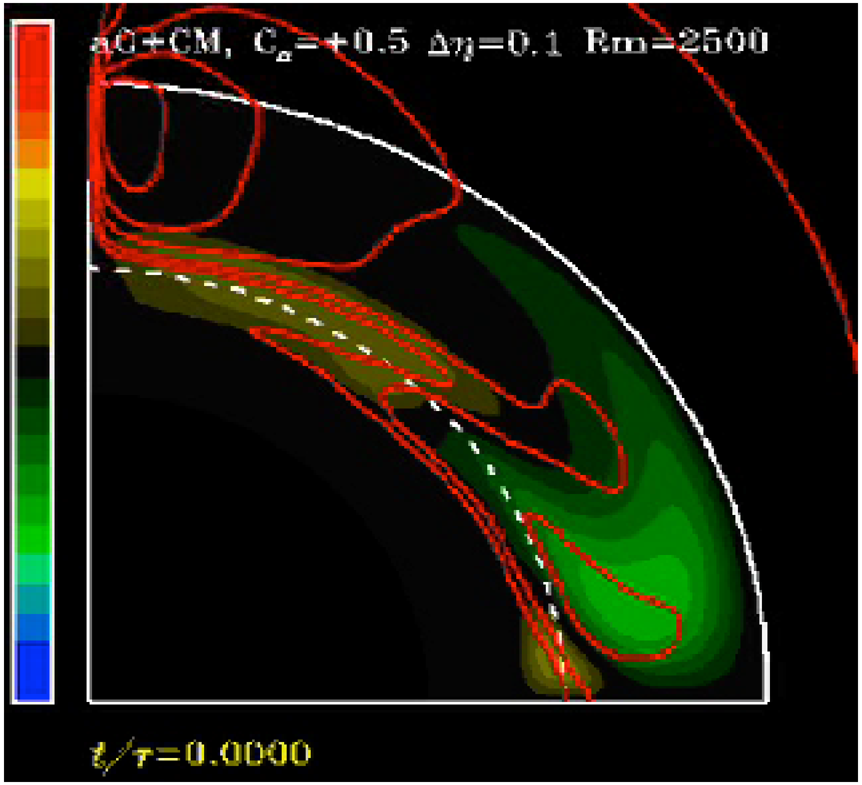
Magnetic fields concentrated at too high latitude;
Try instead a latitudinal dependency for alpha: $g(\theta) = \sin^2 \cos\theta$





Alpha-Omega dynamos with meridional circulation (1)

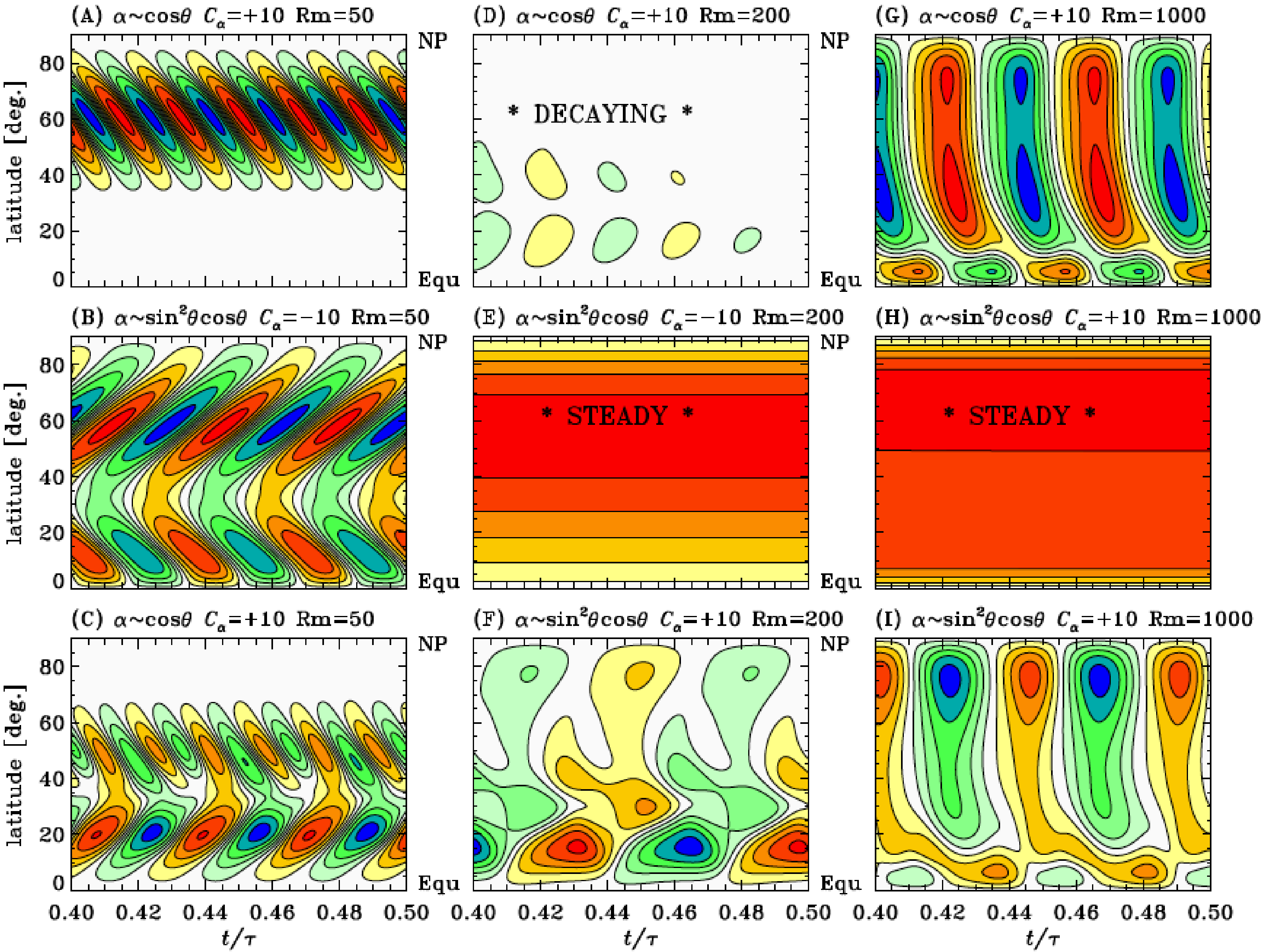




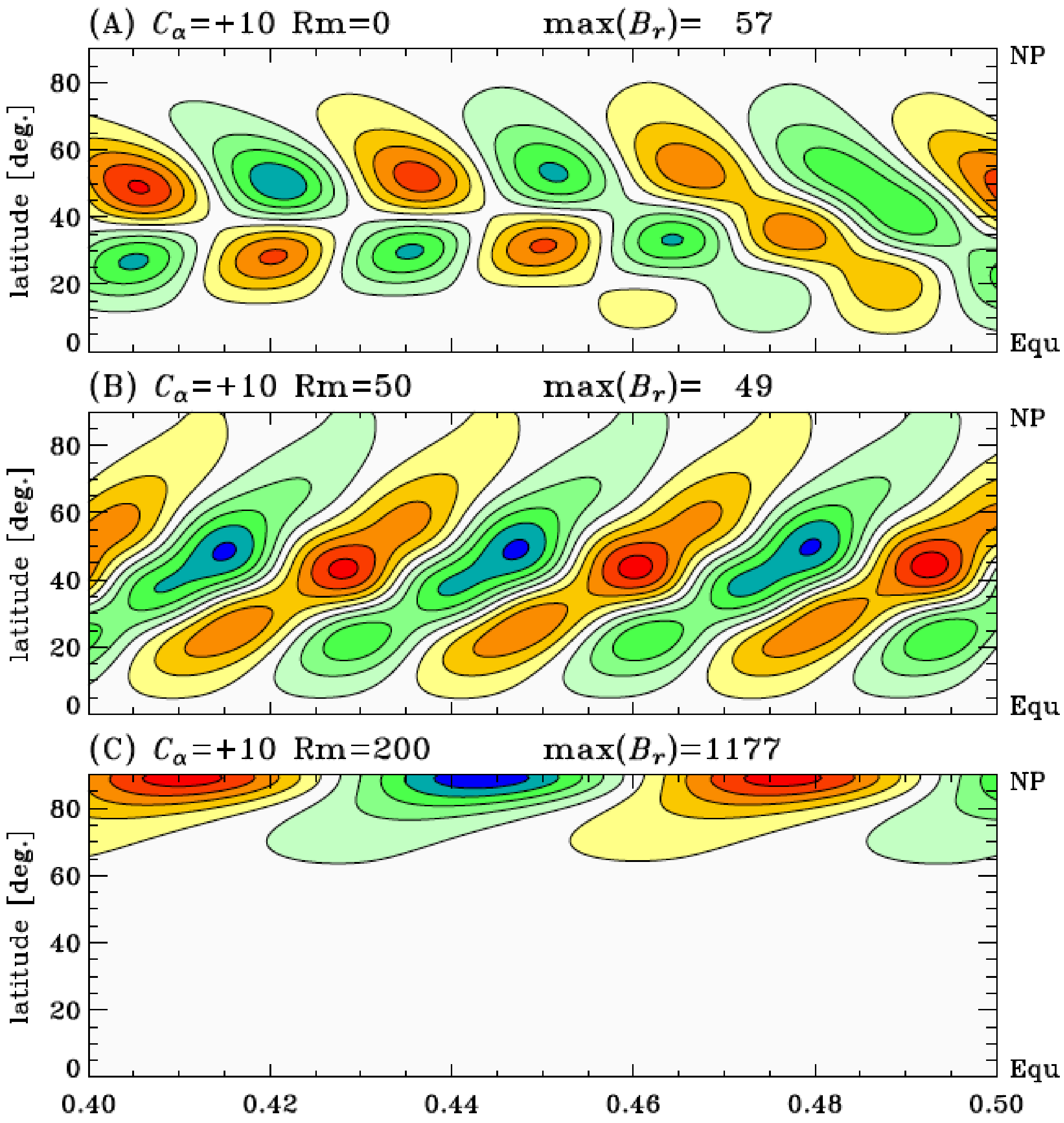
Equatorward propagation of the deep toroidal field is now due to advection by the meridional flow, not « dynamo waves » effect.

