# Shocks in Heliophysics

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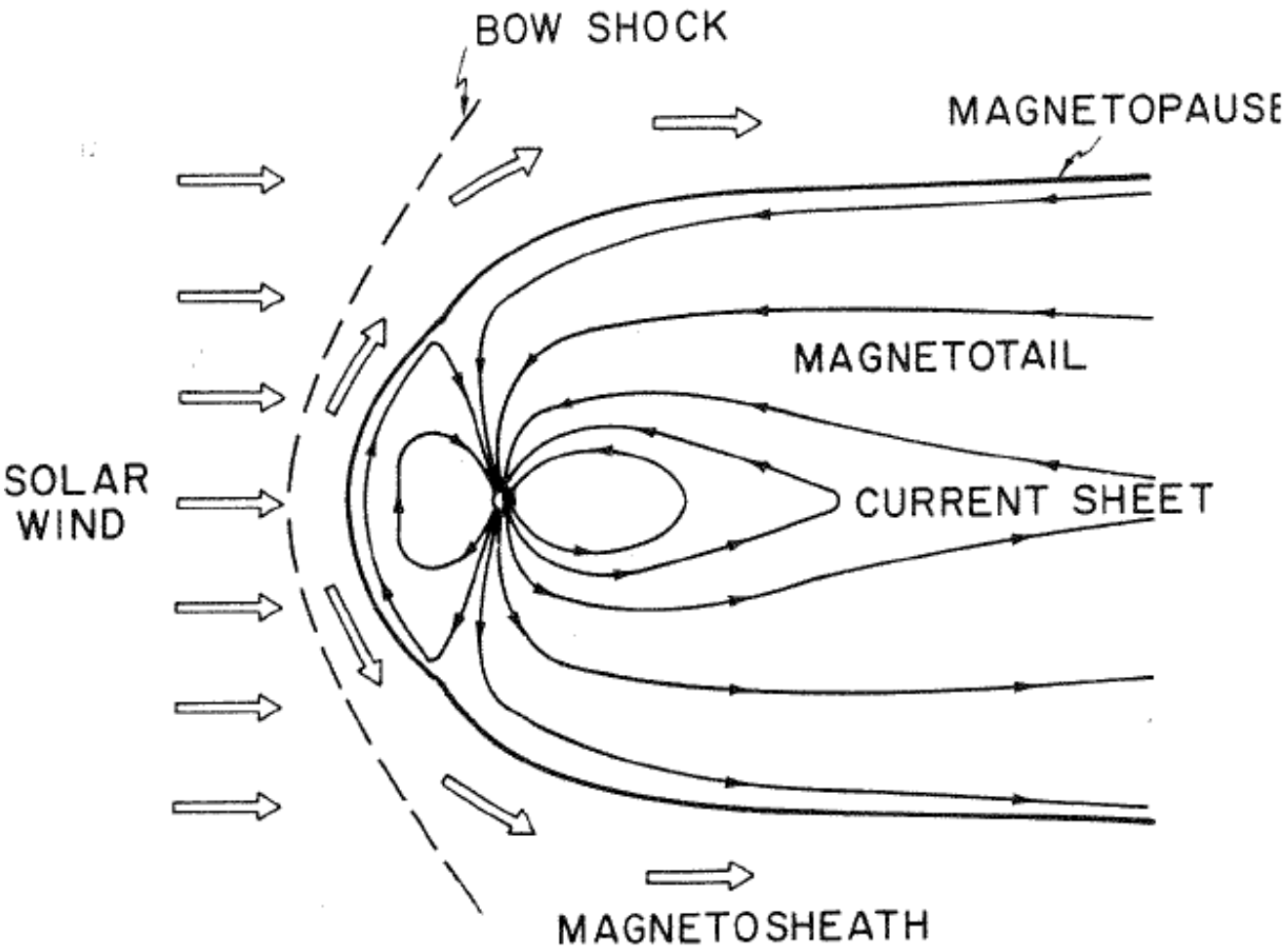
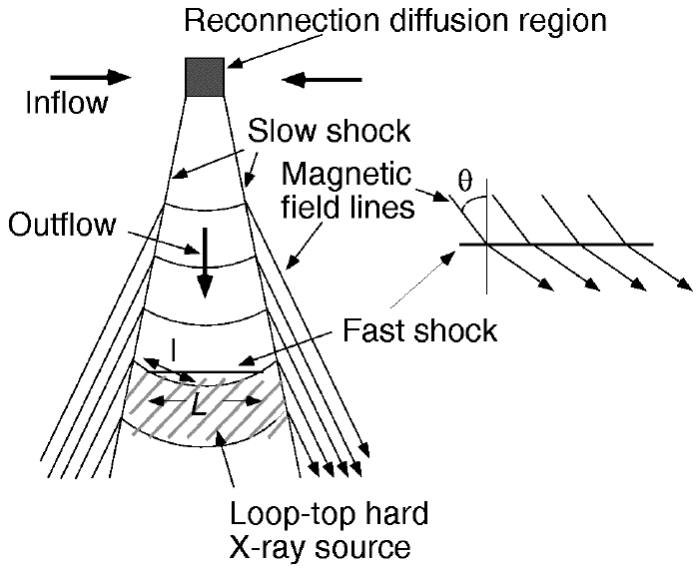
# Shocks in Heliophysics

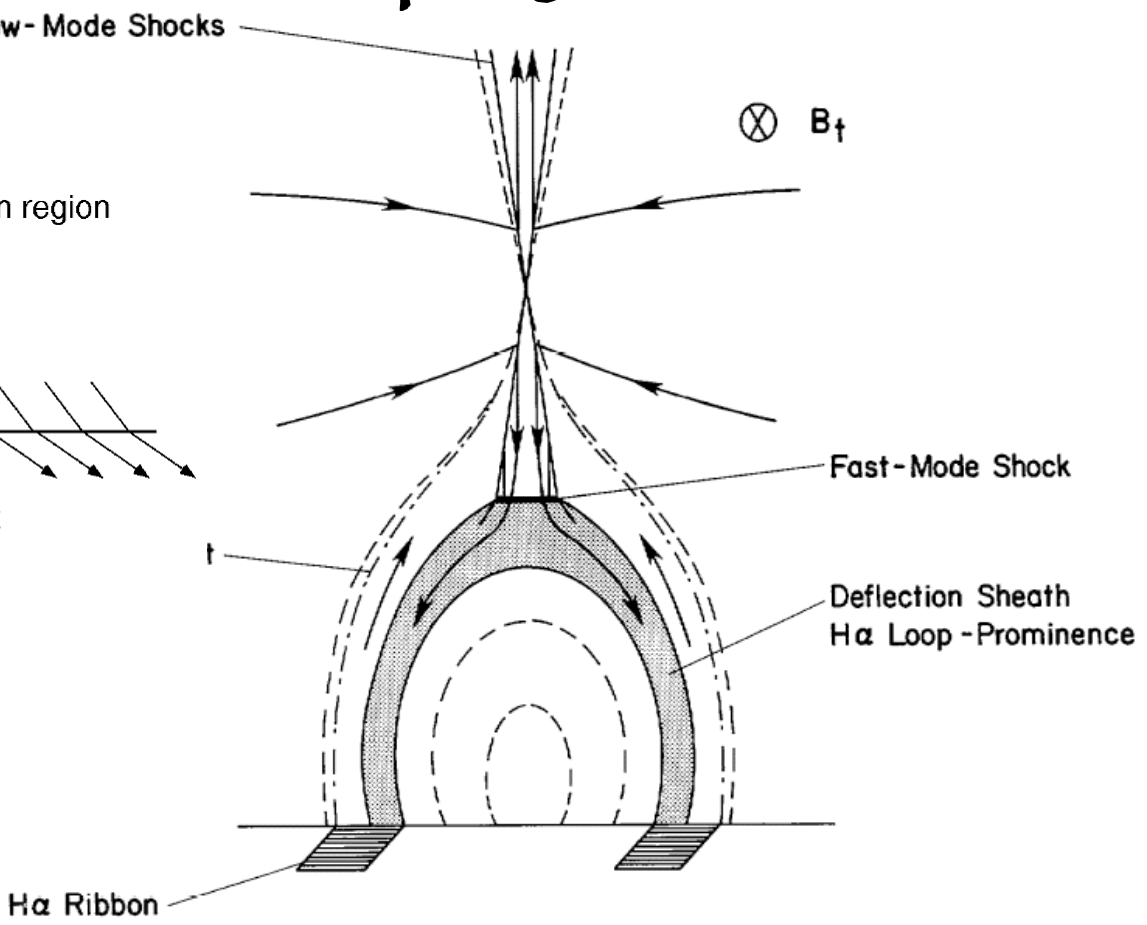



Vasyliunas, v.1 ch.10

# Shocks in Heliophysics

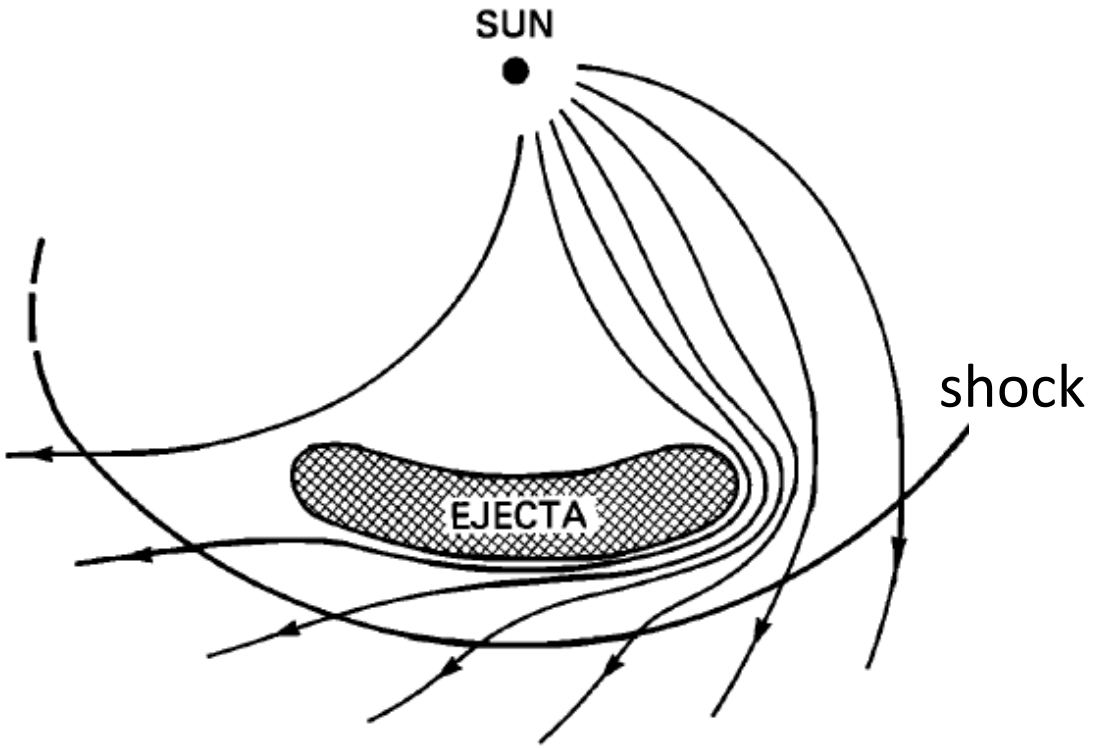



Tsuneta and Naito 1998

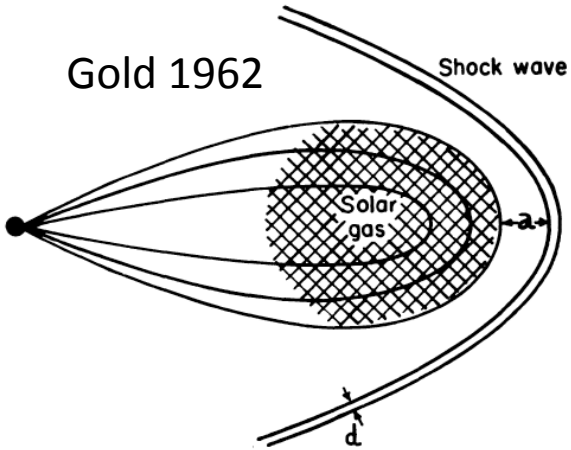



Forbes, T.G., and Malherbe 1986 (v.2 ch. 6)

# Shocks in Heliophysics




Gold 1962




Cane, Reames, and von Rosenvinge 1988 (v. 2 ch. 7)

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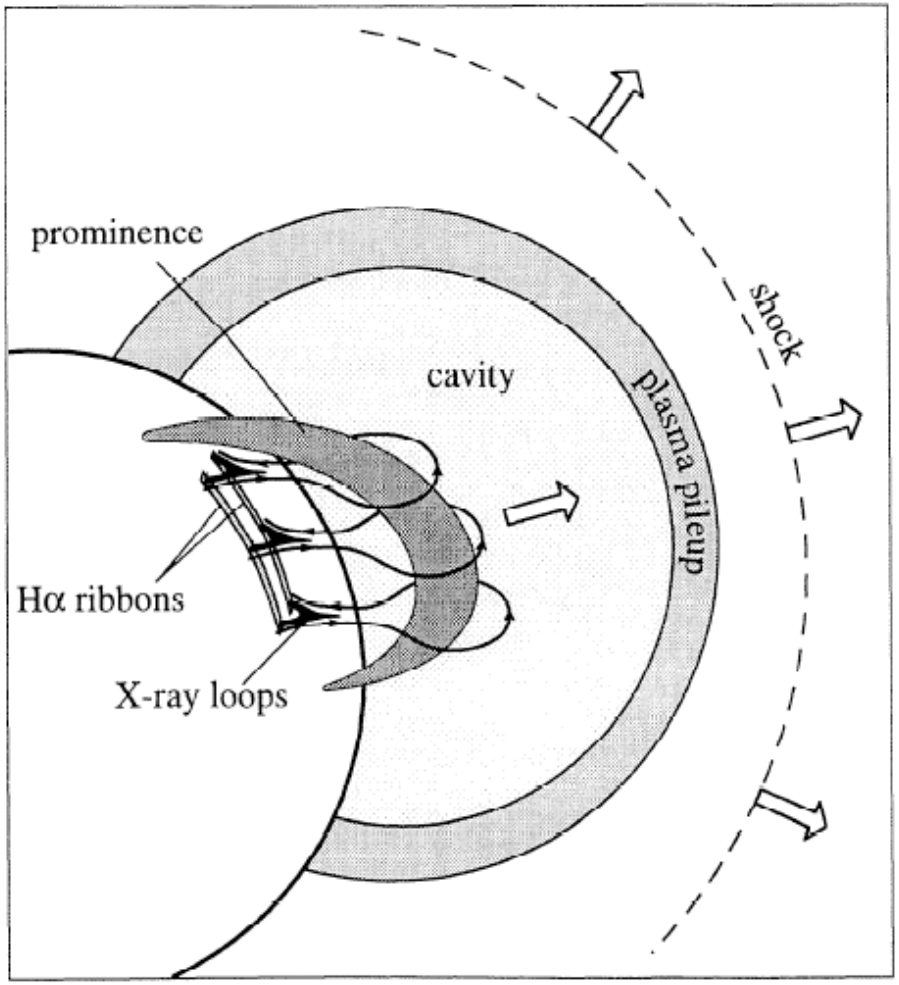
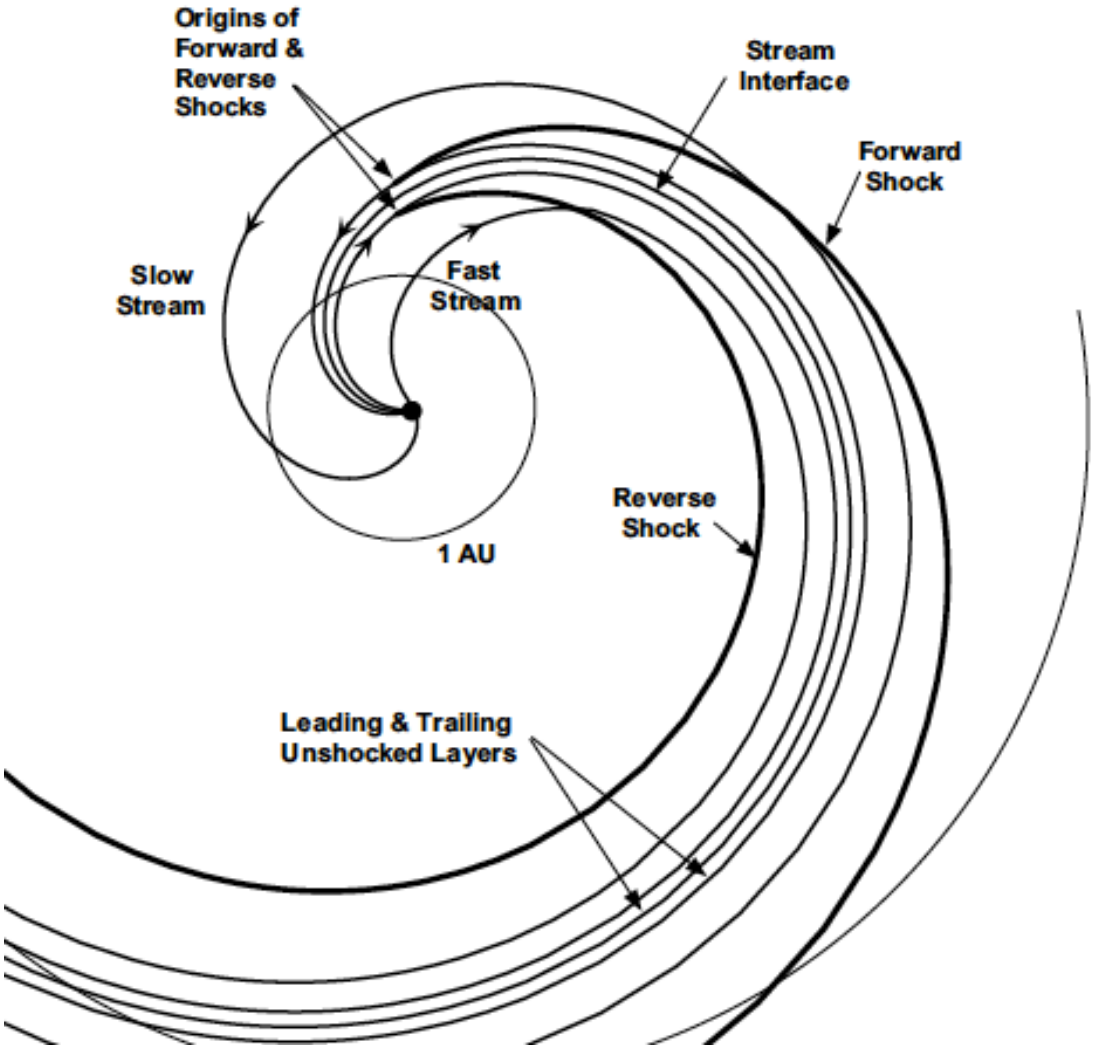


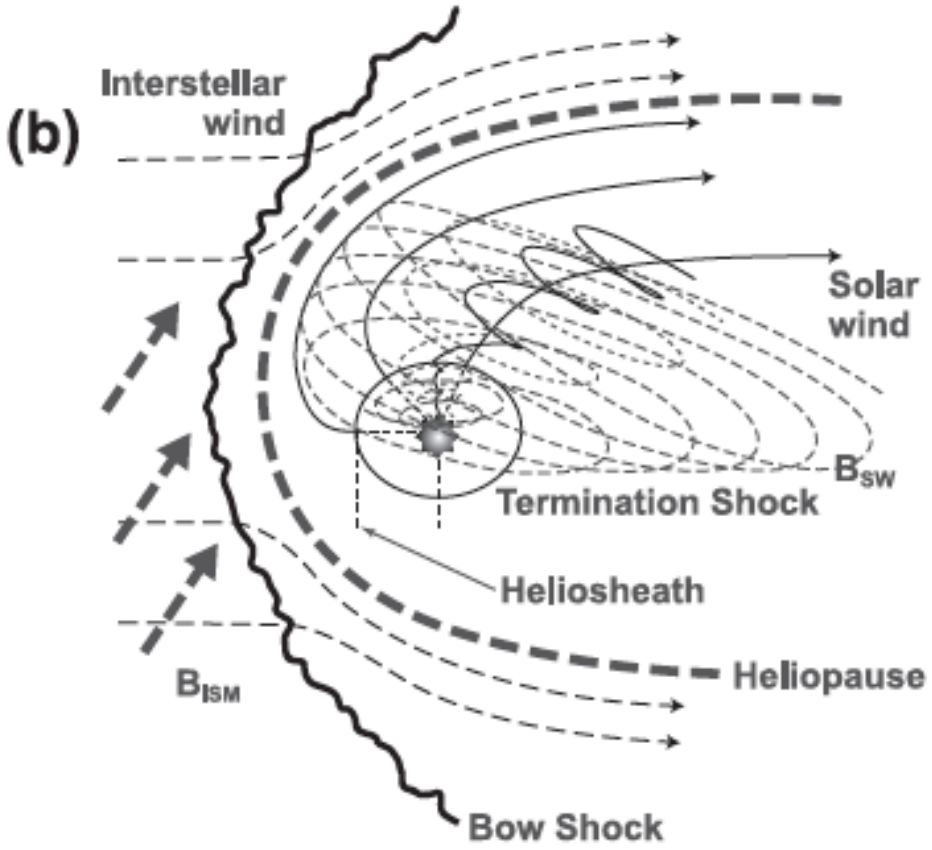

Forbes 2000 (v. 2, ch. 6)

# Shocks in Heliophysics

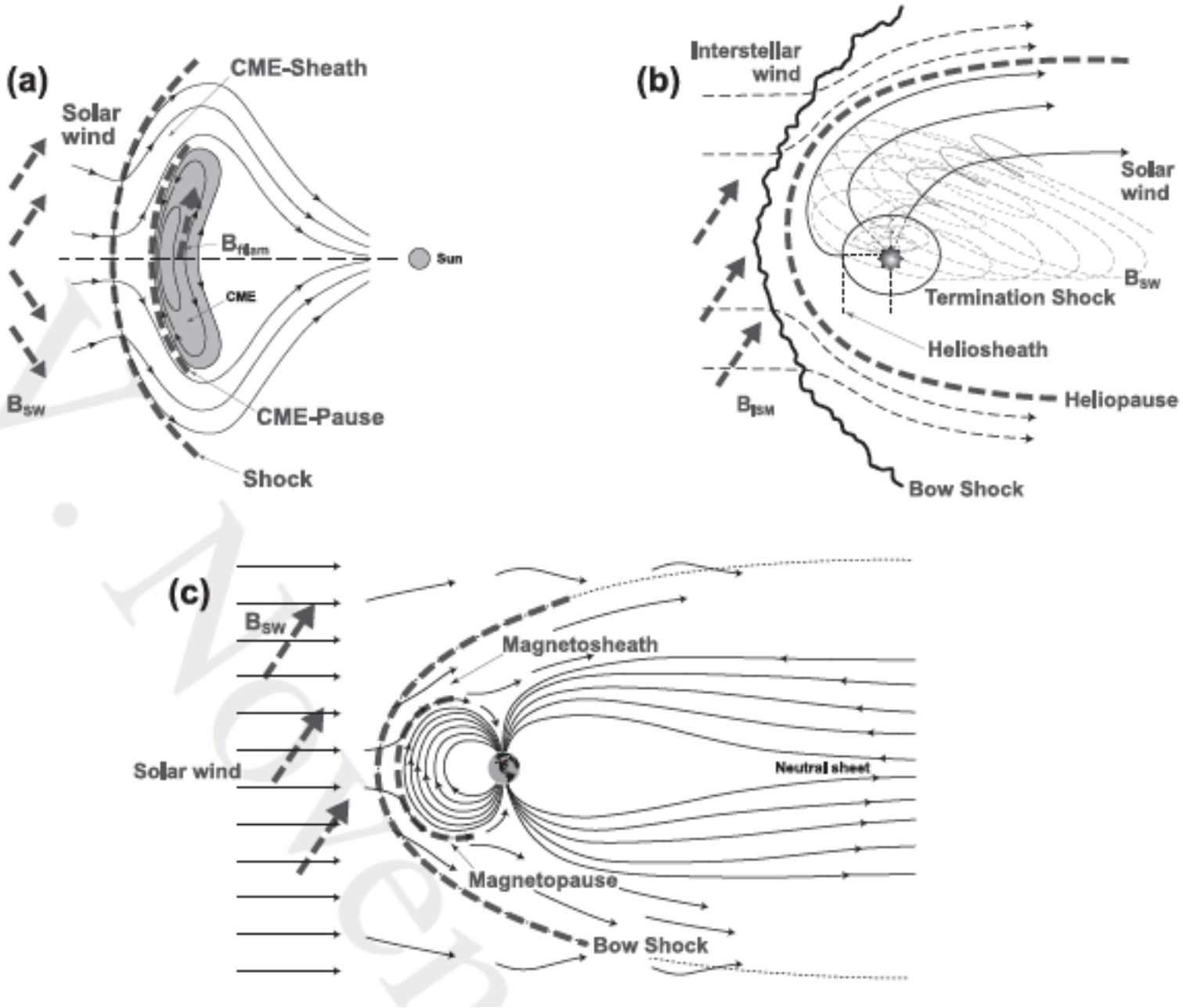



Crooker, Gosling, et al. 1999 (v3, ch. 8)

