

Dynamos in Planets & Stars



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OUTLINE

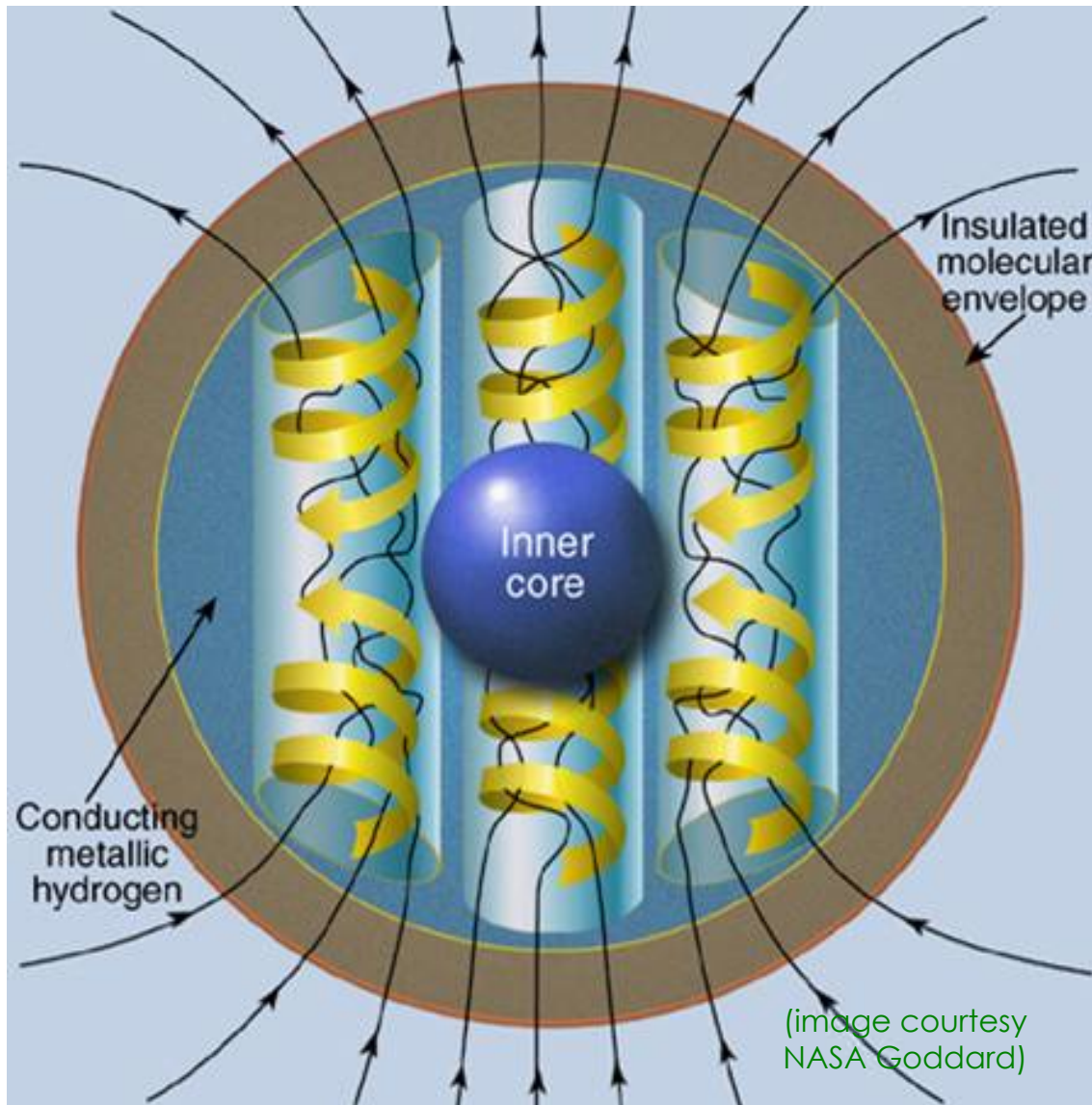
1. Intro to planetary & stellar dynamos
2. Dynamo theory basics
 - MHD approximation
 - Magnetic Induction Equation
 - Magnetic Reynolds #
 - Alfvén Approximation
 - Self-sustained dynamos
3. Planetary & stellar dynamo models
 - Mean field models
 - Macroscopic models
 - Parameter regimes
 - Scaling laws
 - Surveying the models out there

My (not so) hidden agenda:

- planetary & stellar dynamos aren't so different from each other, making it surprising that researchers rarely work on both
- very similar processes, just in different parameter regimes

1. INTRO TO PLANETARY & STELLAR DYNAMOS

- convert mechanical energy into electromagnetic energy producing magnetic fields we can observe



complex motions

+

electrically conducting fluid

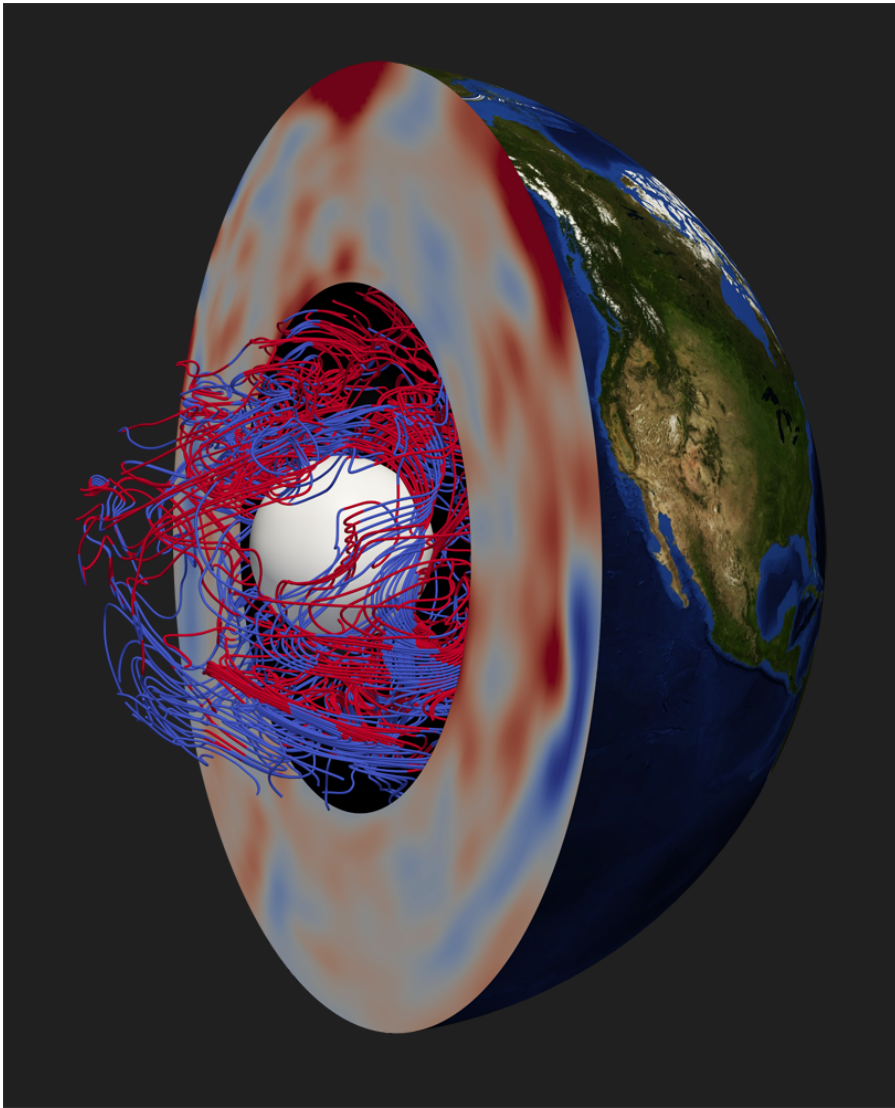
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presence of a magnetic field



maintain field against Ohmic decay

DYNAMO INGREDIENTS

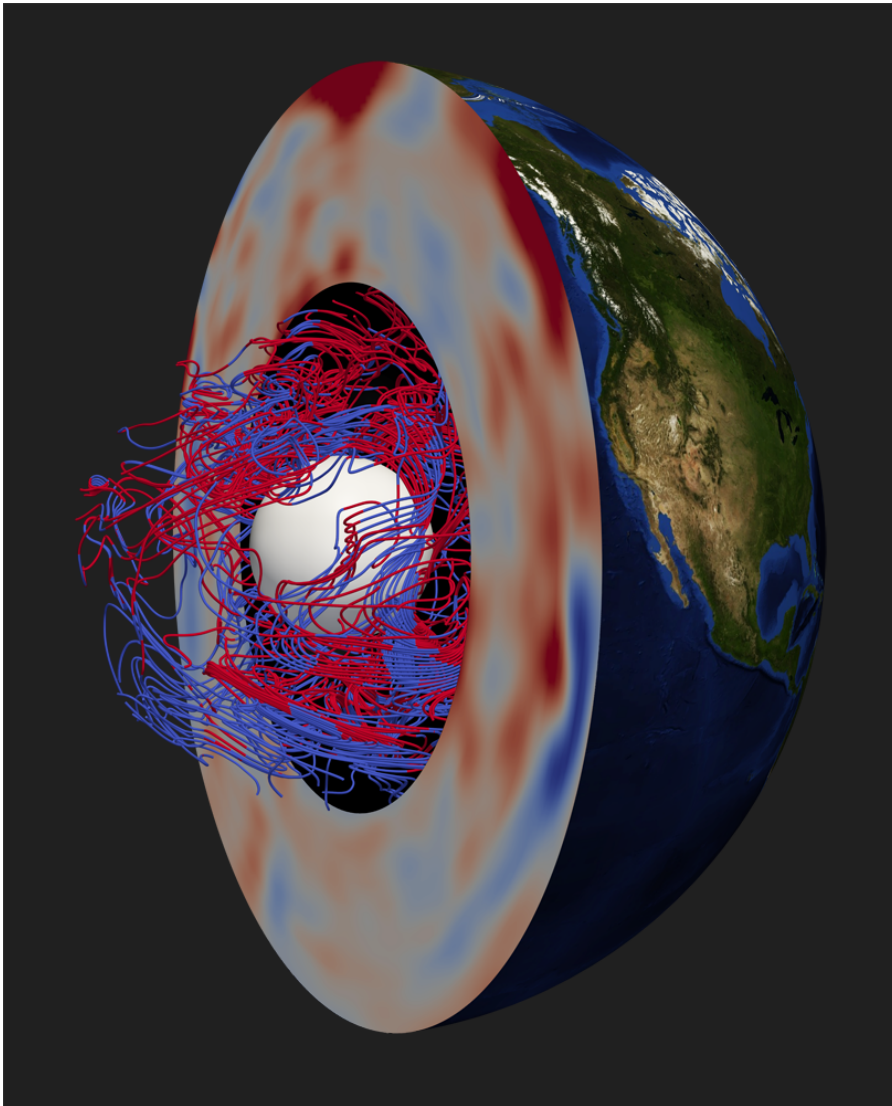


(1) electrically conducting fluid

(2) fluid must have complex motions

(3) motions must be vigorous enough

DYNAMO INGREDIENTS



(1) electrically conducting fluid

- liquid iron (terrestrial planets)
- metallic hydrogen (gas giants)
- ionized water (ice giants)
- hydrogen plasma (stars)

(2) fluid must have complex motions

- lots of twisting, helical flows
- rotation not required, but helpful in producing large-scale fields

(3) motions must be vigorous enough

- $\text{Velocity} * \text{Size} * \text{Conductivity}$ must be big enough
- The “magnetic Reynolds number condition” (see later...)

SOURCE OF COMPLEX MOTIONS

(1) Convection

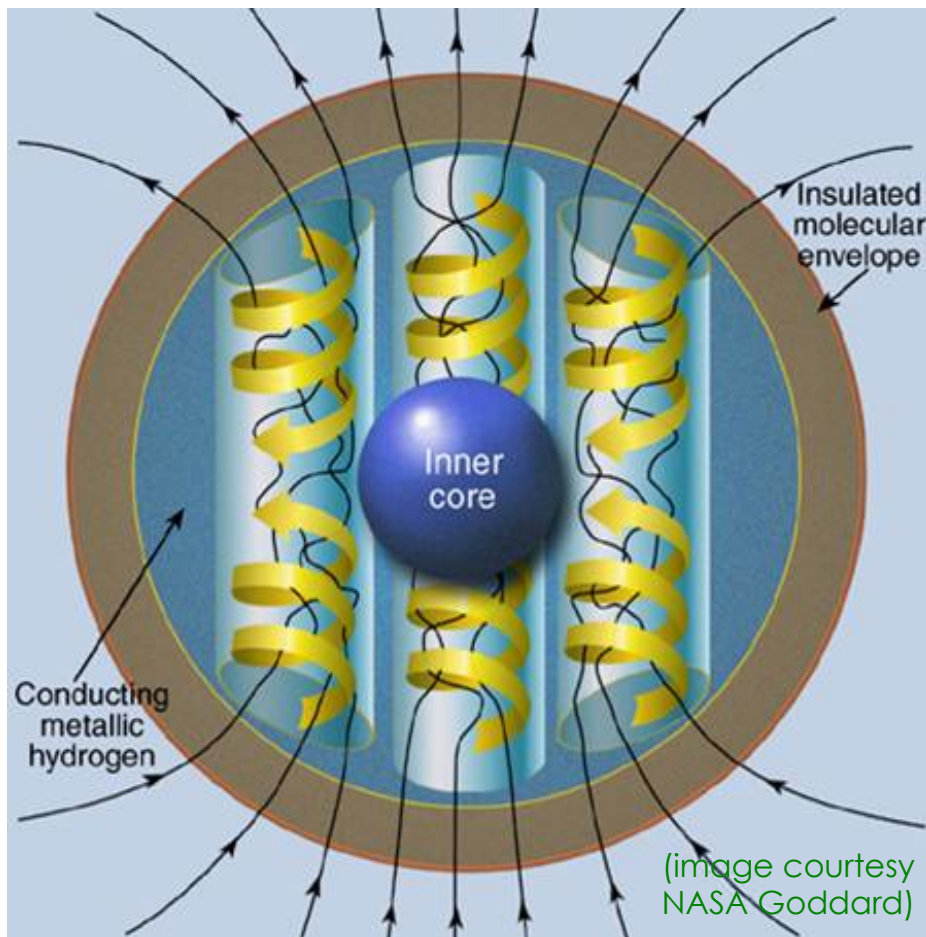
- deep interiors: hot
- surfaces: cold
- if temperature difference large enough: motions transport heat

(2) Shear

- causes magnetic fields to stretch
- good for magnetic field generation

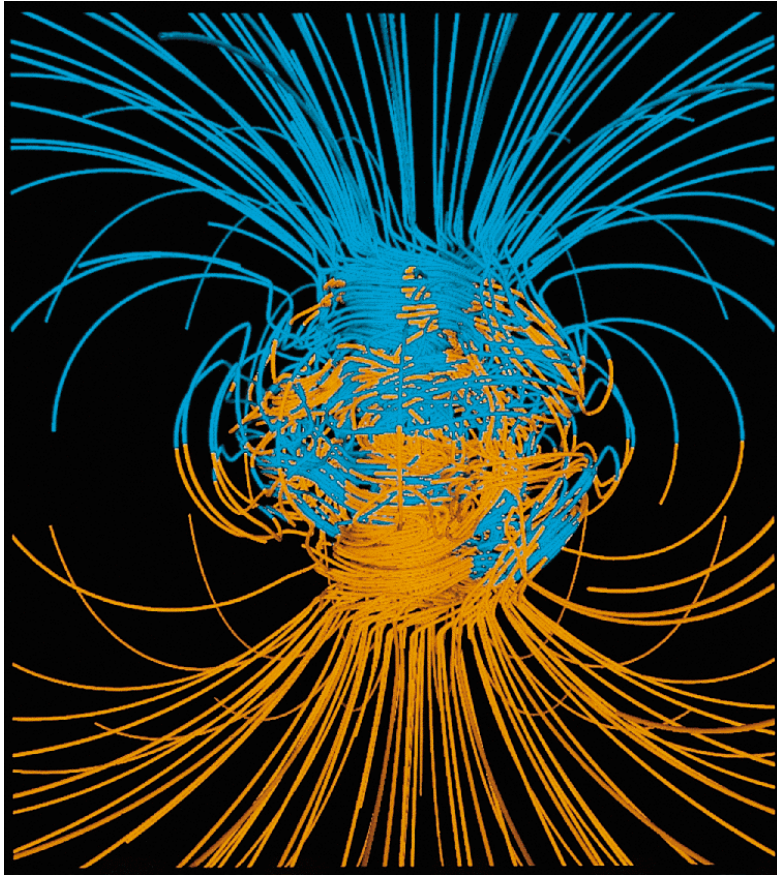
(3) Rotational Constraint

- rapid rotation organizes motions in larger scales



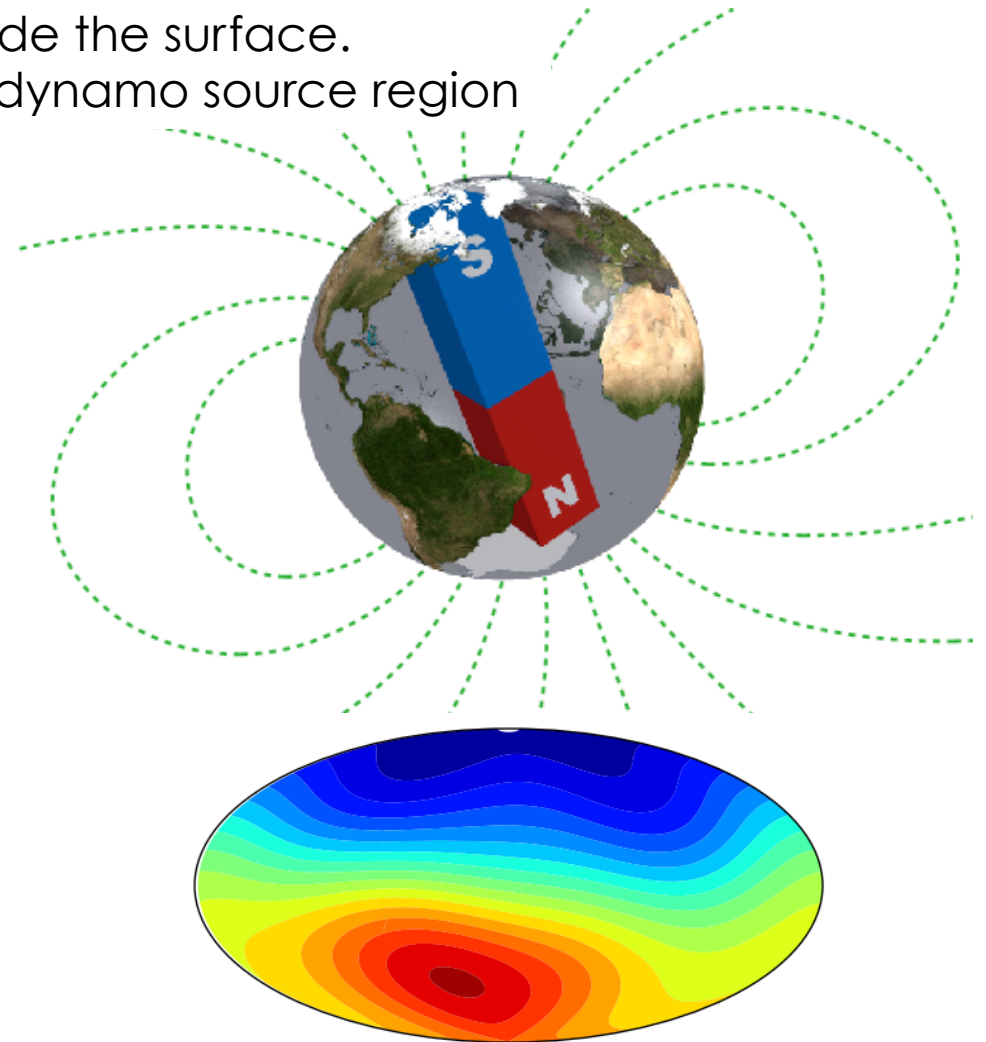
EARTH'S MAGNETIC FIELD

- We can only observe the field outside the surface.
- We try to infer what goes on in the dynamo source region