III.6.2.1.4

Models with meridional circulation (2)

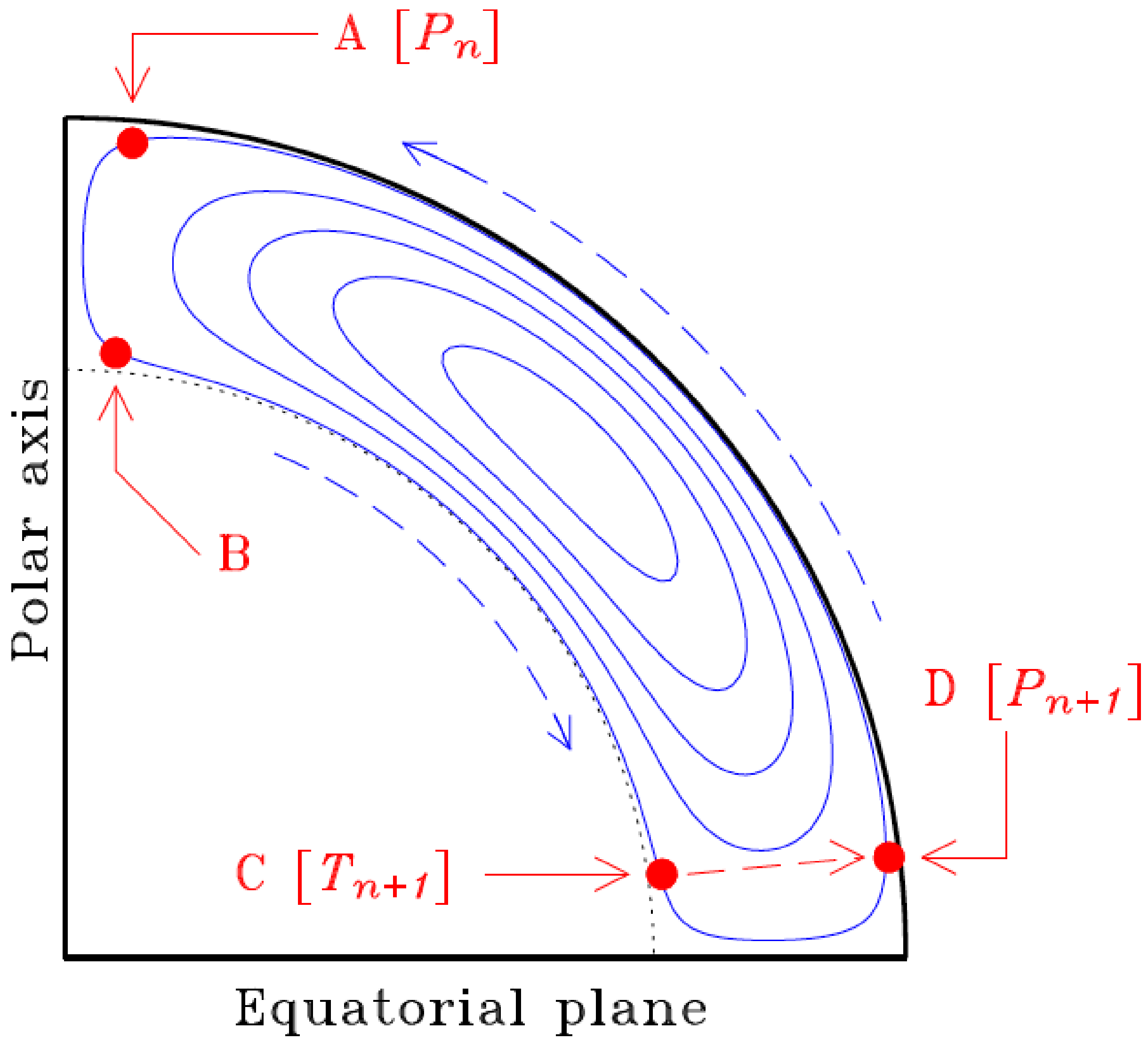


Models with meridional circulation (3)



Models based on the Babcock-Leighton mechanism

Babcock-Leighton dynamo model (1)



Babcock-Leighton dynamo model (2)

A Babcock-Leighton source term for the axisymmetric dynamo equations:

$$S(r,\theta,B(t)) = s_0 f(r) g(\theta)\,\mathrm{erf}\left(\frac{B(r_c,\theta,t)-B_1}{w_1}\right)\left[1-\mathrm{erf}\left(\frac{B(r_c,\theta,t)-B_2}{w_2}\right)\right] B(r_c,\theta,t)$$

$$g(\theta) = \sin\theta\cos\theta\ ,$$

Peaking at mid-latitudes

Non-local in *B*

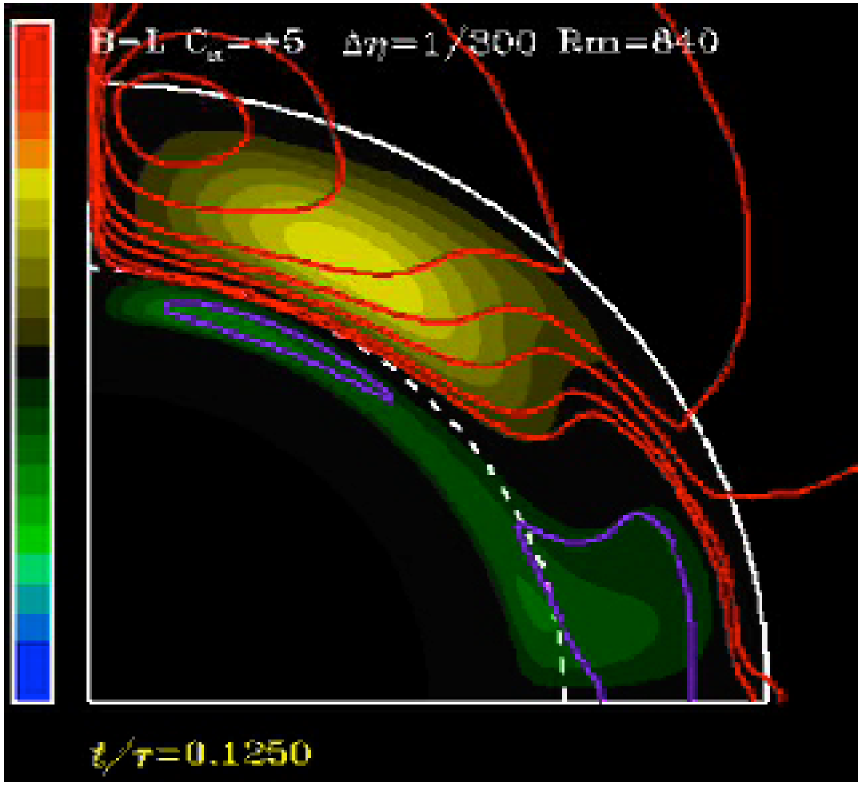
$$f(r) = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{r-r_2}{d_2}\right)\right]\left[1-\mathrm{erf}\left(\frac{r-r_3}{d_3}\right)\right]$$

The source term operates only in a finite range of toroidal field strengths.