# Shocks in Heliophysics



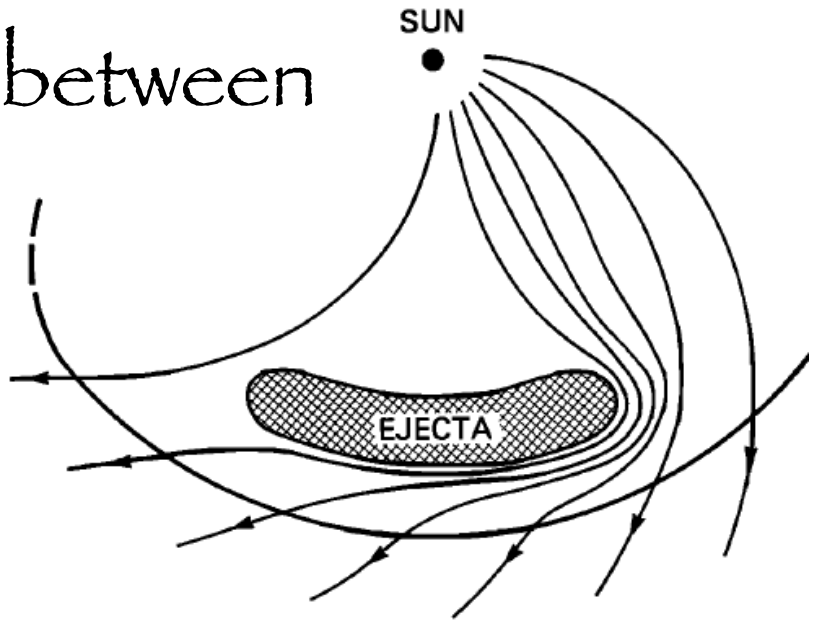

Opher v2, ch. 7


(a)
Solar wind
CME-Sheath
$B_{flux}$
Sun
CME
$B_{SW}$
CME-Pause
Shock
(b)
Interstellar wind
Solar wind
$B_{SW}$
Termination Shock
Heliosheath
Heliopause
$B_{ISM}$
Bow Shock
(c)
$B_{SW}$
Magnetosheath
Solar wind
Neutral sheet
Magnetopause
Bow Shock

# Shocks on Earth

# Shocks are…

- Caused by a collision between 2 different plasmas




- A hydrodynamic surprise:
  one plasma is not ``expecting'' the collision

# Shocks in Hydrodynamics

mass $$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v}\right]$$

momentum $$\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v}\mathbf{v} + p\vec{\vec{\mathrm{I}}} + \vec{\vec{\sigma}}\right]$$

Viscosity, exchanges thermal & kinetic energy

energy $$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \frac{p}{\gamma - 1}\right) = -\nabla \cdot \left[\tfrac{1}{2}\rho v^2 \mathbf{v} + \frac{\gamma}{\gamma - 1} p\mathbf{v} + \vec{\vec{\sigma}} \cdot \mathbf{v}\right]$$

kinetic thermal

viscous stress tensor $$\sigma_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \tfrac{2}{3}\delta_{ij}\nabla \cdot \mathbf{v}\right)$$

mass

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left[ \rho \mathbf{v} \right]$$

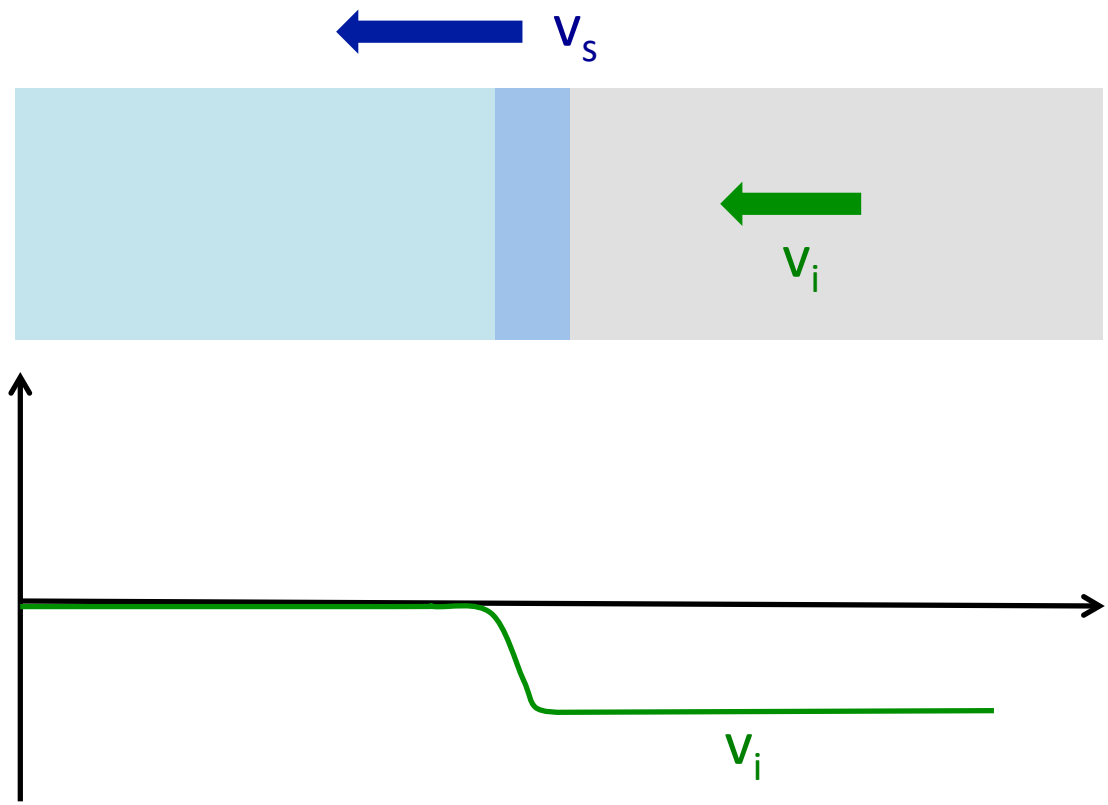
momentum

$$\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot \left[ \rho \mathbf{v}\mathbf{v} + p\vec{\mathbf{I}} + \vec{\sigma} \right]$$

$$\sigma_{ij} = \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \tfrac{2}{3}\delta_{ij} \nabla \cdot \mathbf{v} \right)$$

energy

$$\frac{\partial}{\partial t}\left( \tfrac{1}{2}\rho v^2 + \frac{p}{\gamma - 1} \right) = -\nabla \cdot \left[ \tfrac{1}{2}\rho v^2 \mathbf{v} + \frac{\gamma}{\gamma - 1} p\mathbf{v} + \vec{\sigma} \cdot \mathbf{v} \right]$$



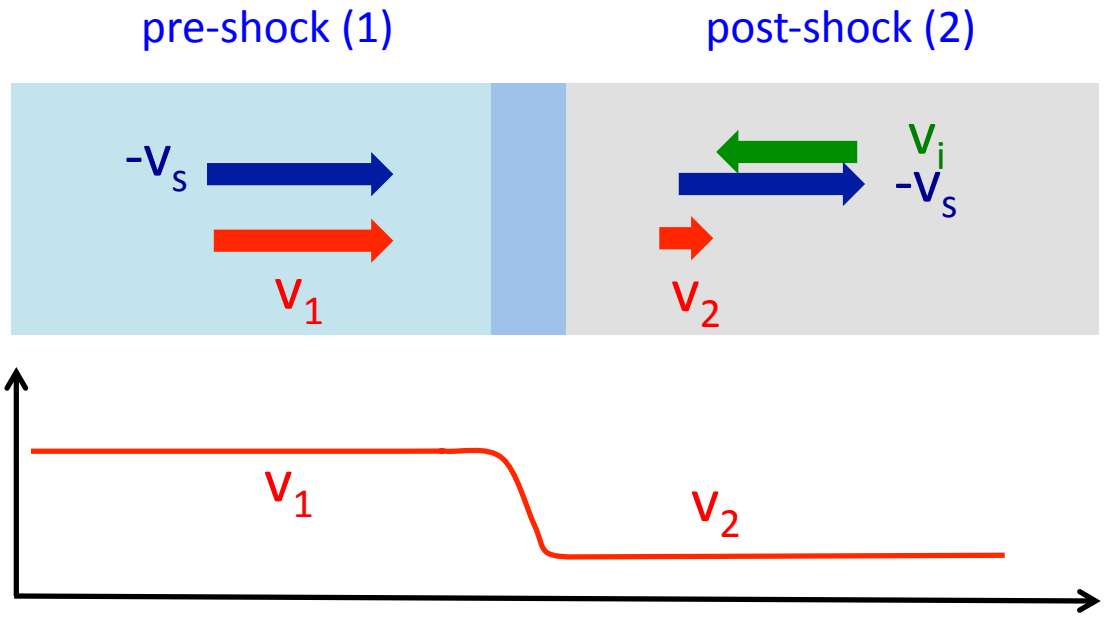

collision

mass $\frac{\partial \rho}{\partial t} = -\nabla \cdot [\rho \mathbf{v}]$

momentum $\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v}\mathbf{v} + p\vec{\mathbf{I}} + \vec{\sigma}\right]$

$$\sigma_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \tfrac{2}{3}\delta_{ij}\nabla \cdot \mathbf{v}\right)$$

energy $\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{p}{\gamma - 1}\right) = -\nabla \cdot \left[\frac{1}{2}\rho v^2 \mathbf{v} + \frac{\gamma}{\gamma - 1} p\mathbf{v} + \vec{\sigma} \cdot \mathbf{v}\right]$




transform to ref. frame co-moving w/ shock **- shock frame**

# Assume

1. Solution is stationary in shock frame

2. Regions 1 & 2 are uniform:

$\rho_1$ $v_1$ $p_1$ $\rho_2$ $v_2$ $p_2$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right]$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} + p\vec{\mathbf{I}} + \vec{\sigma}\right]$$

$$0 = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2\mathbf{v} + \frac{\gamma}{\gamma-1}p\mathbf{v} + \vec{\sigma}\cdot\mathbf{v}\right]$$

jump

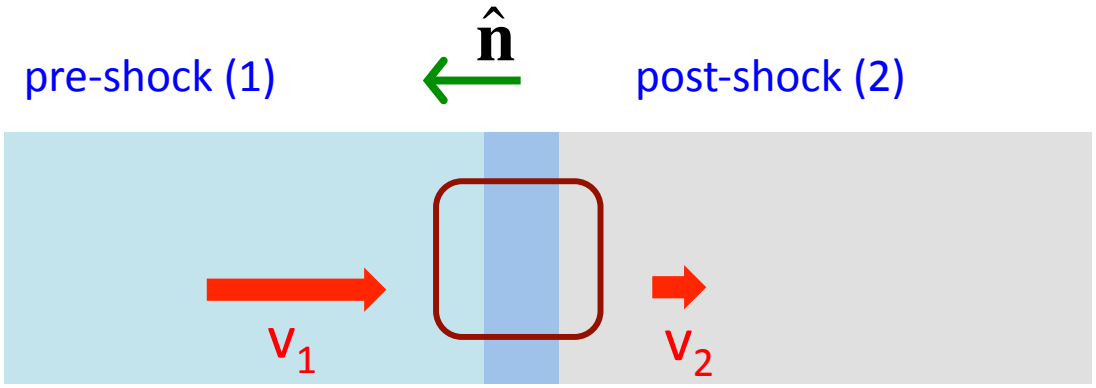
$$[[f]] = f_2 - f_1$$

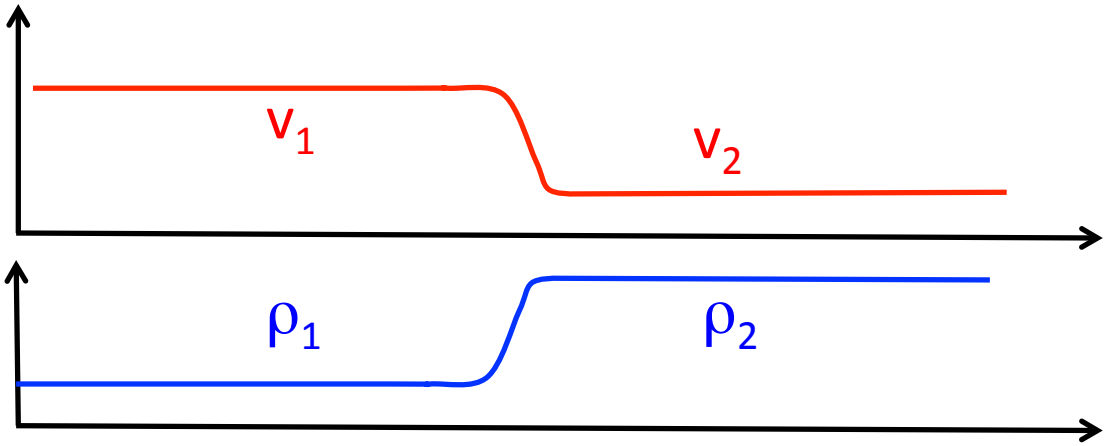
$$0 = [[\rho v_n]]$$

$$0 = [[\rho v_n \mathbf{v} + p\hat{\mathbf{n}}]]$$

$$0 = [[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1}pv_n]]$$

**Jump conditions**
(Rankine-Hugoniot)




2. Regions 1 & 2 are uniform:

$$\sigma_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \tfrac{2}{3}\delta_{ij}\nabla\cdot\mathbf{v}\right) = 0$$

$$0 = [[\rho v_n]]$$

$$0 = [[\rho v_n \mathbf{v} + p\hat{\mathbf{n}}]]$$

$$0 = [[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1} p v_n]]$$

$$\rho v_n = \text{const.}$$

$$\rho v_n [[\mathbf{v}]] + [[p]]\hat{\mathbf{n}} = 0$$

$$\rho v_n [[\tfrac{1}{2} v^2 + \tfrac{\gamma}{\gamma-1}(p/\rho)]] = 0$$

**Case 1:** $\rho v_n = 0$ - no flow across interface

$$0 = [[\rho v_n]]$$

$$0 = [[\rho v_n \mathbf{v} + p\hat{\mathbf{n}}]]$$

$$0 = [[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1} p v_n]]$$

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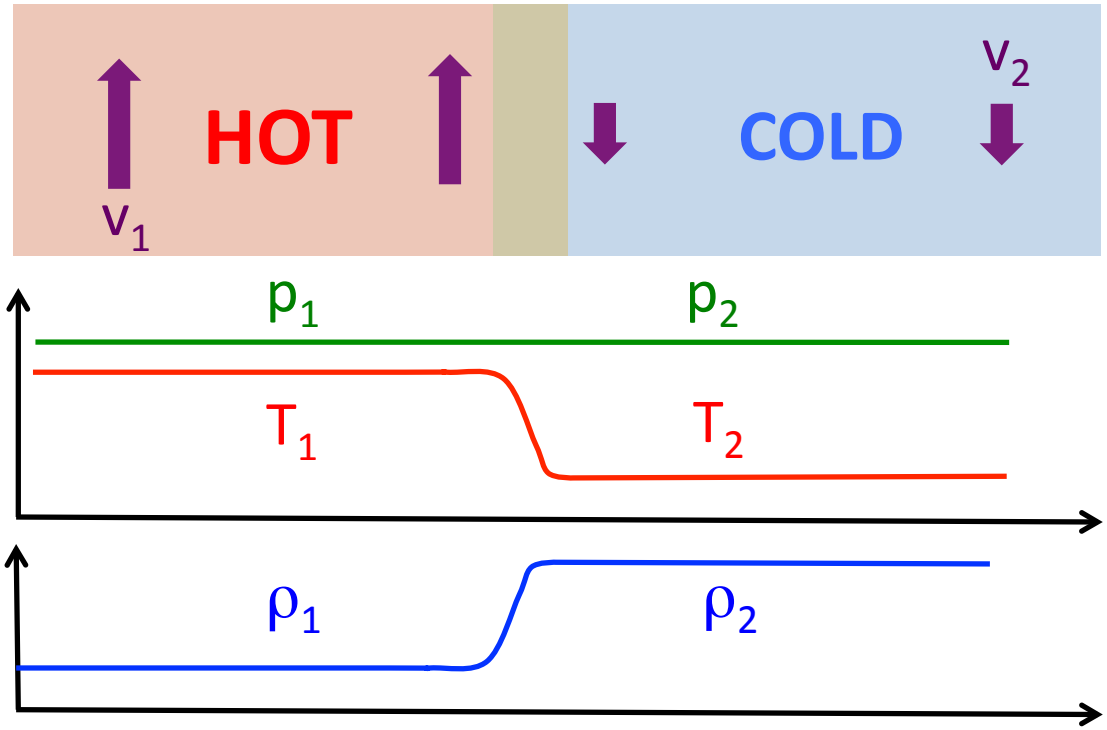
**Case 1:** $\rho v_n = 0$ - no flow across interface

➔ $p_2 = p_1$ pressure balance




## Contact discontinuity

related to linear waves w/ $\omega=0$

entropy mode: $[[\rho]] \neq 0$

shear mode: $[[v_t]] \neq 0$

$$0 = [[\rho v_n]]$$

$$0 = [[\rho v_n \mathbf{v} + p\hat{\mathbf{n}}]] \qquad 0 = [[\rho v_n^2 + p]]$$

$$0 = [[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1} p v_n]]$$

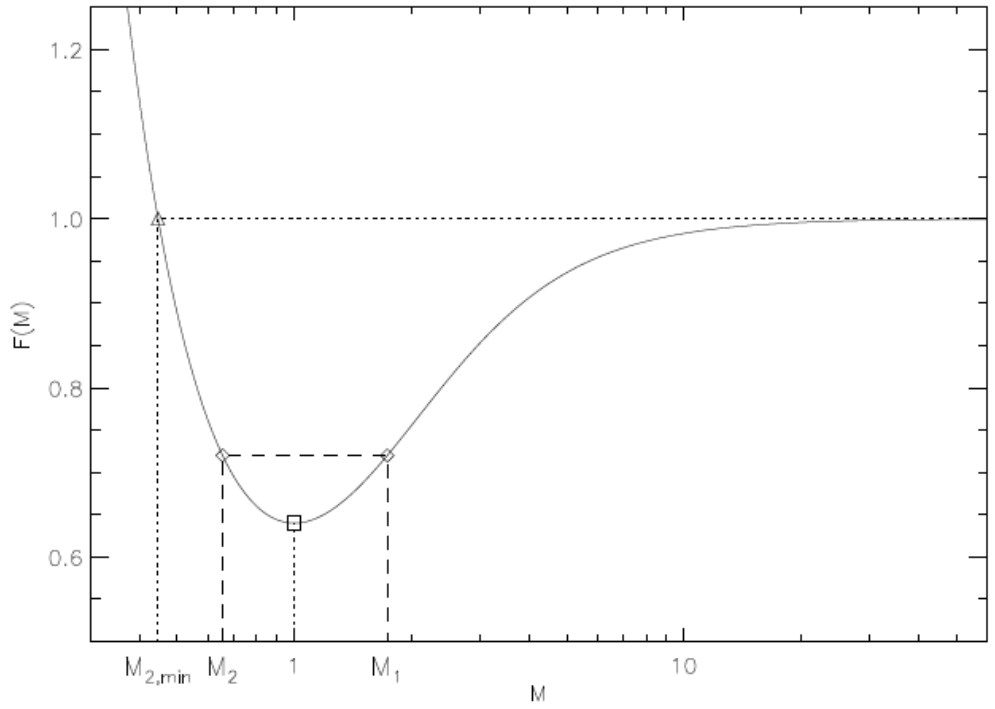
## Case 2: $\rho v_n = \text{const.} \neq 0$

$$\rho v_n \, [[\mathbf{v}_\perp]] = 0$$

$$\Rightarrow \quad \mathbf{v}_{\perp,1} = \mathbf{v}_{\perp,2} = \mathbf{v}_\perp$$

transform away

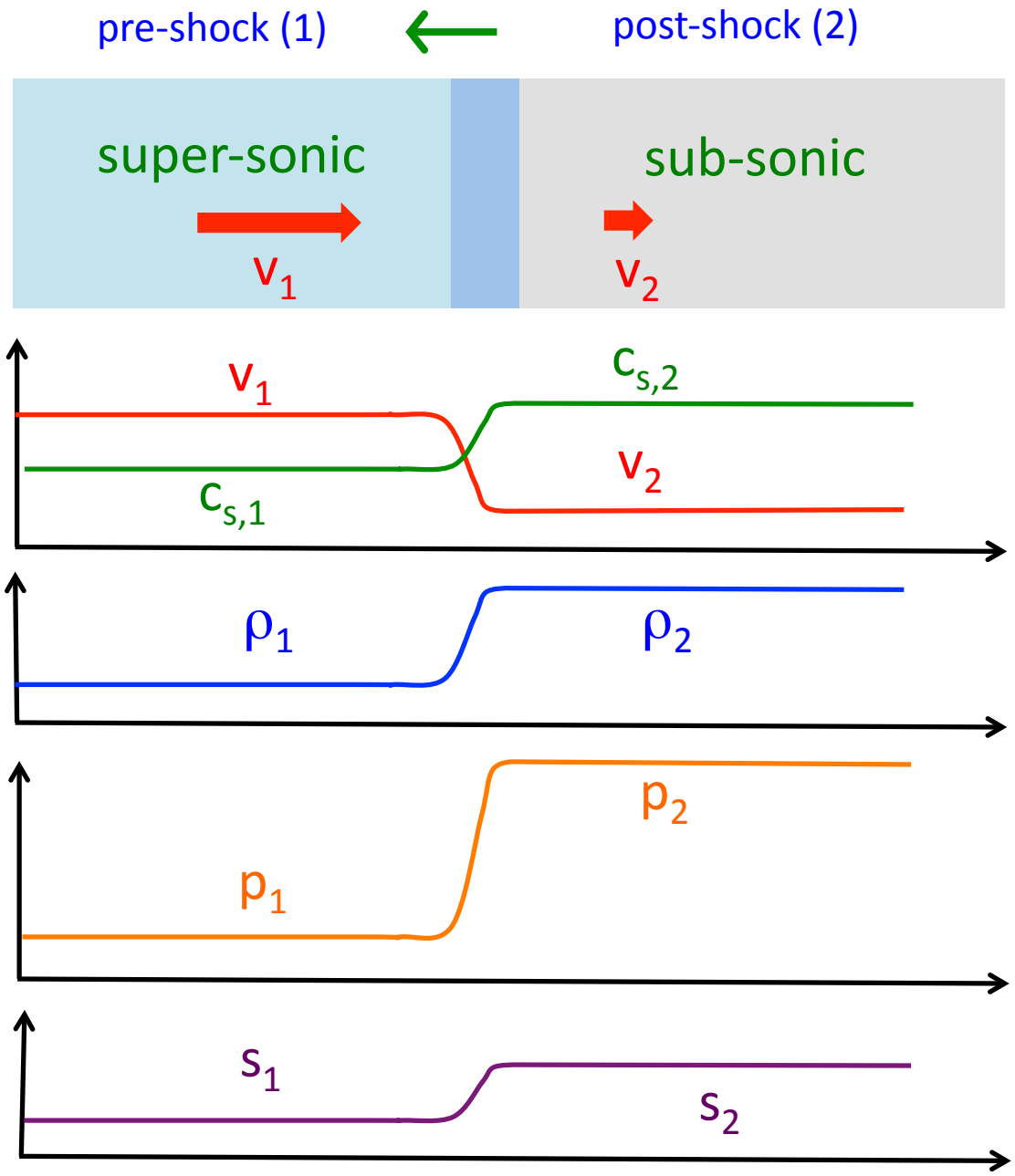
$$\frac{(\rho v^2 + p)^2}{2(\rho v)\left(\frac{1}{2}\rho v^3 + \frac{\gamma}{\gamma-1} p v\right)} = \frac{\gamma - 1}{\gamma^2} \frac{(\gamma M^2 + 1)^2}{M^2\left[(\gamma-1)M^2 + 2\right]} = F(M)$$




$$M = \frac{v}{\sqrt{\gamma p / \rho}}$$ Mach number

$$F(M_1) = F(M_2)$$

$$M_2 = F^{-1}\left(F(M_1)\right)$$




$$M_2 = F^{-1}\left(F(M_1)\right)$$

$$M_2 = \sqrt{\frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-\gamma+1}}$$

$$\frac{v_2}{v_1} = \frac{(\gamma-1)M_1^2+2}{(\gamma+1)M_1^2}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2-\gamma+1}{\gamma+1}$$

$$s_2 - s_1 = \ln\left(\frac{p_2}{p_1}\right) - \gamma\ln\left(\frac{\rho_2}{\rho_1}\right)$$

# Hypersonic limit

$M_1 \gg 1$ $\qquad$ $\gamma = 5/3$

$$M_2 = \sqrt{\frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-\gamma+1}} \qquad M_2 \to \sqrt{\frac{(\gamma-1)}{2\gamma}} = \frac{1}{\sqrt{5}}$$

$$\frac{v_2}{v_1} = \frac{(\gamma-1)M_1^2+2}{(\gamma+1)M_1^2} \qquad \frac{v_2}{v_1} \to \frac{(\gamma-1)}{(\gamma+1)} = \frac{1}{4}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \qquad \frac{\rho_2}{\rho_1} \to \frac{(\gamma+1)}{(\gamma-1)} = 4$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2-\gamma+1}{\gamma+1} \qquad \frac{p_2}{p_1} \to \frac{2\gamma}{\gamma+1}M_1^2 \qquad p_2 \to \tfrac{3}{2} \times \tfrac{1}{2}\rho_1 v_1^2$$

# Entropy's tale

$s_2 \geq s_1$ ➔ pre-shock must be supersonic

➔ fluid goes from super- to sub-sonic

➔ fluid slows down crossing shock

➔ fluid **compresses** crossing shock

viscosity = 0 ➔ $s_2 = s_1$

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \frac{p}{\gamma - 1}\right) = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2 \mathbf{v} + \frac{\gamma}{\gamma - 1} p\mathbf{v} + \vec{\sigma}\cdot\mathbf{v}\right]$$

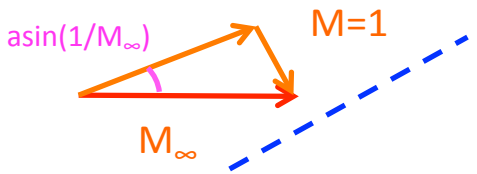
viscosity does not appear in jump conditions (conservation laws) but it is necessary for transition from 1 to 2

Q: what goes wrong if we set $\mu = 0$ ?

