Exploring the Sun and its effects on the Earth's atmosphere and physical environment...

HIGH ALTITUDE OBSERVATORY

Solar Convection and The Solar Dynamo

Mark Miesch

HAO/NCAR

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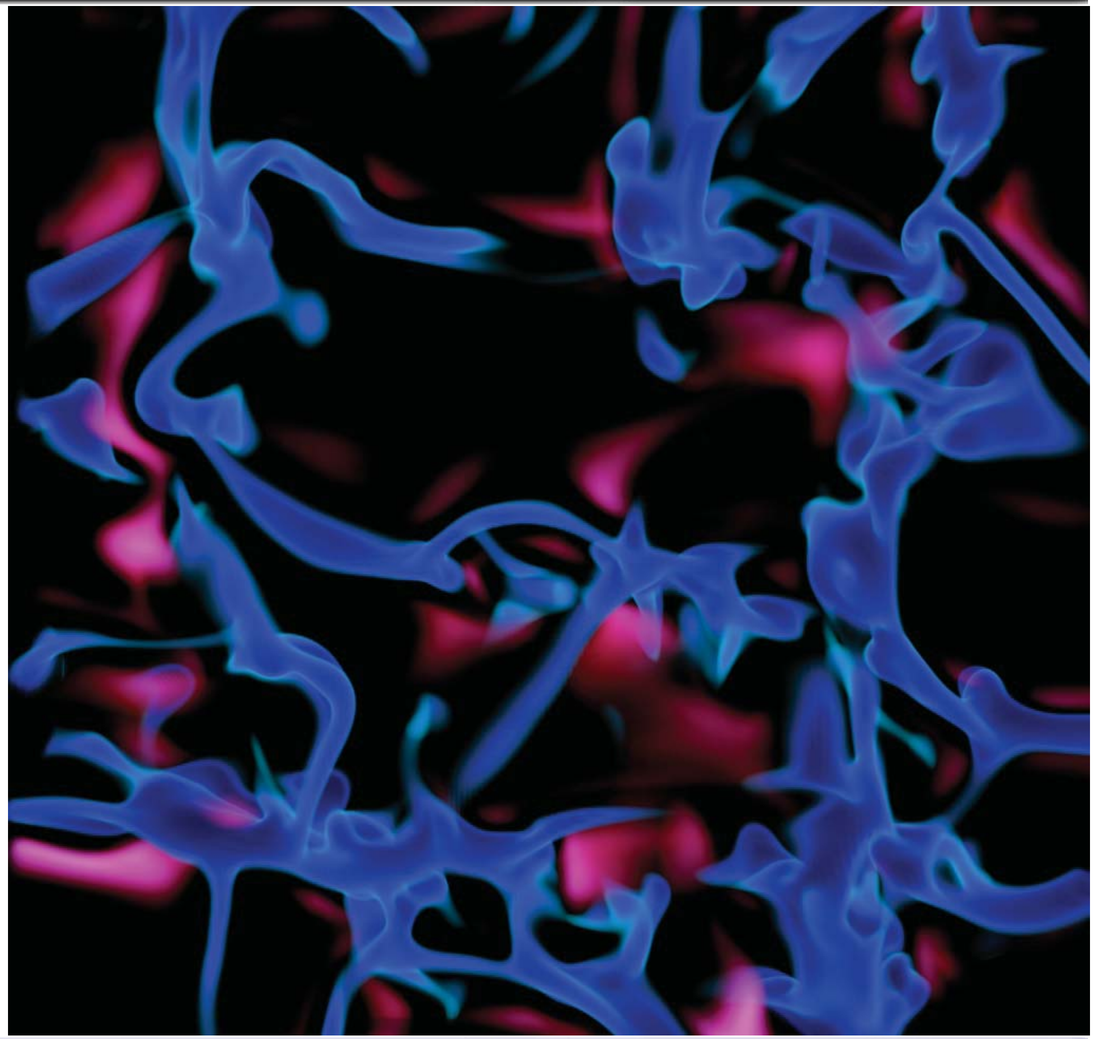