Concentrated in surface layers

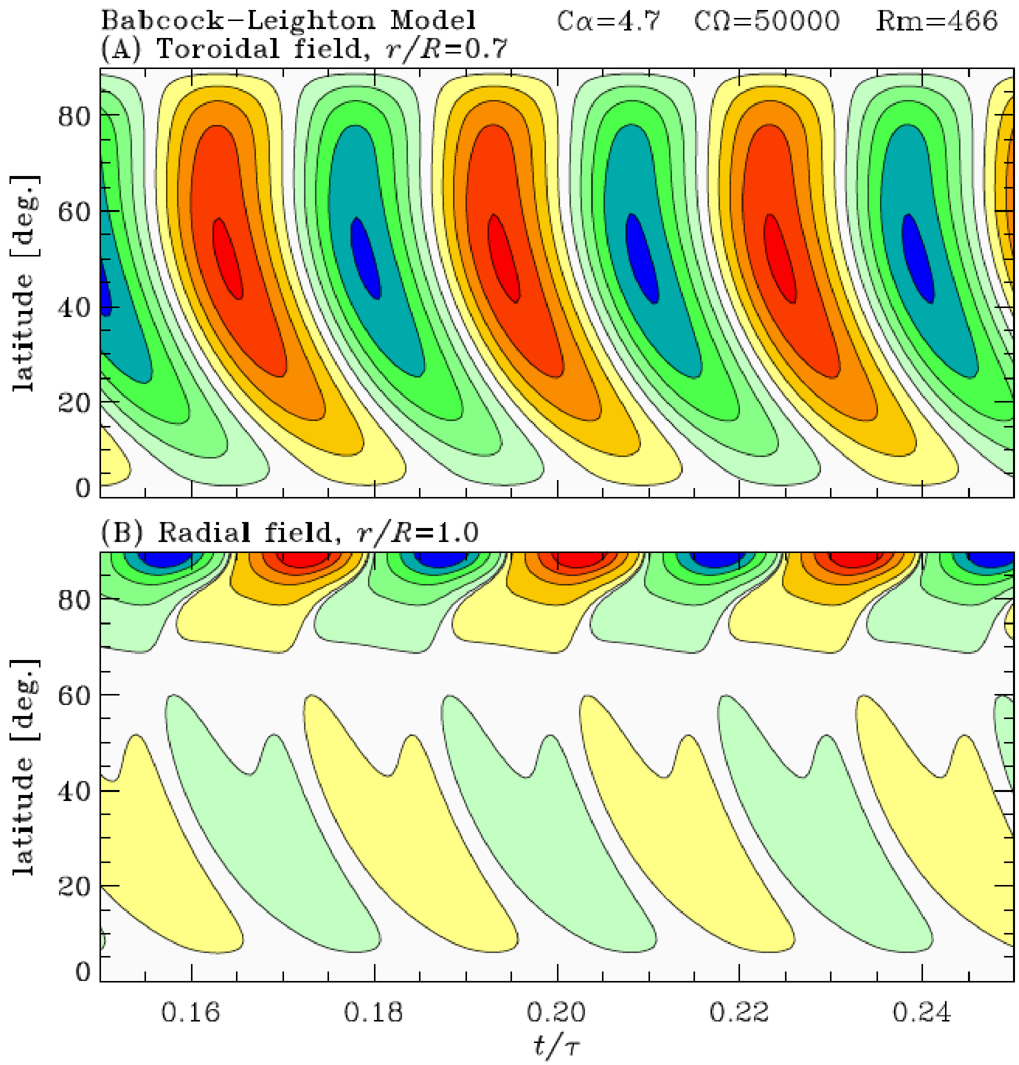
III.6.2.2

Babcock-Leighton dynamo model (3)



The turnover time of the meridional flow is the primary determinant of the cycle period

Babcock-Leighton dynamo model (4)



Babcock-Leighton versus alpha-effect

There are serious potential problems with the operation of the alpha-effect at high field strength; not so with the B-L mechanism

The B-L mechanism operates only in a finite range of field strength; potentially problematic in the presence of large cycle amplitude fluctuations.

Both models can produce tolerably solar-like toroidal field butterfly diagrams, and yield the proper phase relationship between surface poloidal and deep toroidal components (with circulation included in the mean-field model)

The alpha-effect (or something analogous) appears unavoidable in stratified, rotating turbulence.

A decadal period arises « naturally » in B-L models; in mean-field models, it requires tuning the value of the turbulent magnetic diffusivity

Cycle fluctuations

Nonlinear magnetic backreaction through the Lorentz force

Problem: differential rotation and meridional circulation are powered by thermally-driven convective turbulence, for which we are lacking a model simple enough for inclusion in mean-field-like dynamo models.

Trick: large-scale flows are separated into two contributions, with only the second reacting to the Lorentz force:

$$\mathbf{u} = \mathbf{U}(\mathbf{x}) + \mathbf{U}'(\mathbf{x}, t, \mathbf{B})$$

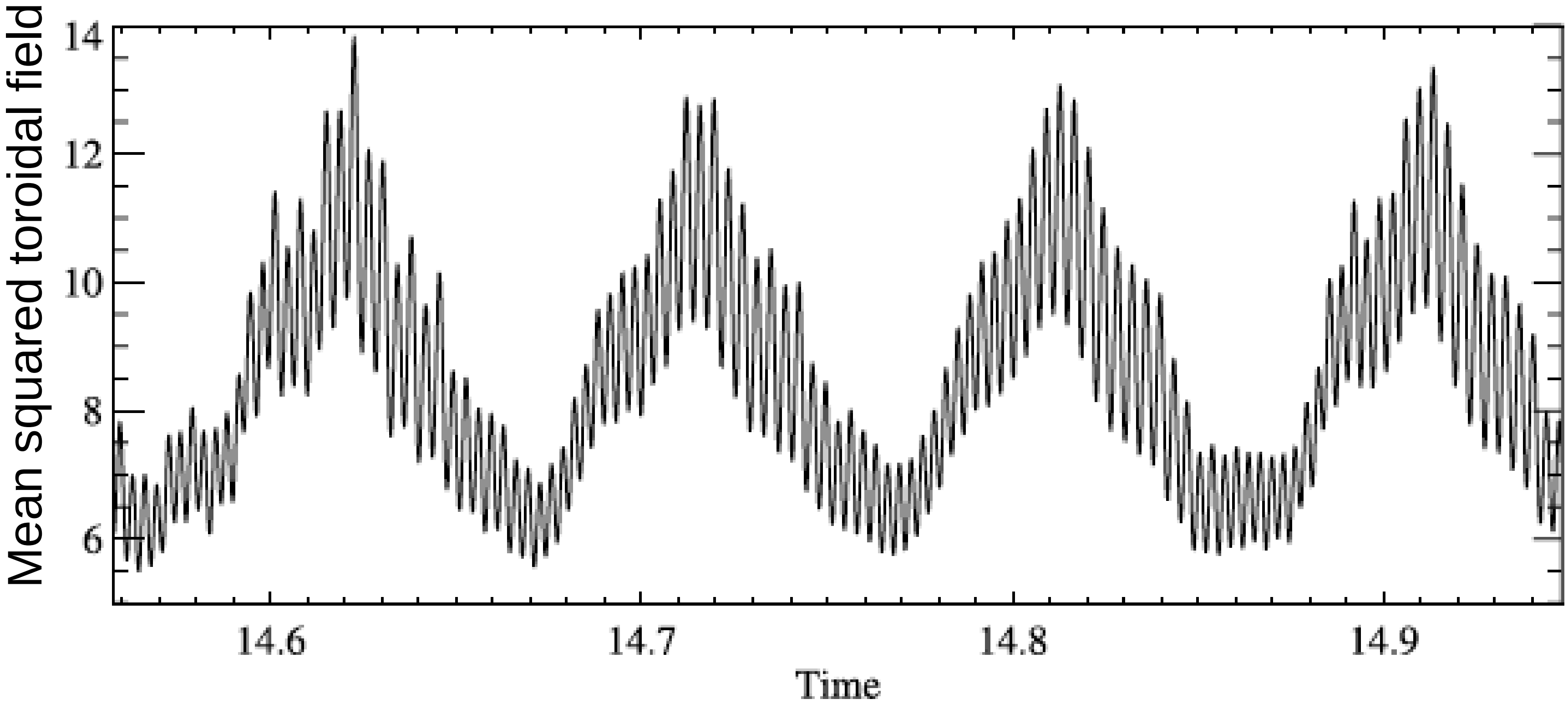
It is now a matter of solving an equation of motion only for this second, time-varying component, together with the usual dynamo equations:

$$\frac{\partial \mathbf{U}'}{\partial t} = \frac{\Lambda}{4\pi\rho}(\nabla \times \mathbf{B}) \times \mathbf{B} + \mathrm{P}_m \nabla^2 \mathbf{U}$$