Q: is that diff. from taking $\mu \to 0$ ?

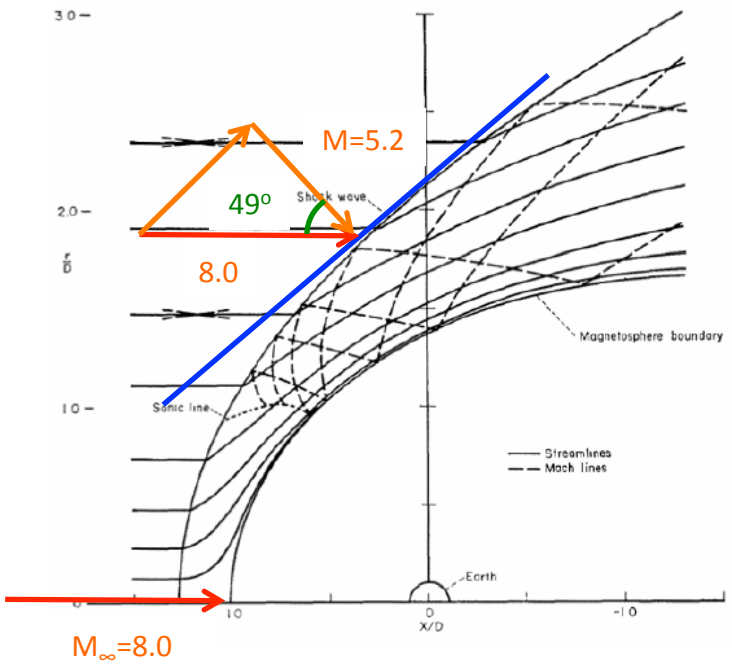
$$\mu \frac{\partial v}{\partial x} \to \text{const.}$$

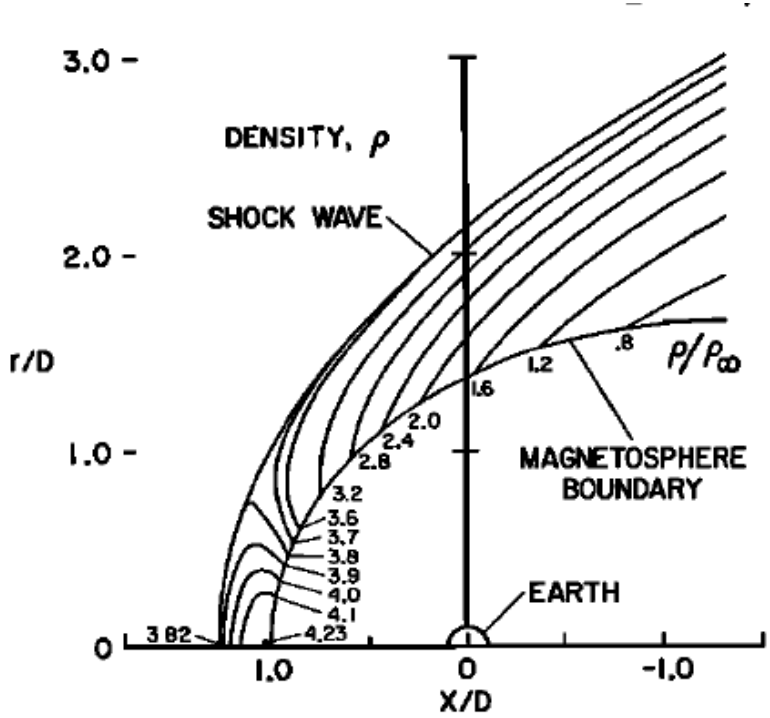
# A nearby shock




Mach cone

$M_\infty=8 \quad \gamma = 5/3$



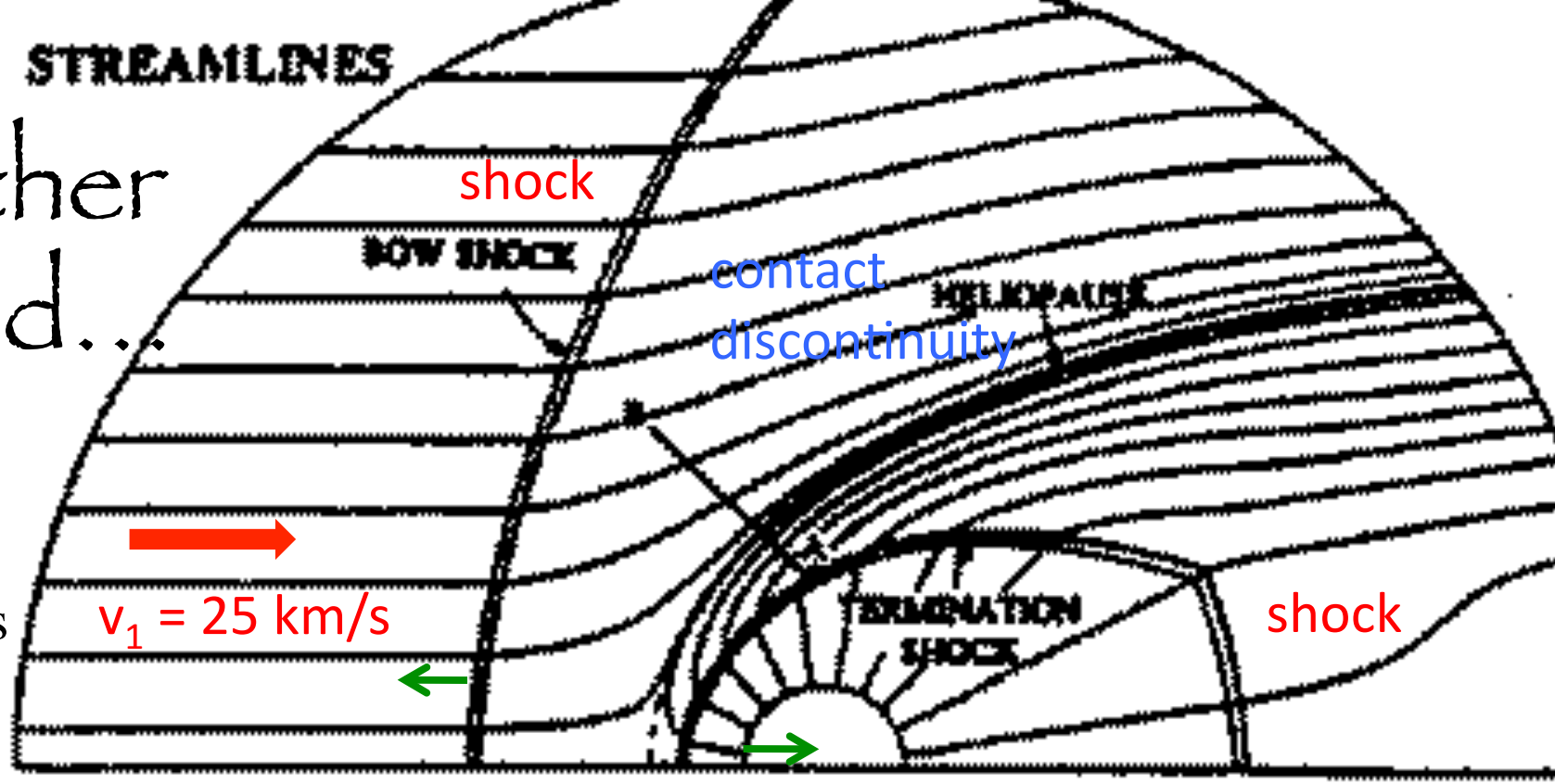

Spreiter, Summers & Alksne 1966

# Farther afield...

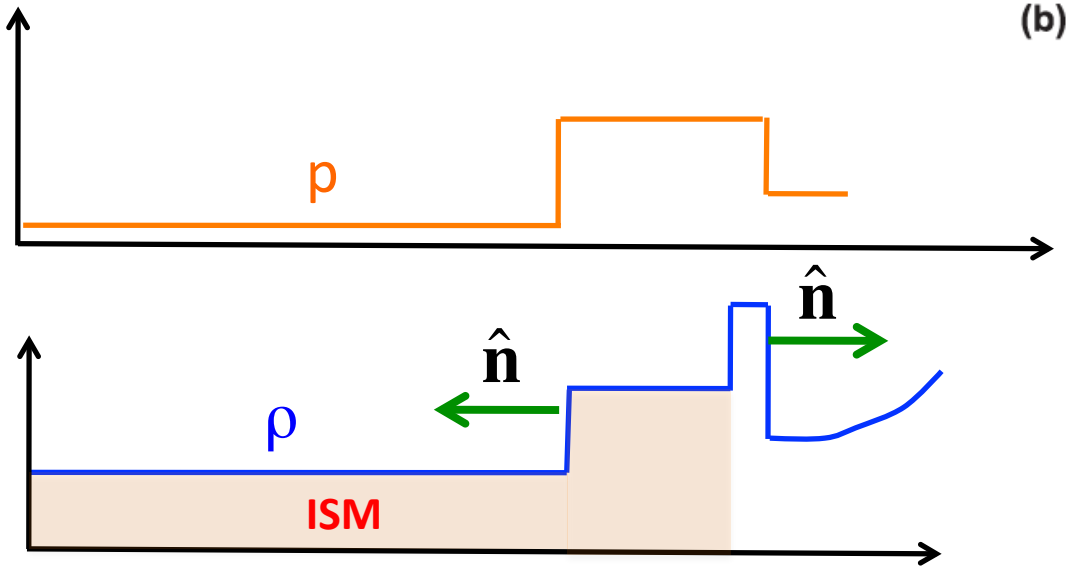
(the picture before May)

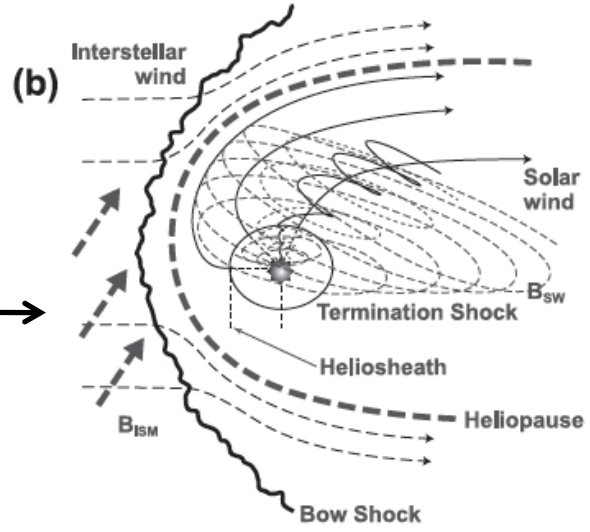
$T_1 = 7000$ K

$c_{s1} = 14$ km/s



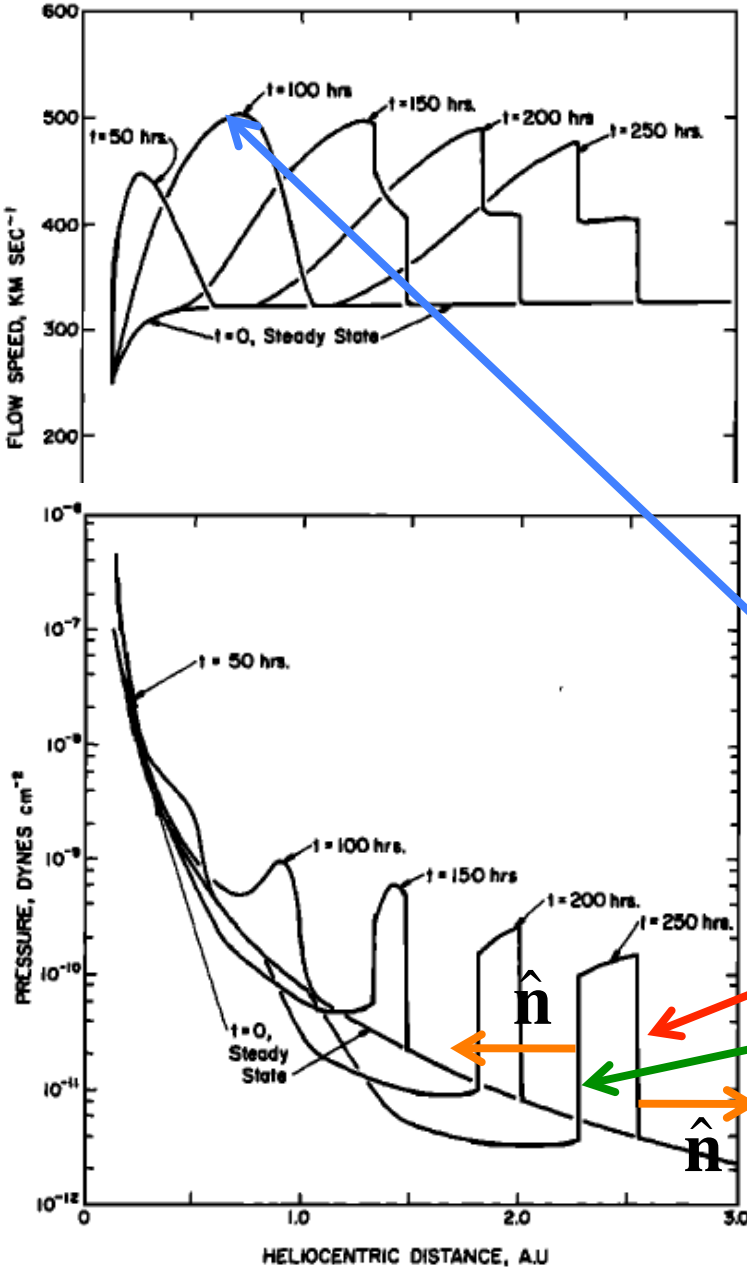

Axford & Suess 1994

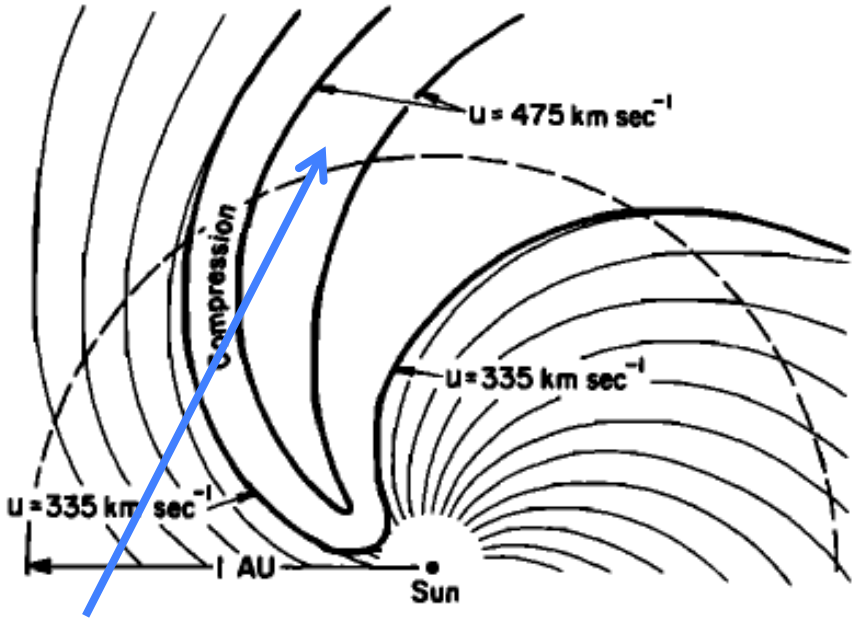



Opher v2, ch. 7

Hundhausen 1973 (v.3, ch.8)




High-speed stream colliding with slower wind creates 2 shocks:

Forward shock

Reverse shock

Both move outward ($v_s$>0)

Q: which way is $\hat{n}$ for each?

# Shocks in MHD

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v}\right]$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v}\mathbf{v} - \frac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \frac{1}{8\pi}B^2\right)\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma}\right]$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{1}{\gamma - 1}p + \frac{1}{8\pi}B^2\right) = -\nabla \cdot \left[\frac{1}{2}\rho v^2 \mathbf{v} + \frac{\gamma}{\gamma - 1}p\mathbf{v} - \frac{1}{4\pi}(\mathbf{v} \times \mathbf{B}) \times \mathbf{B} + \overleftrightarrow{\sigma} \cdot \mathbf{v}\right]$$

kinetic thermal magnetic

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\mathbf{v} \times \mathbf{B}\right]$$

$$0 = \nabla \cdot \mathbf{B}$$

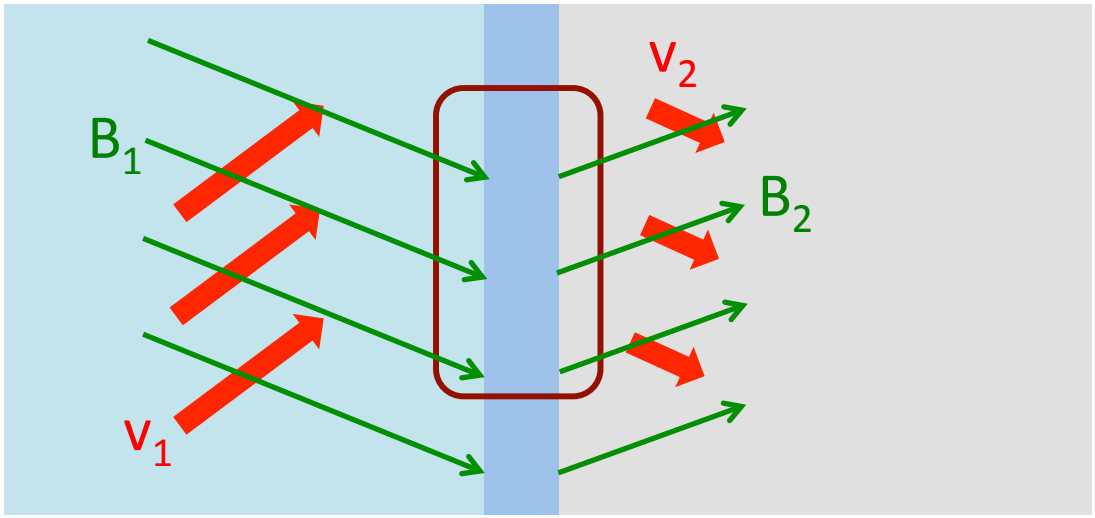
$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right]$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma}\right]$$

$$0 = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2\mathbf{v} + \tfrac{\gamma}{\gamma-1}p\mathbf{v} - \tfrac{1}{4\pi}(\mathbf{v}\times\mathbf{B})\times\mathbf{B} + \overleftrightarrow{\sigma}\cdot\mathbf{v}\right]$$

$$0 = -\nabla\times\left[\mathbf{v}\times\mathbf{B}\right]$$

$$0 = \nabla\cdot\mathbf{B}$$



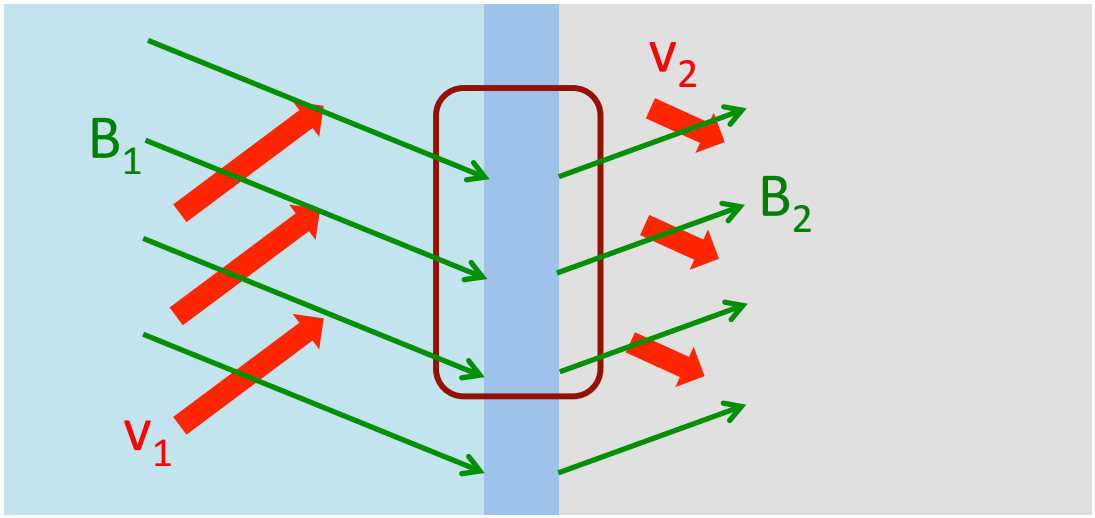
$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right] \qquad 0 = [[\rho v_n]] \qquad \rho v_n = \text{const.}$$

$$0 = \nabla\cdot\mathbf{B} \qquad 0 = [[B_n]] \qquad B_n = \text{const.}$$

$$0 = -\nabla\cdot\left[\rho\mathbf{vv} - \tfrac{1}{4\pi}\mathbf{BB} + \left(p + \tfrac{1}{8\pi}B^2\right)\vec{\vec{\mathrm{I}}} + \vec{\vec{\sigma}}\right]$$

$$\rho v_n\,[[\mathbf{v}]] - B_n[[\mathbf{B}]]/4\pi + [[p + B^2/8\pi]]\hat{\mathbf{n}} = 0$$

$$\rho v_n\,[[\mathbf{v}_\perp]] = B_n[[\mathbf{B}_\perp]]/4\pi$$



$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right] \qquad \rho v_n = \text{const.}$$

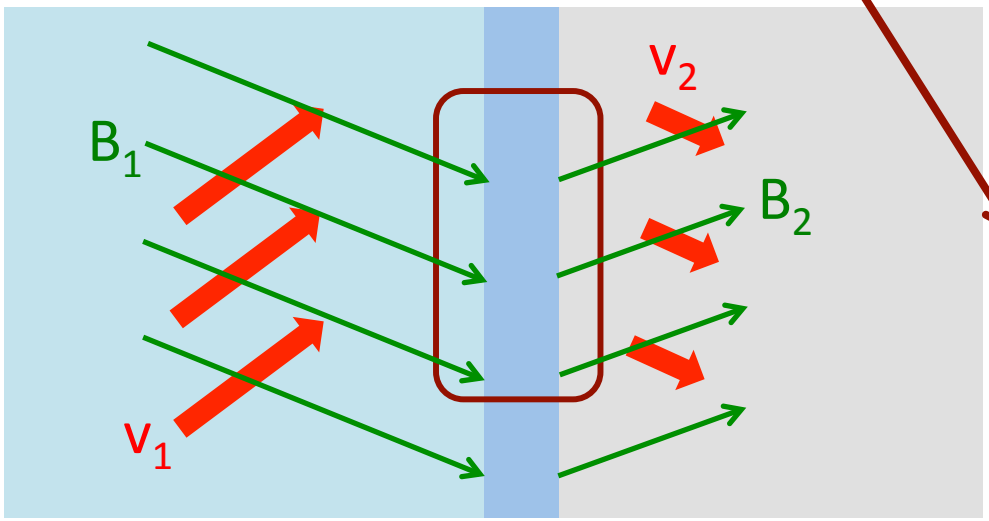
$$0 = \nabla\cdot\mathbf{B} \qquad B_n = \text{const.}$$

$$0 = -\nabla\cdot\left[\rho\mathbf{vv} - \tfrac{1}{4\pi}\mathbf{BB} + \left(p + \tfrac{1}{8\pi}B^2\right)\vec{\vec{\mathrm{I}}} + \vec{\vec{\sigma}}\right]$$

$$\rho v_n [[\mathbf{v}_\perp]] = B_n [[\mathbf{B}_\perp]]/4\pi \qquad 0 = B_n [[\mathbf{B}]]$$

**Case 1:** $\rho v_n = 0$

**Case 2:** $B_n = 0$




**Case 1a:** $B_n = 0$

**Tangential Discontinuity**

**Case 1b:** $[[\,\mathbf{B}\,]] = 0$

**Contact Discontinuity**

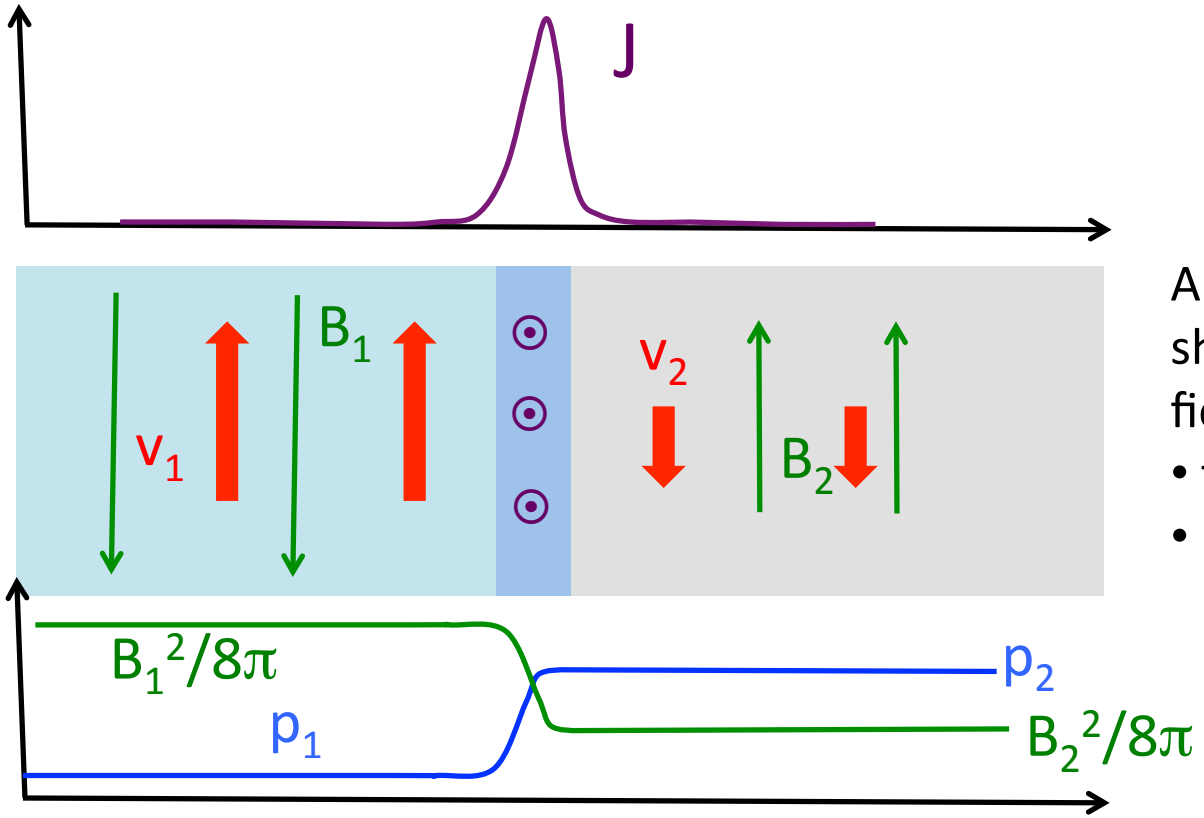
# Case 1a: Tangential Disc.