Outline

I) Convection

- Fundamental Aspects
- Solar Convection

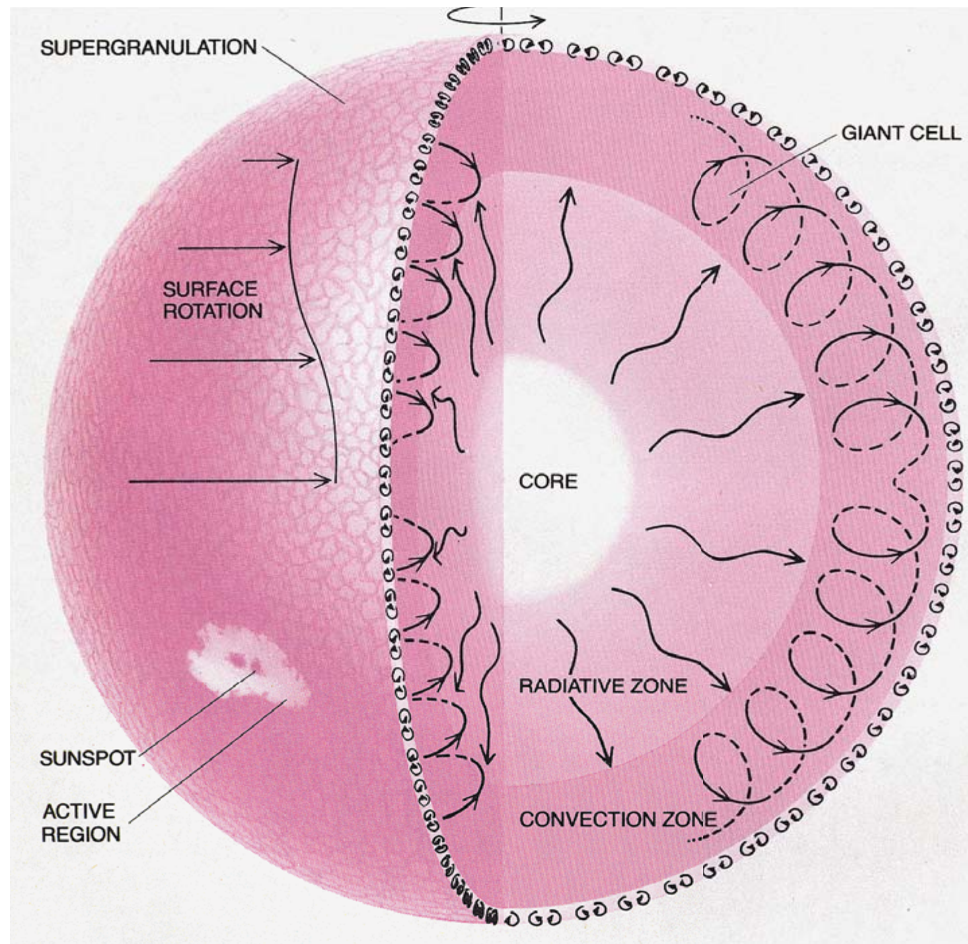
II) Mean Flows

- Differential Rotation
- Meridional Circulation

III) The Solar Dynamo

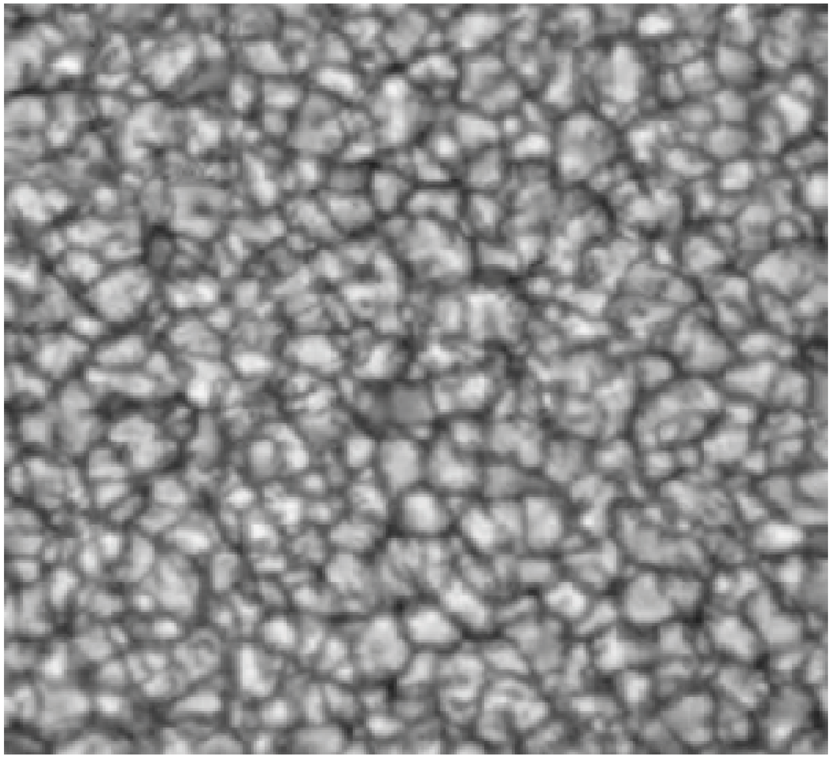
- Convective Dynamos
- Models of the Solar Cycle