<http://www.es.ucsc.edu/~glatz/index.html>

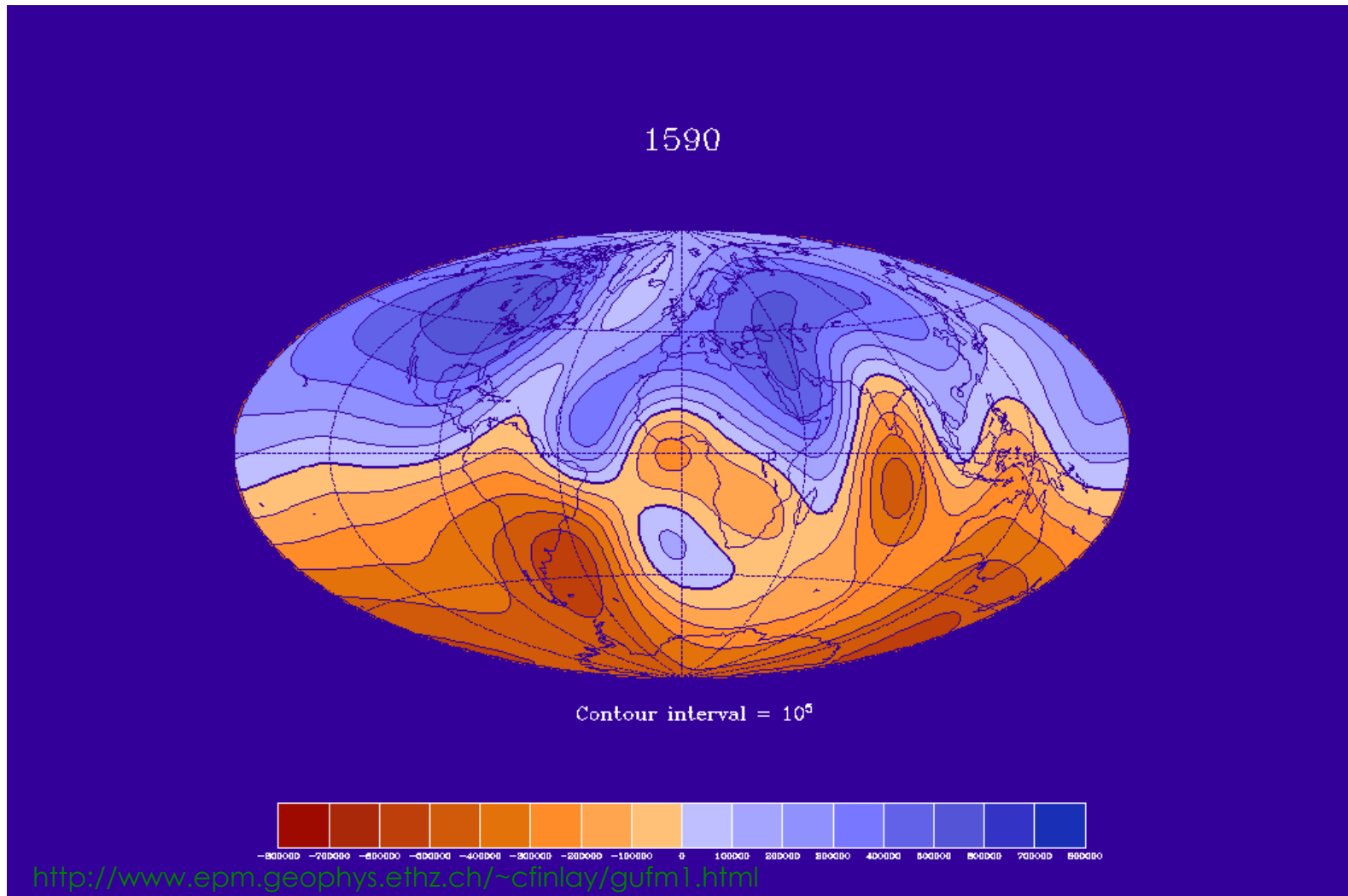
- field in the source region likely very complicated



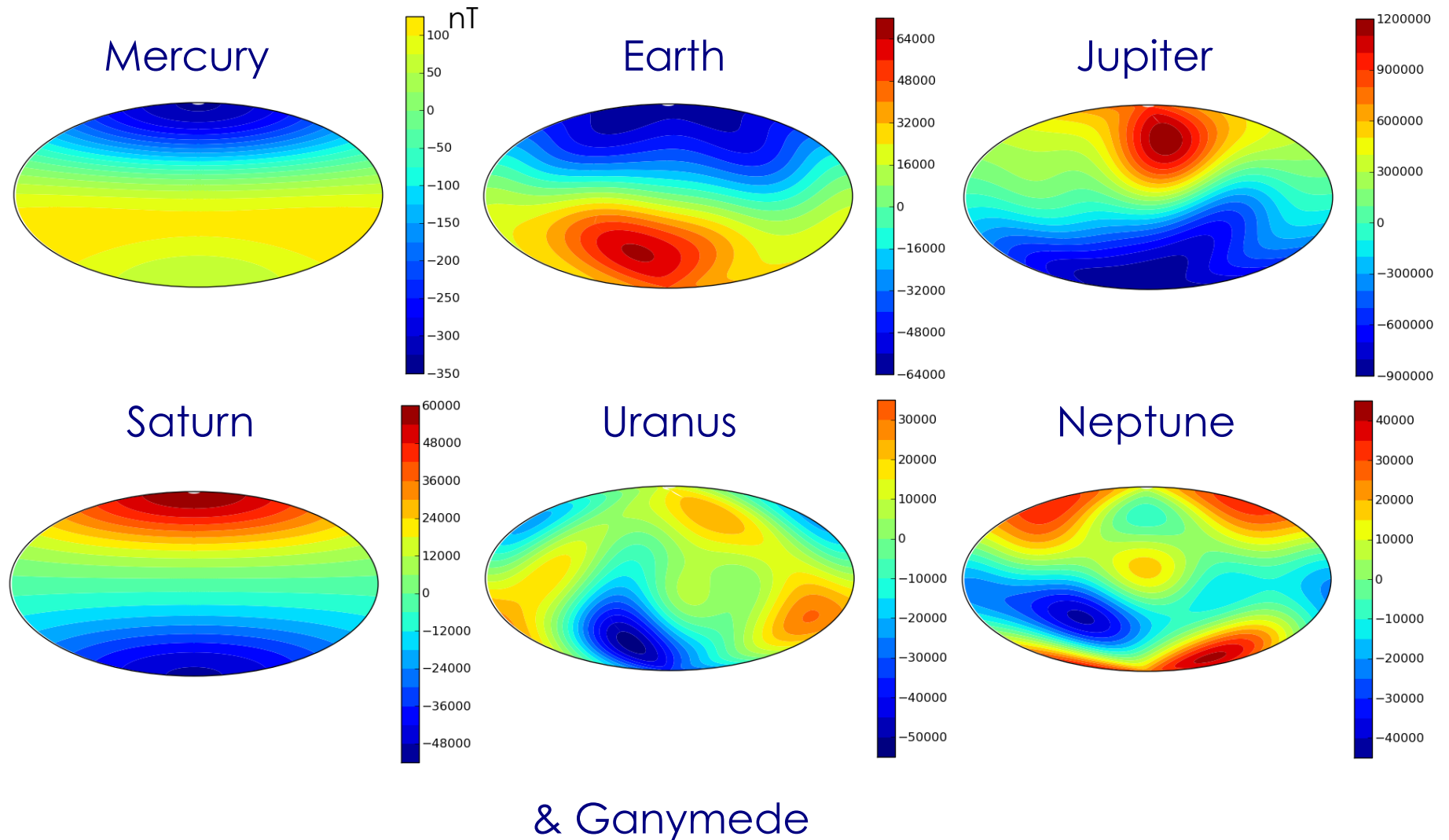
- observed field at the surface very dipolar

OBSERVATIONS

- Earth's field at the core-mantle boundary (CMB):
 - at least 3.5 billion years old (paleomagnetism)
 - polarity reverses chaotically
 - variable on all time scales



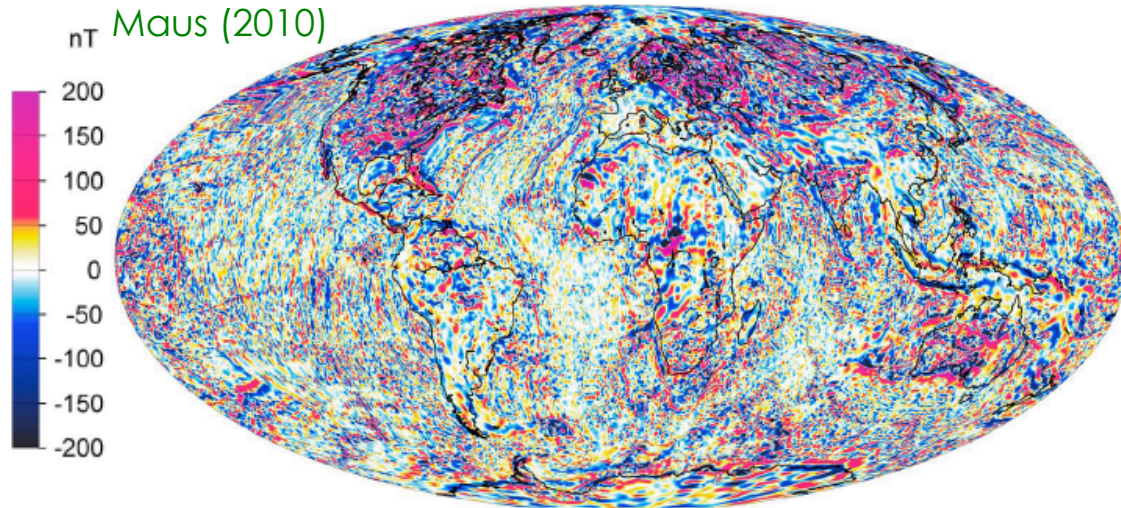
PLANETARY MAGNETIC FIELDS (ACTIVE)



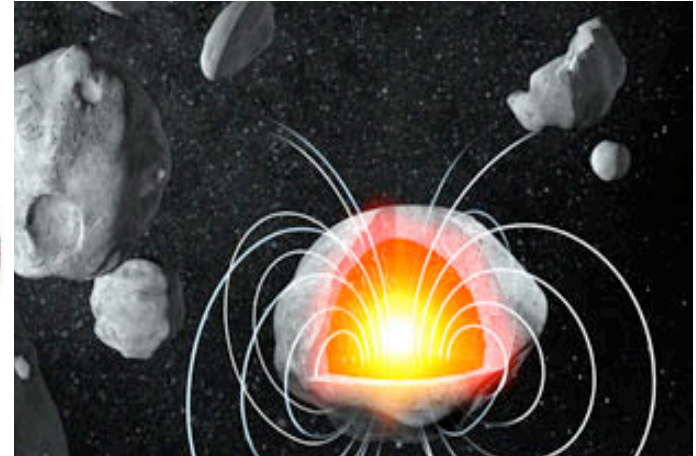
- there are similarities & differences which are linked to interior properties

PLANETARY MAGNETIC FIELDS (PAST)

Earth @ surface

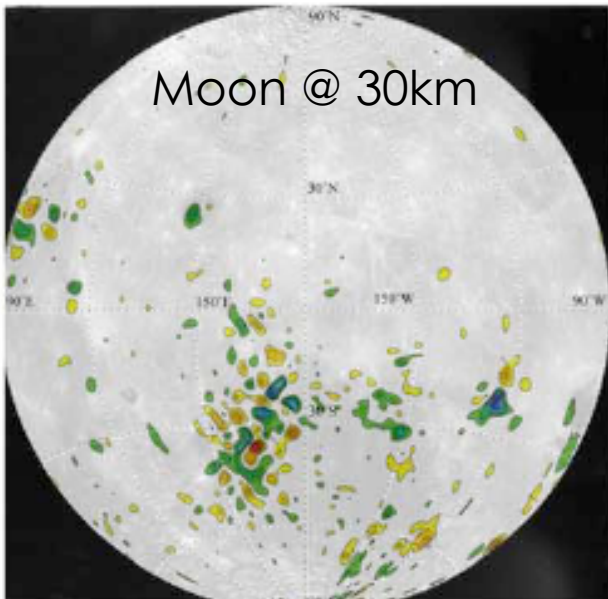


Planetesimals & Asteroids?



Weiss et al. (2008)

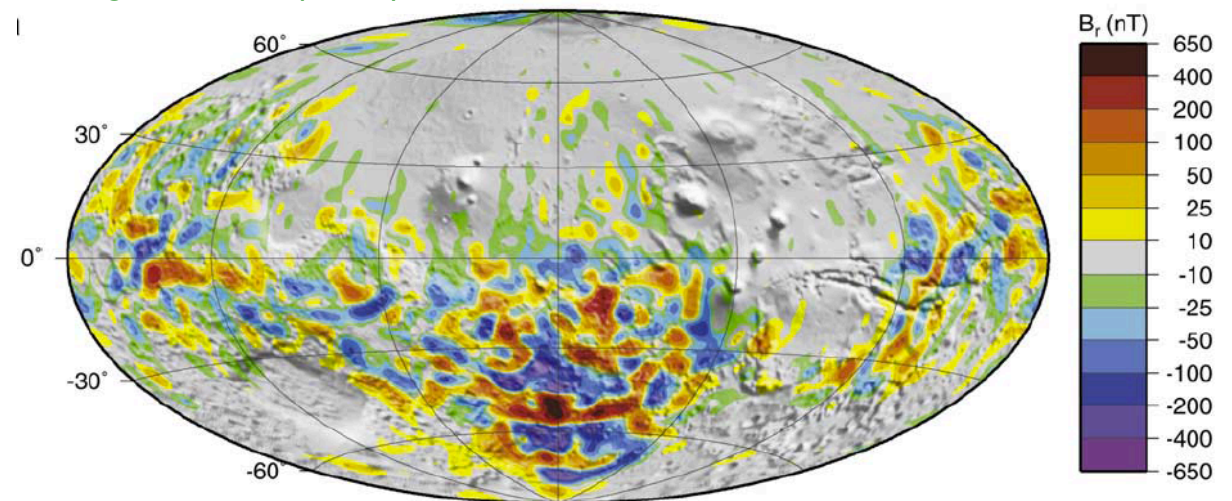
Moon @ 30km



Richmond & Hood (2008)

Mars @ 200km, from magnetometer

Langlais et al. (2004)



SOLAR MAGNETIC FIELD

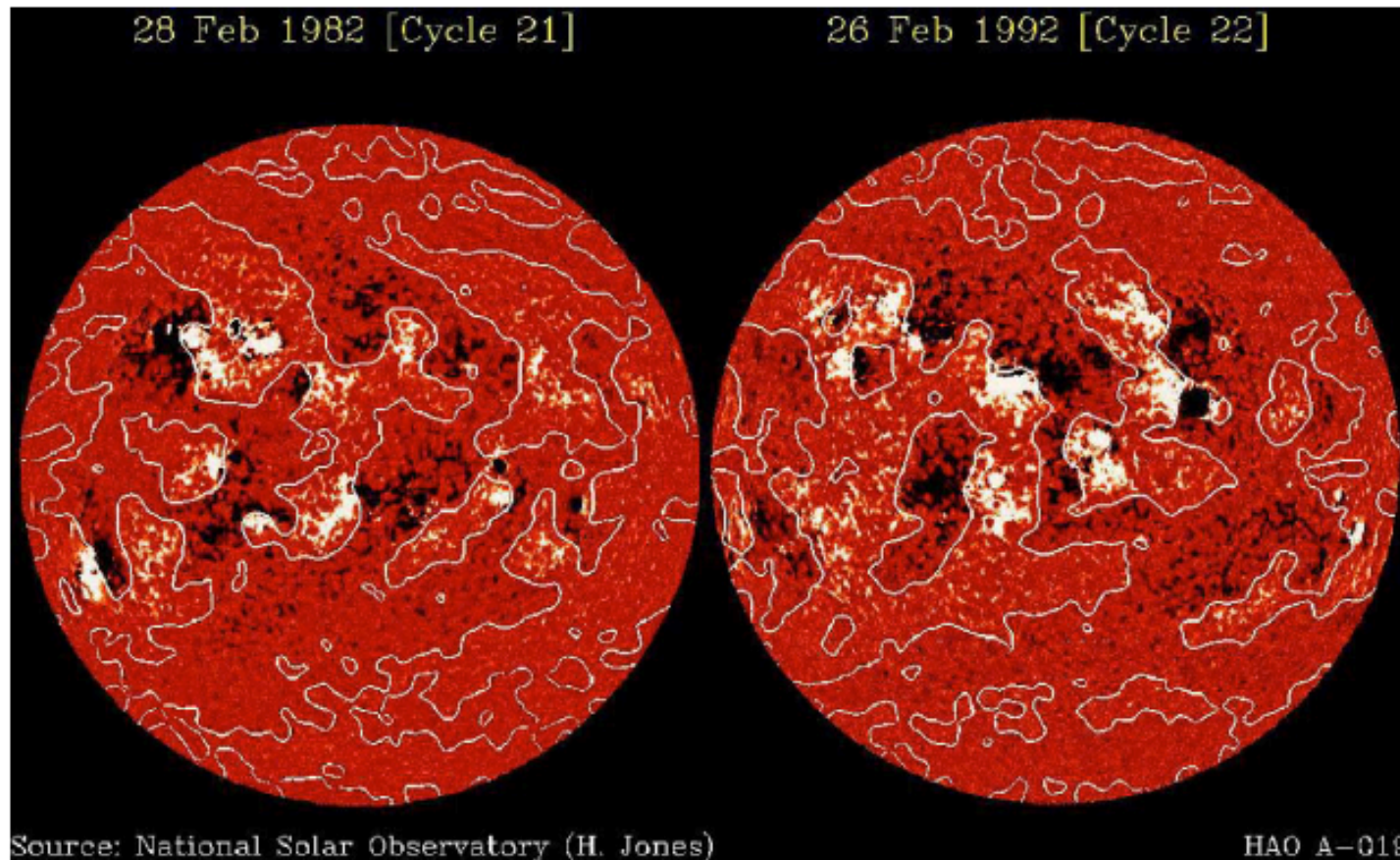
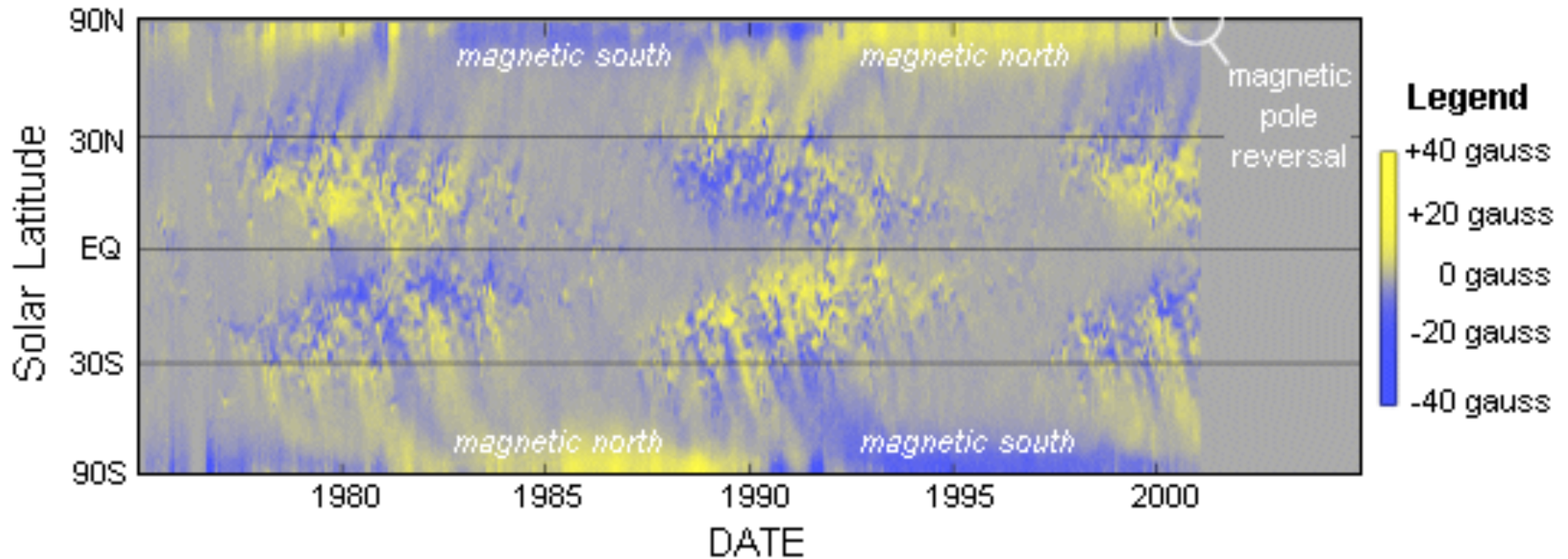


Fig. 6.1. Solar magnetogram showing bipolar regions, their opposite orientation north and south of the equator, and the clockwise tilt in the northern hemisphere and the counter-clockwise tilt in the southern hemisphere. Note that the field orientation has reversed orientation at the next cycle (here after 10 years). Courtesy of the National Solar Observatory.