$v_n = 0, \quad B_n = 0$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right] \qquad \rho v_n = \text{const.}$$

$$0 = \nabla\cdot\mathbf{B} \qquad B_n = \text{const.}$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\vec{\vec{\mathrm{I}}} + \vec{\vec{\sigma}}\right] \qquad [[p + B^2/8\pi]]\hat{\mathbf{n}} = 0$$




An equilibrium current sheet separating uniform **B** fields w/ different angles.

- total pressure balance
- possible shear

# Case 1b: Contact Disc.

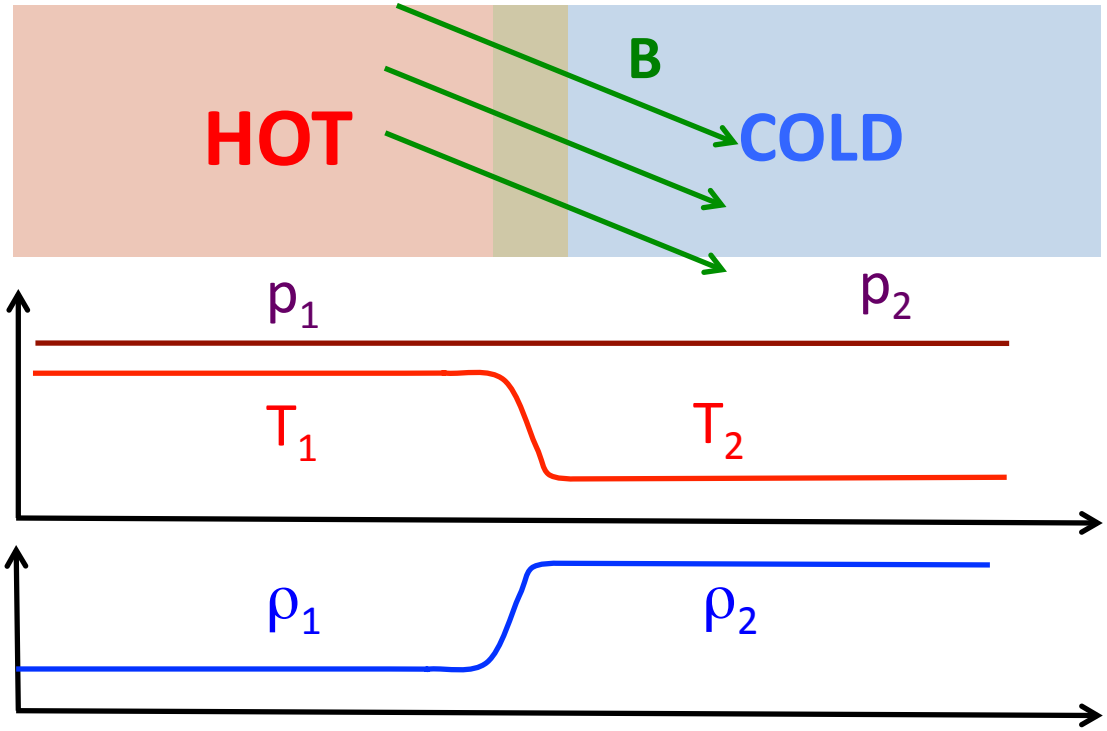
$v_n = 0$, $[[\,\mathbf{B}\,]] = 0$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right] \qquad \rho v_n = \text{const.}$$

$$0 = \nabla\cdot\mathbf{B} \qquad B_n = \text{const.}$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\vec{\vec{\mathrm{I}}} + \vec{\vec{\sigma}}\right] \qquad [[p]]\hat{\mathbf{n}} = 0$$

$$0 = -\nabla\times\left[\mathbf{v}\times\mathbf{B}\right] \qquad [[\mathbf{v}_\perp]] = 0$$




related to linear waves w/ $\omega=0$

entropy mode: $[[\rho]]\neq 0$

~~shear mode: $[[v_t]]\neq 0$~~

# Case 2: perpendicular shock

$B_n = 0, \quad v_n \neq 0$

$$\rho v_n \, [[\mathbf{v}_\perp]] = B_n [[\mathbf{B}_\perp]]/4\pi$$

$$\Rightarrow \quad \mathbf{v}_{\perp,1} = \mathbf{v}_{\perp,2} = \mathbf{v}_\perp$$

transform away

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\vec{\mathrm{I}} + \vec{\sigma}\right]$$

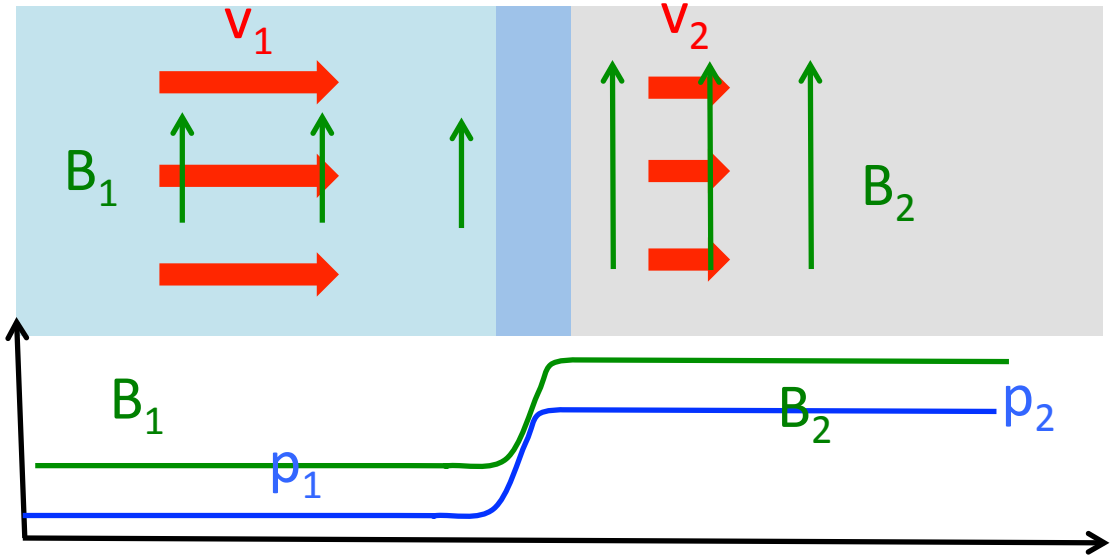
$$[[\rho v^2 + p + B^2/8\pi]] = 0$$

$$0 = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2\mathbf{v} + \tfrac{\gamma}{\gamma-1}p\mathbf{v} - \tfrac{1}{4\pi}(\mathbf{v}\times\mathbf{B})\times\mathbf{B} + \vec{\sigma}\cdot\mathbf{v}\right]$$

$$[[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1}p v_n + \tfrac{1}{4\pi}B^2 v_n]] = 0$$

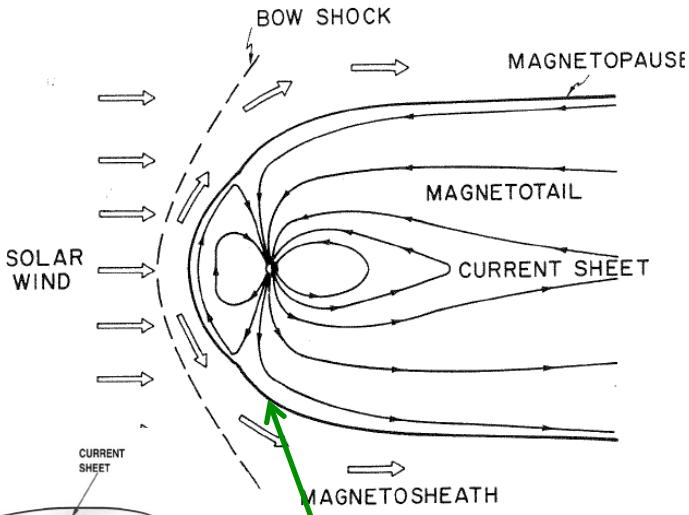
$$0 = -\nabla\times[\mathbf{v}\times\mathbf{B}]$$

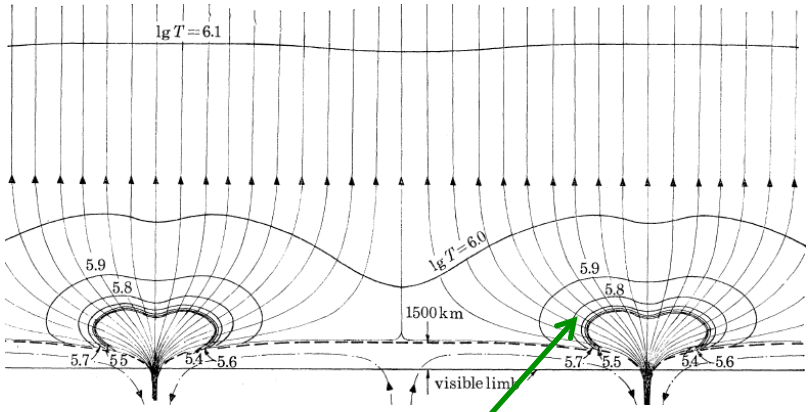
$$[[B v_n]] = 0$$



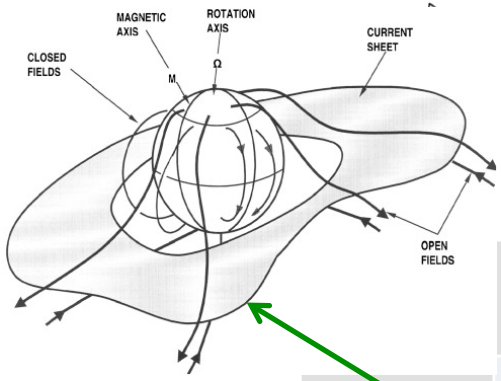

HD shock w/ 2 ``fluids''

- plasma: $\gamma=5/3$
- mag. field: $\gamma=2$

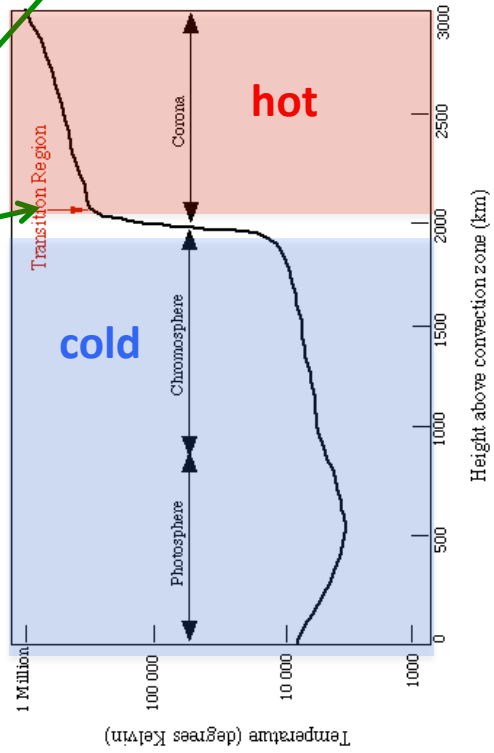



Gabriel 1976




| | $B_n = 0$ | $B_n \neq 0$ |
|---|---|---|
| $v_n = 0$ | Tangential discontinuity | Contact discontinuity |
| $v_n \neq 0$ | Perpendicular shock | |



# Case 3: General MHD shock

$B_n \neq 0$, $v_n \neq 0$

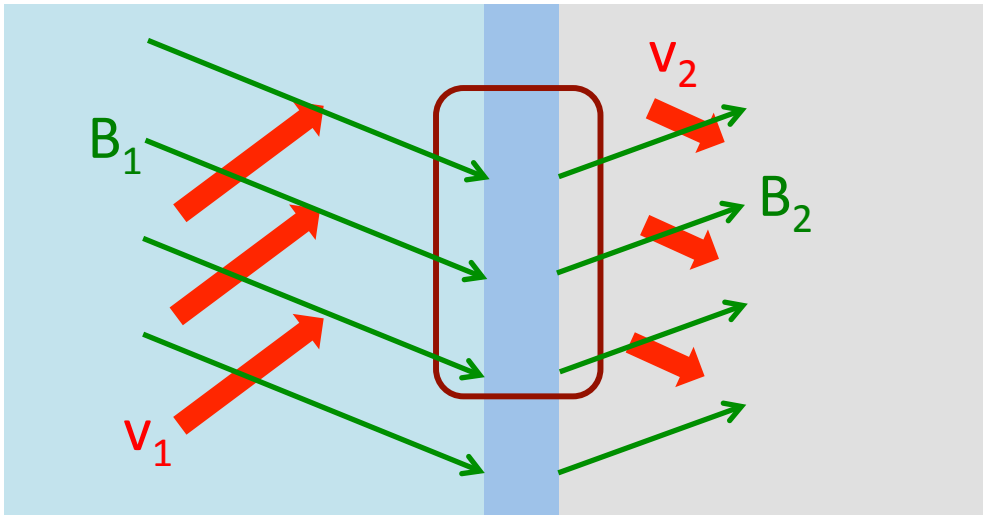
$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right] \qquad \rho v_n = \text{const.} \neq 0$$

$$0 = \nabla\cdot\mathbf{B} \qquad B_n = \text{const.} \neq 0$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\vec{\vec{I}} + \vec{\vec{\sigma}}\right]$$

$$0 = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2\mathbf{v} + \tfrac{\gamma}{\gamma-1}p\mathbf{v} - \tfrac{1}{4\pi}(\mathbf{v}\times\mathbf{B})\times\mathbf{B} + \vec{\vec{\sigma}}\cdot\mathbf{v}\right]$$

$$0 = -\nabla\times\left[\mathbf{v}\times\mathbf{B}\right]$$




$$\rho v_n\,[[\mathbf{v}_\perp]] = B_n[[\mathbf{B}_\perp]]/4\pi$$

$$\Rightarrow \quad \mathbf{v}_{\perp,1} \neq \mathbf{v}_{\perp,2}$$

**cannot** transform away $\mathbf{v}_\perp$

**can** transform to deHoffman-Teller frame

$$\mathbf{v}_{\text{dHT}} = \mathbf{v}_{\perp,1} - \frac{v_{n,1}}{B_{n,1}}\mathbf{B}_{\perp,1}$$

**In dH-T frame:**

- $\mathbf{v}_1$ || $\mathbf{B}_1$
- $\mathbf{E}_1 = \mathbf{v}_1 \times \mathbf{B}_1 = 0$
- $\mathbf{E}_2 = \mathbf{v}_2 \times \mathbf{B}_2 = 0$
- $\mathbf{v}_2$ || $\mathbf{B}_2$
- flow is parallel to field lines on both sides

# Case 3: General MHD shock

$B_n \neq 0$, $v_n \neq 0$

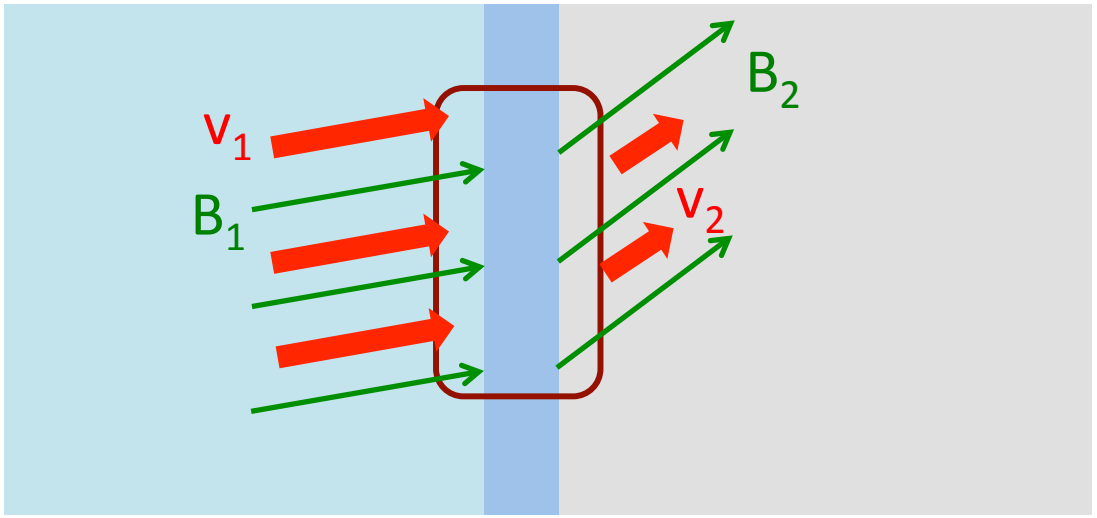
$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right] \qquad \rho v_n = \text{const.} \neq 0$$

$$0 = \nabla\cdot\mathbf{B} \qquad B_n = \text{const.} \neq 0$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\vec{\vec{\mathrm{I}}} + \vec{\vec{\sigma}}\right]$$

$$0 = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2\mathbf{v} + \tfrac{\gamma}{\gamma-1}p\mathbf{v} - \tfrac{1}{4\pi}(\mathbf{v}\times\mathbf{B})\times\mathbf{B} + \vec{\vec{\sigma}}\cdot\mathbf{v}\right]$$

$$0 = -\nabla\times\left[\mathbf{v}\times\mathbf{B}\right]$$




$$\rho v_n\,[[\mathbf{v}_\perp]] = B_n[[\mathbf{B}_\perp]]/4\pi$$

$$\Rightarrow \quad \mathbf{v}_{\perp,1} \neq \mathbf{v}_{\perp,2}$$

**cannot** transform away $\mathbf{v}_\perp$

**can** transform to deHoffman-Teller frame

$$\mathbf{v}_{\mathrm{dHT}} = \mathbf{v}_{\perp,1} - \frac{v_{n,1}}{B_{n,1}}\mathbf{B}_{\perp,1}$$

**In dH-T frame:**

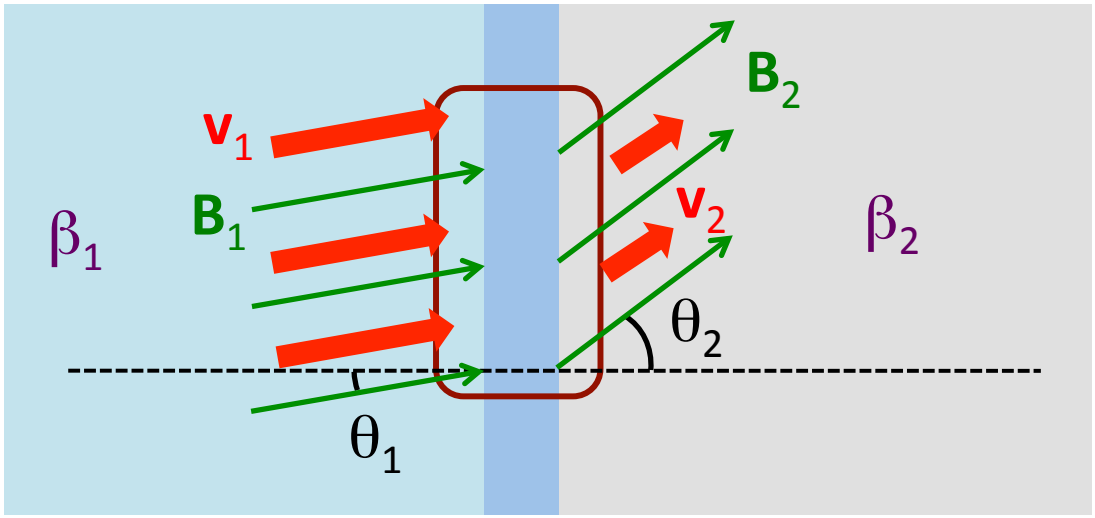
- $\mathbf{v}_1$ || $\mathbf{B}_1$
- $\mathbf{E}_1 = \mathbf{v}_1 \times \mathbf{B}_1 = 0$
- $\mathbf{E}_2 = \mathbf{v}_2 \times \mathbf{B}_2 = 0$
- $\mathbf{v}_2$ || $\mathbf{B}_2$
- flow is parallel to field lines on both sides

## Jump conditions in de Hoffman-Teller frame

$$\rho v_n\,[[\mathbf{v}_\perp]] - B_n[[\mathbf{B}_\perp]]/4\pi = 0$$

$$[[\rho v_n^2 + p + B_\perp^2/8\pi]] = 0$$

$$[[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1} p v_n]] = 0$$




## Shock characterized by 3 dimensionless pre-shock parameters:

$$M_{1A} = \frac{|\mathbf{v}_1|}{|\mathbf{B}_1|/\sqrt{4\pi\rho_1}} = \frac{v_{1n}}{B_{1n}/\sqrt{4\pi\rho_1}}$$

$$\theta_1 = \tan^{-1}\left(\frac{B_{\perp 1}}{B_{n1}}\right) = \tan^{-1}\left(\frac{v_{\perp 1}}{v_{n1}}\right)$$

$$\beta_1 = \frac{p_1}{B_1^2/8\pi}$$

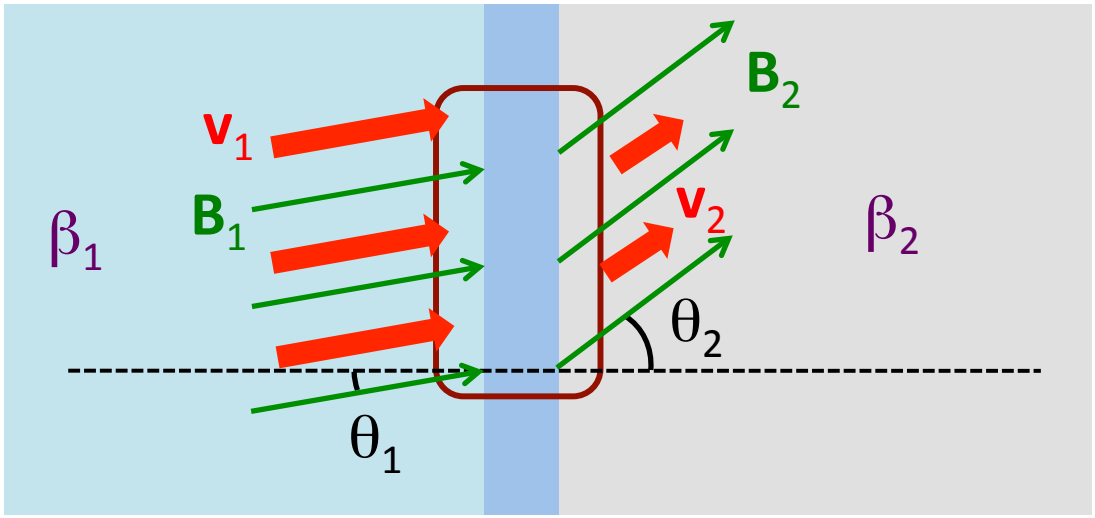
**Use 3 jump. conds to solve for downstream parameters**