Solar & Stellar!



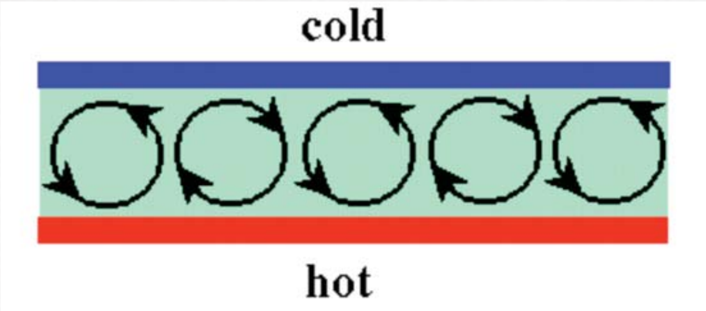
I) Solar Convection

- **Fundamental Aspects**
  - Plumes & Lanes
  - Boundary Layers
  - Rotation
  - Stratification
  - Magnetism
  - Spherical Geometry
- **Application to the Sun**
  - Granulation
  - Mesogranulation
  - Supergranulation
  - Giant Cells



Rayleigh-Bénard Convection

See Lab Exercise!



Bénard (1900)
Rayleigh (1916)

Chandrasekhar (1961)

Ahlers, Grossman & Lohse
(2009, Rev. Mod. Phys, 81, 503)

$$\mathrm{Ra} = \frac{\alpha \Delta g D^3}{\nu \kappa} \qquad \mathrm{Pr} = \frac{\nu}{\kappa}$$

Question: What happens as you decrease ν,κ while keeping everything else the same, including Pr?

Ra = 10^8, Pr = 0.7, 6.4

Zhong et al (2009)

Rayleigh-Bénard Convection

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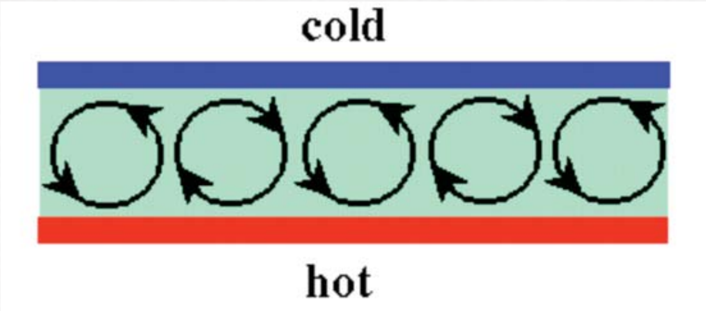