III.6.3.2

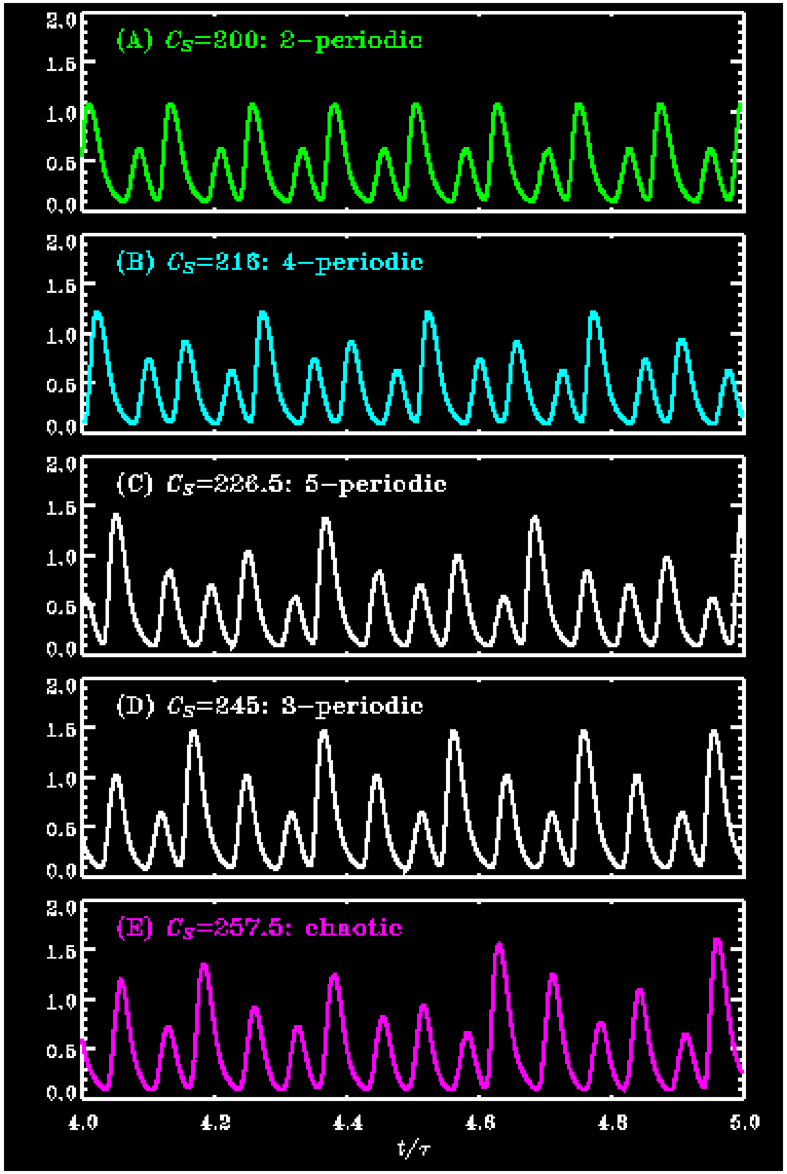
Amplitude modulation (1)

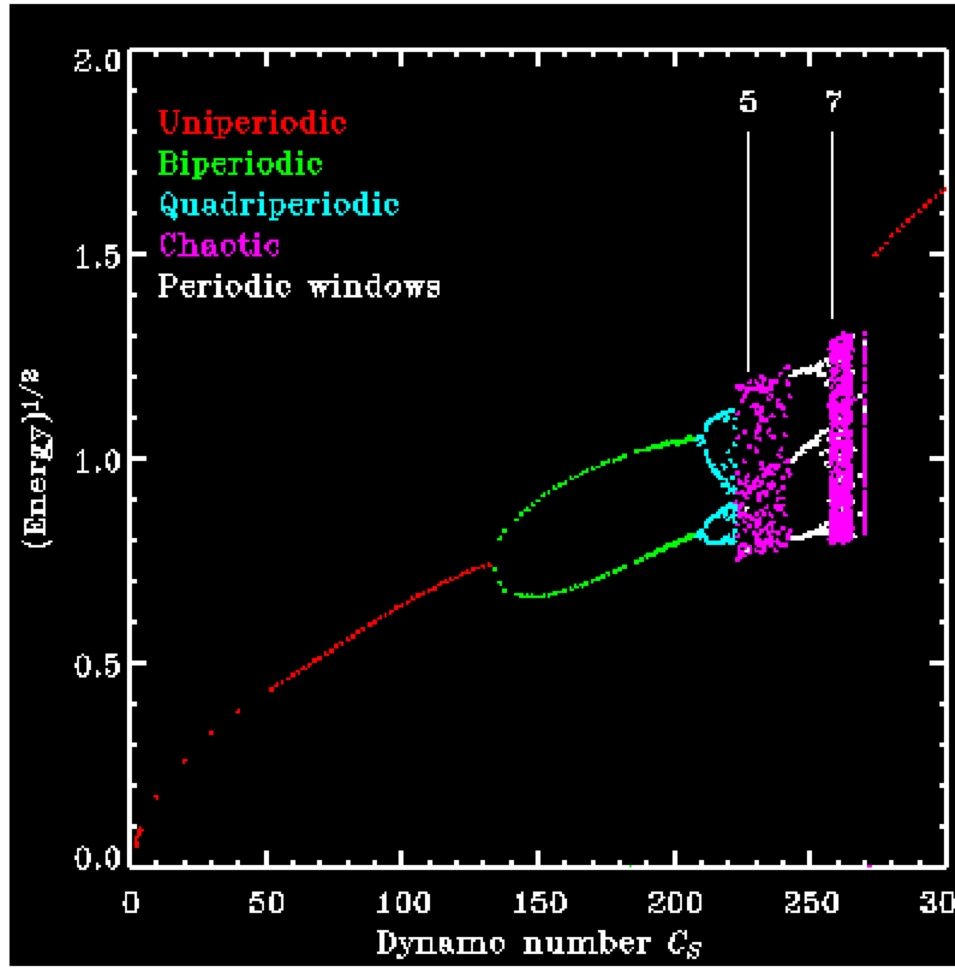
The primary cycle picks up a longer modulation, with period controlled by the magnetic Prandtl number (ratio of viscosity to magnetic diffusivity; ~0.01 for microscopic values)



Bushby & Tobias 2007, ApJ, **661**, 1289.

Bifurcations in numerical solutions





Charbonneau et al., ApJ, **619**, 613 (2005)

Stochastic forcing

The solar dynamo operates in part or in totality in a strongly turbulent environment; all large-scale flows contributing to field amplification will be characterized by strong fluctuations about the mean.