$M_{A2}$ $\theta_2$ $\beta_2$

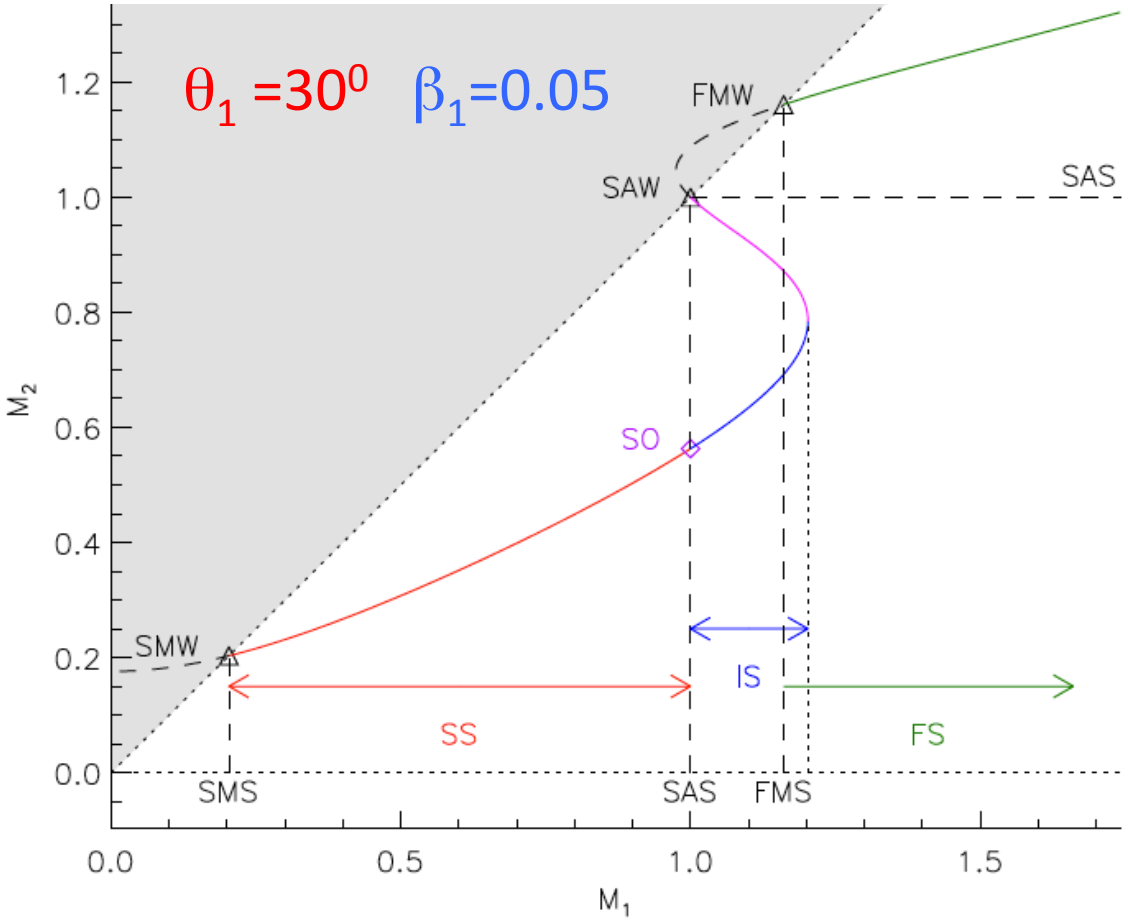
$$\rho v_n\,[[\mathbf{v}_\perp]] - B_n[[\mathbf{B}_\perp]]/4\pi = 0 \qquad\qquad [[(M_A^2 - 1)\tan\theta]] = 0$$

eliminate $\theta_2$ in terms of $\theta_1$, $M_{A1}^2$ and $M_{A2}^2$
etc. until…

$$\begin{aligned} F(M_1, M_2) \;=\; & -(\gamma+1)M_2^6 \;+\; \left[(\gamma-1)M_1^2 + 2(\gamma+1) + \frac{\gamma\beta_1}{\cos^2\theta_1} + \gamma\tan^2\theta_1\right] M_2^4 \\ & - \left\{2(\gamma-1)M_1^2 + \frac{(\gamma+1)+2\gamma\beta_1}{\cos^2\theta_1} + (\gamma-2)M_1^2\tan^2\theta_1\right\} M_2^2 \\ & + \frac{(\gamma-1)M_1^2 + \gamma\beta_1}{\cos^2\theta_1} \;=\; 0 \end{aligned}$$

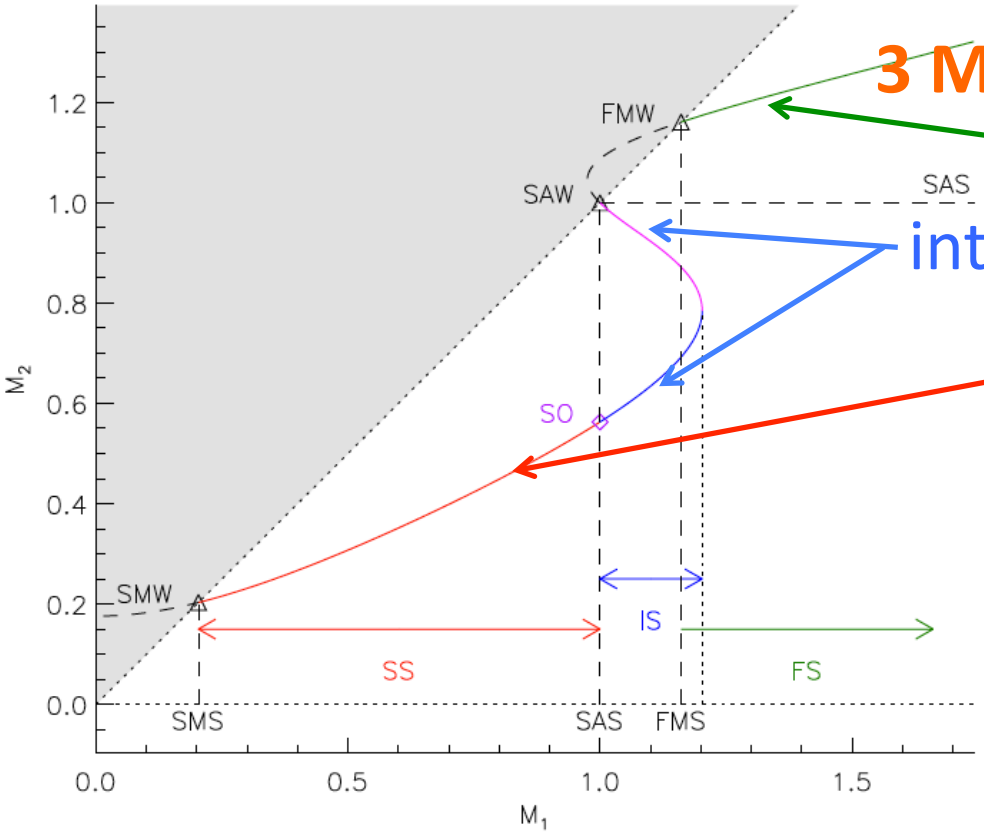


$$F(M_1, M_2) = -(\gamma+1)M_2^6 + \left[(\gamma-1)M_1^2 + 2(\gamma+1) + \frac{\gamma\beta_1}{\cos^2\theta_1} + \gamma\tan^2\theta_1\right] M_2^4$$
$$- \left\{2(\gamma-1)M_1^2 + \frac{(\gamma+1)+2\gamma\beta_1}{\cos^2\theta_1} + (\gamma-2)M_1^2\tan^2\theta_1\right\} M_2^2$$
$$+ \frac{(\gamma-1)M_1^2 + \gamma\beta_1}{\cos^2\theta_1} = 0$$




**NB:**

- $\rho_2/\rho_1 = M_{A1}^2 / M_{A2}^2$
- region $M_{A2} > M_{A1}$ ($\rho_2 < \rho_1$) disallowed by 2nd law thermo.
- $M_{A2} = M_{A1}$ reps. 3 linear waves of MHD
- each shock: flow **faster (slower)** than lin. wave in region **1 (2)**

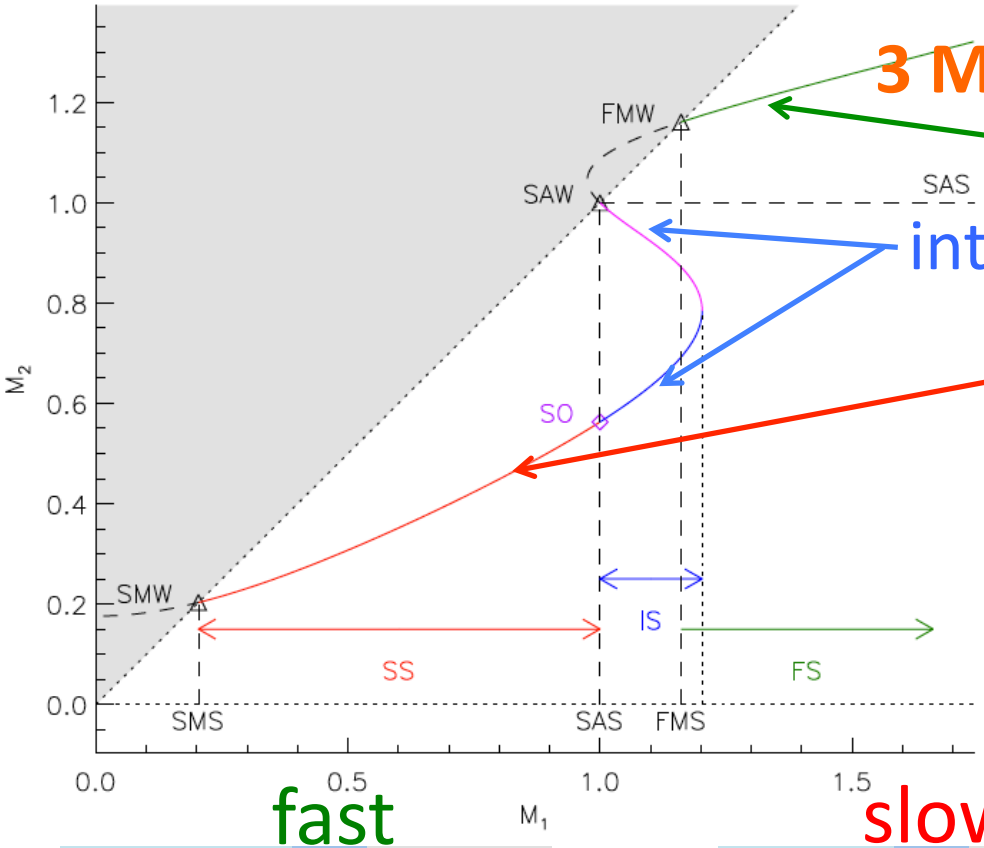



**3 MHD shocks**

fast: $1 < M_{A2} < M_{A1}$

intermediate: $M_{A2} < 1 < M_{A1}$

slow: $M_{A2} < M_{A1} < 1$




## 3 MHD shocks

fast: $1 < M_{A2} < M_{A1}$

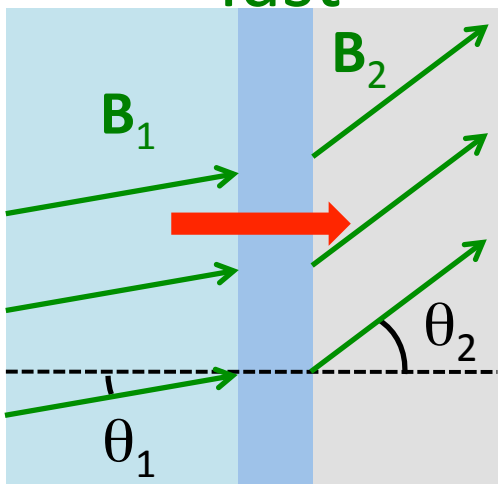
intermediate: $M_{A2} < 1 < M_{A1}$

slow: $M_{A2} < M_{A1} < 1$

$$[[(M_A^2 - 1)\tan\theta]] = 0$$

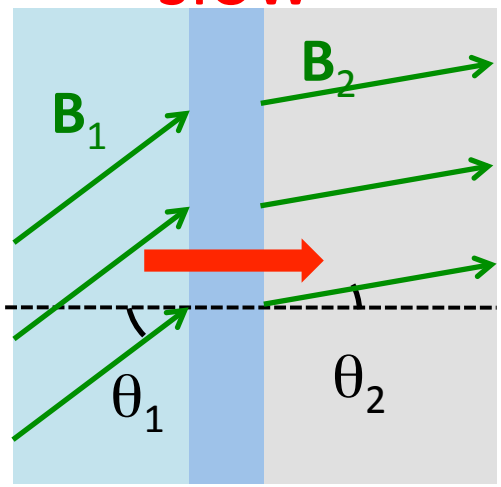
$$\frac{M_{A1}^2 - 1}{M_{A2}^2 - 1} = \frac{\tan\theta_2}{\tan\theta_1} = \frac{B_{\perp 2}}{B_{\perp 1}}$$

### fast



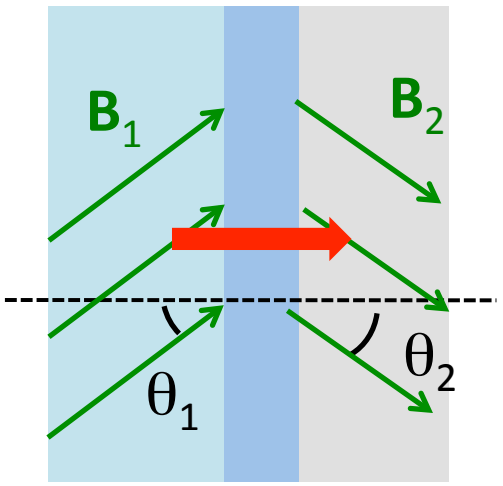

$\theta_2 > \theta_1 > 0$ $B_1 < B_2$

### slow

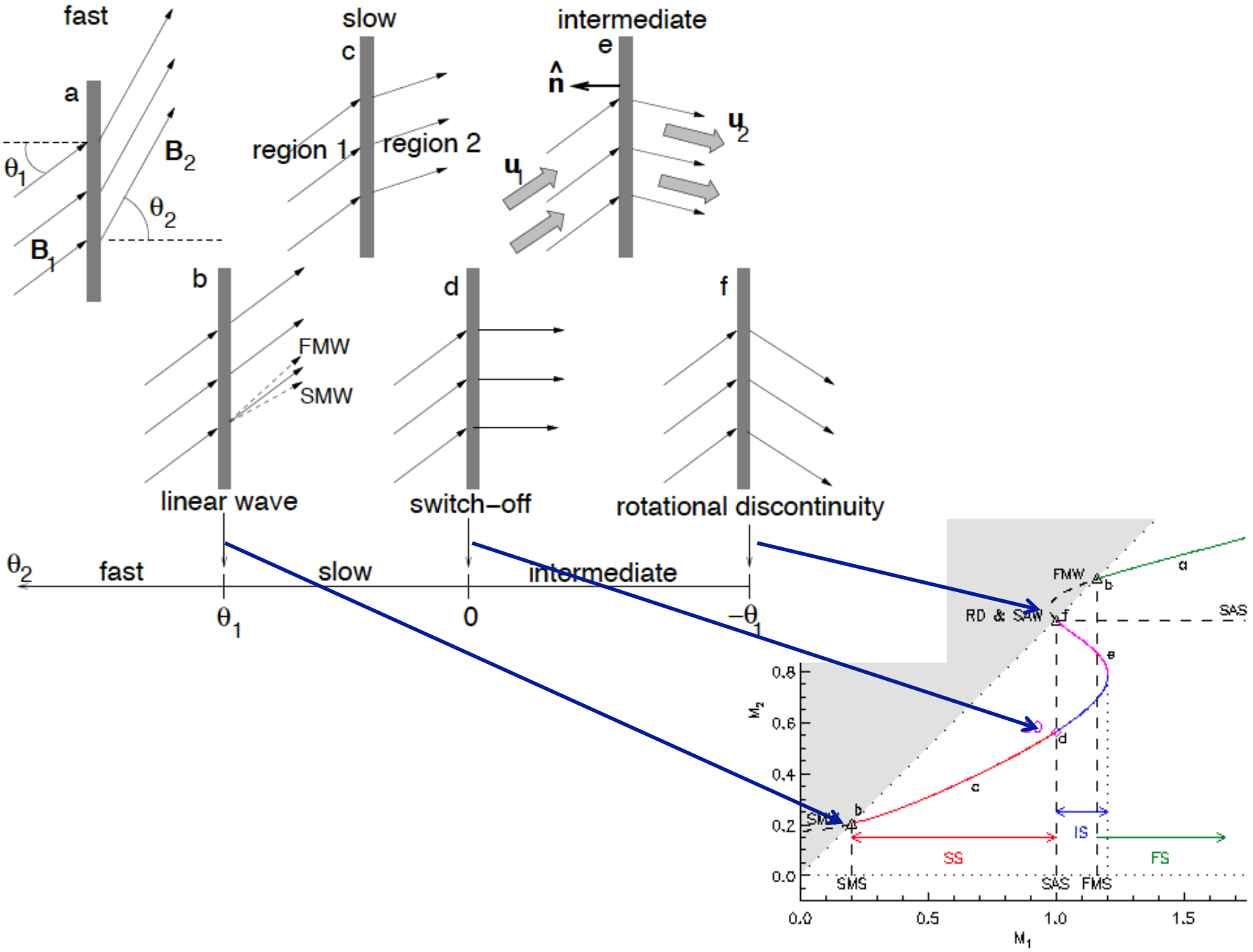


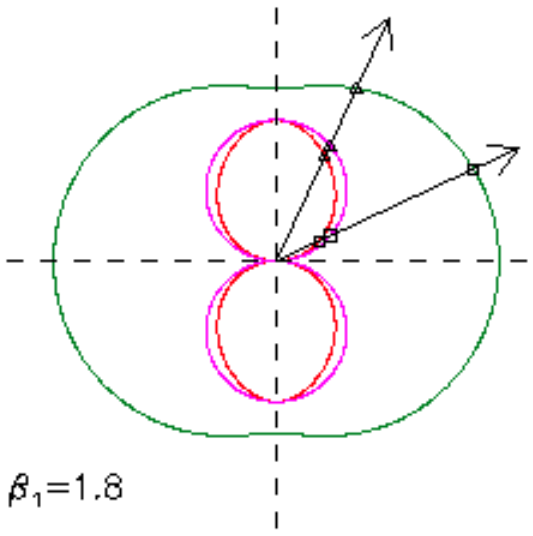

$\theta_1 > \theta_2 > 0$ $B_1 > B_2$

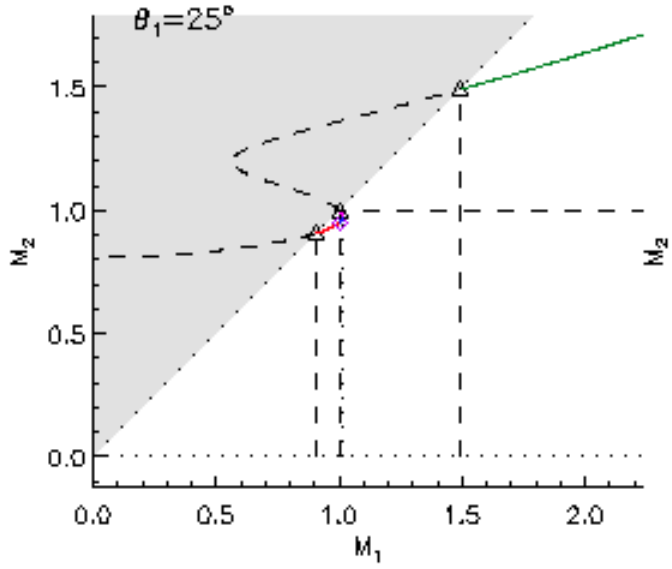
### intermediate

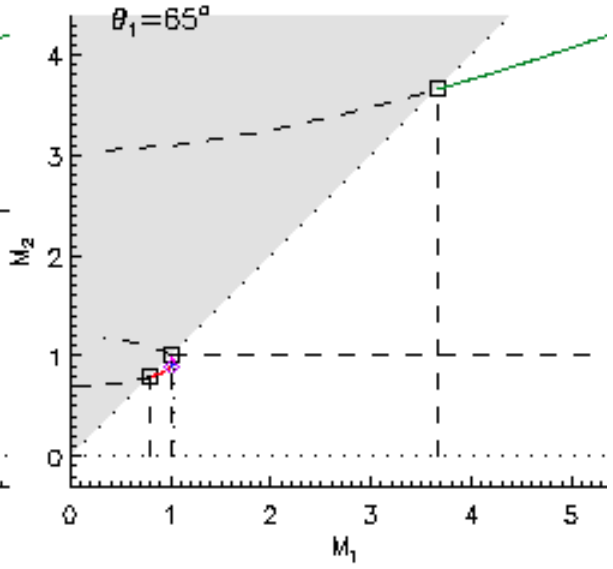


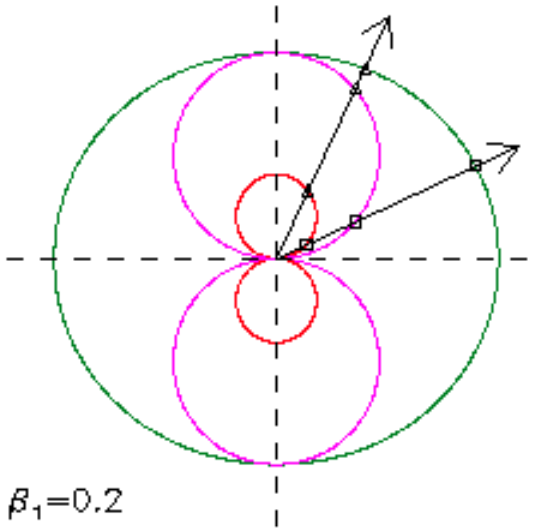

$\theta_1 > 0 > \theta_2$

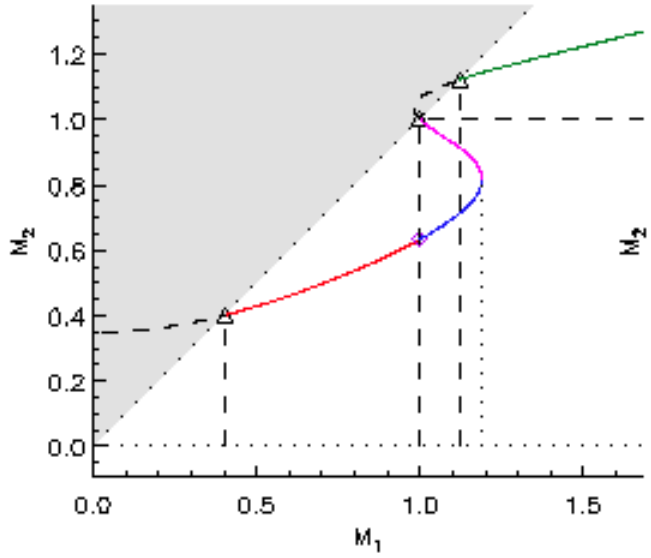

fast
slow
intermediate
a
c
e
$\hat{n}$
$\theta_1$
$B_2$
$\theta_2$
$B_1$
region 1
region 2
$u_1$
$u_2$
b
d
f
FMW
SMW
linear wave
switch–off
rotational discontinuity
$\theta_2$
fast
slow
intermediate
$\theta_1$
0
$-\theta_1$
FMW
RD & SMW
SAS
IS
SS
FS
SMS
SAS
FMS
$M_2$
$M_1$
0.0
0.2
0.4
0.6
0.8
0.5
1.0
1.5