- strong magnetic field associated with sunspots (~ 0.2 T or 2 kG)
- global field of $\sim 10^{-4}$ T or 1 G, approximately dipolar

SOLAR MAGNETIC FIELD

The Magnetic Butterfly Diagram
average magnetic fields at the Sun's surface

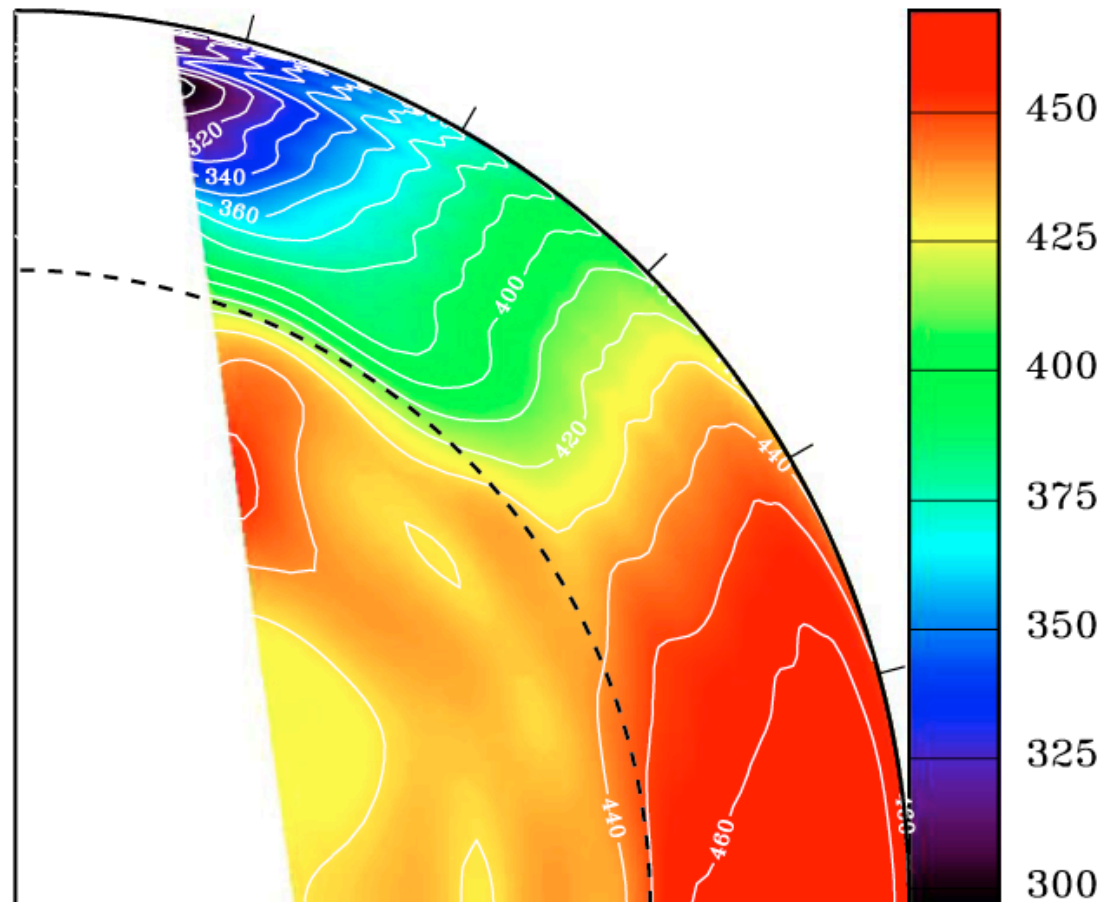


- field reverses polarity every ~ 11 years ("solar cycle")
- solar cycle: sunspots appear at mid-latitudes, then region migrates toward equator. Then new group appears at mid-latitudes with flip of polarity
- sunspots have been continuously observed since time of Galileo
- Maunder minimum: time period (1647-1715) when sunspots were absent

CONVECTION ZONE & TACHOCLINE

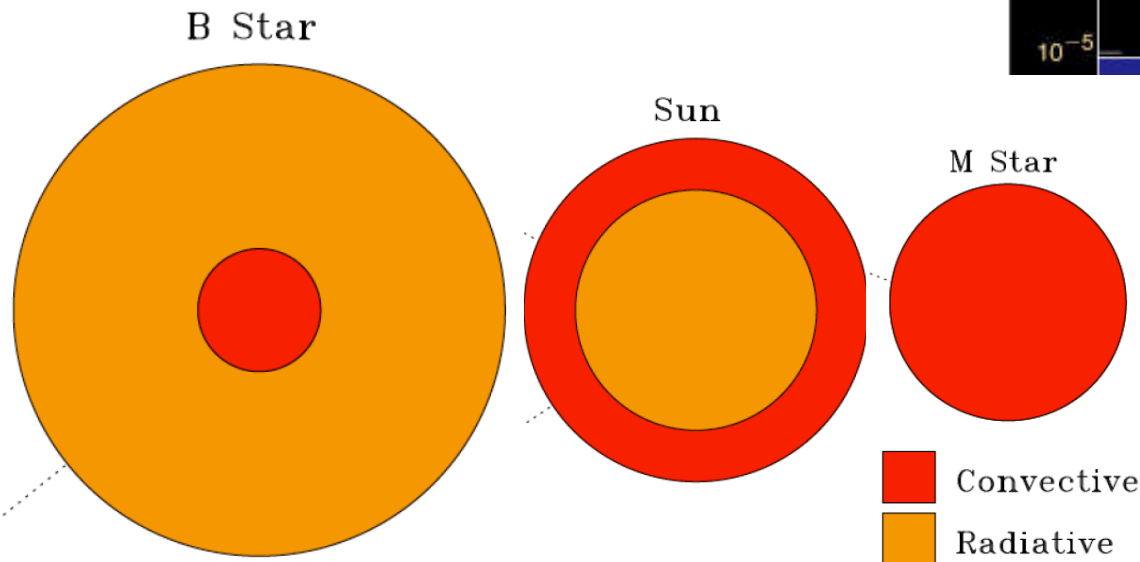
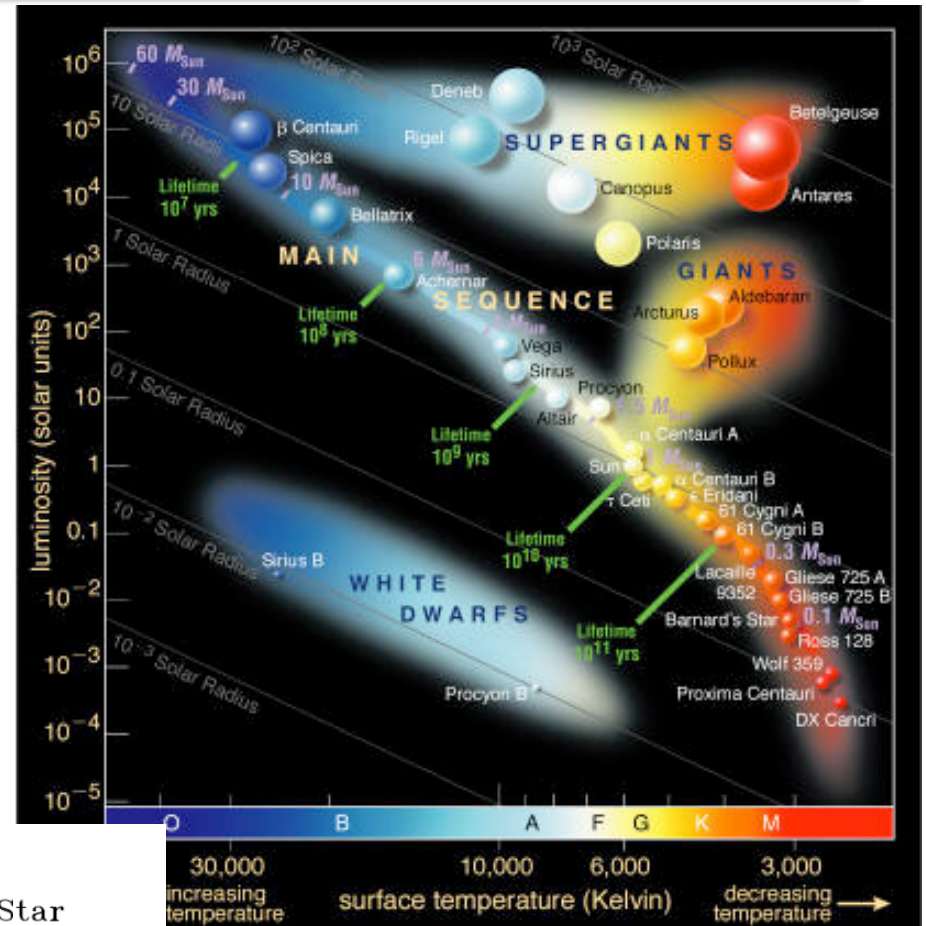
- helioseismology tells us rotation profile in convection zone
- strong shear layer at base of convection zone may be very important for storing magnetic fields, producing cycles & spots

Fig. 2 The internal rotation of the Sun, as determined from observations by the MDI instrument on board the SOHO satellite. The solar equator is along the horizontal axis, the pole along the vertical axis. Values of $\Omega/2\pi$ are shown, in nHz. *The dashed line indicates the base of the convection zone, and tick marks are at 15° intervals in latitude (from Thompson et al. 2003)*



STELLAR MAGNETIC FIELDS

- correlations between stellar types and magnetic field properties, probably due to geometry of convection zones
- stars with outer convection zones (late-type stars) have observed magnetic fields whose strength tends to increase with their angular velocity
- Cyclic variations are known to exist only for spectral types between G0 and K7).



EFFECT OF ROTATION

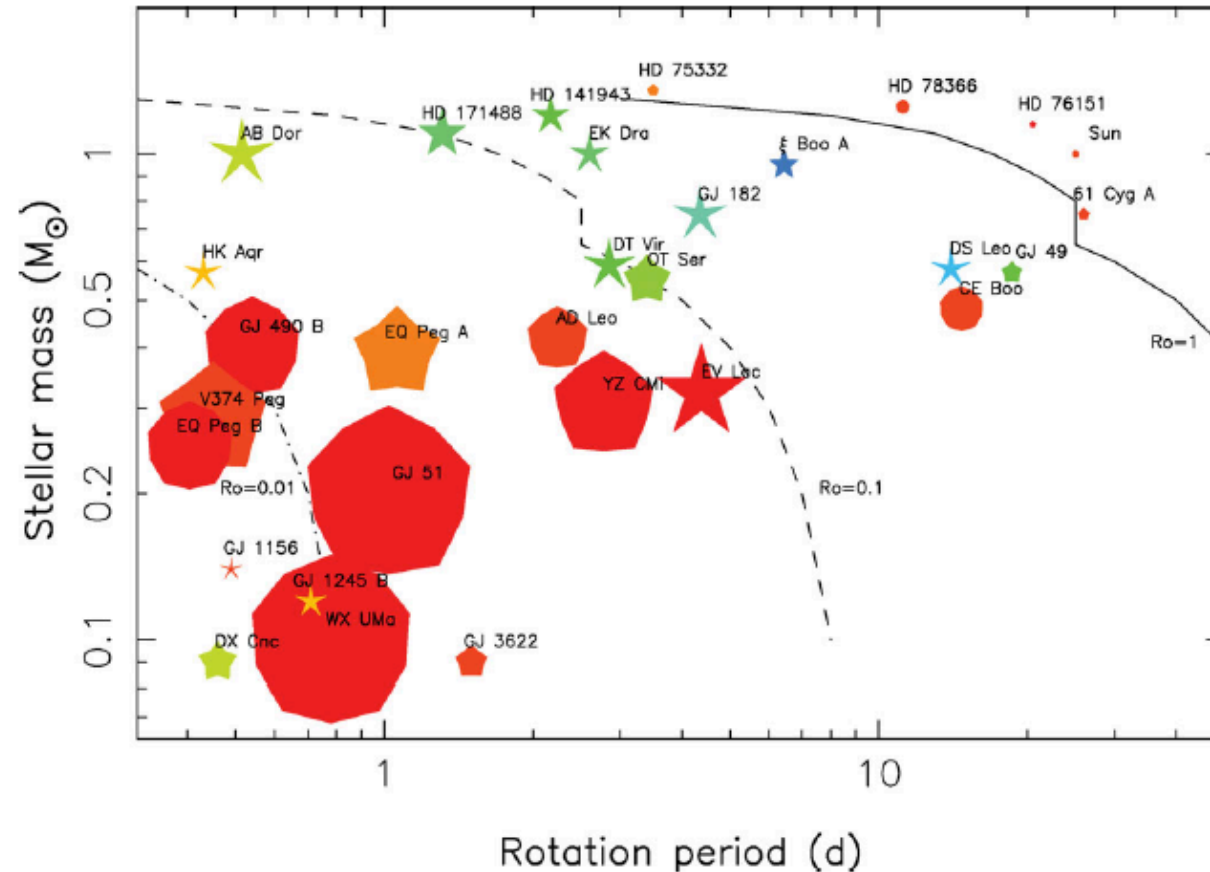
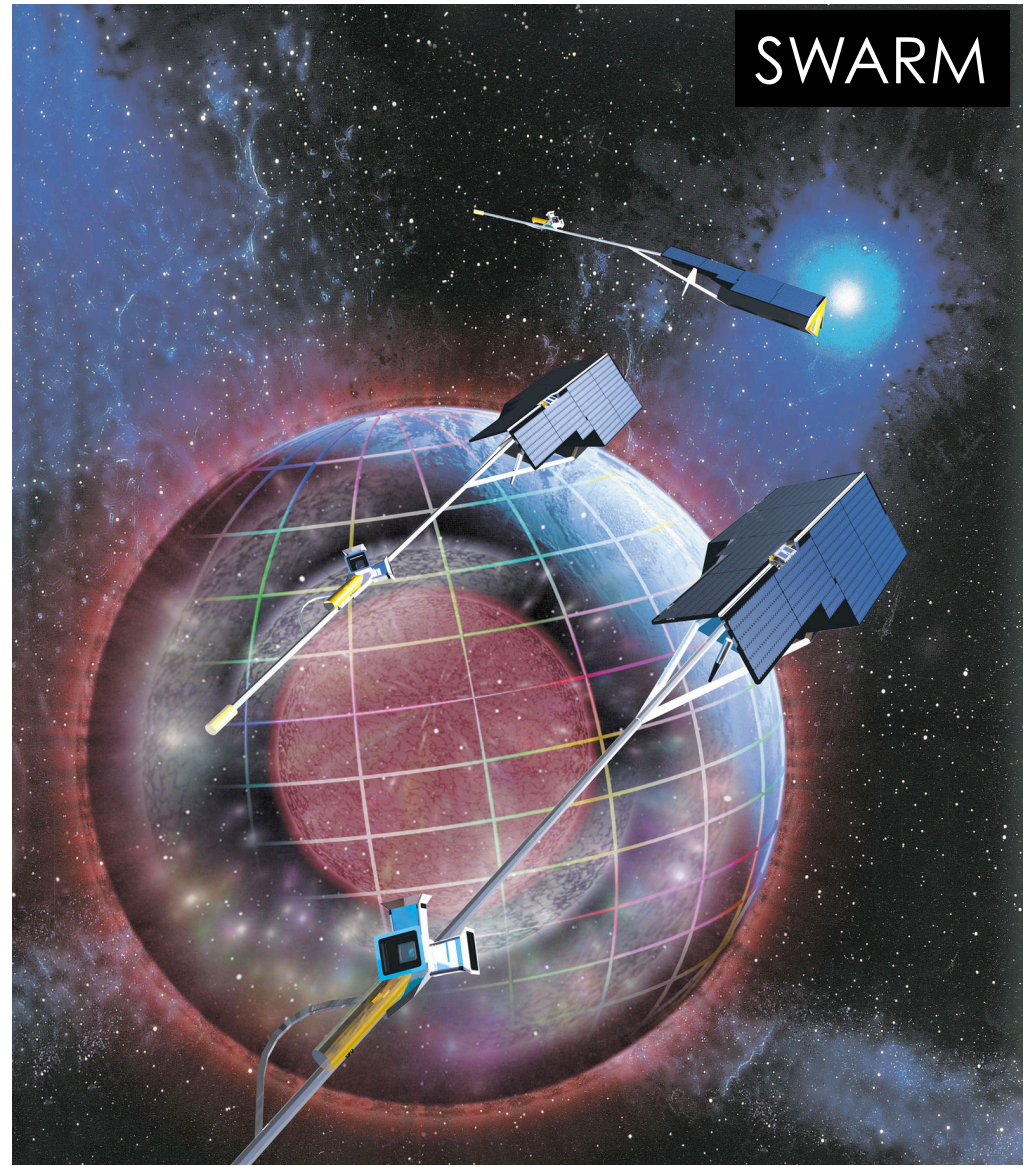
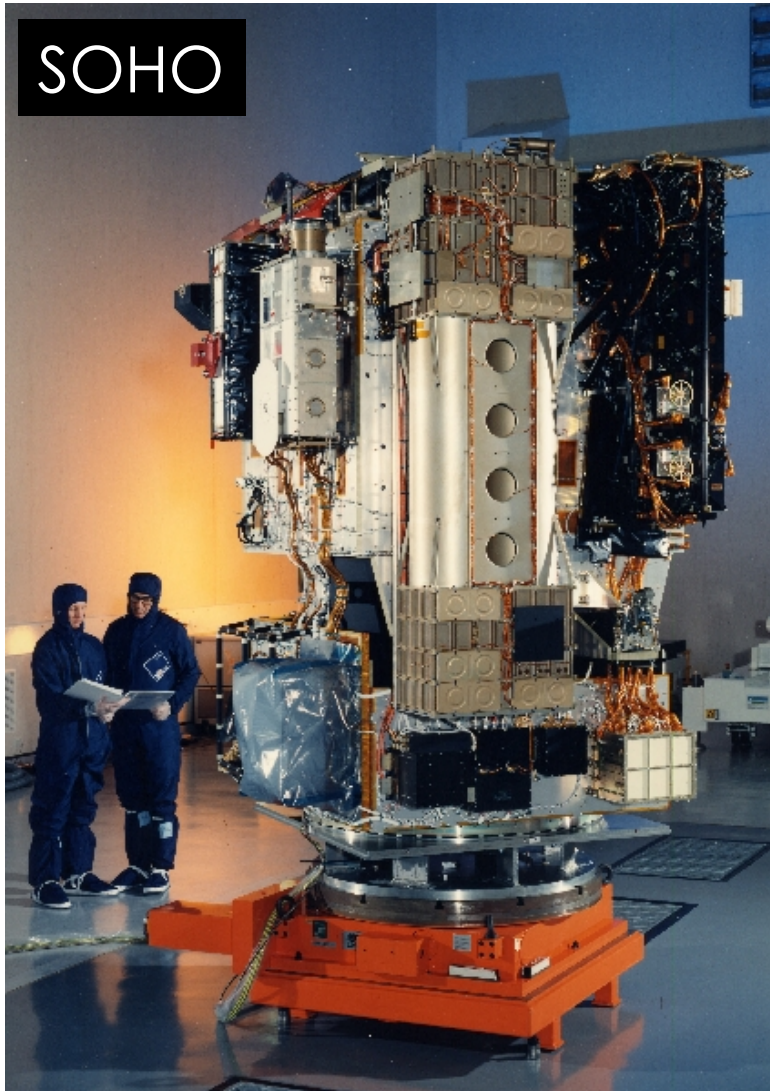


Figure 23: Properties of the large-scale magnetic geometries of cool stars (Donati, 2011) as a function of rotation period and stellar mass. Symbol size indicates magnetic densities with the smallest symbols corresponding to mean large-scale field strengths of 3 G and the largest symbols to 1.5 kG. Symbol shapes depict different degrees of axisymmetry of the reconstructed magnetic field (from decagons for purely axisymmetric fields to sharp stars for purely non-axisymmetric fields). Colors illustrate field configuration (dark blue for purely toroidal fields, dark red for purely poloidal fields, intermediate colors for intermediate configurations). Full, dashed, and dash-dot lines trace lines of equal Rossby number $Ro = 1$, 0.1, and 0.01, respectively (from Donati, 2011, reproduced by permission of Cambridge University Press).

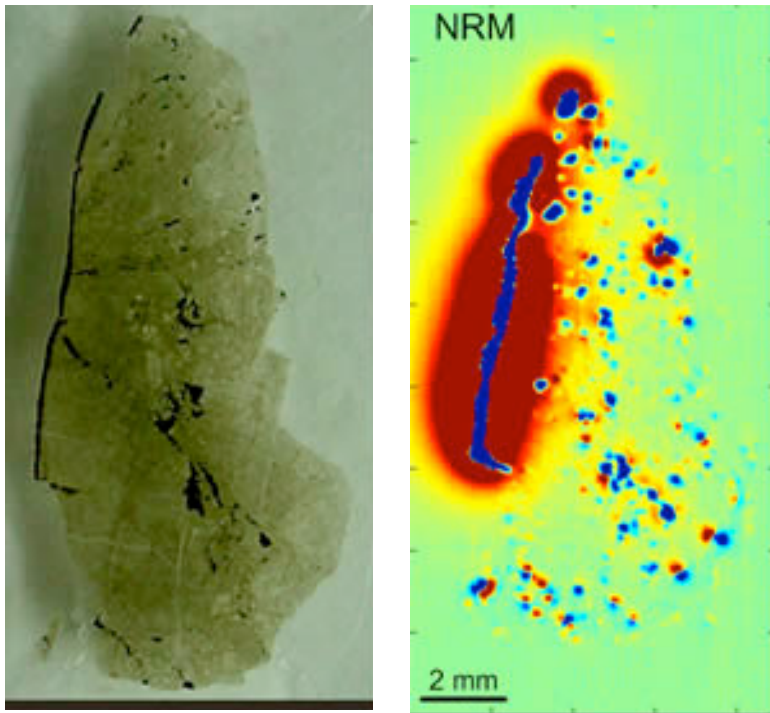
STUDYING MAGNETIC FIELDS

- observations from spacecraft and telescopes



STUDYING MAGNETIC FIELDS

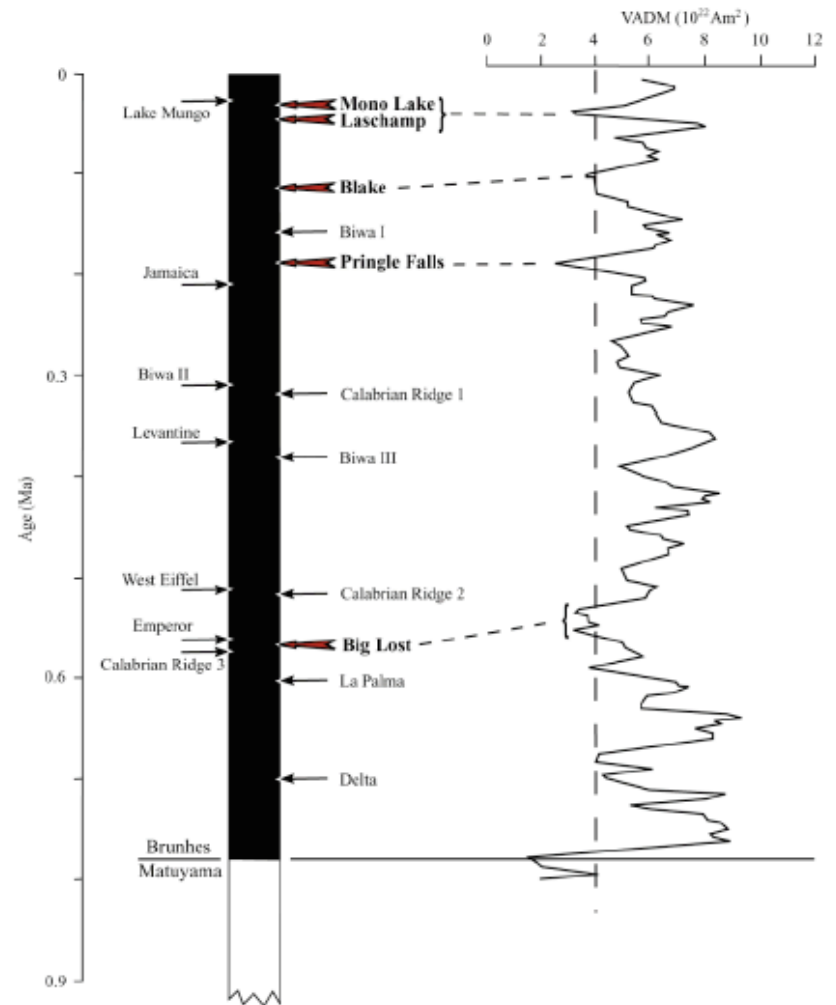
- paleomagnetism: investigating magnetic fields frozen into rocks



Weiss et al. (2008)

Left: sample of Martian meteorite ALH84001

Right: Squid microscope scan of magnetic field in sample

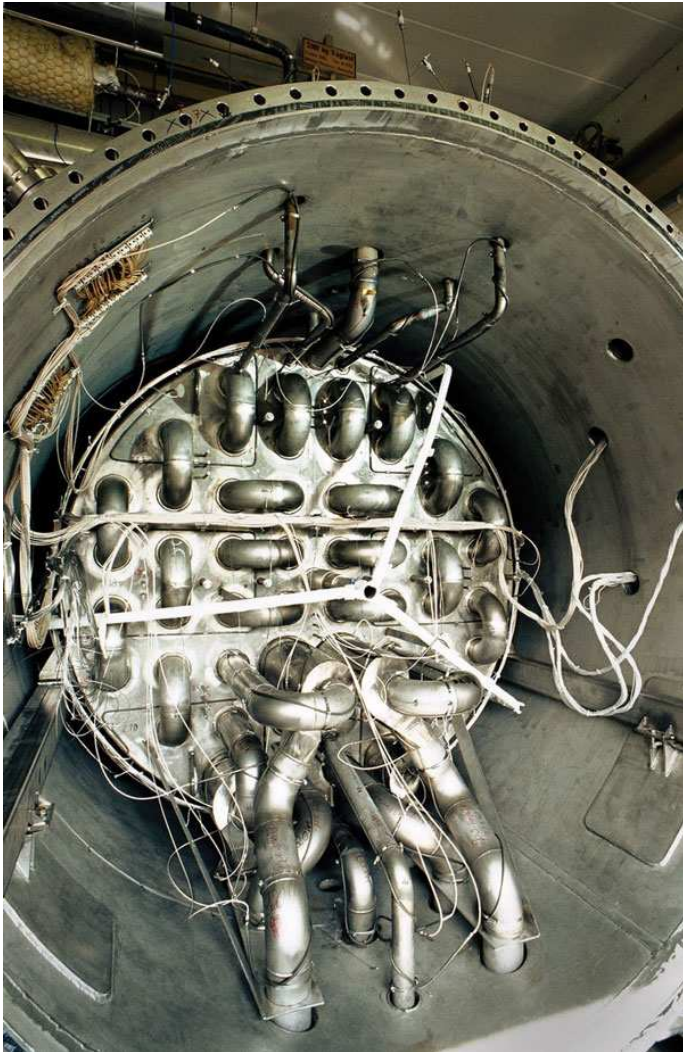


Earth's dipole moment vs. time

STUDYING PLANETARY MAGNETIC FIELDS

- experiments: build your own dynamo!