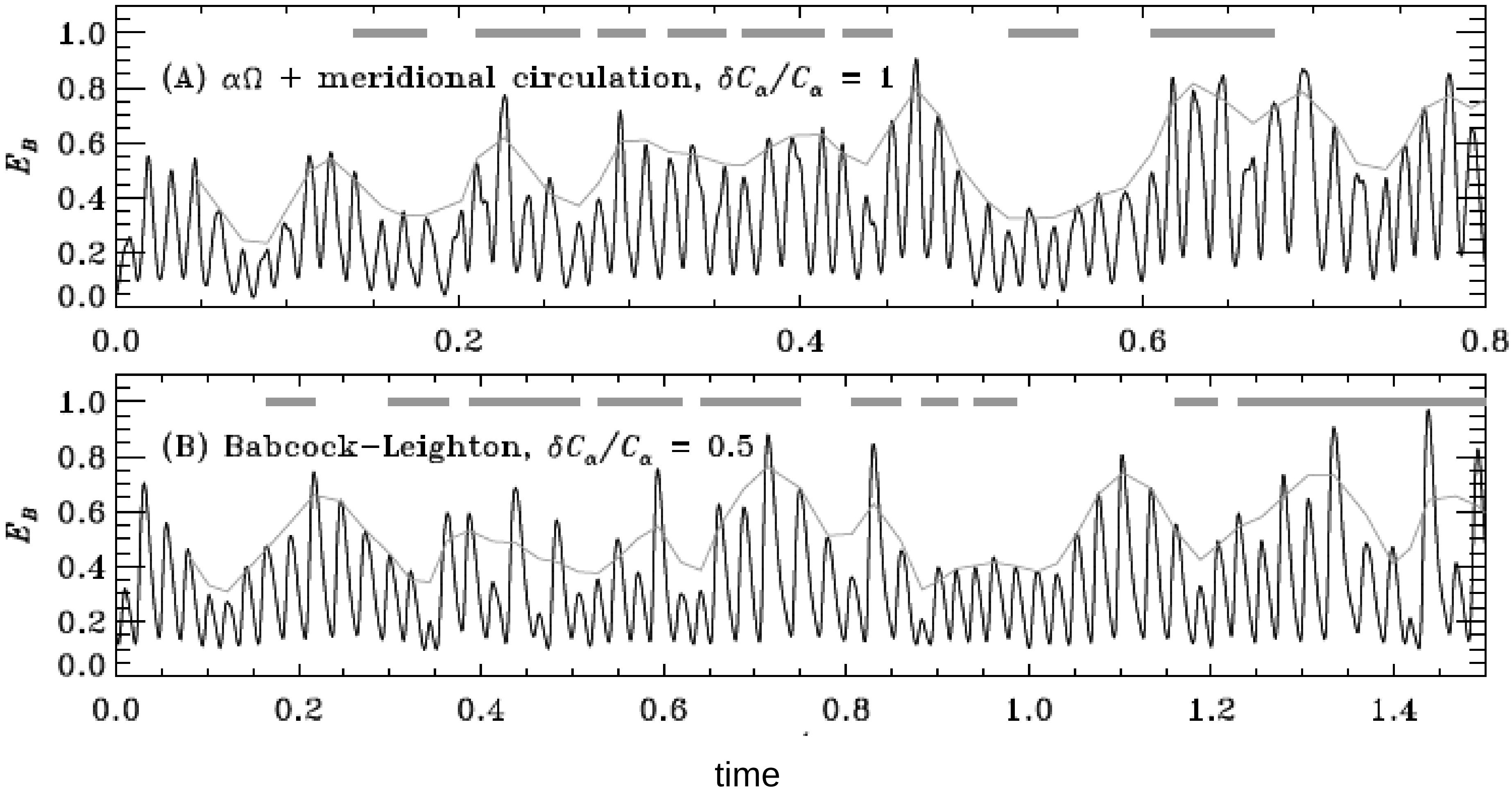
Also, mean-field coefficients or other source terms result from a process of averaging over many elementary « events », and therefore will also fluctuate in time about their mean.

Introduce this latter effect in mean-field-like models as:

$$C_\alpha \to \bar{C}_\alpha + \rho \times \delta C \ , \qquad \rho \in [-1,1] \ , \qquad \mathrm{if}(t \bmod \tau_c) = 0 \ .$$

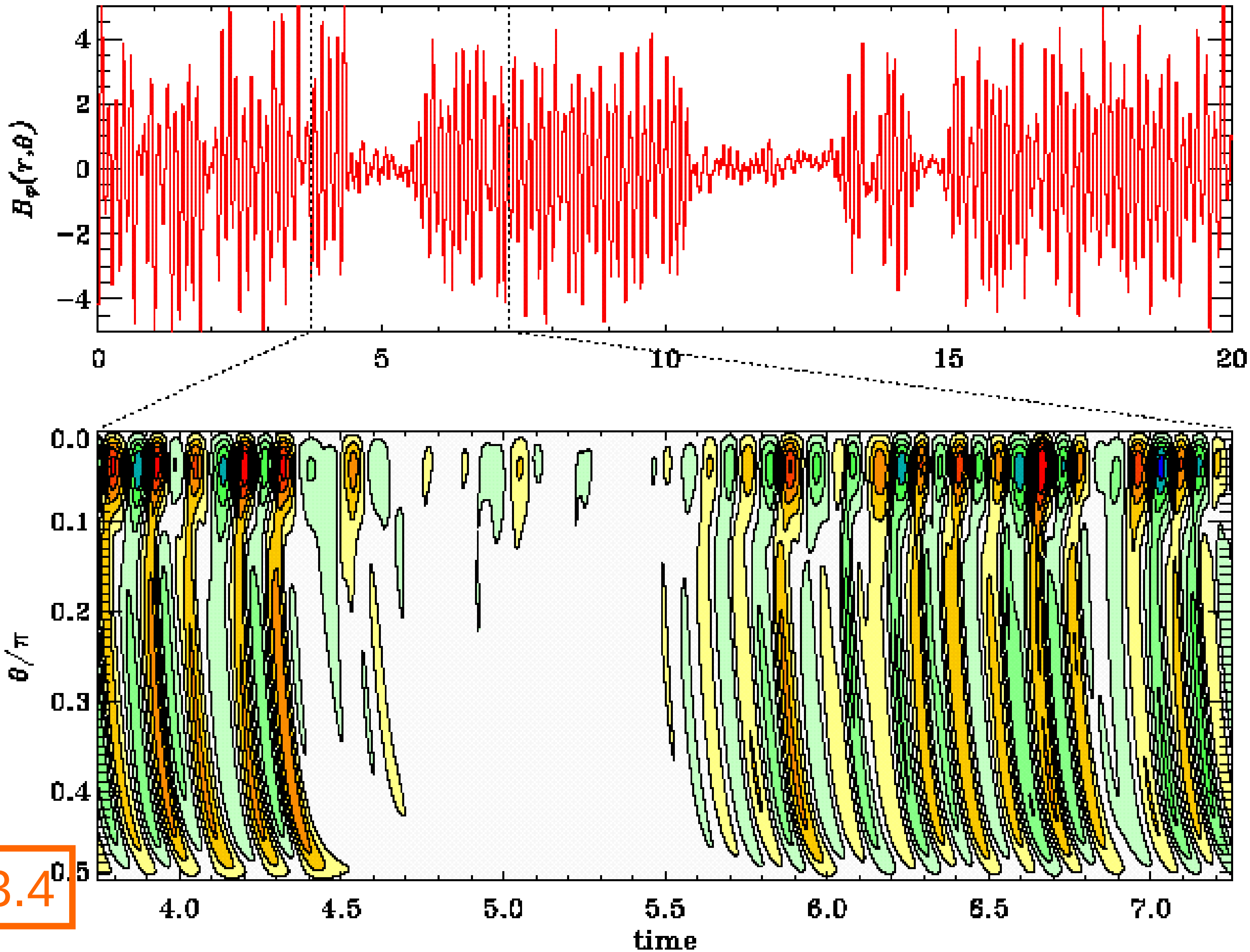
III.6.3.1

Amplitude modulation by stochastic forcing (1)



Intermittency (1)

[Charbonneau et al., ApJ **616**, L183 (2004)]



III.6.3.4

Stellar dynamos

Back to basics…

What have we learned?

1. When rotation is present, a turbulent flow in a stratified environment can produce a large-scale magnetic field

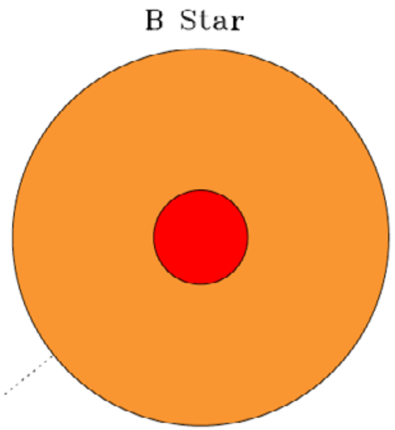
2. Differential rotation is an excellent mechanism to produce magnetic fields organised on large spatial scales

This is good because:

1. Most stars convect somewhere in their interior

2. Most stars rotate significantly

Early-type stars

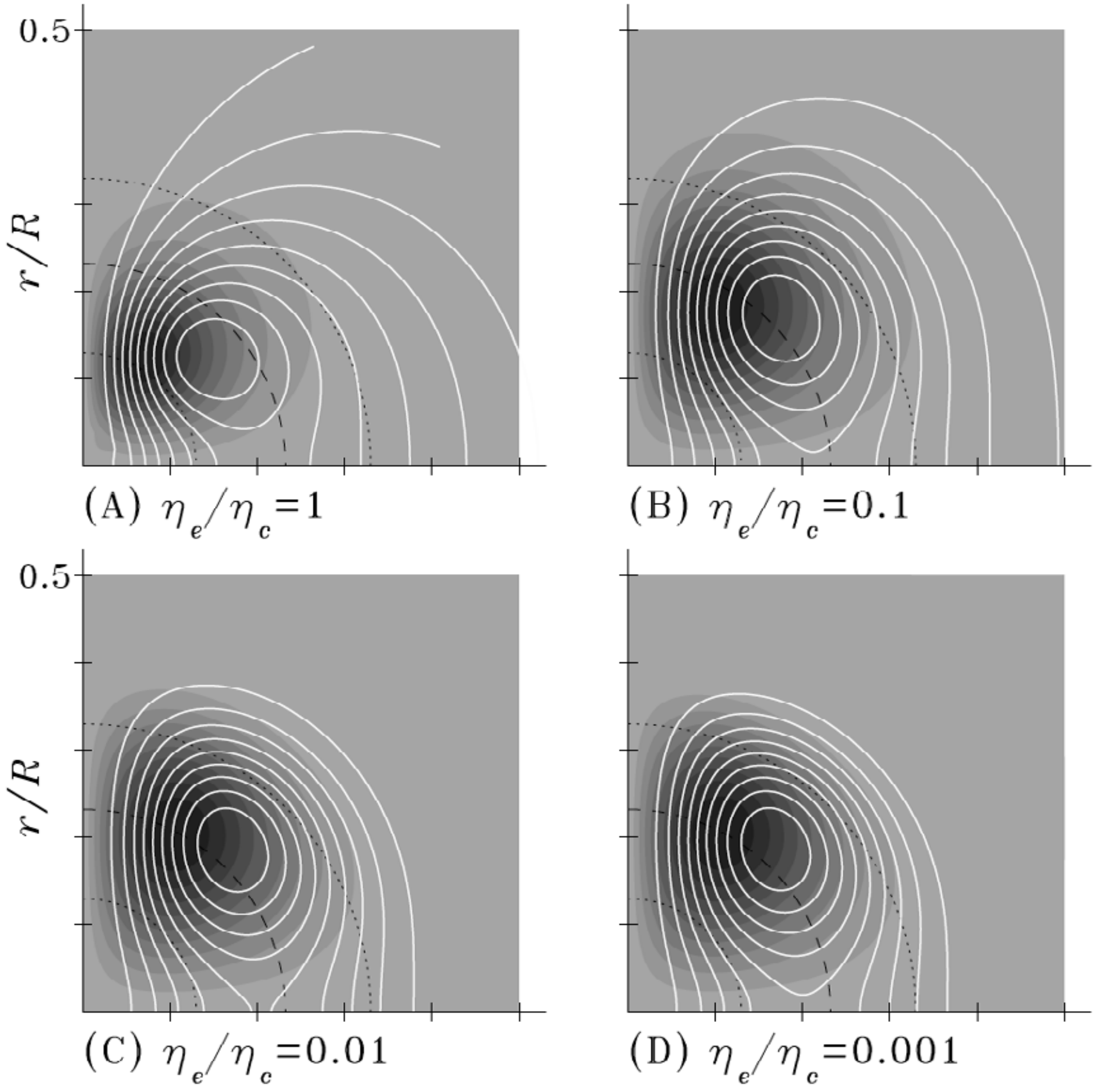


alpha² dynamo solutions

The magnetic field produced by a kinematic alpha^2 dynamo is usually steady in time.

The magnetic field remains « trapped » in the deep interior if a strong magnetic diffusivity contrast exists between the core and envelope.

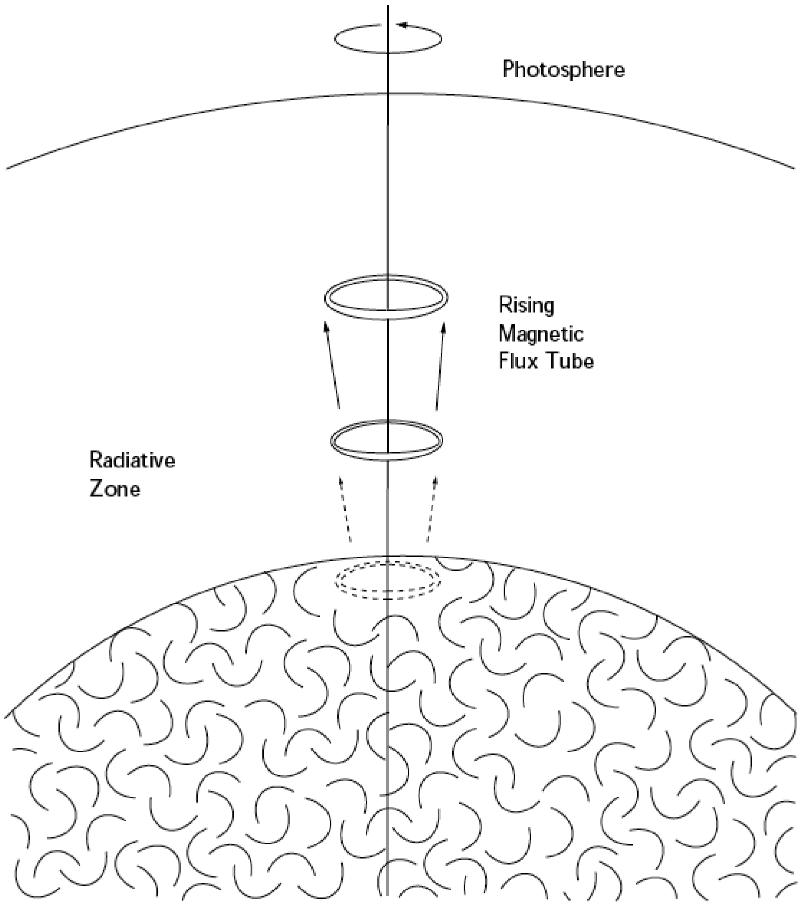
A very strong field may exist in the deep interior, without being visible at the surface !!



III.6.4.1

From the core to the surface (2)

[MacGregor & Cassinelli, ApJ, 586, 480 (2003)]

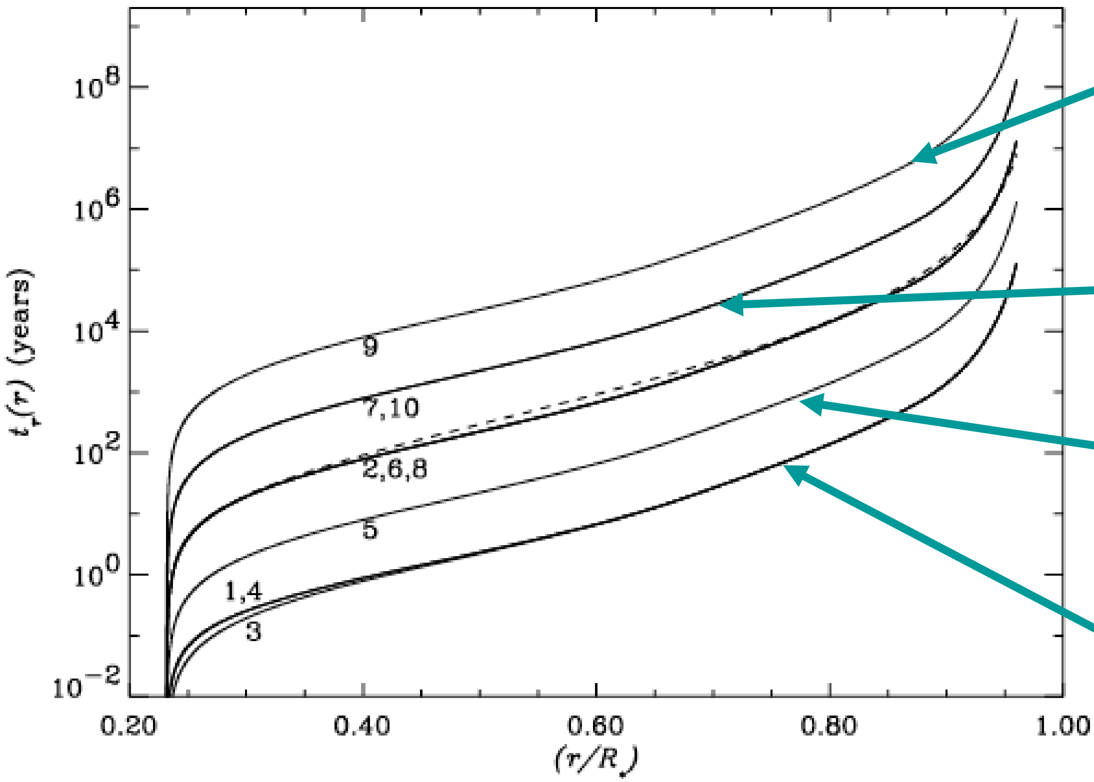


In analogy to what we think happens at the solar core-envelope interface, could toroidal flux ropes here also form at the core boundary, and if so could they rise buoyantly to the surface