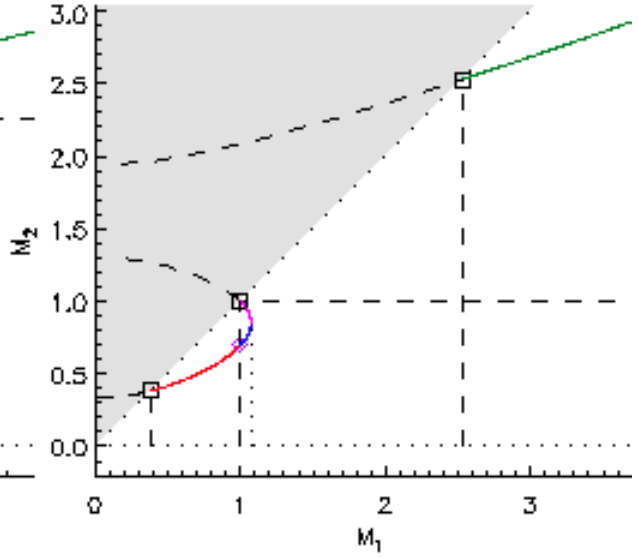

$\beta_1$=1.8


$\theta_1$=25°
M2
M1
0.0
0.5
1.0
1.5
2.0


$\theta_1$=65°
M2
M1
0
1
2
3
4
5


$\beta_1$=0.2


M2
M1
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.5
1.5


M2
M1
0.0
0.5
1.0
1.5
2.0
2.5
3.0
1
2
3

# Hypersonic limit

$$
\begin{aligned}
F(M_1, M_2) \;=\; & -(\gamma+1)M_2^6 \;+\; \left[(\gamma-1)M_1^2 + 2(\gamma+1) + \frac{\gamma\beta_1}{\cos^2\theta_1} + \gamma\tan^2\theta_1\right] M_2^4 \\
& - \left\{2(\gamma-1)M_1^2 + \frac{(\gamma+1)+2\gamma\beta_1}{\cos^2\theta_1} + (\gamma-2)M_1^2\tan^2\theta_1\right\} M_2^2 \\
& + \frac{(\gamma-1)M_1^2 + \gamma\beta_1}{\cos^2\theta_1} \;=\; 0
\end{aligned}
$$

$$
F(M_1, M_2) \;\simeq\; -(\gamma+1)M_2^6 \;+\; (\gamma-1)M_1^2 M_2^4 \;+\; \cdots \;=\; 0 \;\; .
$$

$$
\frac{M_1^2}{M_2^2} \;\simeq\; \frac{\gamma+1}{\gamma-1} \;\simeq\; \frac{\rho_2}{\rho_1} \;\simeq\; \frac{B_{t,2}}{B_{t,1}} \;\; , \quad \rightarrow 4
$$

Magnetic pressure: only x16

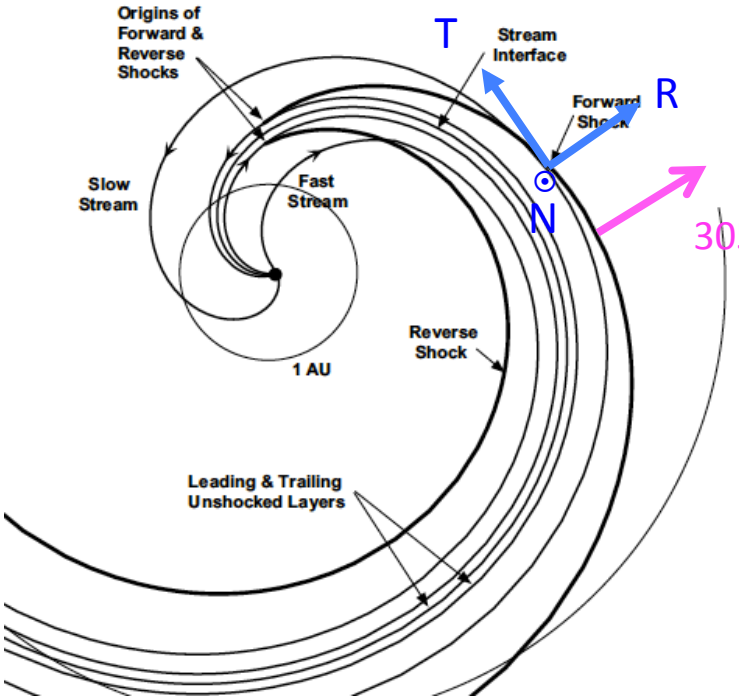
$$
p_2 \;\simeq\; p_1 + \frac{B_n^2}{4\pi}\left(M_1^2 - M_2^2\right) \;\simeq\; p_1 + \frac{2}{\gamma+1}\,\rho_1\, u_{n,1}^2
$$

Plasma pressure, no limit

# Fast Shocks Observed

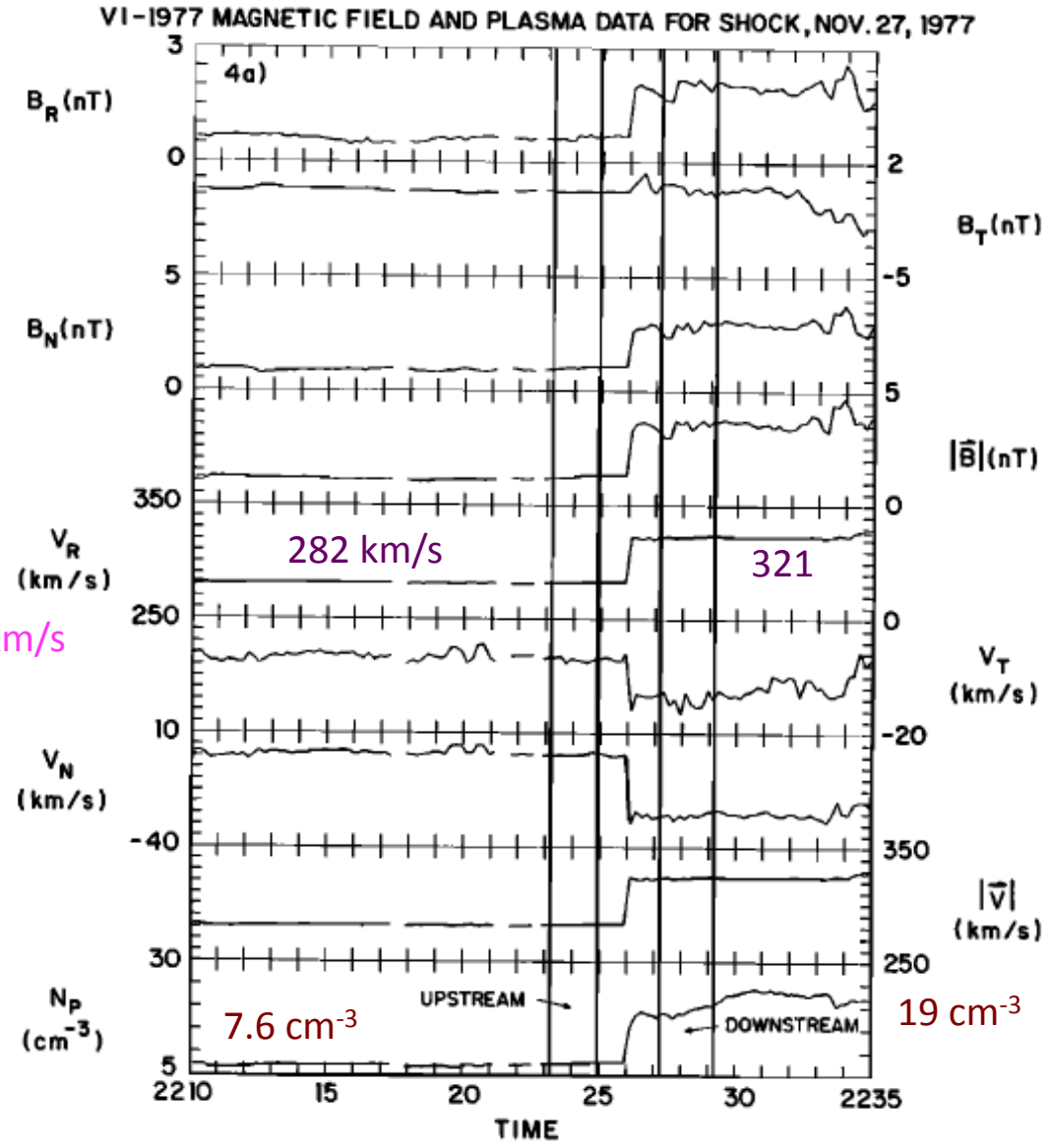
## Voyager I @ r=1.6 AU

($v_{esc}$ = 34 km/s)