Karlsruhe dynamo

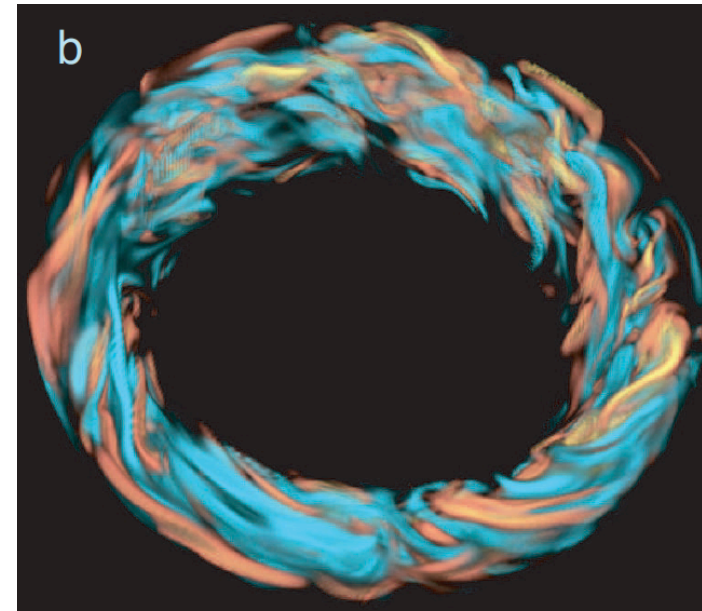
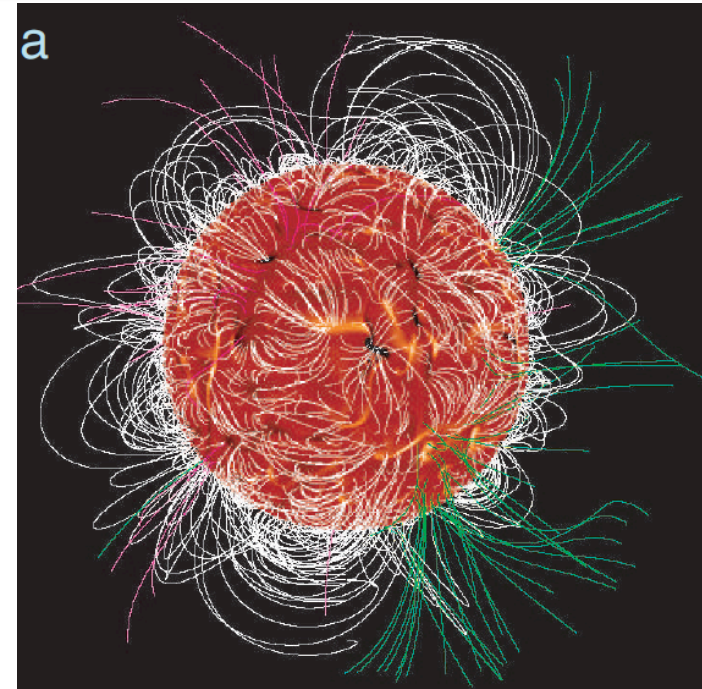
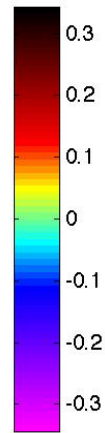
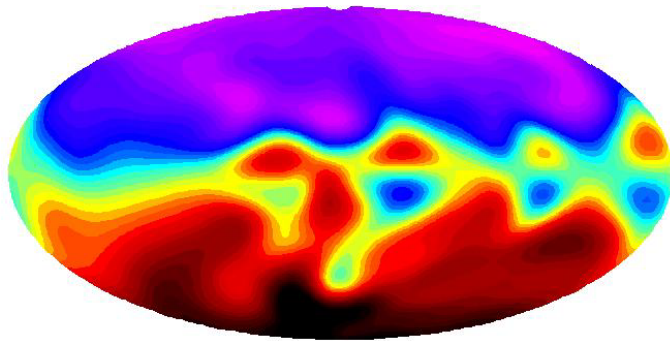


Maryland dynamo experiment



STUDYING PLANETARY MAGNETIC FIELDS

- computer simulations: have computers solve the governing equations



2. DYNAMO THEORY BASICS

- Start with the EM stuff:

Maxwell's equations

Law	integral form	differential form	description
Gauss' Law	$\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}$	$\nabla \cdot \vec{E} = \frac{\rho_e}{\epsilon_0}$	charged particles create an electric field
Gauss' Magnetism Law	$\oint \vec{B} \cdot d\vec{S} = 0$	$\nabla \cdot \vec{B} = 0$	there are no magnetic monopoles
Faraday's Law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	a changing magnetic field creates an electric field
Ampere-Maxwell Law	$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{th} + \epsilon_0 \mu_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{S}$	$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$	currents and changing electric fields create magnetic fields

Lorentz force: $\vec{F}_L = \rho_e \vec{E} + \vec{J} \times \vec{B}$

Ohm's Law: $\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B})$
(simplified)

- combined electric and magnetic force/volume
- how charges respond (i.e. move therefore producing a current) to EM forces

MHD APPROXIMATION

- Magneto-Hydro-Dynamic (MHD) approximation is an approximation made to Maxwell's equations when velocities \ll speed of light.
- The approximations are determined by considering which terms are so small that they can be neglected.
- In problem set, you will show the following results:
 - displacement current neglected in Ampere's law
 - simplifications in Lorentz transformations between reference frames
 - Electric force neglected in Lorentz force
- Summary of the MHD approximation equations:

$$\begin{aligned} \nabla \cdot \vec{B} &= 0 & \nabla \times \vec{B} &= \mu_0 \vec{J} & \nabla \cdot \vec{E} &= \frac{\rho_e}{\epsilon_0} & \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{E}' &= \vec{E} + \vec{u} \times \vec{B} & \vec{B}' &= \vec{B} & \vec{J}' &= \vec{J} & \vec{J}' &= \sigma \vec{E}' = \sigma(\vec{E} + \vec{u} \times \vec{B}) \\ & & & & \vec{F}_L &= \vec{J} \times \vec{B} & & \end{aligned}$$

MAGNETIC INDUCTION EQUATION

- We don't normally work directly with all the EM equations. Instead, we use the Lorentz force in the momentum equation plus we derive an equation from the EM equations called the **Magnetic Induction Equation (MIE)**.

- Here is how:

- Start with Ohm's law:

$$\vec{J} = \sigma [\vec{E} + \vec{u} \times \vec{B}]$$

- Take its curl: (assuming constant conductivity)

$$\nabla \times \vec{J} = \sigma \nabla \times \vec{E} + \sigma \nabla \times (\vec{u} \times \vec{B})$$

- Take the curl of Ampere's law:

$$\nabla \times \nabla \times \vec{B} = \mu_0 \nabla \times \vec{J}$$

- Plug in the expression from the curl of Ohm's law:

$$\nabla \times \nabla \times \vec{B} = \mu_0 \sigma \nabla \times \vec{E} + \mu_0 \sigma \nabla \times (\vec{u} \times \vec{B})$$

continued...

MAGNETIC INDUCTION EQUATION CONT.

- Substitute using Faraday's law:

$$\nabla \times \nabla \times \vec{B} = -\mu_0\sigma \frac{\partial \vec{B}}{\partial t} + \mu_0\sigma \nabla \times (\vec{u} \times \vec{B})$$

- Use the following standard vector identity along with Gauss' law for magnetism:

$$\nabla \times \nabla \times \vec{B} = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} = -\nabla^2 \vec{B}$$

- Combining the last 2 equations gives the magnetic induction equation:

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \lambda \nabla^2 \vec{B}$$

where $\lambda = \frac{1}{\mu_0\sigma}$ is called the 'magnetic diffusivity'.

Similar to other diffusivities, it has units of $m^2 s^{-1}$

- Notice that the equation has the form of a source/diffusion equation. The growth or decay of B (LHS) is governed by its creation through induction processes (1st term on RHS) and diffusion processes (2nd term on RHS)

WORKING WITH THE MIE

- Is a dynamo mechanism necessary for planets and stars?
- e.g. could Earth's field today just be a remnant field created during formation?
- If the field is remnant, there is no regeneration process $\rightarrow u = 0$. Plug into the MIE:

$$\frac{\partial \vec{B}}{\partial t} = \lambda \nabla^2 \vec{B}$$

- How long does it take for a given field to decay? (i.e. the "magnetic diffusion time")

$$\frac{B}{\tau_B} = \lambda \frac{B}{L^2} \Rightarrow \tau_B = L^2 / \lambda$$

- What do we use for L? For planetary cores, L is sometimes taken to be the radius of the core "R". Other times (the more appropriate choice), L is taken to be the length scale of the slowest decaying eigenmode which is: $L = R/\pi$.
- Using: $R = 3 \times 10^6$, $\sigma = 3 \times 10^5$ mho/m gives $\lambda = 2m^2/s$.
 $\rightarrow \tau_B = (3 \times 10^6 / \pi)^2 / 2 = 5 \times 10^{12} s \approx 15000$ yrs.
- if the Earth's field was remnant, its e-folding time would be 15000 yrs, but paleomag tells us B ~ constant for past 3.5 billion years \rightarrow not a remnant field
- (for stars: 10^9 years, so its not so easy to dismiss a remnant field hypothesis)

INTERPRETING THE MIE

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \lambda \nabla^2 \vec{B}$$

- Using standard vector identity, can rearrange as:

$$\frac{\partial \vec{B}}{\partial t} = (\vec{B} \cdot \nabla) \vec{u} - (\vec{u} \cdot \nabla) \vec{B} - \vec{B}(\nabla \cdot \vec{u}) + \vec{u}(\nabla \cdot \vec{B}) + \lambda \nabla^2 \vec{B}$$

- Using Gauss' magnetism law and a bit of rearranging then gives:

$$\left[\frac{\partial}{\partial t} + \vec{u} \cdot \nabla \right] \vec{B} = (\vec{B} \cdot \nabla) \vec{u} - \vec{B}(\nabla \cdot \vec{u}) + \lambda \nabla^2 \vec{B}$$

- This has a nice physical interpretation:
 - LHS: Rate of change of field moving with the fluid parcel (i.e the Lagrangian derivative)
 - RHS:
 - 1st term: stretching of field lines due to gradients in velocity
 - 2nd term: change in field due to compression/dilatation of fluid parcels
 - 3rd term: diffusion

MIE: THE MAGNETIC REYNOLDS NUMBER

- In order to generate a dynamo the driving force must be larger than the dissipative force

$$\frac{\partial \vec{B}}{\partial t} = \underbrace{\nabla \times (\vec{u} \times \vec{B})}_{\text{driving force}} + \underbrace{\lambda \nabla^2 \vec{B}}_{\text{dissipative force}}$$

- The ratio of these two terms is called the magnetic Reynolds number:

$$Re_m = \frac{\nabla \times (\vec{u} \times \vec{B})}{\lambda \nabla^2 \vec{B}} \approx \frac{UB/L}{\lambda B/L^2} \approx \frac{UL}{\lambda} \quad \lambda = \frac{1}{\mu_0 \sigma}$$

- Notice that it increases with increasing U and L and as the conductivity increases.
- Re_m must be larger than a critical value (~ 10) for dynamo action to occur

object	λ (m/s)	L(m)	u (m/s)	Re_m
Earth's core	2	3×10^6	3×10^{-4}	450
Star	0.5	10^9	1	2×10^9
Cu sphere	0.15	1	1	6

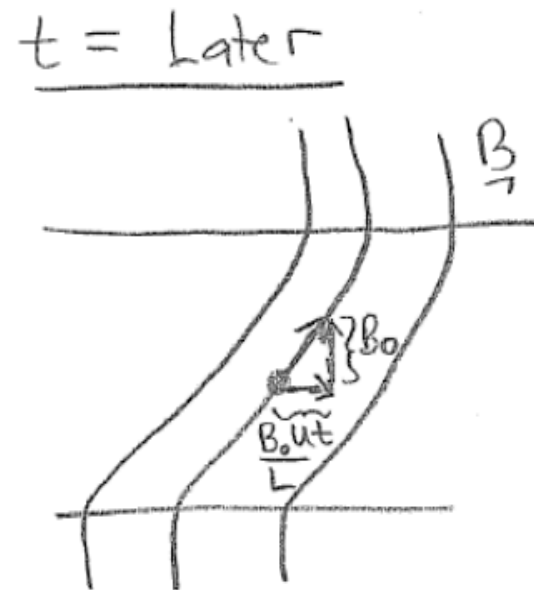
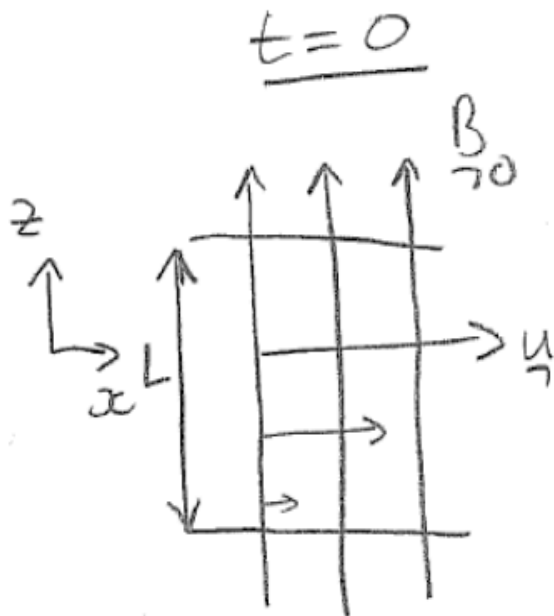
- Its very hard to generate laboratory dynamos b/c length scales so small. Its easy in very large bodies.

MIE: ALFVEN'S THEOREM

- planets and stars have large magnetic Reynolds number: $Re_m = \frac{UL}{\lambda}$
- In order to understand properties of MHD in high Re_m flows, look at extreme case: The limit as $Re_m \rightarrow \infty$
- This limit can be interpreted as the perfectly conducting limit since as:
$$\sigma \rightarrow \infty, Re_m \rightarrow \infty$$
- In this case, the magnetic diffusivity $\lambda \rightarrow 0$ so we can ignore the diffusion term in the MIE
- Alfven's theorem results in the following 3 facts about magnetic fields in this limit:
 1. magnetic flux through a surface moving with the fluid is constant
 2. magnetic flux tubes move with the fluid
 3. if 2 material particles are on the same field line at time 0, then they remain on that field line for all time $t > 0$.
- For a fluid that is not perfectly conducting, the effects of diffusion permit the field to slip through the fluid.
- Alfven's theorem can also be used for dynamo processes whose time scales are much shorter than the magnetic diffusion time at that length scale

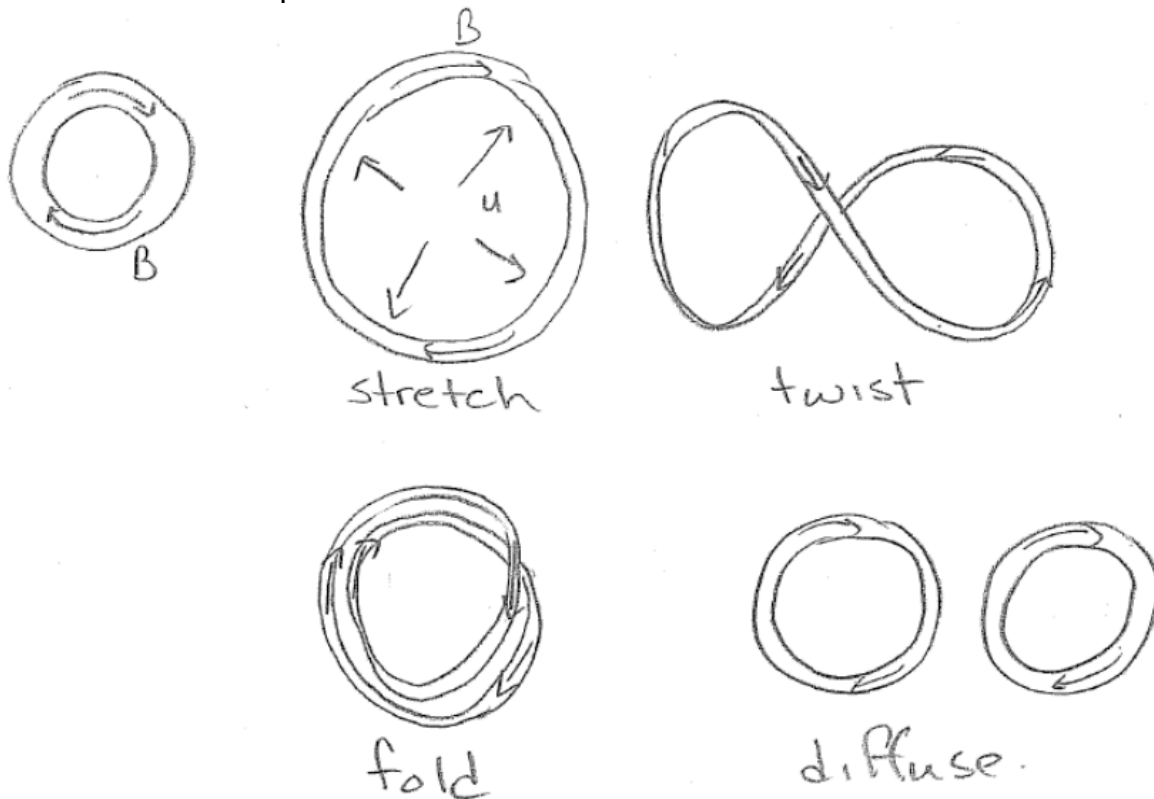
MIE: ALFVEN'S THEOREM EXAMPLE

- Consider a perfect conductor and a constant vertical magnetic field: $\vec{B} = B_0 \hat{z}$ and a simple horizontal shear flow: $\vec{u} = U z \hat{x}$.
- Can solve for B from the MIE: (prove on problem set): $\vec{B} = \frac{U t B_0}{L} \hat{x} + B_0 \hat{z}$
- Notice in this example, that the action of the flow on the original field generates new field (in the x direction). BUT, we don't regenerate the original field. This is not a "self sustaining dynamo" (i.e. after we've started, if we remove the original field, the total field decays).