From the core to the surface (4)

[MacGregor & Cassinelli, ApJ, 586, 480 (2003)]

If toroidal magnetic flux ropes do form at the boundary of the convective core, magnetic buoyancy can lift them up to a tenth of a stellar radius under the surface, **under the most optimistic working hypotheses.**



Curve « 9 »: thick tube, 58T, without drag

Curve « 7 »: thick tube, 18T, with drag

Curve « 5 »: thin tube, 18T, with drag

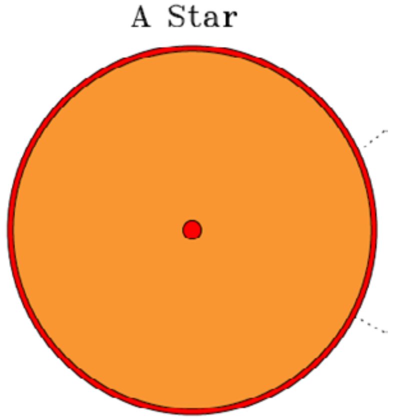
Curve « 1 »: thin tube, 58T, with drag

Alternatives to core dynamo action

1. Dynamo action powered by MHD instabilities in the radiative envelope (e.g., Spruit-Tayler); could contribute to internal angular momentum redistribution and to chemical mixing

2. Dynamo action in outer convective layers produced by iron opacities

Intermediate mass stars



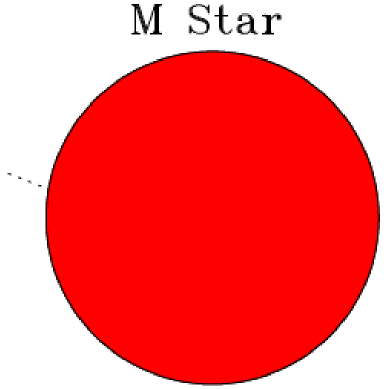
Fossil fields versus dynamo action

The absence of observed temporal variability is compatible with the idea that a fossil field, **OR** a field produced during a convective phase during pre-main-sequence evolution

There exists dynamo mechanisms driven by MHD instabilities of large-scale internal fossil fields, which could operate in stellar radiative envelopes if significant differential rotation is present.

III.6.4.2

Fully convective stars



Dynamo modelling in fully convective stars

To « extrapolate » solar dynamo models to fully convective stars, we encounter the following complications:

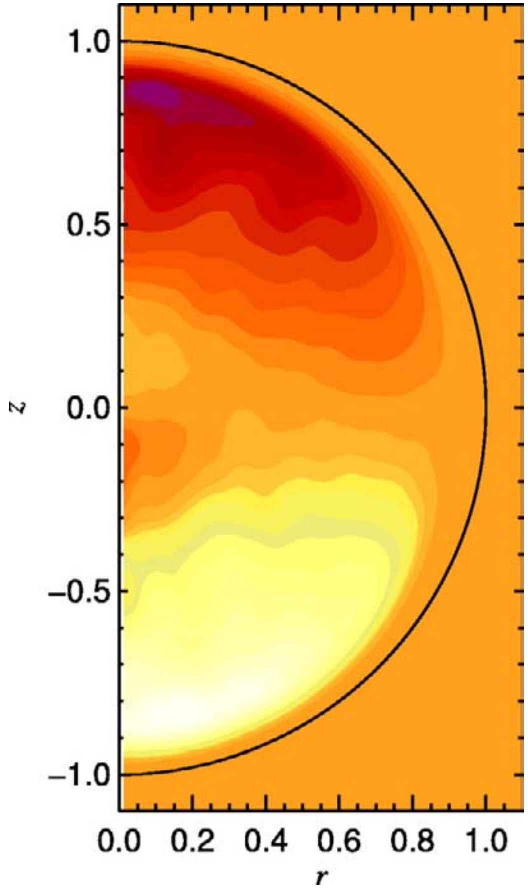
1. What is the star's internal structure ?
2. What happens to differential rotation in the absence of a tachocline ?
3. As the convective envelope reaches the center, are there « transitions » in dynamo operating modes (alpha-Omega to alpha^2-Omega to alpha^2) ?
4. Without a tachocline, is the Babcock-Leighton mechanism still possible? Are there still starspots?
5. As the photosphere becomes ever cooler, what happens to photospheric flux evolution ?
6. How do dynamical nonlinearities play into all this ?

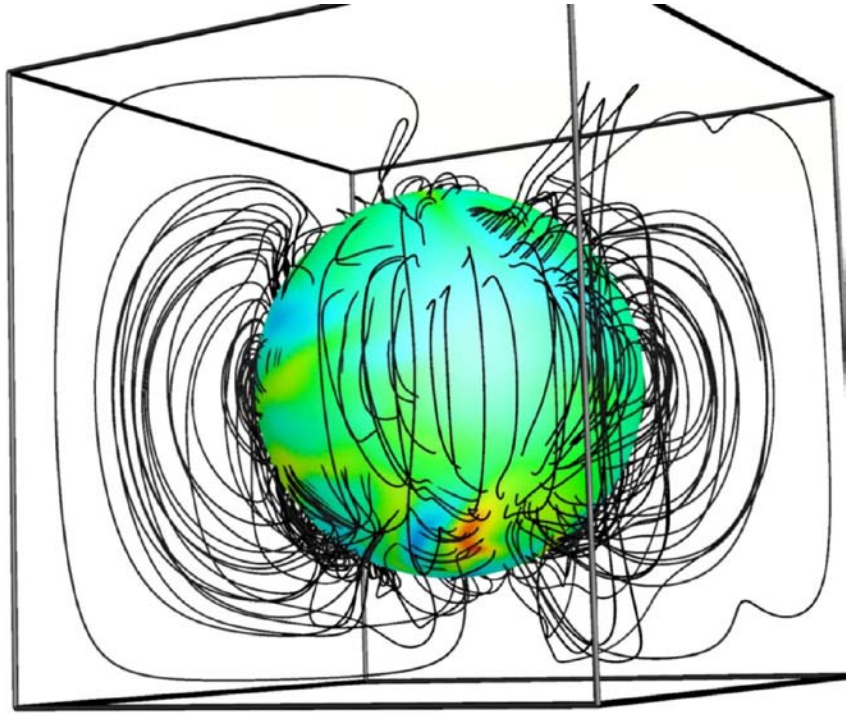
III.6.4.4

Fully convective stars

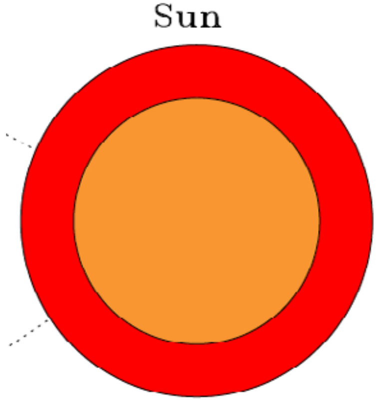
[Dobler, Stix & Brandenburg 2006, *Astrophys. J.*, **638**, 336]