$M_{A1} = 8.0/\cos(\theta_1)$

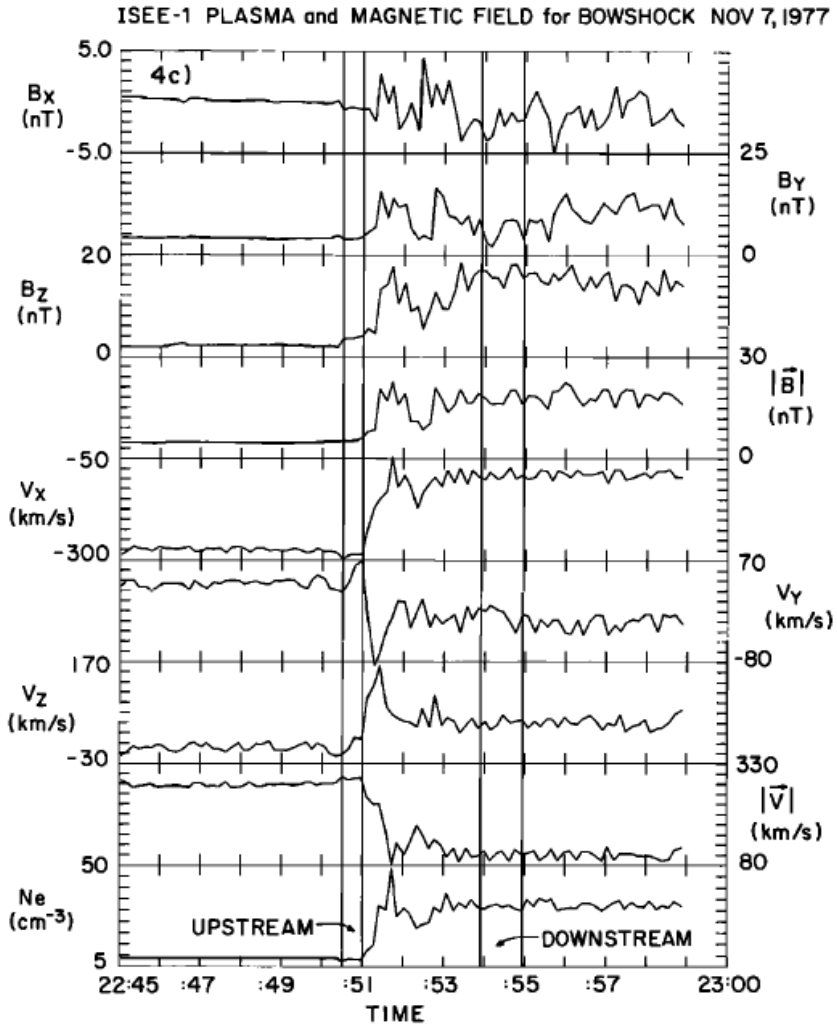
H-sphere: $\theta_1$=84°, $\beta_1$=15




Vinas & Scudder 1986

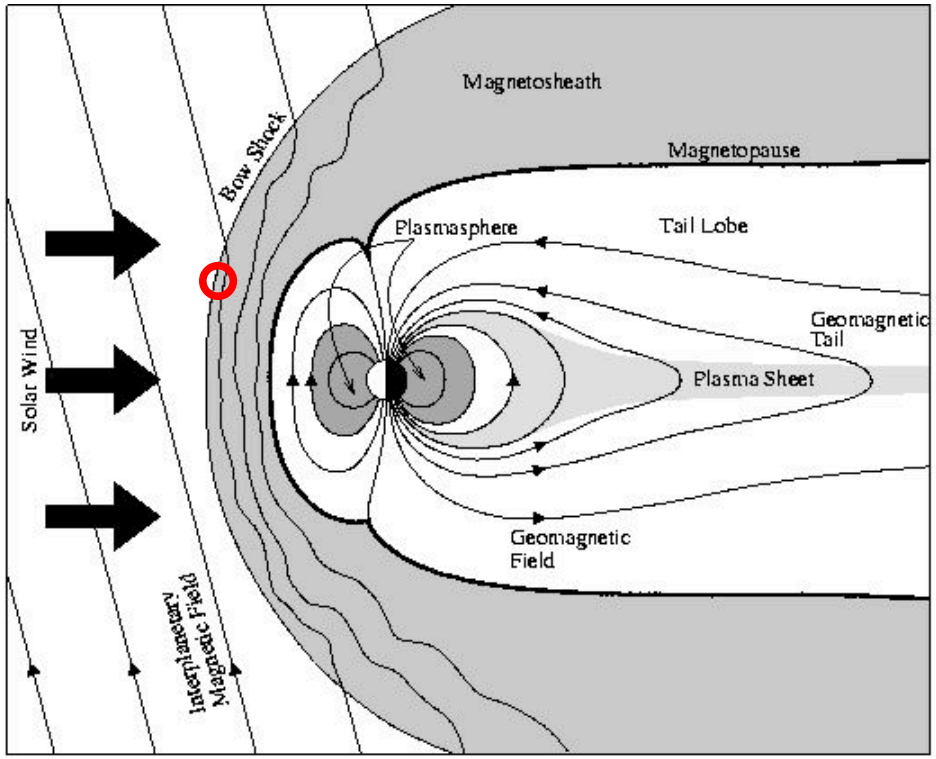
# ISEE-1 crosses bow shock

$M_{A1} = 8.1/\cos(\theta_1)$



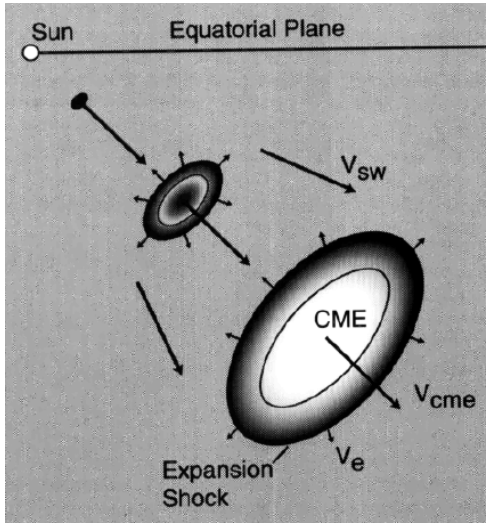

Vinas & Scudder 1986

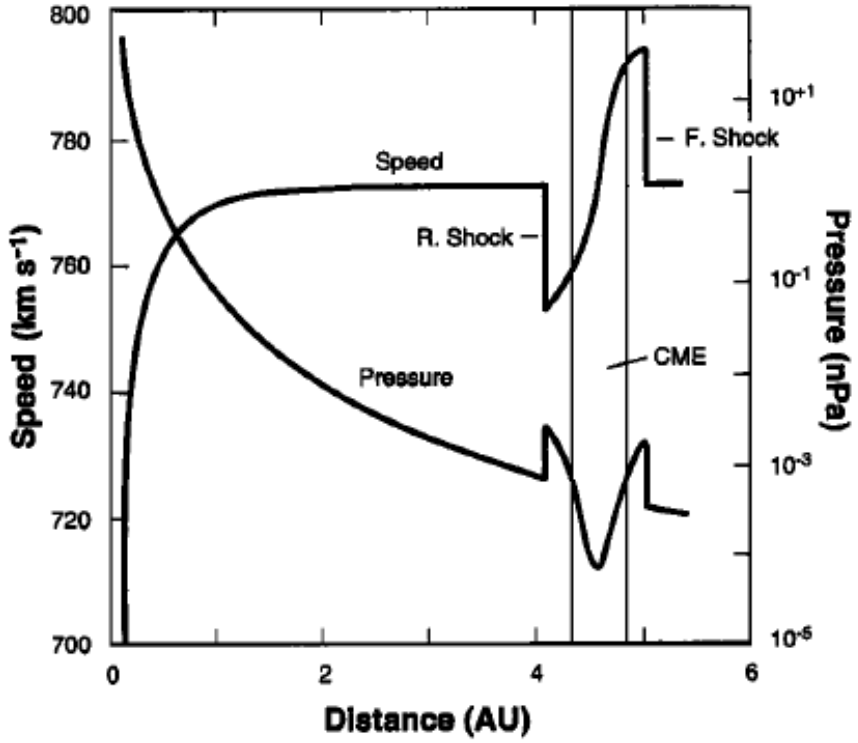
$\theta_1=74^o$, $\beta_1=6$

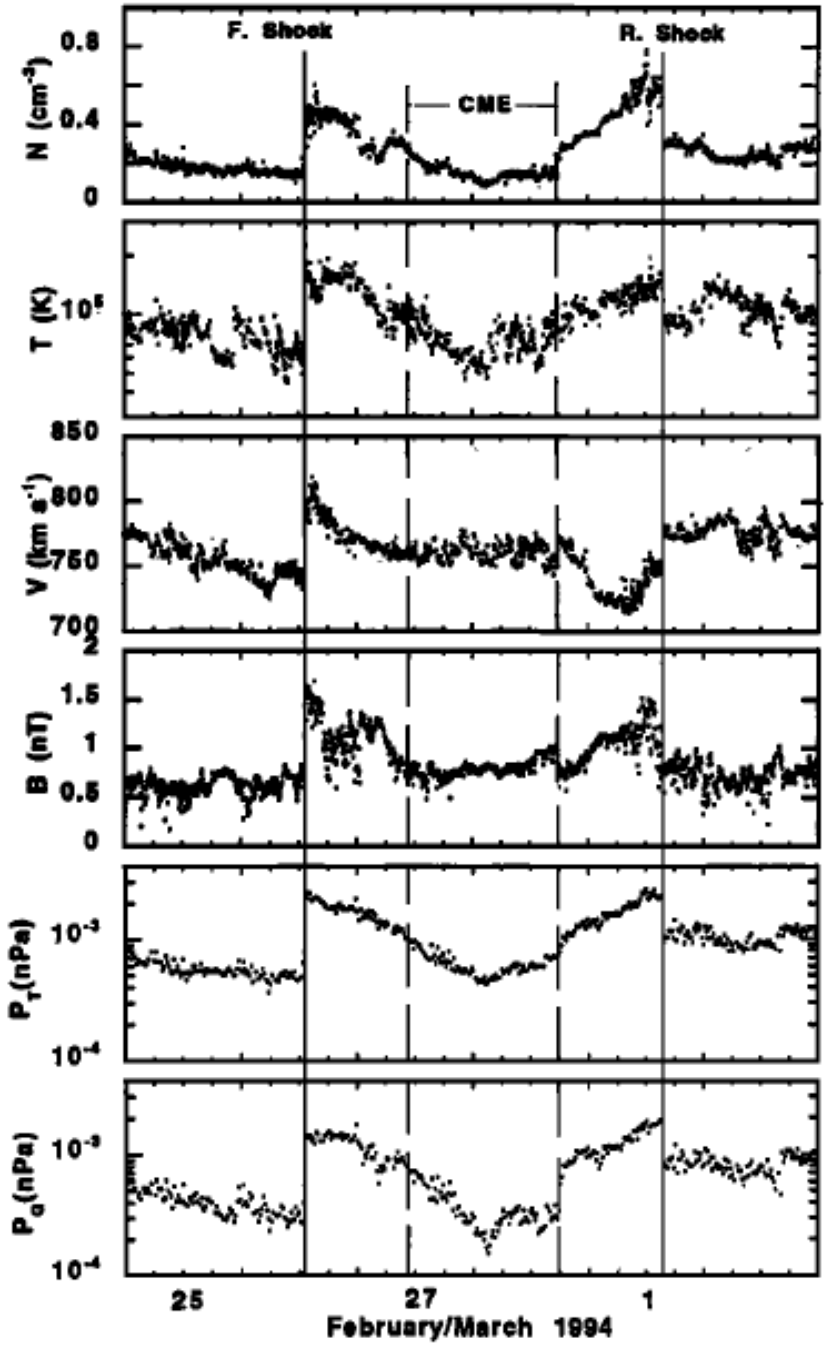


2 FM shocks
2 (1?) TDs

Ulysses
@ r=3.5 AU

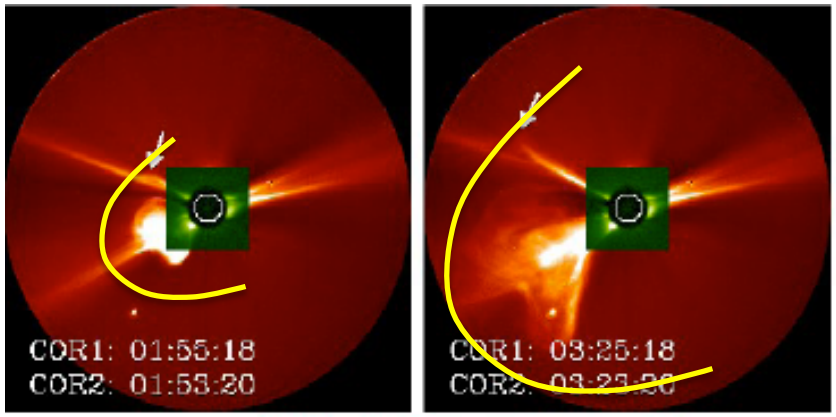


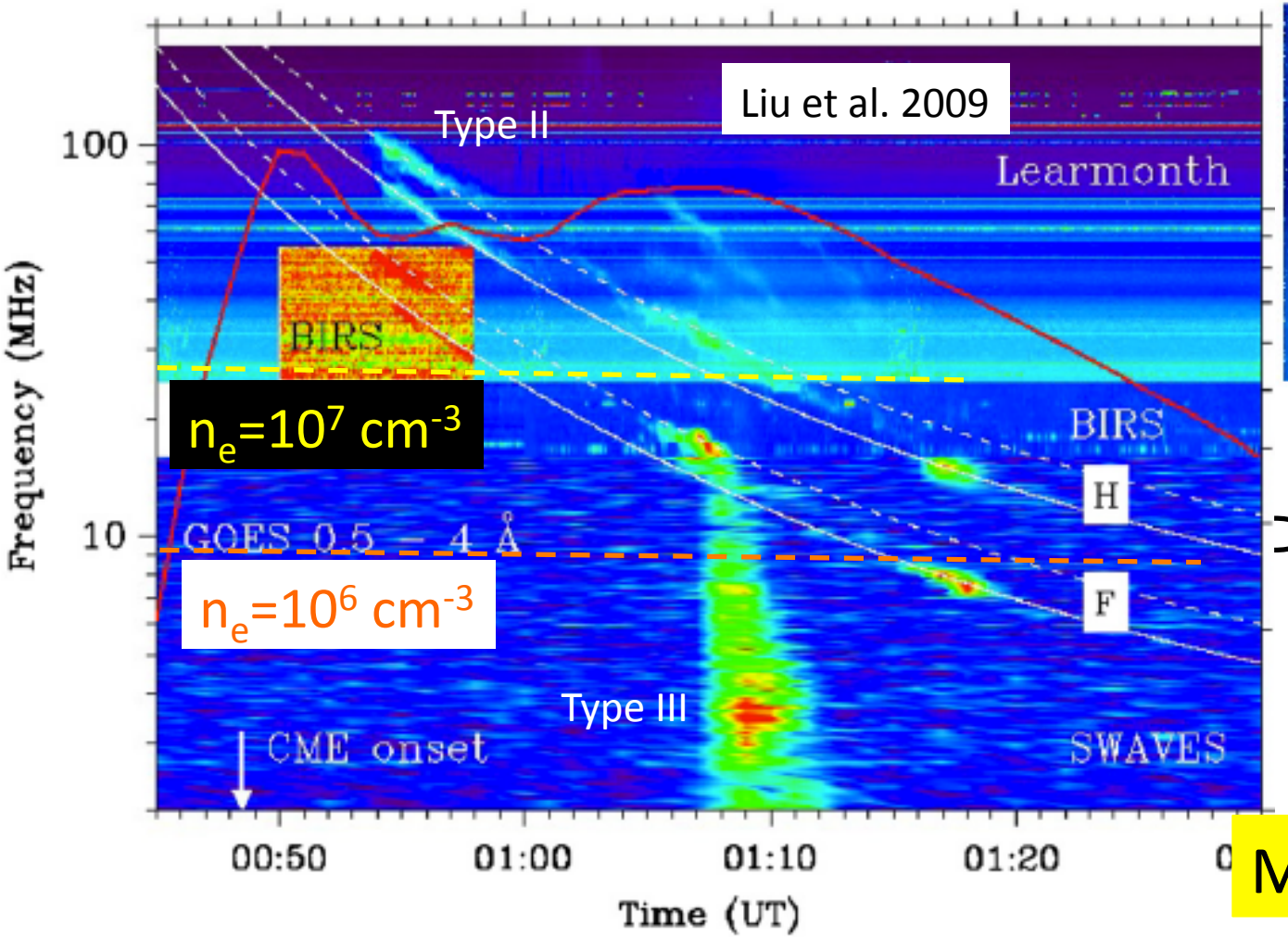



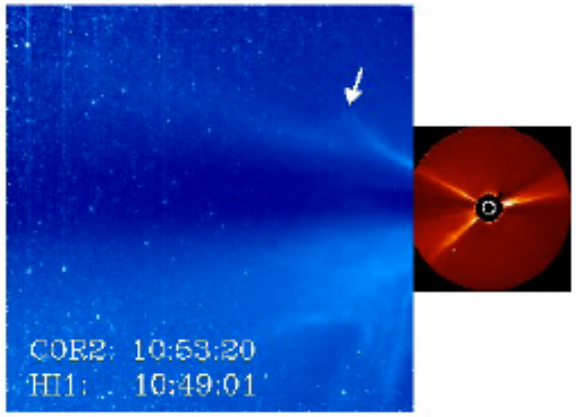




Gosling et al. 1994

# Solar Fast Shocks



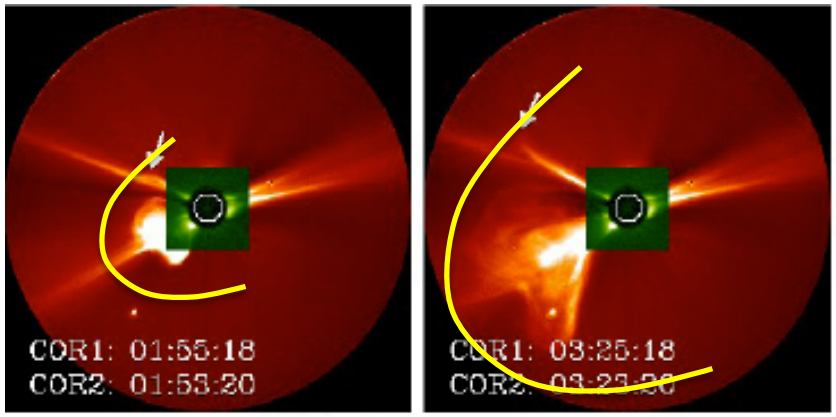



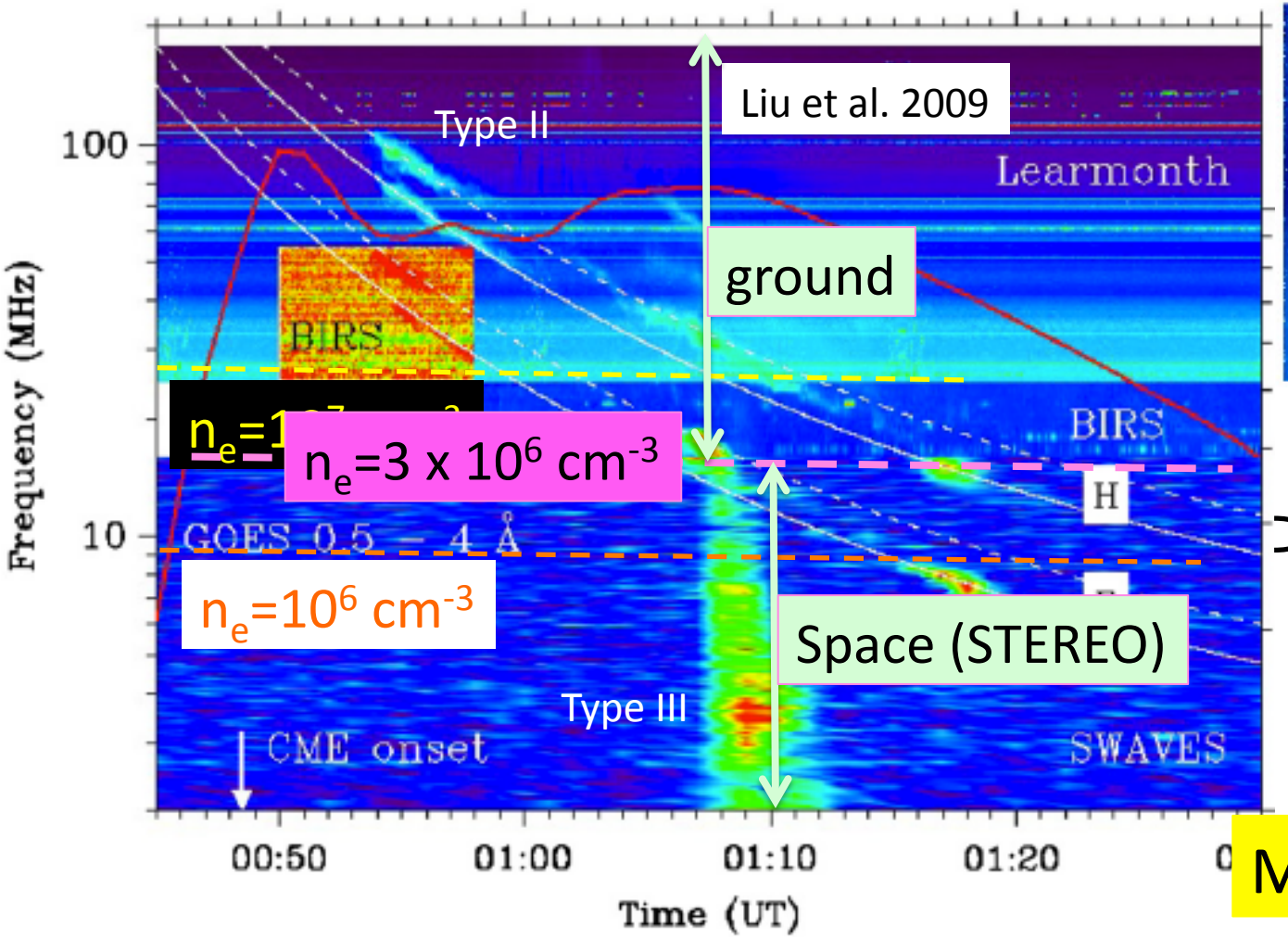

Band splitting reveals **both** $n_{e1}$ & $n_{e2}$

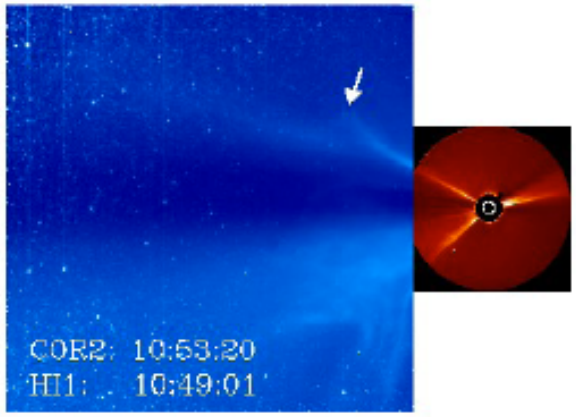
$n_{e2}/n_{e1}=1.59$

$M_{A1}$~ 1.26-1.47

# Solar Fast Shocks




Band splitting reveals **both** $n_{e1}$ & $n_{e2}$

$n_{e2}/n_{e1}$=1.59

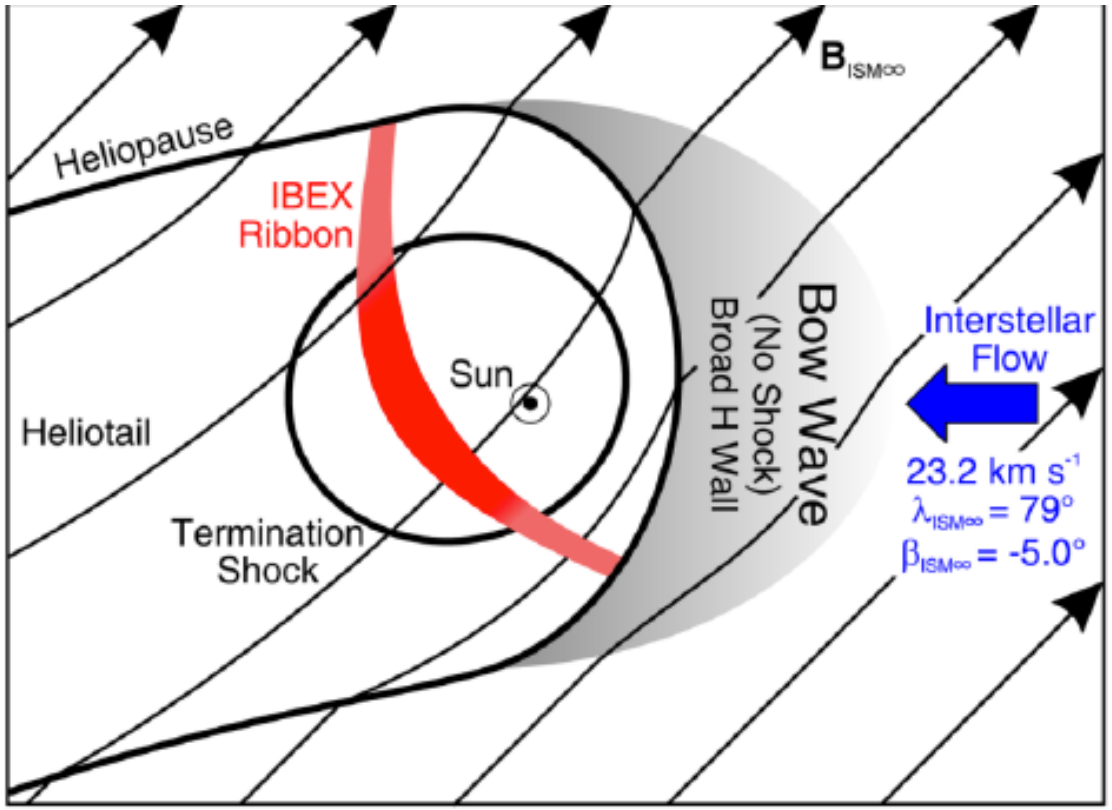
$M_{A1}$~ 1.26-1.47

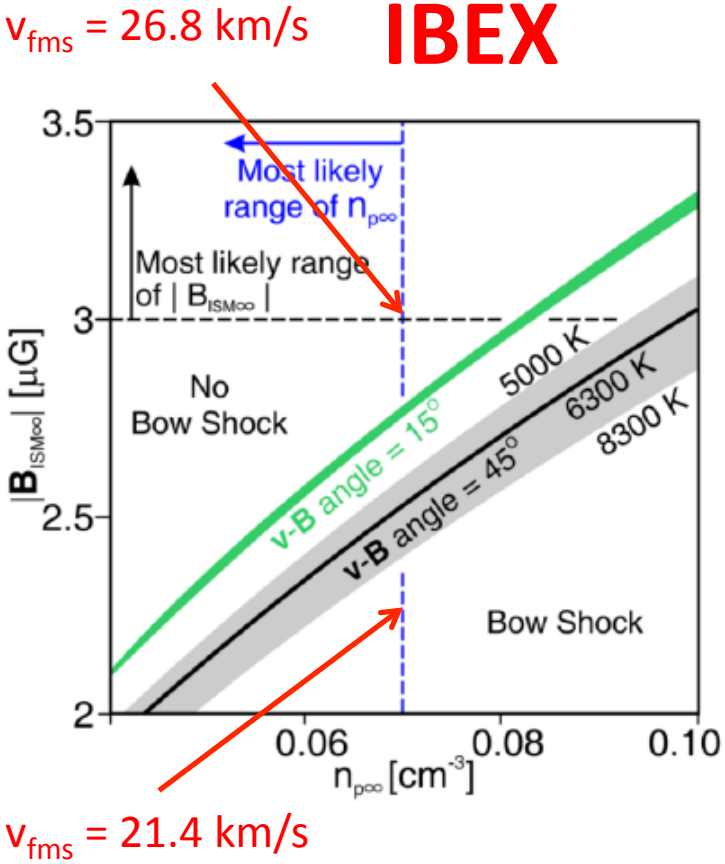
# The Heliosphere's Interstellar Interaction: No Bow Shock

*Science* May 10, 2012

D. J. McComas,[1,2*] D. Alexashov,[3] M. Bzowski,[4] H. Fahr,[5] J. Heerikhuisen,[6] V. Izmodenov,[3] M. A. Lee,[7] E. Möbius,[7,8] N. Pogorelov,[6] N. A. Schwadron,[7] G. P. Zank[6]

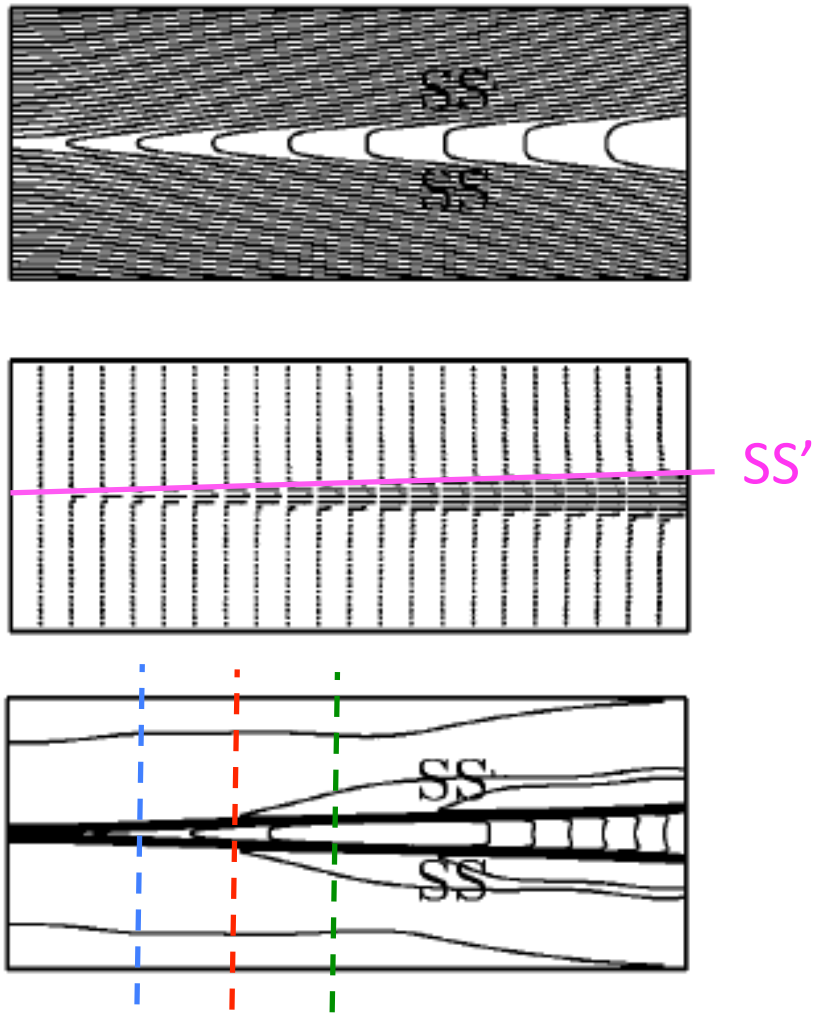
**Breaking news from IBEX**

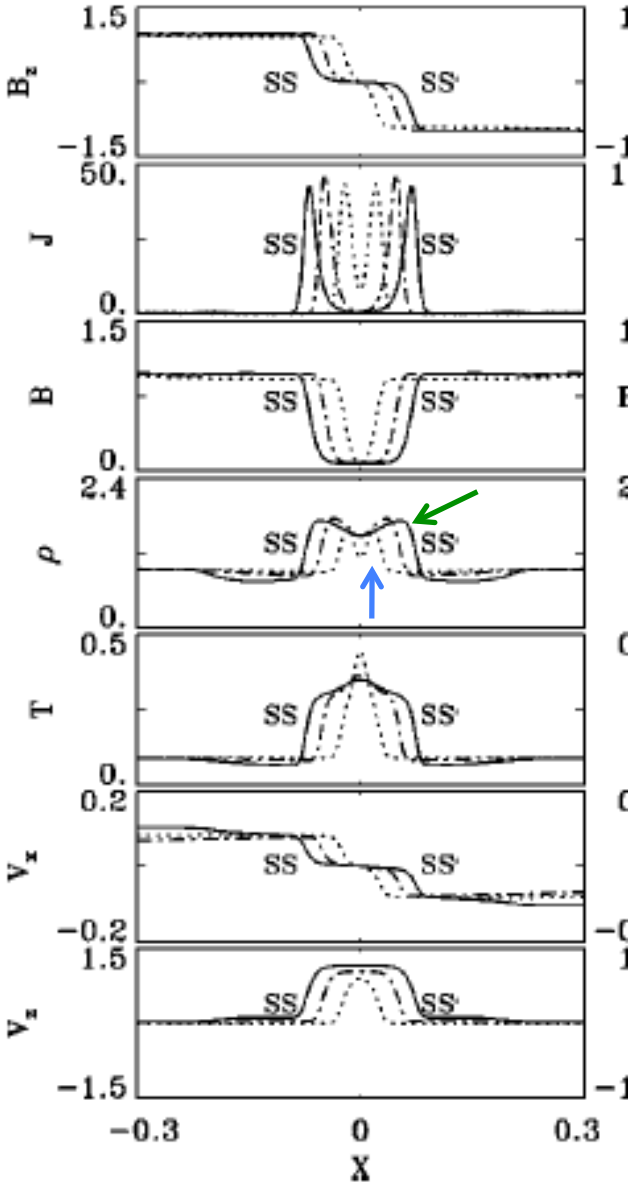


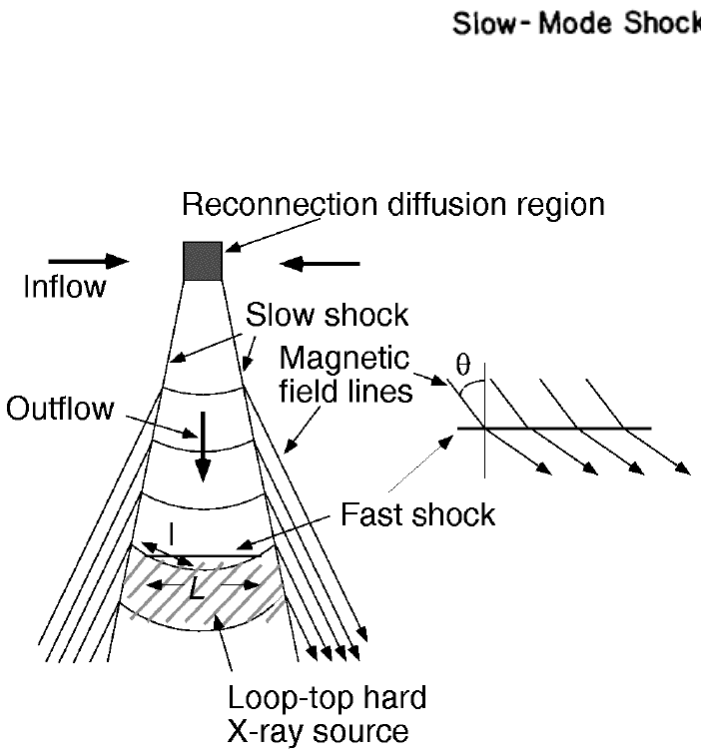
# Slow shocks: reconnection

Lin & Lee 1999

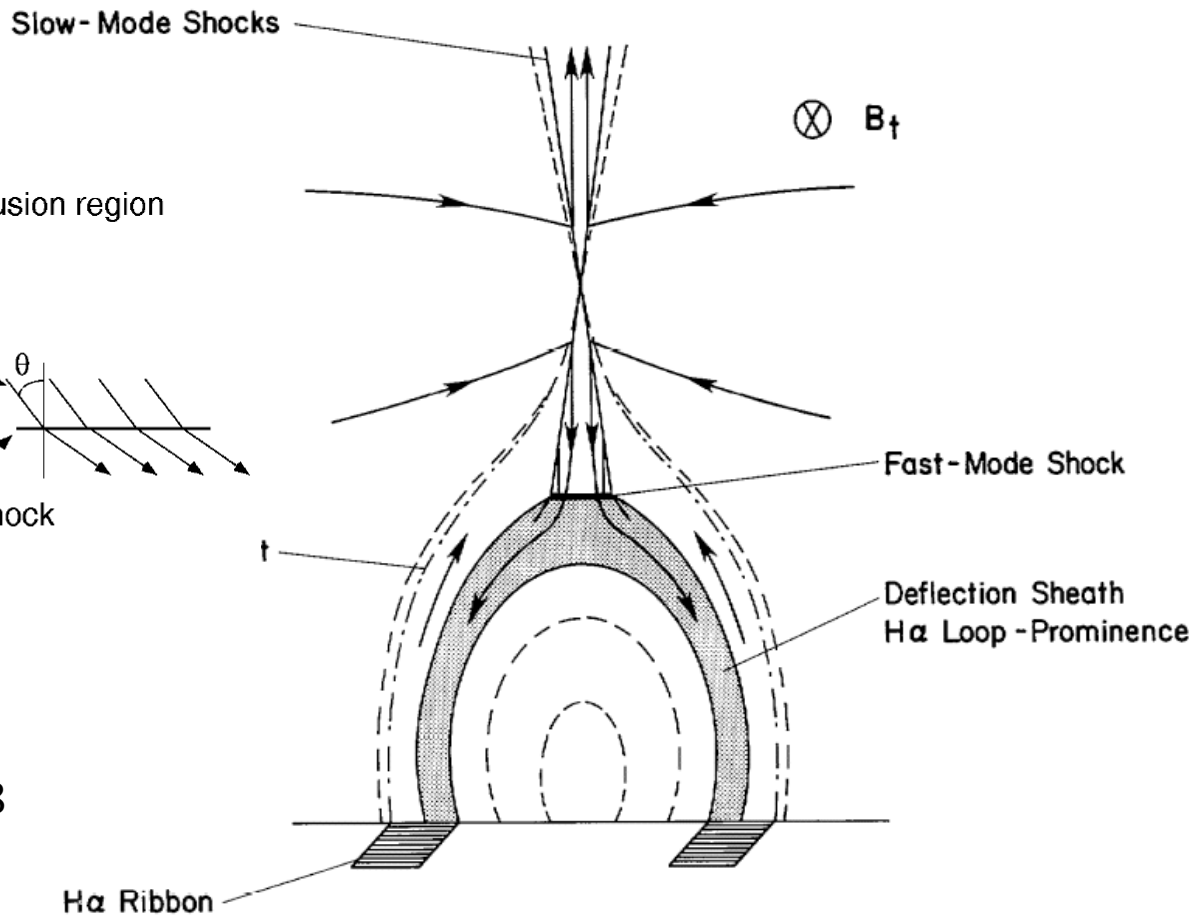


# Model reconnection

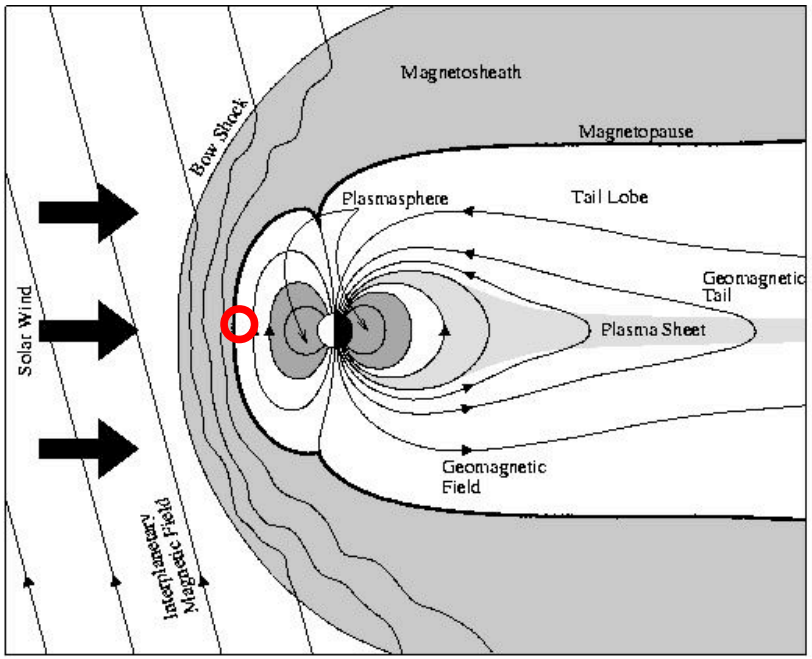


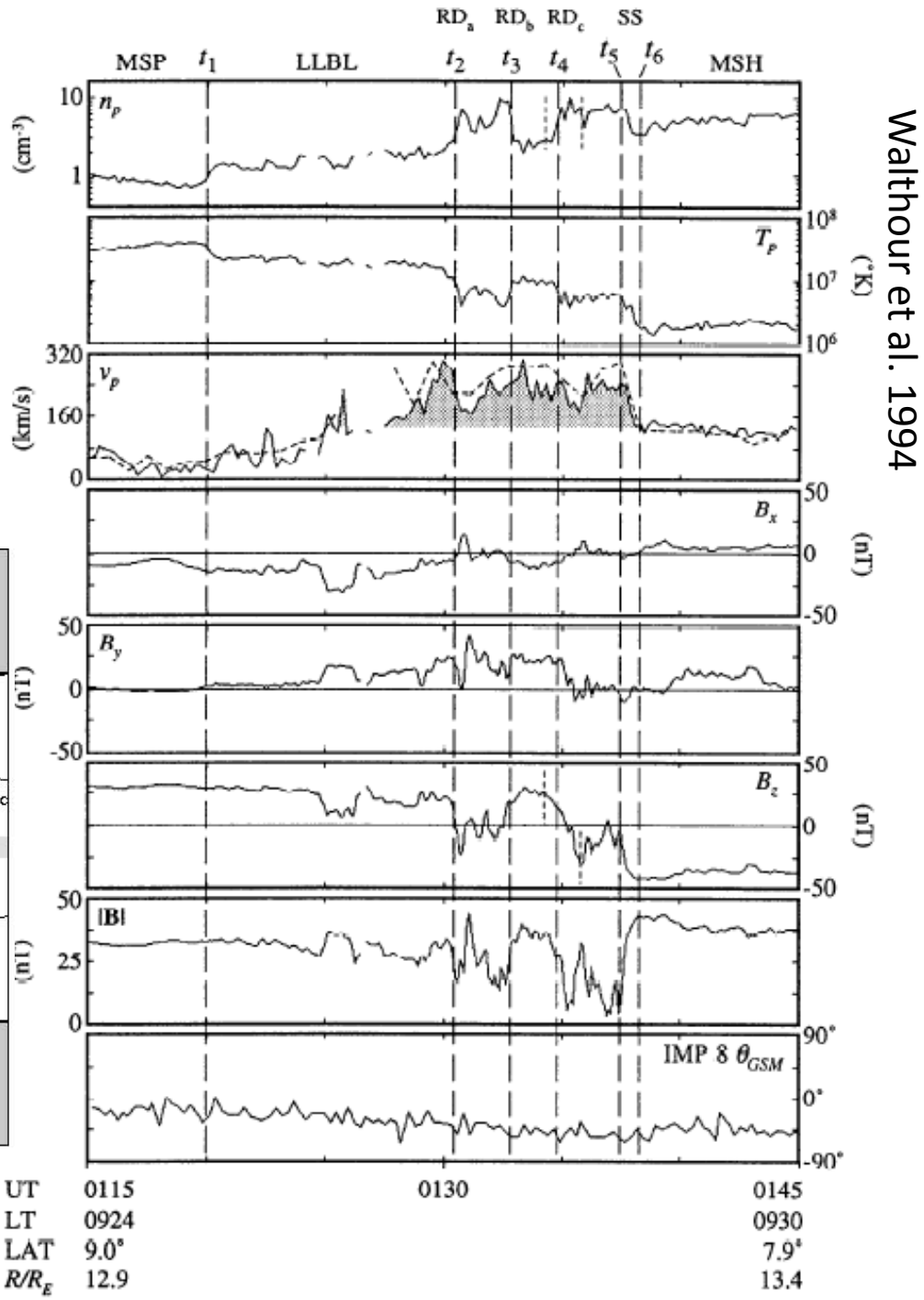

Tsuneta and Naito 1998




Forbes, T.G., and Malherbe 1986 (v.2 ch. 6)