MIE: SELF SUSTAINING DYNAMO EXAMPLE

- Consider a good, but not perfect conductor, so Re_m is large but finite.
- Diffusion is most effective where the gradients of B are large.
- In the following example, large gradients only occur in small regions, elsewhere, Alfven's theorem holds well.
- This example is known as the "Vainshtein-Zel'dovich rope dynamo":



- Left with twice as much field
- Could remove initial 'seed' field and the flow would continue to regenerate it →

Self-sustained dynamo

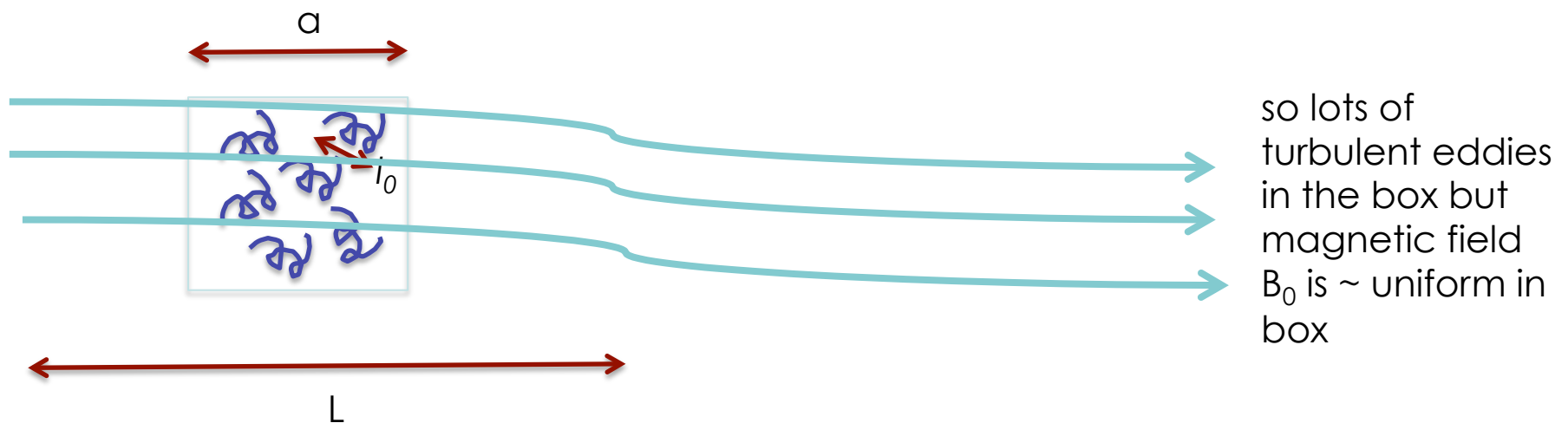
FLUID FLOWS IN DYNAMOS

- force balances determine the fluid motions in dynamo regions
- notice that in the previous heuristic examples, the flows were all fairly simple
- in stars & planets, the flow is very turbulent (i.e. lots of length and velocity scales, large amount of disorganization)
- Usually we are interested in how these flows generate the **large scale** magnetic fields that we see.
- 2 possible ways to proceed:
 1. attempt to parameterize the effects of the small scale turbulence on the large scales ("**mean field models**")
 2. ignore the small scales, only investigate the effects of the larger scale flows on the dynamo ("**macroscopic models**")
- Both of these methods are used and provide us with useful information.

3. PLANETARY & STELLAR DYNAMO MODELS

- **MEAN FIELD MODELS:** Work described here mostly done by Steenbeck, Krause & Radler (1966, etc) and Parker (1955).
- Setup: Assume turbulent velocity field of characteristic length scale l_0 which is much smaller than the length scale L associated with the mean magnetic field B_0
- Only consider statistical properties of the turbulent velocity and magnetic fields (e.g. the mean of the flow or field)
- Define “mean” of flow or field as average of quantity over a box of size a where $l_0 \ll a \ll L$:

$$\langle f \rangle = \frac{3}{4\pi a^3} \int_a f dV$$



MEAN FIELD MODELS

- Write the magnetic field and velocity as the sum of the mean and perturbations about the mean:

$$\vec{B}_0(\vec{r}, t) = \langle \vec{B}(\vec{r}, t) \rangle$$

$$\vec{B}(\vec{r}, t) = \vec{B}_0(\vec{r}, t) + \vec{b}(\vec{r}, t)$$

$$\langle \vec{b}(\vec{r}, t) \rangle = 0$$

(For now assume mean velocity is 0
and flow is incompressible)

$$\langle \vec{u} \rangle = 0$$

$$\nabla \cdot \vec{u} = 0$$

- Plug these forms of B and u into the magnetic induction equation:

$$\frac{\partial(\vec{B}_0 + \vec{b})}{\partial t} = \nabla \times [\vec{u} \times (\vec{B}_0 + \vec{b})] + \lambda \nabla^2 (\vec{B}_0 + \vec{b})$$

- Take the average of this equation over the box of length a to get the mean magnetic induction equation... (will do on problem set). Result...

MEAN FIELD MODELS

$$\frac{\partial \vec{B}_0}{\partial t} = \nabla \times (\langle \vec{u} \times \vec{b} \rangle) + \lambda \nabla^2 \vec{B}_0$$

evolution of mean B

generated by induction due to
perturbation fields u and b

- the induction term is usually referred to as an **emf**: $\varepsilon = \langle \vec{u} \times \vec{b} \rangle$
- now for the parameterization: on the problem set you will show that with certain assumptions, the emf can be written in terms of the mean field B_0 :

$$\varepsilon_i = \alpha_{ij} B_{0j} + \beta_{ijk} \frac{\partial B_{0j}}{\partial x_k} + \gamma_{ijkl} \frac{\partial^2 B_{0j}}{\partial x_k \partial x_l} + \dots$$

- all of the info about the small scale stuff is parameterized by the choice of tensor coefficients.
- various theories, wanted behavior can be implemented in the coefficients.

MACROSCOPIC MODELS

- numerically solve the equations for the fluid motions, BUT, work with modest parameter values such that the fluid motions are not very turbulent and hence can be resolved directly
- this is probably a “better” approach for planets than for stars because of the physical properties of the dynamo regions, i.e. planets have Re_m that can be reached in models
- to solve for the velocity and magnetic fields self-consistently, need equations for:
 - conservation of momentum
 - conservation of mass
 - conservation of energy
 - MIE

MACROSCOPIC MODELS

- Magnetic induction equation:

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \lambda \nabla^2 \vec{B}$$

- Conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

- Conservation of momentum:

$$\left(\frac{\partial}{\partial t} + \vec{u} \cdot \nabla \right) \vec{u} + 2\vec{\Omega} \times \vec{u} = -\nabla P + \frac{\delta \rho}{\rho} \vec{g} + \nu \nabla^2 \vec{u} + \frac{1}{3} \nu \nabla (\nabla \cdot \vec{u}) + \frac{1}{\rho \mu_0} (\nabla \times \vec{B}) \times \vec{B}$$

\vec{u} : velocity

$\vec{\Omega}$: rotation vector

P : modified pressure

\vec{g} : gravity

ν : coefficient of viscosity

\vec{B} : magnetic field

σ : electrical conductivity

μ_0 : magnetic permeability

ρ : density

- Driving force (buoyancy due to gravity) due to thermal effects → need an equation of state and an energy equation to complete the system:

MACROSCOPIC MODELS

- Equation of state (EOS):

Planets: (linear relation)

$$\delta\rho(T) = -\rho_0\alpha_T(T - T_0)$$

Stars: (linearized ideal-gas)

$$\frac{\rho}{\bar{\rho}} = \frac{P}{\bar{P}} - \frac{T}{\bar{T}}$$

- Conservation of energy:

(Notes: could also be written in terms of entropy S using thermodynamic relations, I've also assume constant thermal conductivity)

$$\rho C_p \left(\frac{DT}{Dt} - \frac{\alpha_T T}{\rho C_p} \frac{DP}{Dt} \right) = \frac{J^2}{\sigma} + \vec{\tau} : \nabla \vec{u} + H + k \nabla^2 T$$

C_p : specific heat

α_T : coefficient of thermal expansion

H : internal heat sources

T : temperature

$\vec{\tau}$: deviatoric stress tensor

\vec{j} : current density

k : thermal conductivity

NON-DIMENSIONAL EQUATIONS

- common in planetary dynamo models to non-dimensionalize the equations.
- e.g.: using “Boussinesq” approximation:

- Momentum:
$$EP_m^{-1}(\partial_t + \vec{v} \cdot \nabla)\vec{v} + \hat{z} \times \vec{v} = -\nabla P + \vec{J} \times \vec{B} + RaT\vec{r} + E\nabla^2\vec{v}$$

- Magnetic Induction:
$$(\partial_t - \nabla^2)\vec{B} = \nabla \times (\vec{v} \times \vec{B})$$

- Energy:
$$(\partial_t - P_m P_r^{-1} \nabla^2)T = -\vec{v} \cdot \nabla T + Q$$

- Mass:
$$\nabla \cdot \vec{v} = 0$$

Non-dimensional Numbers:

$$\text{Rayleigh \#}: Ra \equiv \frac{\alpha g_0 \Delta T r_0}{2\Omega\eta}$$

$$\text{Prandtl \#}: P_r \equiv \frac{\nu}{\kappa}$$

$$\text{Ekman \#}: E \equiv \frac{\nu}{2\Omega r_0^2}$$

$$\text{Magnetic Prandtl \#}: P_m \equiv \frac{\nu}{\eta}$$

PLANETARY PARAMETER REGIME

	Control Parameters			
	Ra^* / Ra_c^*	E	Pm	Pr
Earth's core	≈ 5000	$10^{-15} - 10^{-14}$	$10^{-6} - 10^{-5}$	0.1–1
Models	1–100	$10^{-3} - 10^{-6}$	$0.1 - 10^3$	$0.1 - 10^3$
	Diagnostic numbers			
	Rm	Re	Ro	Λ
Earth's core	$\approx 10^3$	$10^8 - 10^9$	$\approx 10^{-6}$	0.1–10
Models	$50 - 10^3$	<2000	$3 \times 10^{-4} - 10^{-2}$	0.1–100

Christensen et al. 2008

What can't we do right:

- viscosity & thermal diffusivity too large compared to magnetic diffusivity
- rotation too slow, much less turbulent

Hope that if we are getting the force balances right, then models might be telling us something about core dynamics

- scaling laws suggest this is happening (e.g. Christensen 2010)

SOLAR PARAMETER REGIME

		Base of convection zone	Photosphere
Rayleigh number	$Ra = g\alpha\beta L^4 \rho / \kappa \mu$	10^{20}	10^{16}
Reynolds number	$Re = UL/\nu$	10^{13}	10^{12}
Magnetic Reynolds number	$Rm = UL/\eta$	10^{10}	10^6
Prandtl number	$Pr = \nu/\kappa$	10^{-7}	10^{-7}
Magnetic Prandtl number	$Pm = \nu/\eta$	10^{-3}	10^{-6}
Rossby number	$Ro = U/2\Omega L$	0.1–1	10^{-3} –0.4
Ekman number	$Ek = \nu/2\Omega L^2$	10^{-14}	10^{-15} – 10^{-13}

- much larger Reynolds number than planetary dynamos → extremely turbulent
- much larger magnetic Reynolds number than planetary dynamos → magnetic field generation on extremely small scales
- Rossby number larger (rotation less important)

PLANETARY & STELLAR MODELS

Aside from the fact that we can't numerically simulate the right parameter regime...

Positives:

Planet models:

- magnetic Reynolds #: ~ 1000 , can get to these values in models without mean field parameterizations
- rotation acts to organize flow which promotes large scale fields
- solid rocks at surface record past fields (at least some info like pole location and intensity). This is how we know reversal timescales

Solar/stellar models:

- can study reversals in human lifetime
- have observations of flows in convection zone from helioseismology
- dynamo region is near the surface so we see smaller scales

Negatives:

Planet models:

- reversal timescale $\sim 500,000$ years, time for reversals to occur ~ 1000 years \rightarrow can't observe reversals in a human life span
- can't observe flows in dynamo region
- dynamo regions far removed from surface

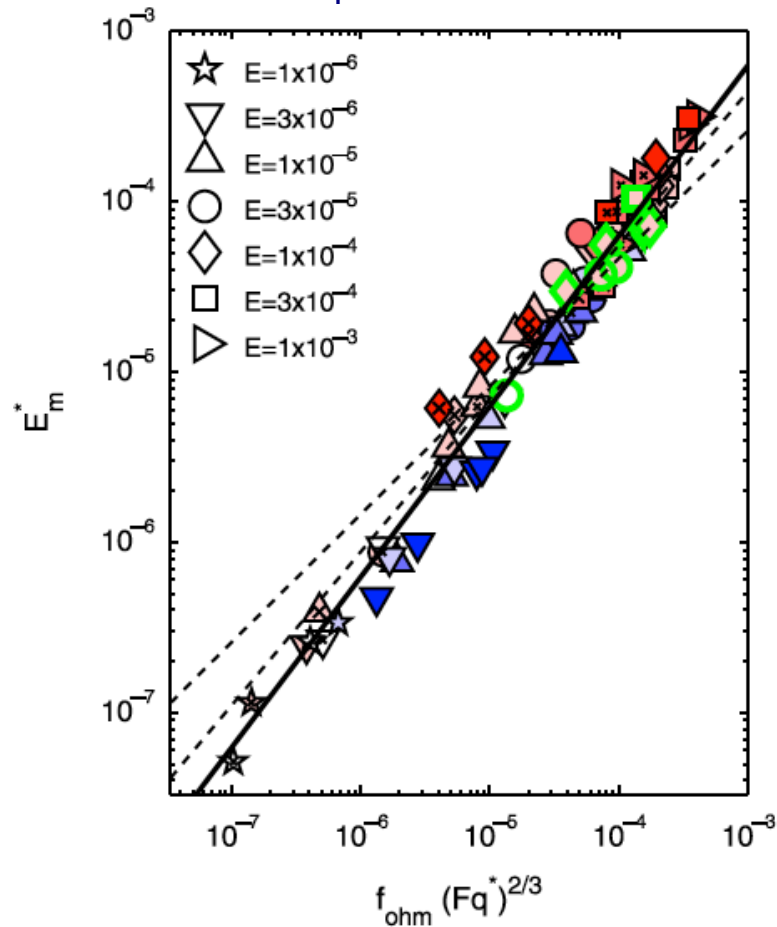
Solar/stellar models:

- $Re_m \sim 10^9$, cannot get these values without mean field parameterizations
- flow more turbulent
- physical properties vary significantly over convection zone

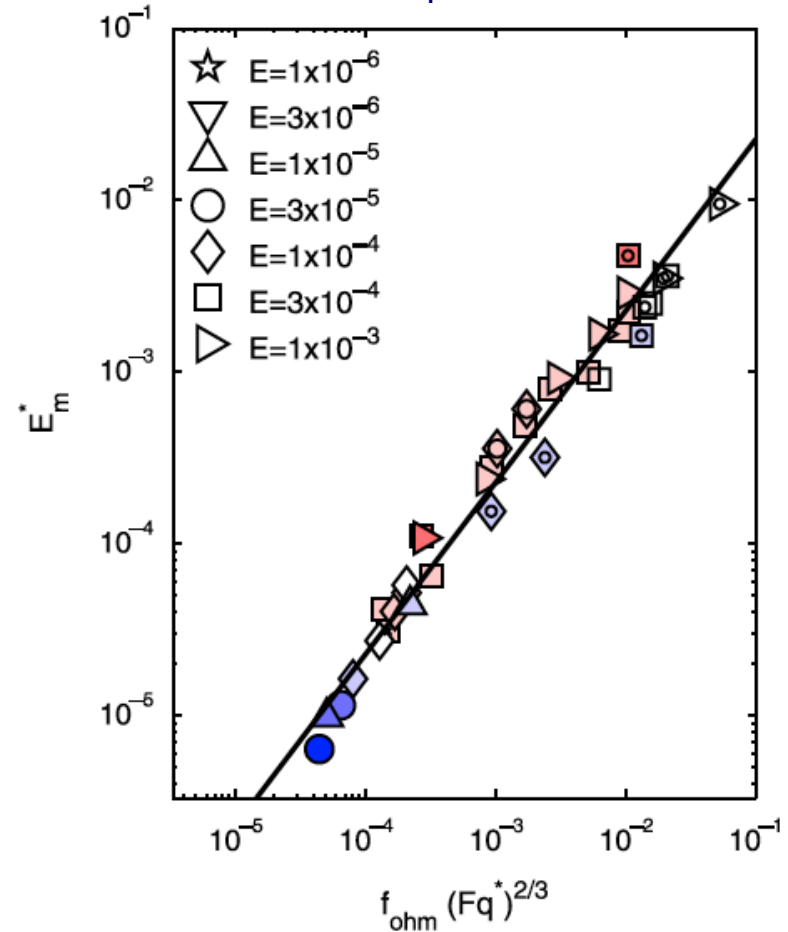
SCALING LAWS

- although we don't work in the appropriate parameter regime, can we use models to determine scaling laws and hence properties of planetary & stellar dynamos
- e.g.: magnetic energy scales with power available to drive dynamo:

dipolar fields

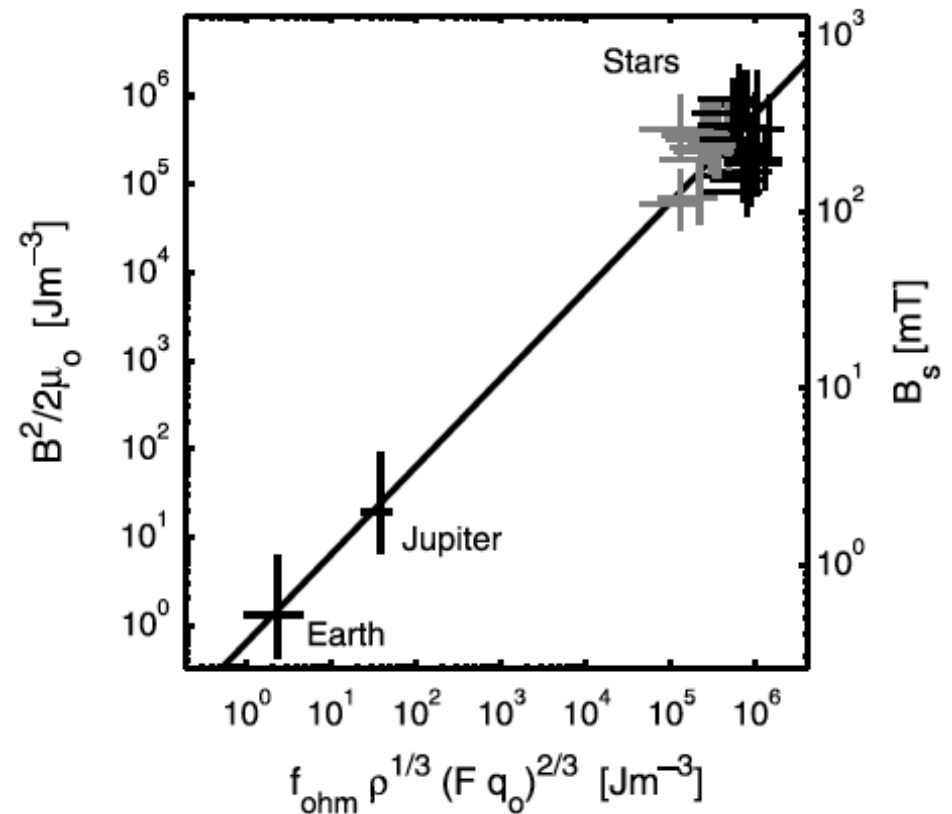
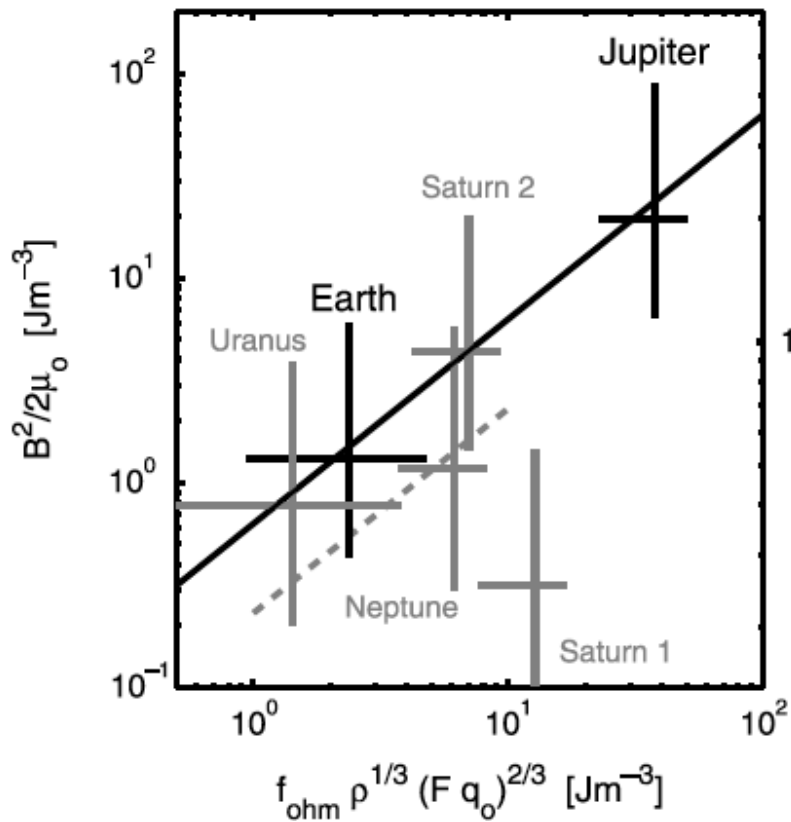


nondipolar fields



SCALING LAWS

- scaling laws derived from numerical models seem to work well for observations

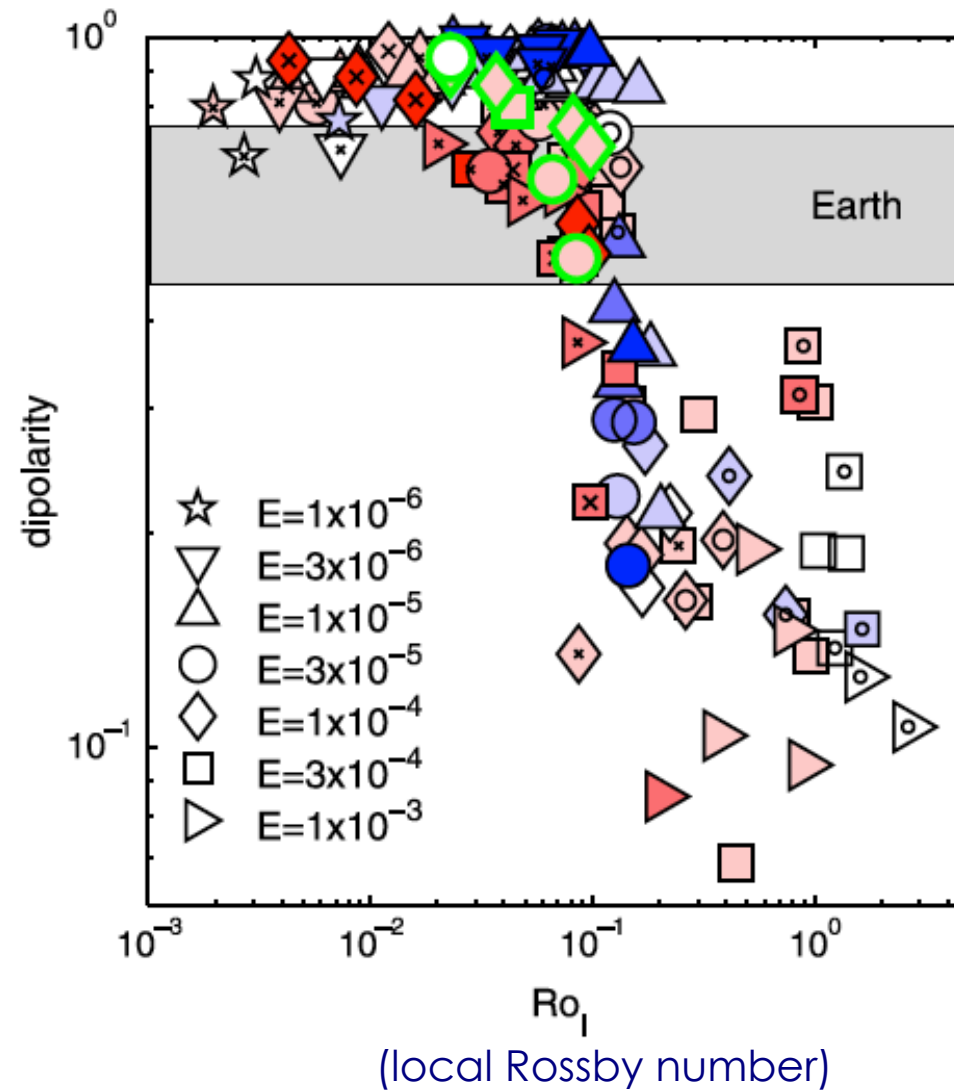


(energy density available to drive dynamo)