Kinetic helicity





Solar-type stars

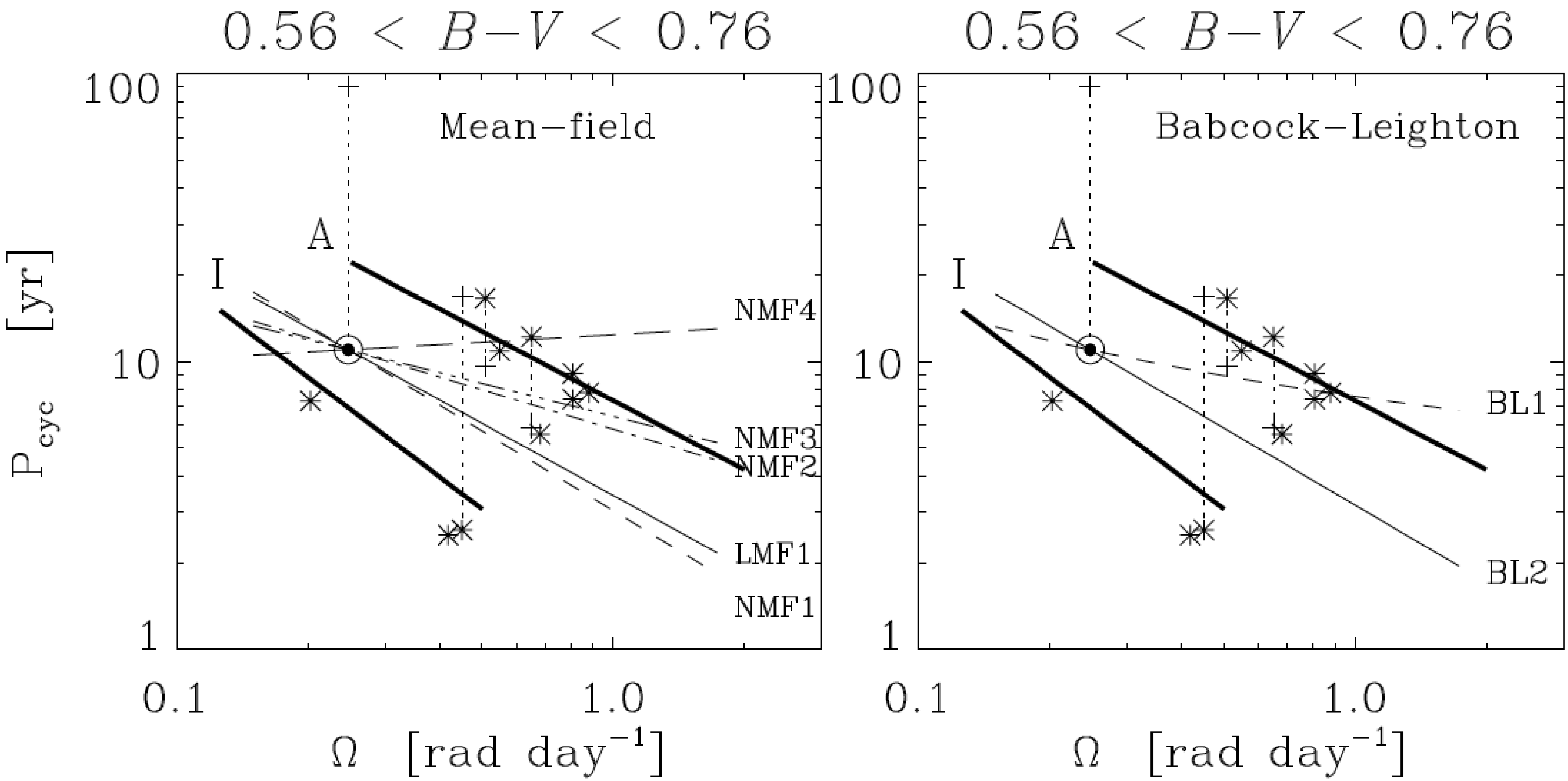


Dynamo modelling in solar-type stars (1)

To « extrapolate » solar dynamo models to solar-type stars of varying masses, luminosities and rotation rate, we must specify:

1. What is the mechanism responsible for poloidal field regeneration, and in what regime is it operating?
2. What is the star's internal structure (convection zone depth, etc)
3. How do the form and magnitude of differential rotation vary with stellar parameters (rotation, luminosity, etc) ?
4. How does meridional circulation vary with stellar parameters?
5. How do the alpha-effect, turbulent diffusivity, Babcock-Leighton source term, etc, vary with stellar parameters
6. Which nonlinear effect quenches the growth of the dynamo magnetic field?

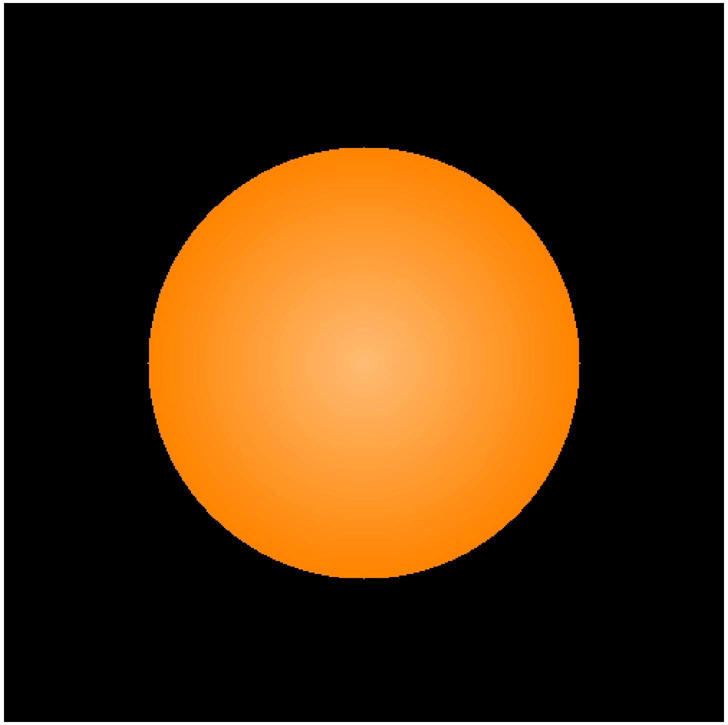
Dynamo modelling in solar-type stars (3)

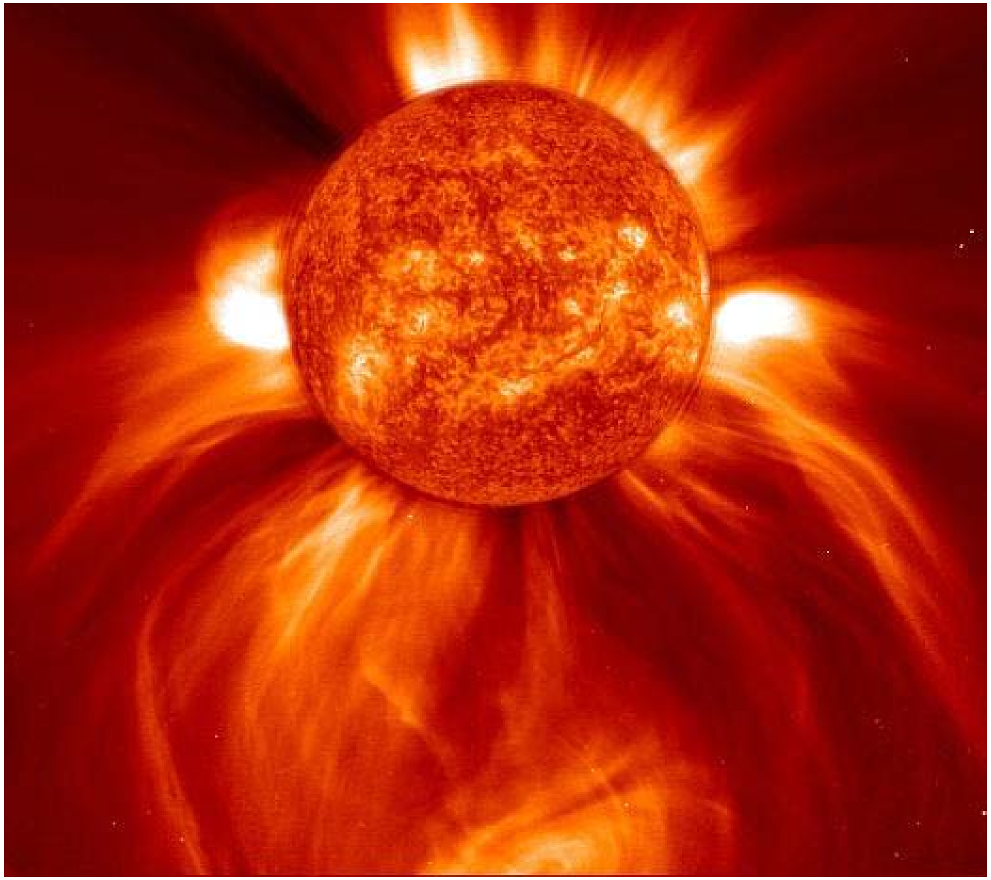


[Joint work with S. Saar, Harvard/CfA]

…some answers in the Lab ! (?)

Solar/stellar magnetism





« If the sun did not have a magnetic field, it would be as boring a star as most astronomers believe it to be »

(Attributed to R.B. Leighton)