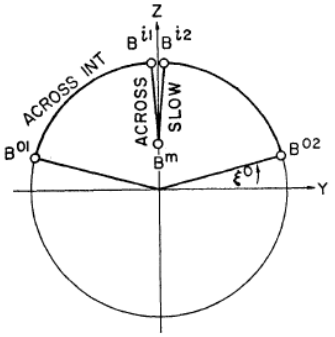
# ISEE-1 at the magnetopause:
## SS from reconnection

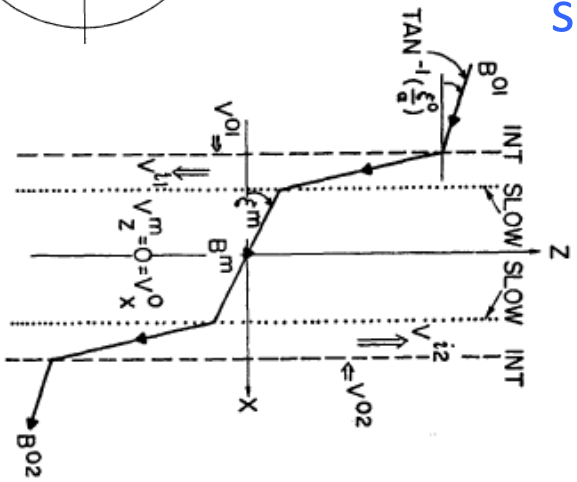


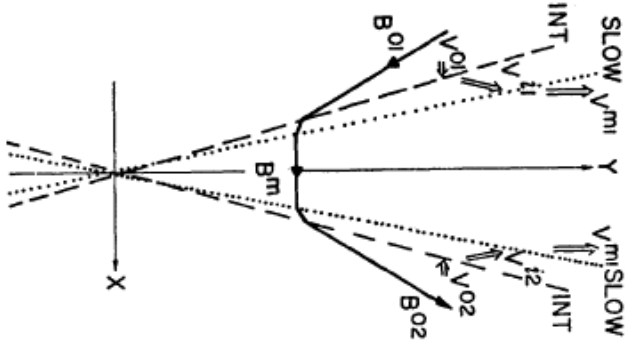




Walthour et al. 1994

# Intermediate Shocks

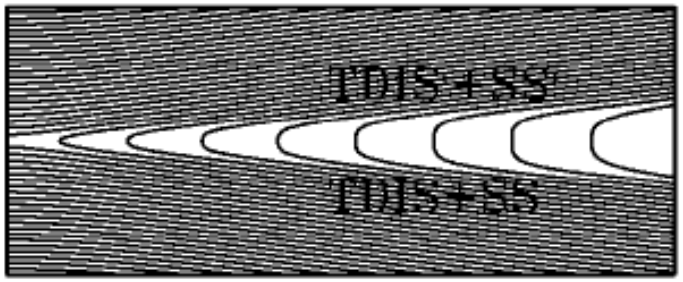


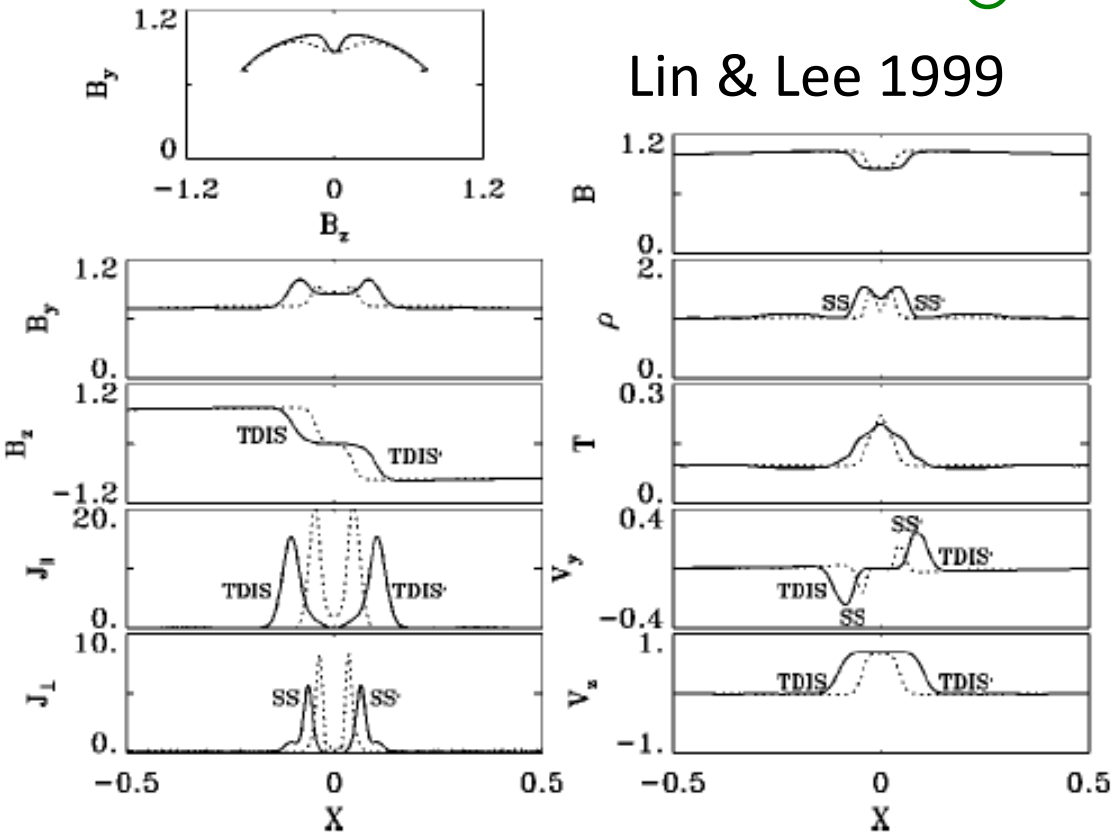




Petschek & Thorne 1967

Reconnection between skewed fields




Lin & Lee 1999

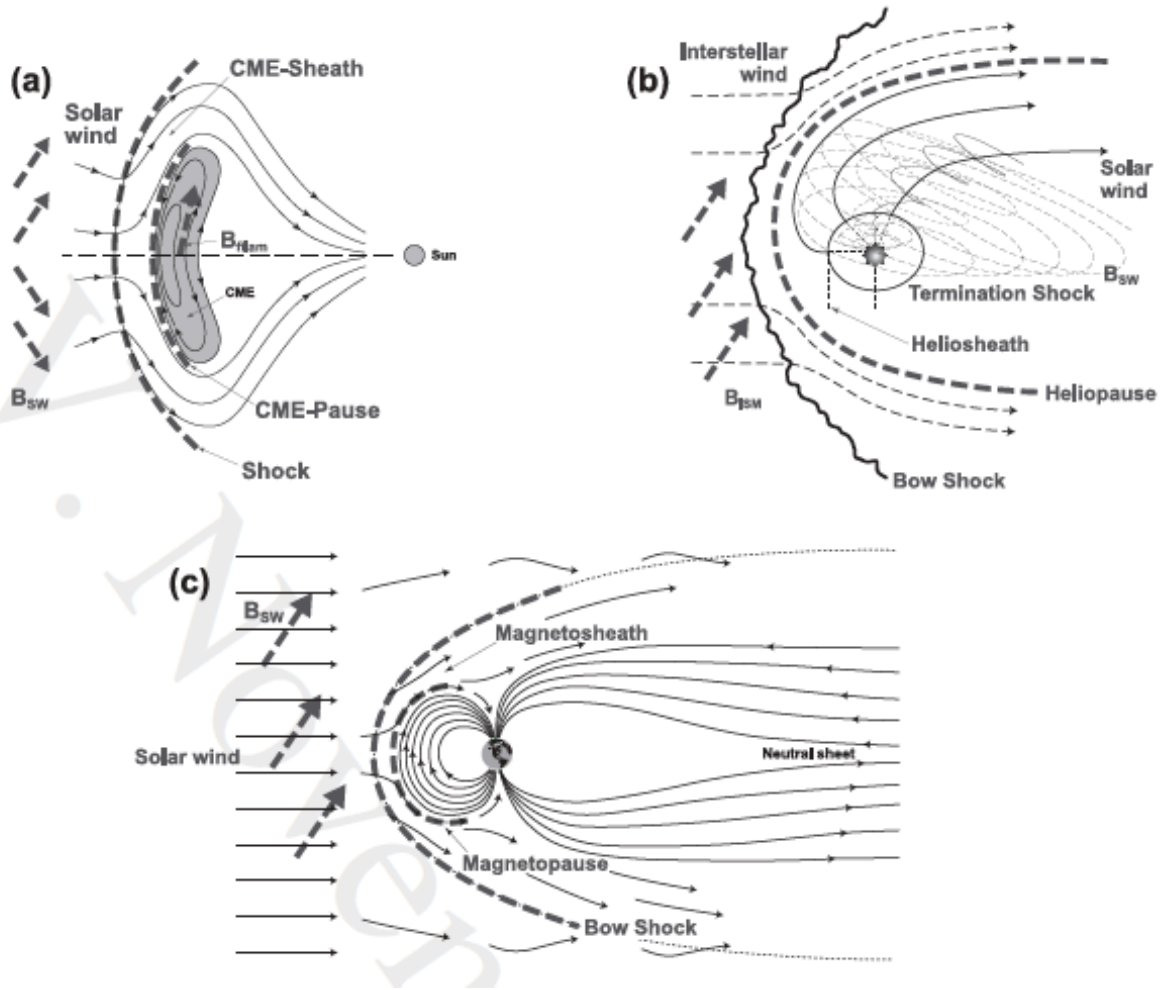


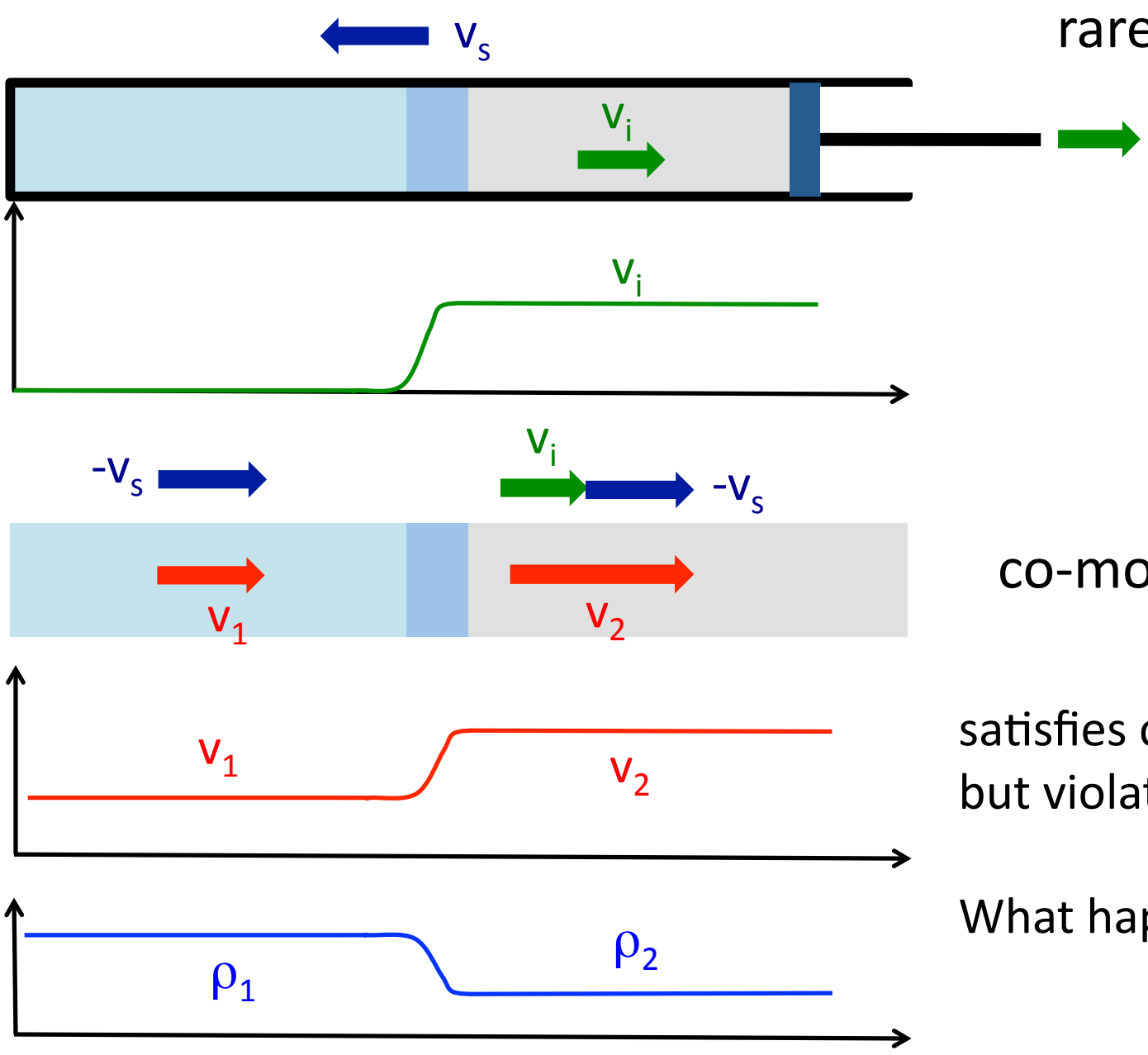
# Entropy's Tale … the sequel

- A shock converts kinetic to thermal energy
- How? What is the μ-physics?
  - Collisions (i.e. viscosity) – high enough density
  - Otherwise… (story not finished yet)
    - Instabilities, chaotic particle motion, …
    - Related to acceleration of non-thermal particles
    - Easier if particles gyrate near shock:
      quasi-perp. thermalizes easier than quasi-parallel

# Summary




(a)
CME-Sheath
Solar wind
$B_{fldsm}$
Sun
CME
$B_{SW}$
CME-Pause
Shock
(b)
Interstellar wind
Solar wind
$B_{SW}$
Termination Shock
Heliosheath
Heliopause
$B_{ISM}$
Bow Shock
(c)
$B_{SW}$
Magnetosheath
Solar wind
Neutral sheet
Magnetopause
Bow Shock

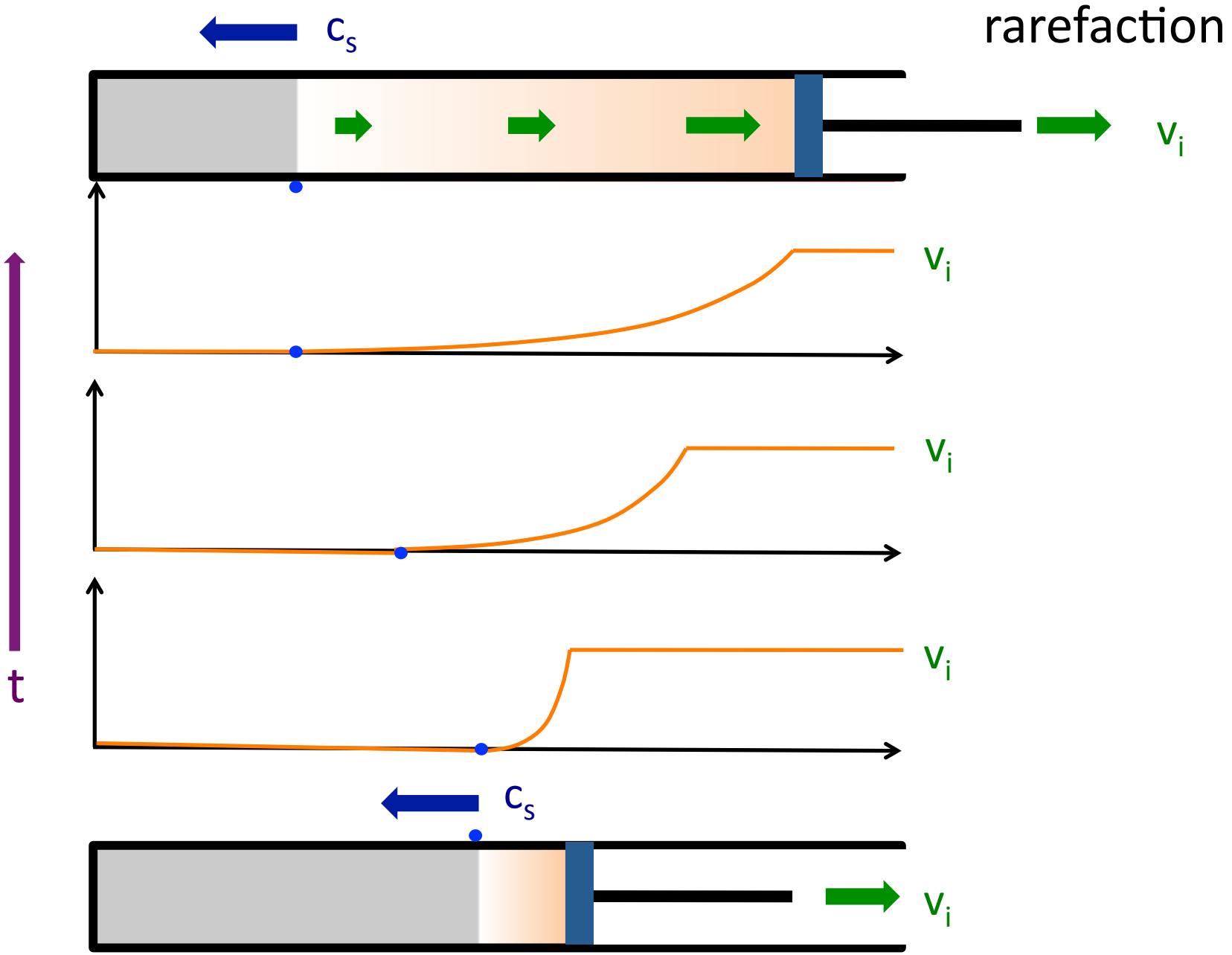



rarefaction

co-moving frame

satisfies conservation
but violates 2nd law TD

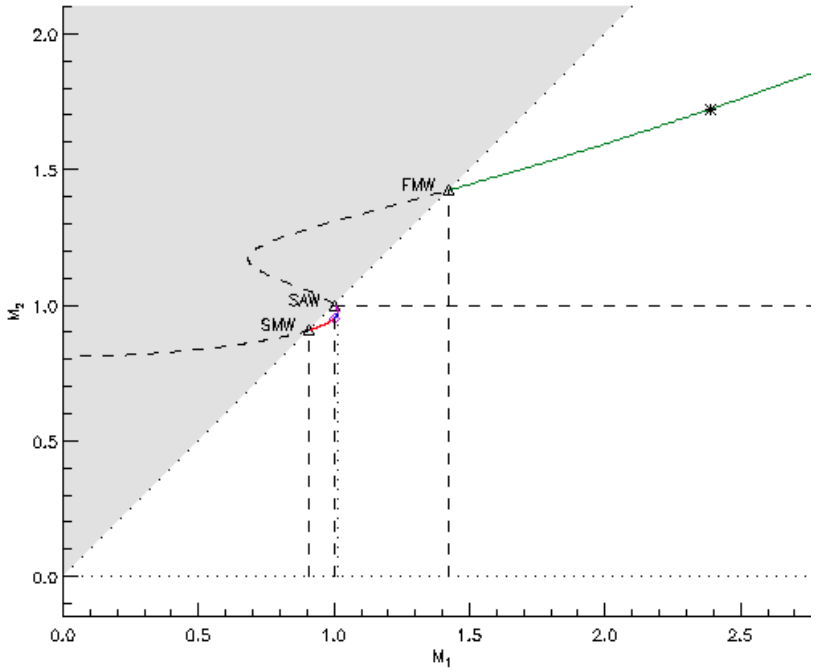
What happens instead?

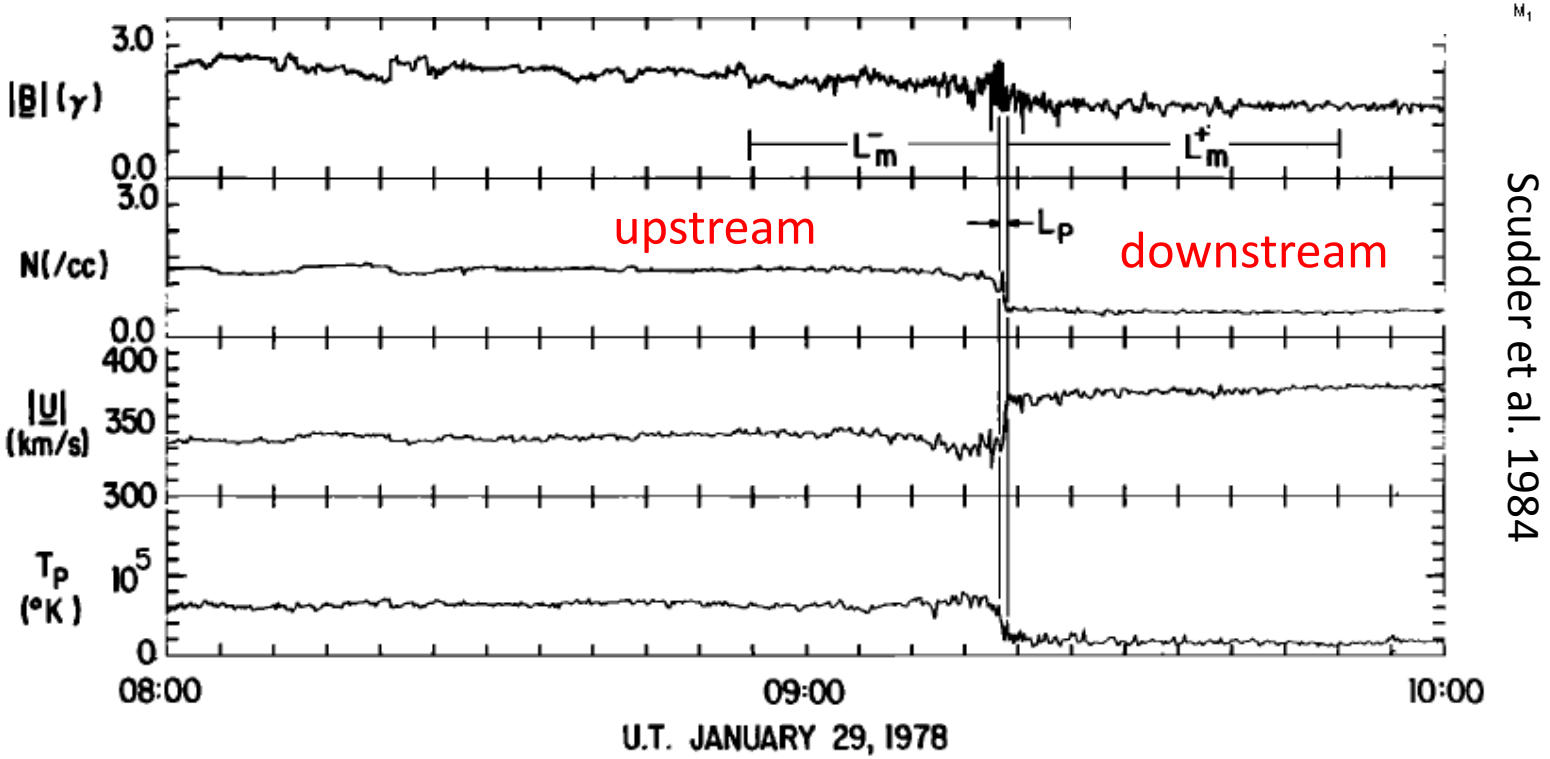

rarefaction
$c_s$
$v_i$
$v_i$
$v_i$
$v_i$
t
$c_s$
$v_i$

## Forward shock

## Voyager 2 at r=2.17 AU

$\theta_1=23^{o,}$ $\beta_1=1.7$, $M_{A1}=2.4$




Scudder et al. 1984