Christensen (2010)

SCALING LAWS

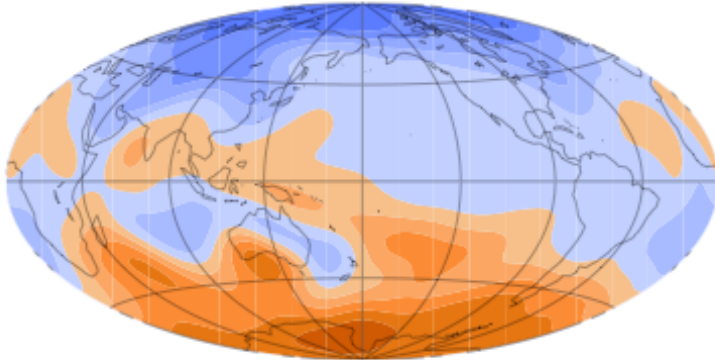
- e.g.: field dipolarity depends on rotation/inertia balance:



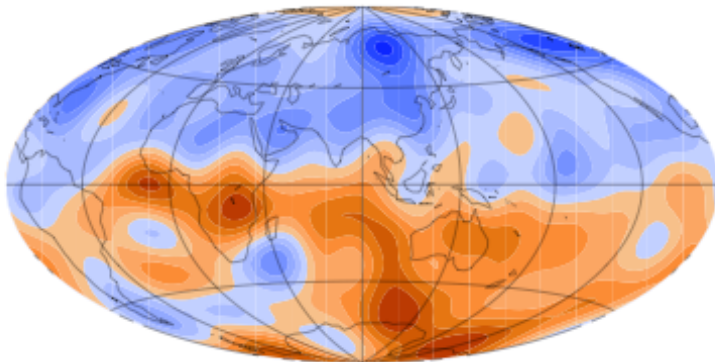
MACROSCOPIC EARTH MODELS

Earth:

Model



Observations

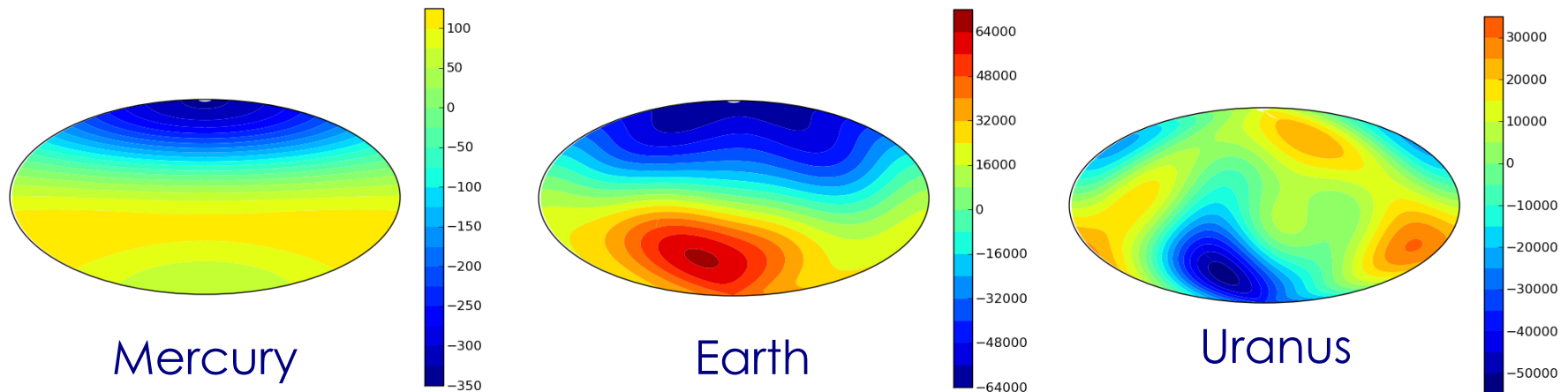


- Able to reproduce main field characteristics even though not working in Earth-like parameter regime:

- axial-dipole dominance
- surface field intensity
- reversals
- secular variation
- core surface flow

PLANETARY MACROSCOPIC MODELS

- Can't work at parameters for other planets either, but there are some planetary features that we can get right, e.g.:
 - core geometry
 - force balances
 - buoyancy sources
 - external influences
- Explaining major differences between planetary magnetic fields is possible by appealing to differences in these features



MERCURY

Observables a model should explain:

(1) WEAK intensity of the field

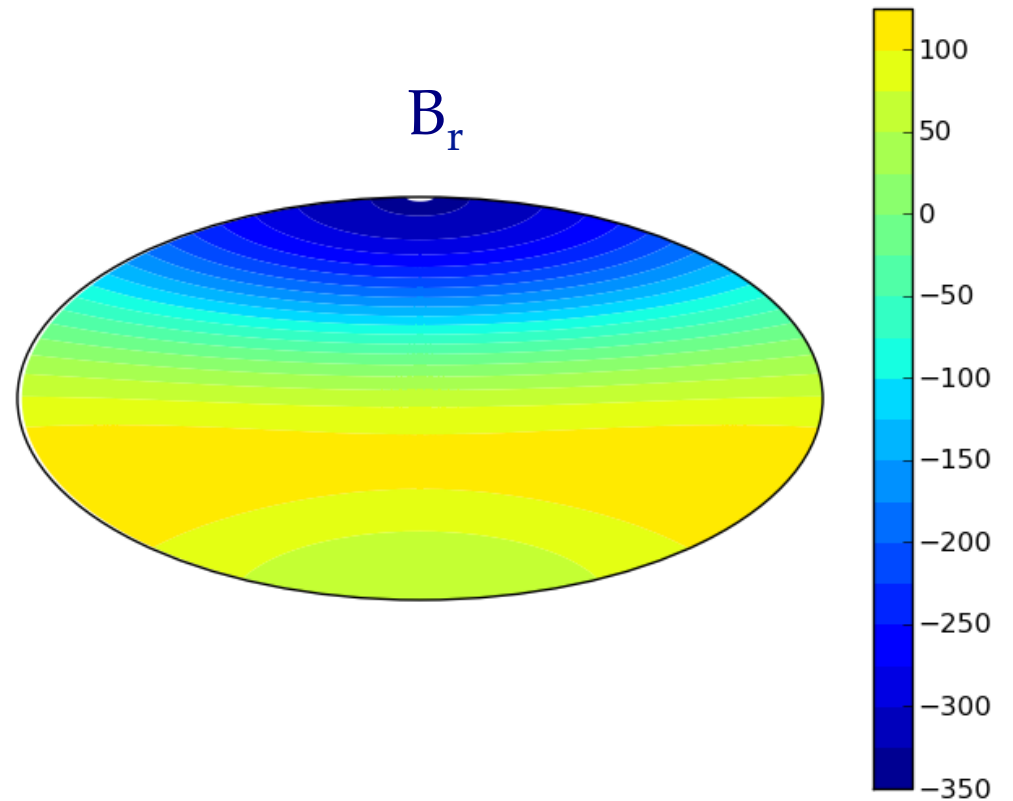
$$g_1^0 = -195 \pm 10 \text{ nT}$$

(2) LARGE quadrupole

$$g_2^0/g_1^0 = 0.38$$

(3) SMALL tilt

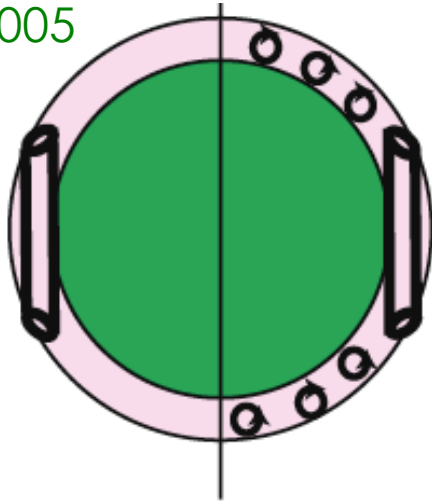
$$\text{dipole tilt} < 1^\circ$$



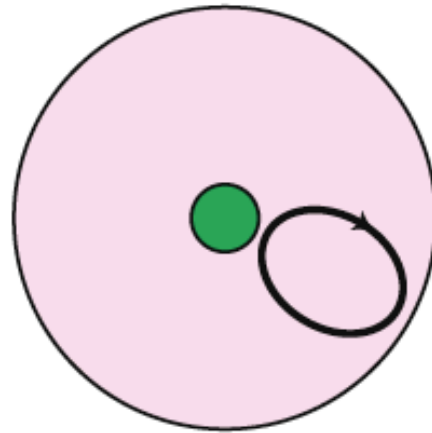
(Anderson et al. 2011)

CONTESTANTS FOR MERCURY DYNAMO MODEL

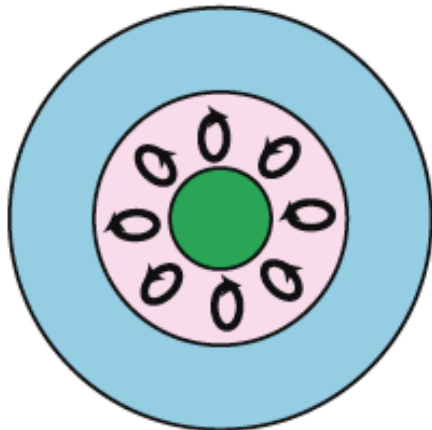
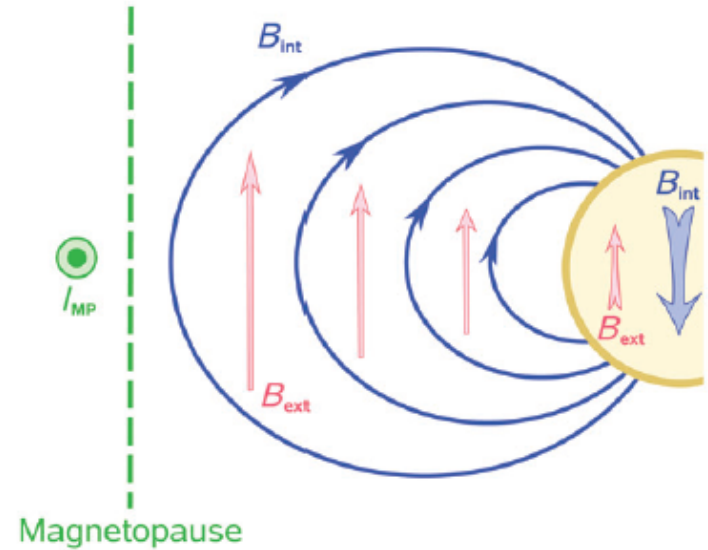
Stanley et al.
2005



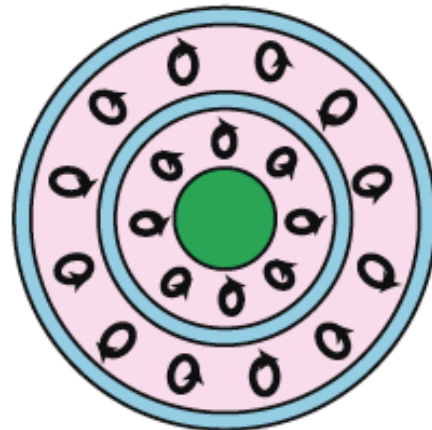
Heimpel et al.
2005



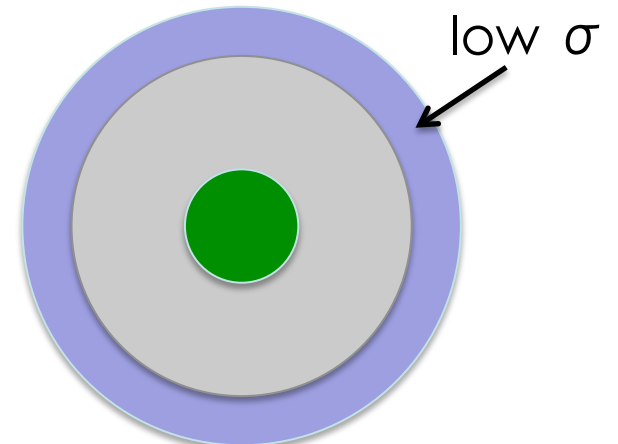
Gomez-Perez & Solomon 2010,
Gomez-Perez & Wicht 2010,



Christensen 2006,
Christensen & Wicht 2008
Manglik et al. 2010



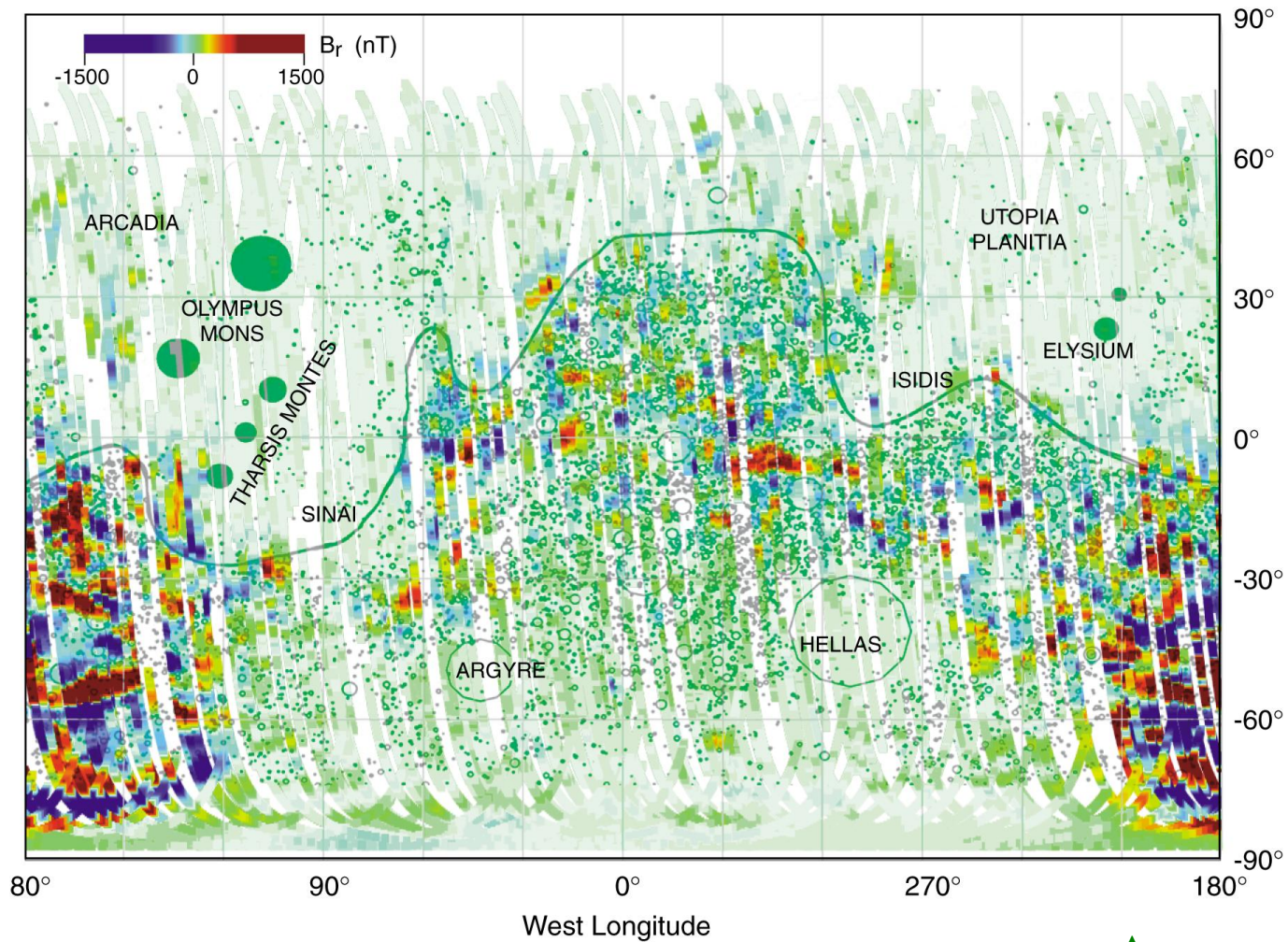
Vilim et al. 2010



Gomez-Perez et al., 2010

MARS ANCIENT DYNAMO

- Why is the crustal field concentrated in the southern hemisphere and correlated with the crustal dichotomy?



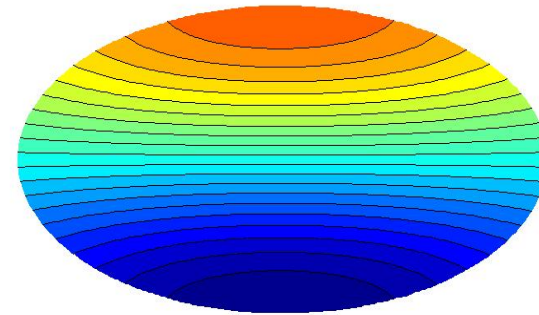
Acuna et al. (1999)

SINGLE HEMISPHERE DYNAMO

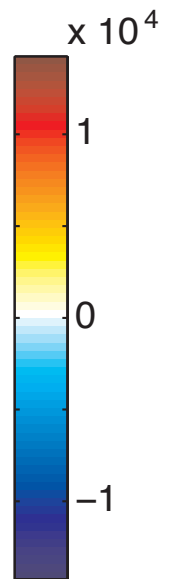
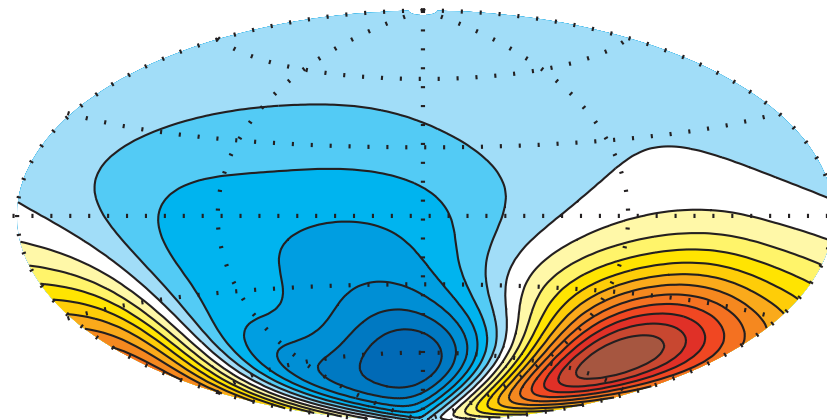
- Crustal Dichotomy formation mechanism → thermal variations at CMB

[Roberts & Zhong \(2007\)](#)

hemispheric CMB thermal variation



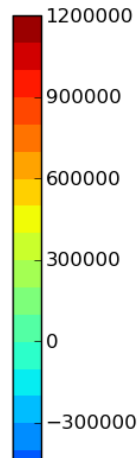
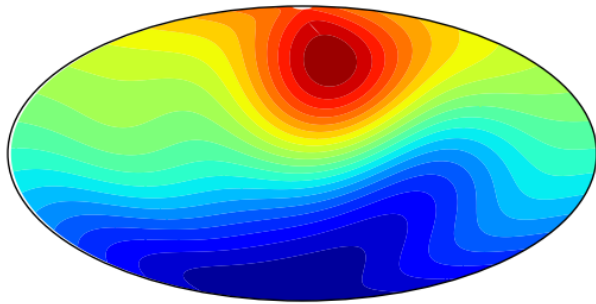
“Single-hemisphere dynamo”
[\(Stanley et al., 2008\)](#)