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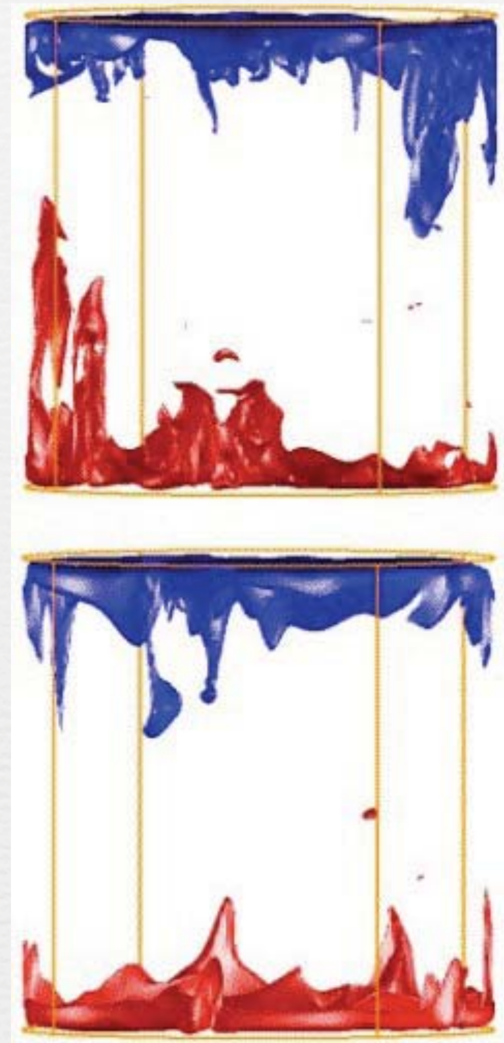

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$$\mathrm{Ra} = \frac{\alpha \Delta g D^3}{\nu\kappa}$$

$$\mathrm{Pr} = \frac{\nu}{\kappa}$$

Question: What happens as you decrease ν,κ while keeping everything else the same, including Pr?

Answer: Re, Nu increase

$$\mathrm{Re} = \frac{UD}{\nu}$$

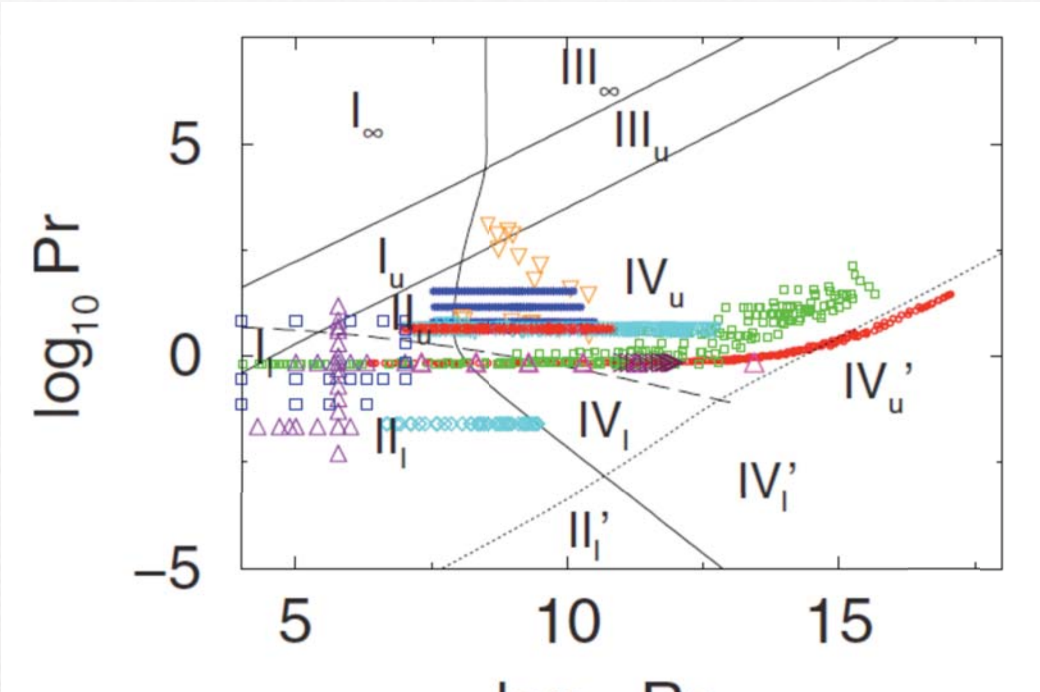
turbulent intensity

$$\mathrm{Nu} = \frac{H}{k\Delta D^{-1}}$$

turbulent heat flux

$$k = C_P \rho \kappa$$

Plumes and Boundary Layers!