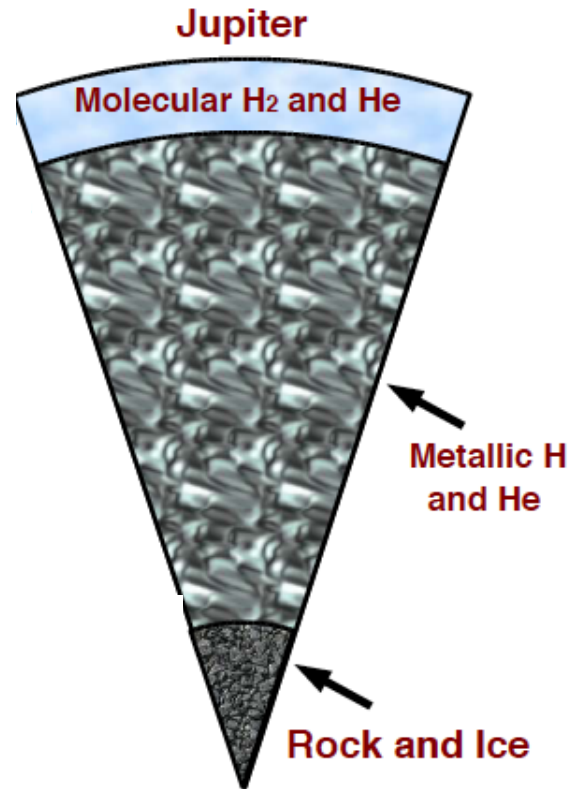
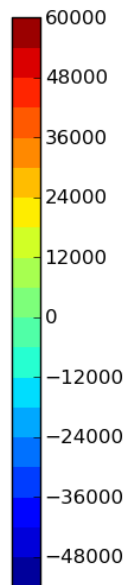
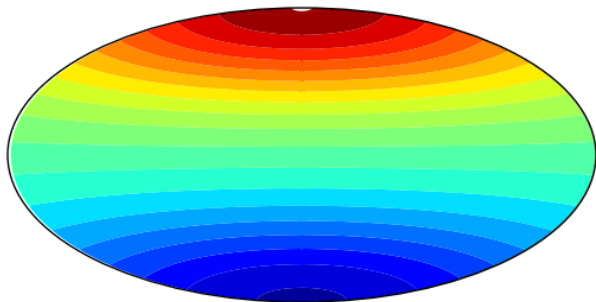
- Death of the Dynamo:
 - subcritical dynamos: [Kuang et al. \(2008\)](#)
 - impacts: [Arkani-Hamed & Olson \(2010\)](#)

GAS GIANTS

Jupiter



Saturn



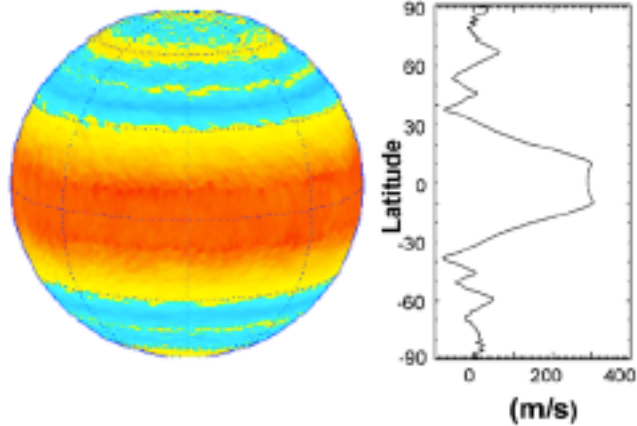
- large pressure range → radially variable properties can be important

SURFACE ZONAL FLOWS

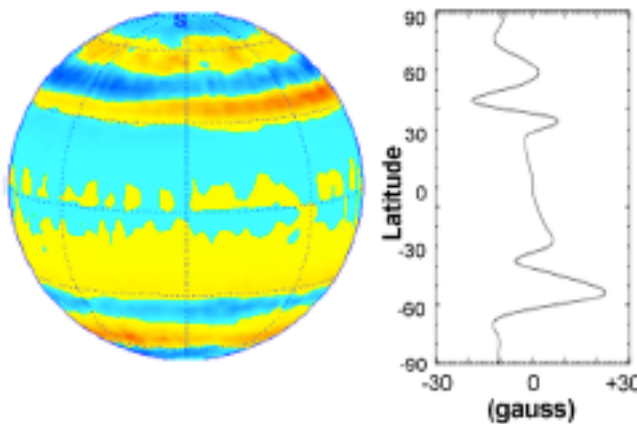
$\sigma(r)$ & $\rho(r)$

Glatzmaier 2005, reviewed in Stanley & Glatzmaier 2010)

Surface longitudinal winds

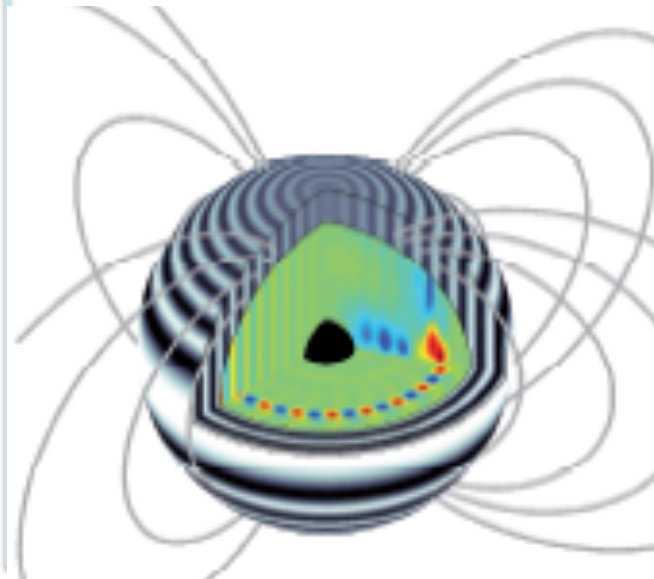
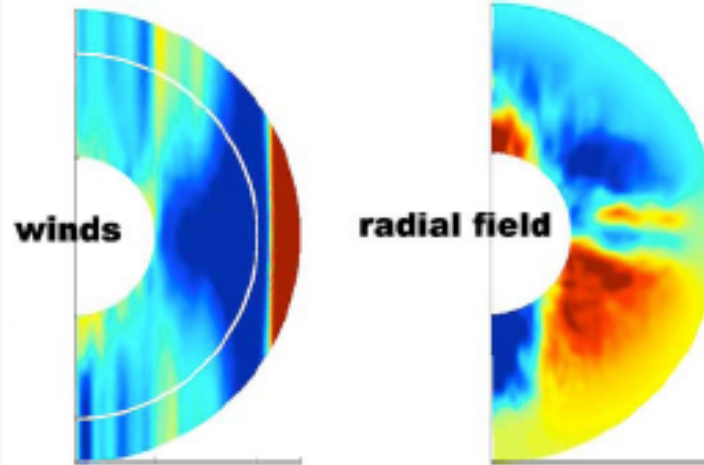


Surface radial magnetic field



$\sigma(r)$

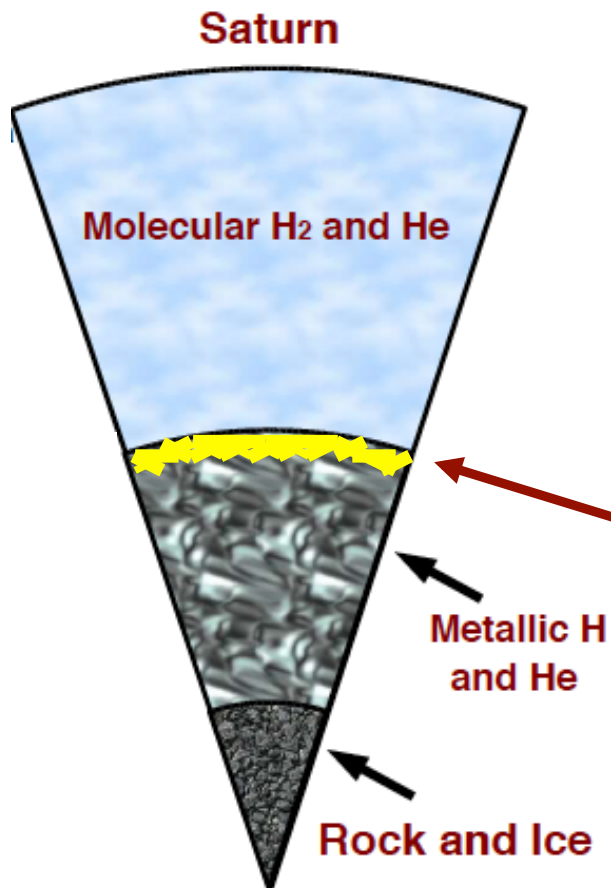
Heimpel & Gomez-Perez 2011



Guervilly et al. 2011
instabilities of narrow zonal jets results in axially-dipolar fields

SATURN'S AXISYMMETRY

Cao et al. 2011: dipole tilt < 0.06 degrees

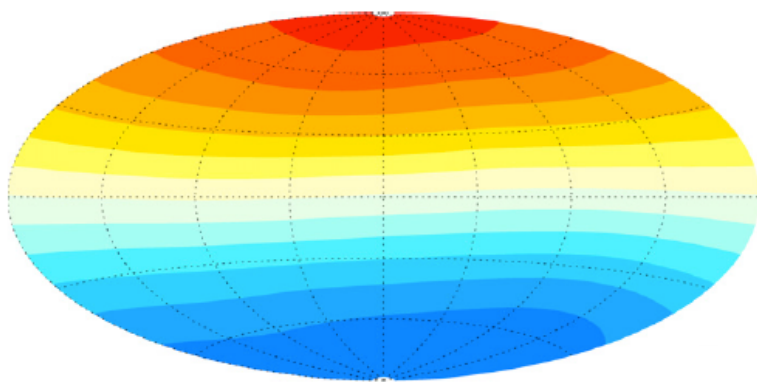


- Although dynamo models dominated by axial dipole component, this level of axisymmetry is not seen.
- Take non-axisymmetric field and axisymmetrize it somehow between the dynamo region and the surface

Helium rain-out layer
(Stevenson & Salpeter 1977)

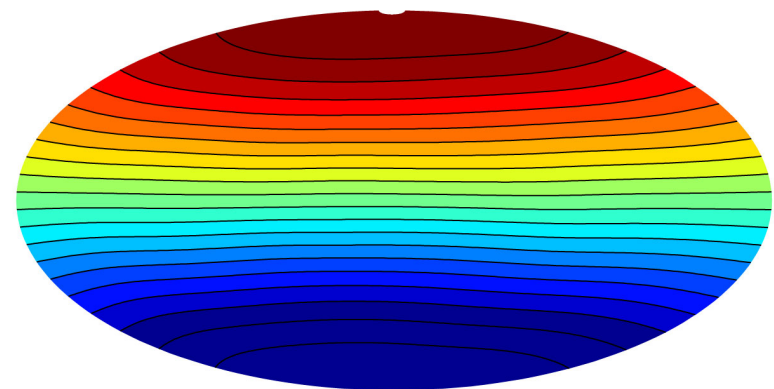
CONTESTANTS FOR SATURN DYNAMO MODEL

- **Thick** stably-stratified layer ~ 0.4 core radii
- Average dipole tilt: 1.5 degrees



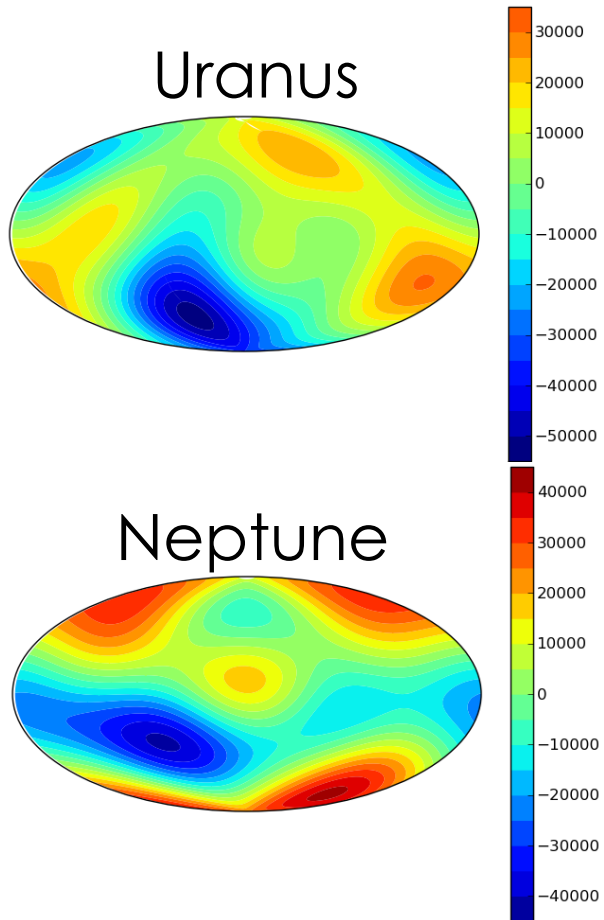
(Christensen & Wicht 2008)

- **Thin** stably-stratified layer ~ 0.1 core radii + bdy thermal variations
- Average dipole tilt: 0.8 degrees



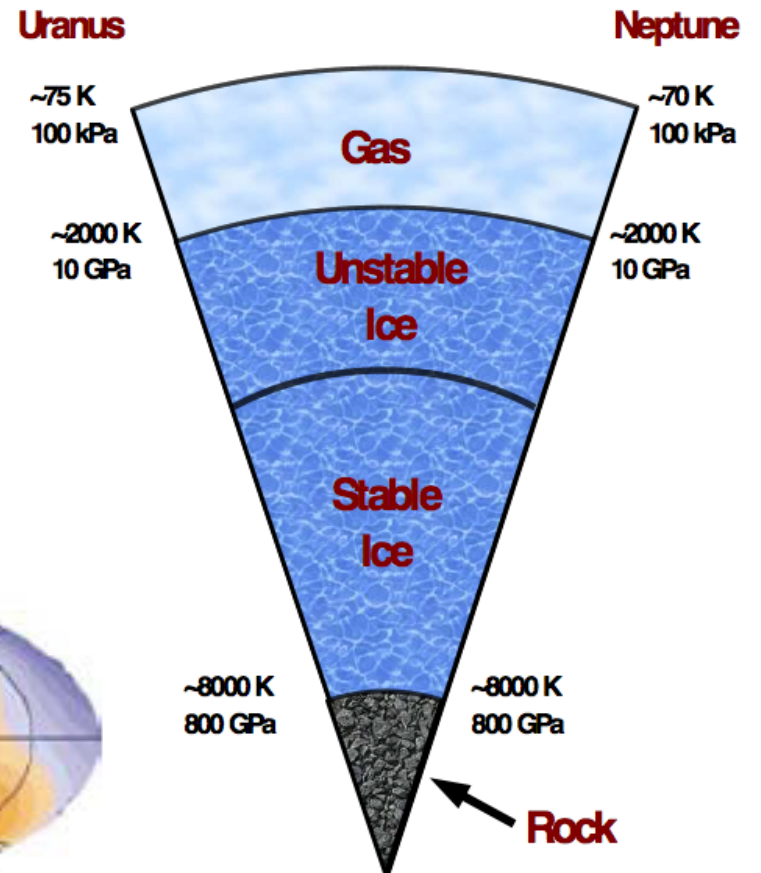
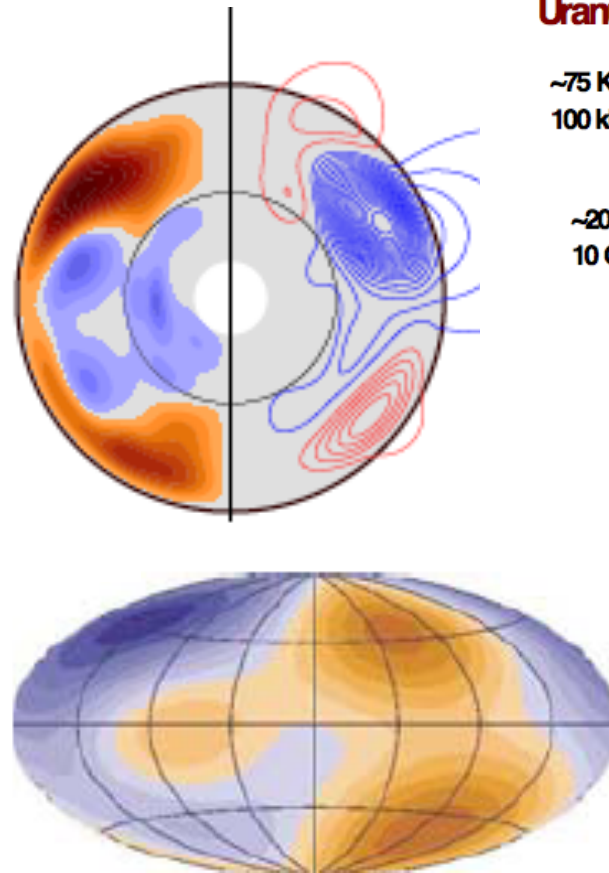
(Stanley 2010)

ICE GIANTS

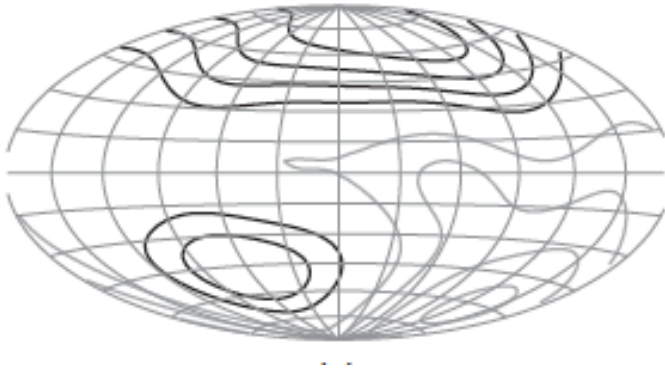


Stanley & Bloxham 2004, 2006

- work in a geometry suggested by low heat flow observations

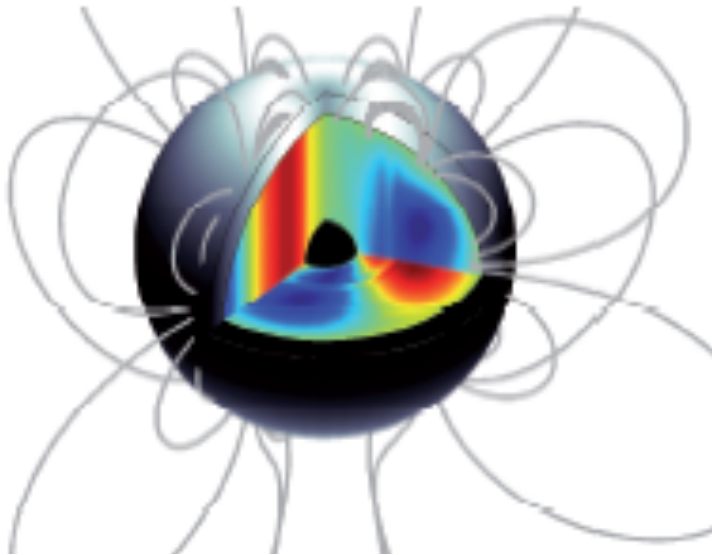


ICE GIANTS



Gomez-Perez & Heimpel 2007

- zonal flow dynamos



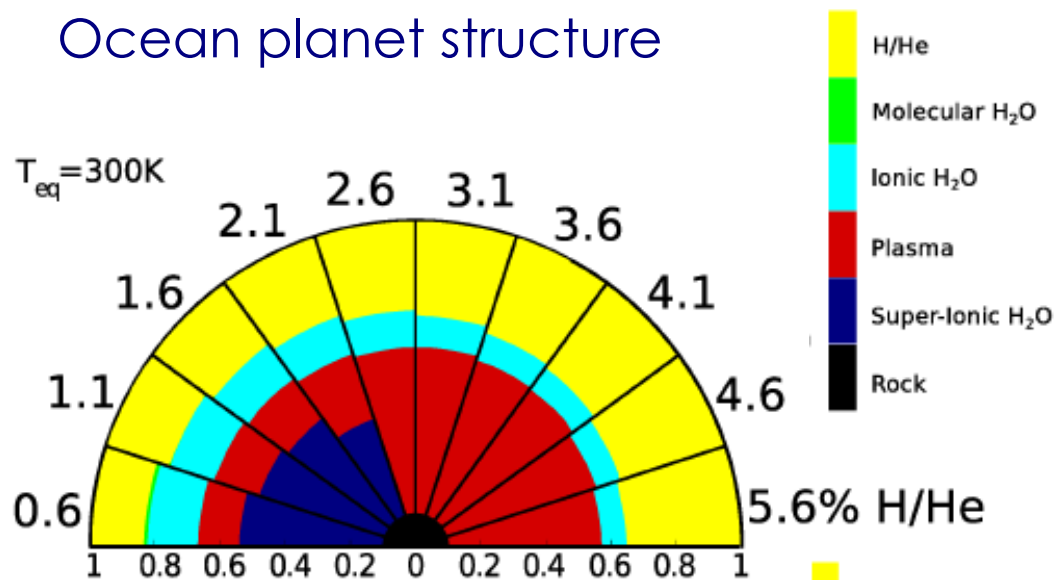
Guervilly et al. 2011

- instabilities of wide zonal jets results in non-dipolar non-axisymmetric fields
- width of jets controls topology of magnetic field (explains difference of Jupiter & Neptune)

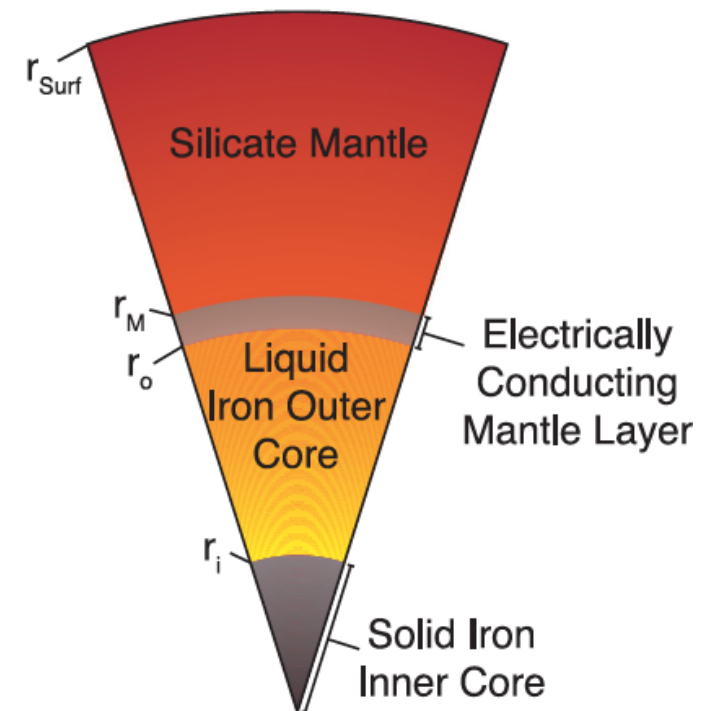
EXTRASOLAR PLANET DYNAMOS

- **Gas giants:** magnetic fields in ionized atmospheric layers can affect flows (Batygin et al., 2010, 2011, 2013, Perna et al. 2010, Rauscher & Menou, 2012, 2013)
- **Ocean planets:** small variations in interior properties can lead to large changes in magnetic field (Tian & Stanley, 2013)
- **Rocky planets:** metallic mantles can shield strong fields from reaching surfaces (Vilim et al., 2013)

Ocean planet structure

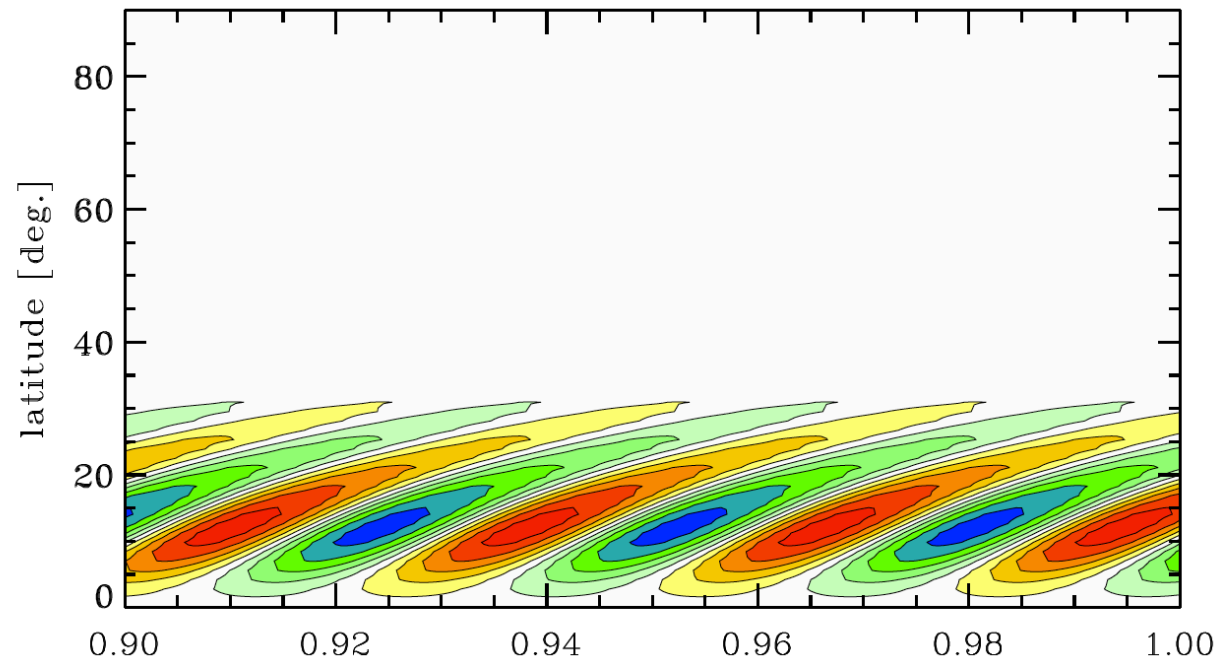


Terrestrial planet structure



SOLAR DYNAMO MODELS

1. Mean field models: (velocity imposed):
 - use differential rotation profile from helioseismology
 - meridional (N-S) transport model
 - mean field parameterization for small scales
2. Interface models:
 - same as 1. plus also include tachocline (shear layer at base of convection zone)
3. Flux Transport models:
 - same as 2. plus include large scale meridional (N-S) circulation to transport fields from tachocline to surface

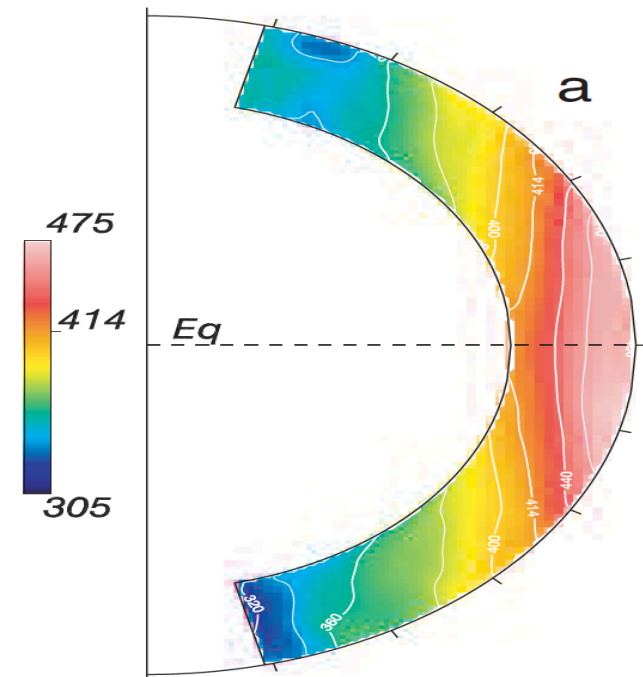
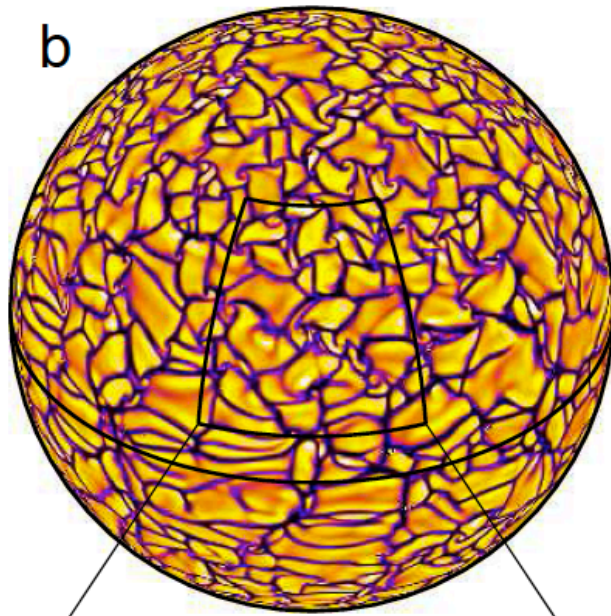


SOLAR DYNAMO MODELS:

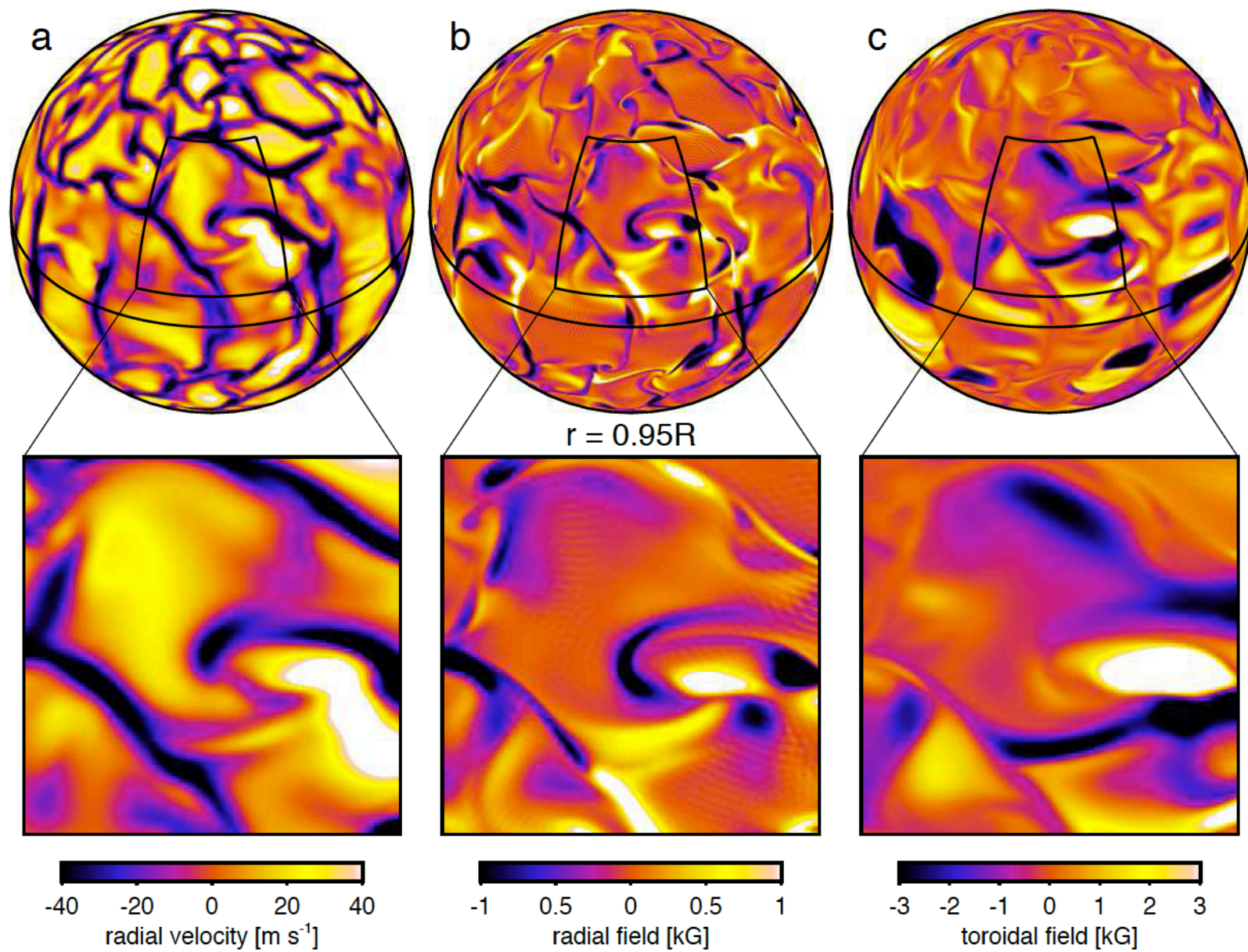
4. Macroscopic models:

e.g.: Anelastic Spherical Harmonic (ASH) code:

- uses realistic values for solar radius, luminosity and mean rotation rate
 - reference state is based on 1D solar structure models
 - upper boundary placed below photosphere ($0.96-0.98 R_s$)
 - lower boundary: base of convection zone, but some simulations allow penetration into the radiative interior
-
- This model tries to simulate solar convection properly and see what magnetic fields you get out of it. Reproduces solar differential rotation profile

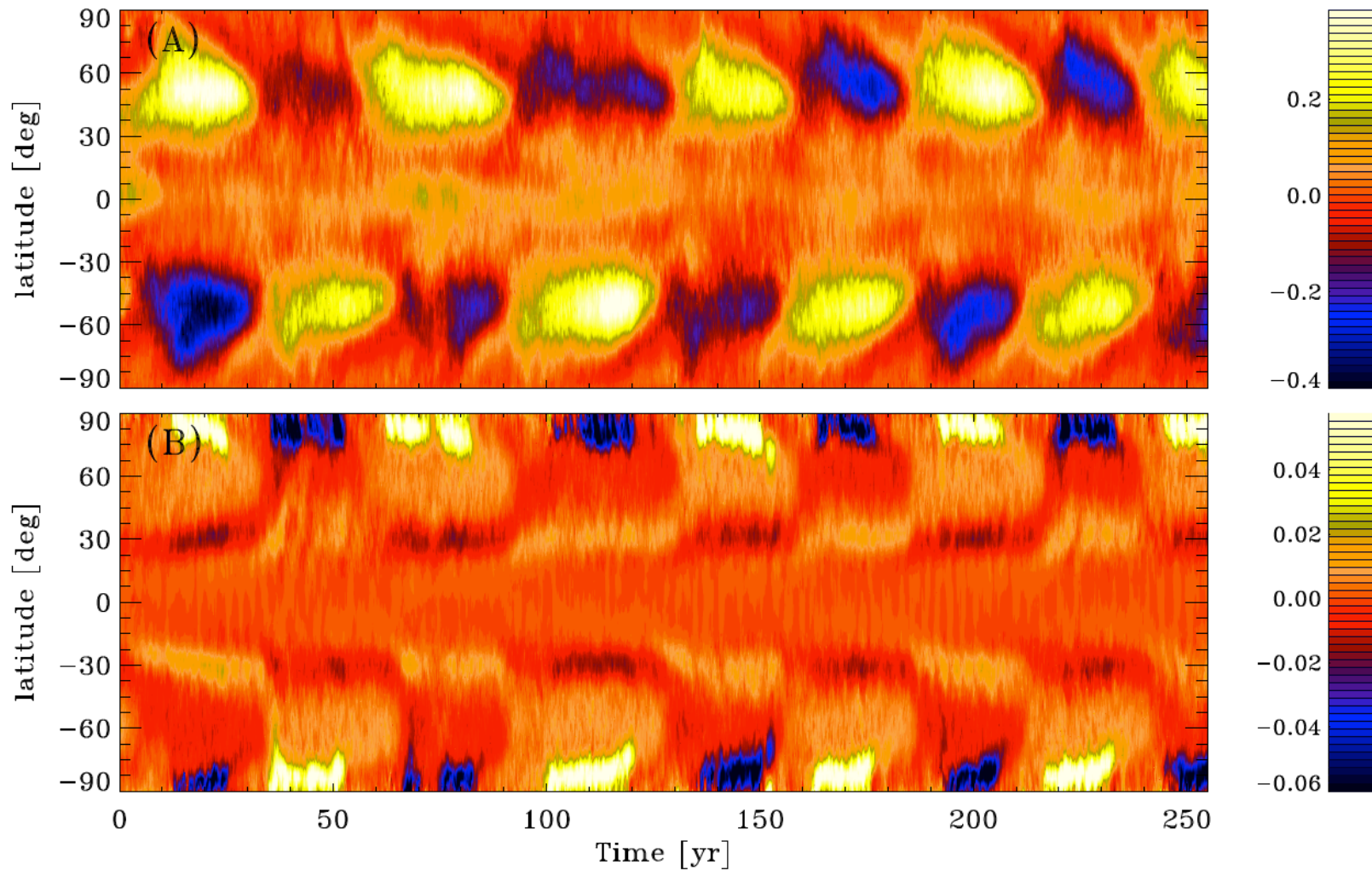


MAGNETIC FIELDS IN MACROSCOPIC MODELS



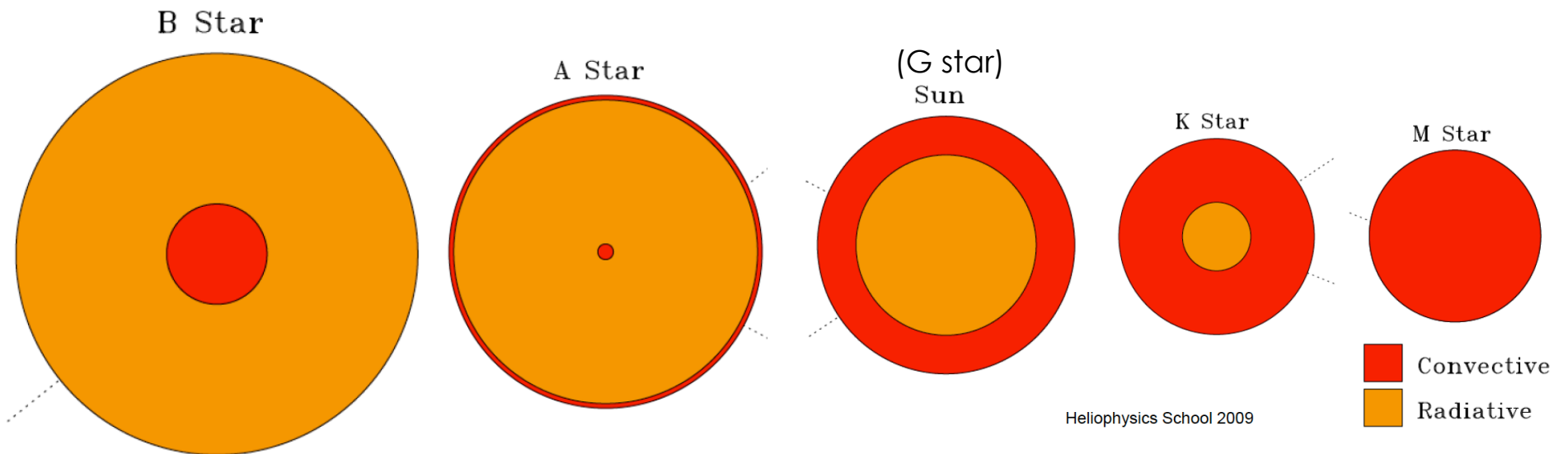
SPOTS & CYCLES IN MACROSCOPIC MODELS

- models don't produce a tachocline region, but one can be added artificially
- recent models starting to produce cycles:



STELLAR DYNAMO MODELS

- Convection zone geometry likely plays an important role

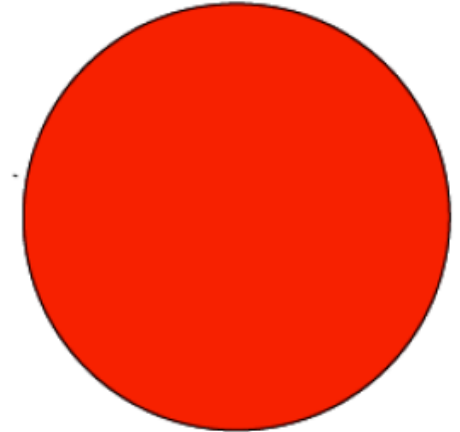


- In cooler stars, the convective envelope deepens as the surface temperature/mass decreases
- In hotter stars, the convective envelope disappears, but a convective core builds up as the surface temperature mass/effective temperature increases

FULLY CONVECTIVE STARS

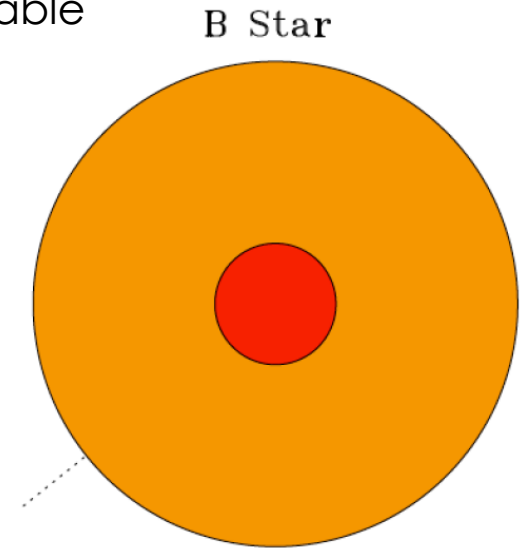
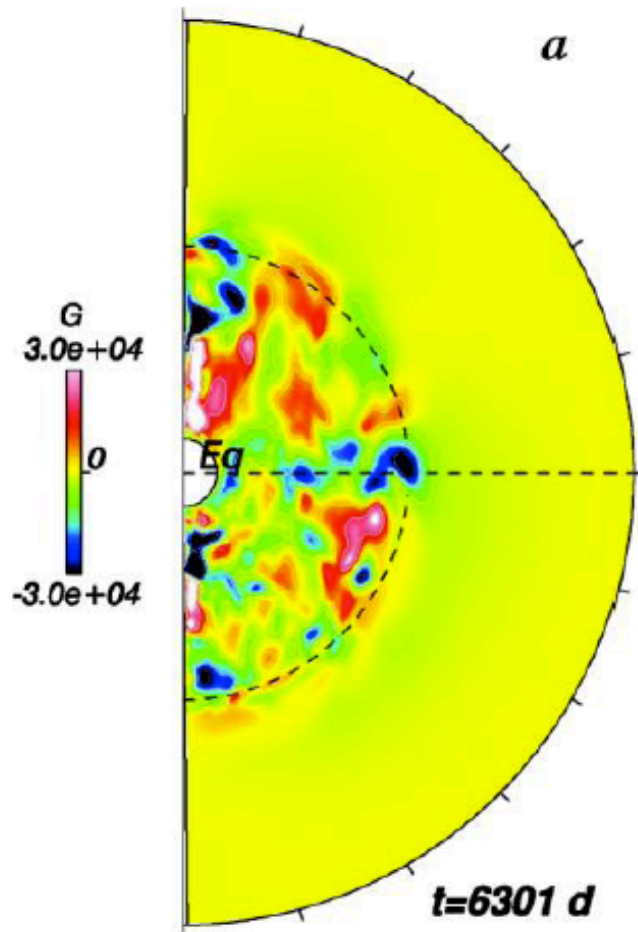
- dynamos may be fundamentally different than solar-type stars
- no tachocline to store fields resulting in interface dynamos
- many fully convective stars observed to have strong chromospheric H-alpha emission & FeH line ratios, indicative of strong magnetic fields
- unclear how dynamos in fully convective stars depend on rotation rate
- Doppler imaging of rapidly rotating M-dwarfs reveals dark patterns at low and moderate latitudes (star spots) (Barnes et al. 2001).

M Star



EARLY-TYPE STARS

- magnetic fields may be trapped inside core \rightarrow not observable



Brun et al. (2005)

SUMMARY

- lots of variety in planetary and stellar dynamos, but the fundamental principles, mechanisms and problems are similar (i.e. they are not so different)
- models and observations suggest that magnetic fields depend strongly on dynamo region geometry, stratification, rotation and other properties
- models seem to be on the right track, but we should be careful and vigilant because of the vast distance between model and planet/star parameters

Thank You