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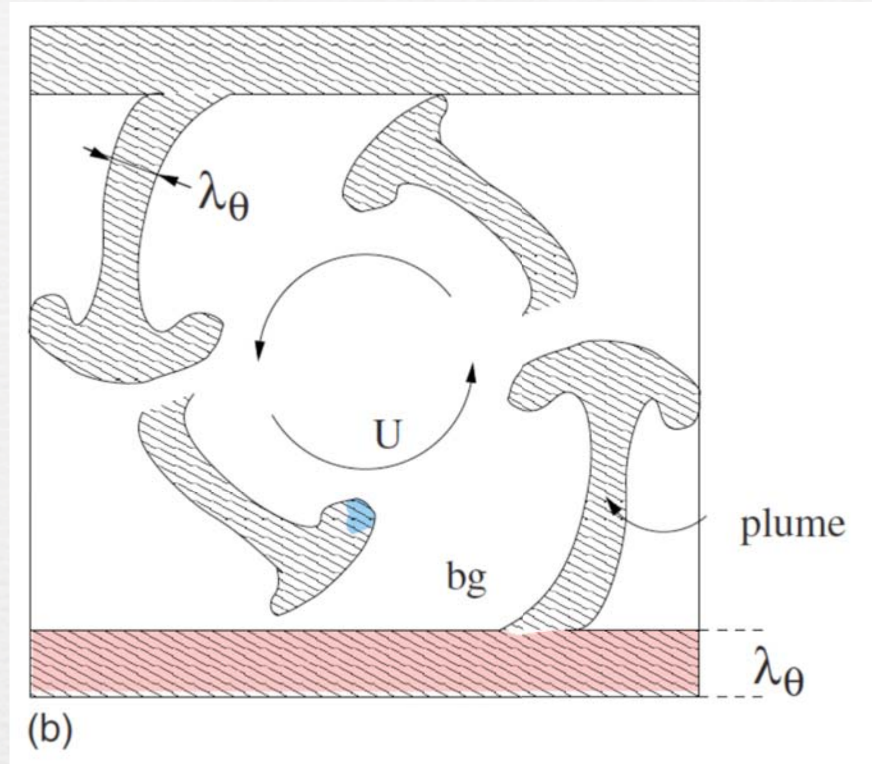


Regime	Dominance of	BLs	Nu	Re
I_l	$\epsilon_{u,BL}$, $\epsilon_{\theta,BL}$	$\lambda_u<\lambda_\theta$	$Ra^{1/4}Pr^{1/8}$	$Ra^{1/2}Pr^{-3/4}$
I_u		$\lambda_u>\lambda_\theta$	$Ra^{1/4}Pr^{-1/12}$	$Ra^{1/2}Pr^{-5/6}$
I_∞		$\lambda_u=L/4>\lambda_\theta$	$Ra^{1/5}$	$Ra^{3/5}Pr^{-1}$
II_l	$\epsilon_{u,bulk}$, $\epsilon_{\theta,BL}$	$\lambda_u<\lambda_\theta$	$Ra^{1/5}Pr^{1/5}$	$Ra^{2/5}Pr^{-3/5}$
II_u		$\lambda_u>\lambda_\theta$	$Ra^{1/5}$	$Ra^{2/5}Pr^{-2/3}$
III_u	$\epsilon_{u,BL}$, $\epsilon_{\theta,bulk}$	$\lambda_u>\lambda_\theta$	$Ra^{3/7}Pr^{-1/7}$	$Ra^{4/7}Pr^{-6/7}$
III_∞		$\lambda_u=L/4>\lambda_\theta$	$Ra^{1/3}$	$Ra^{2/3}Pr^{-1}$
IV_l	$\epsilon_{u,bulk}$, $\epsilon_{\theta,bulk}$	$\lambda_u<\lambda_\theta$	$Ra^{1/2}Pr^{1/2}$	$Ra^{1/2}Pr^{-1/2}$
IV_u		$\lambda_u>\lambda_\theta$	$Ra^{1/3}$	$Ra^{4/9}Pr^{-2/3}$

Grossmann & Lohse (2000,2001, 2002, 2004)

For L ~ 7m
(Barrel of Ilmenau)

λ ~ 1 mm
for Ra ~ 10^{14}

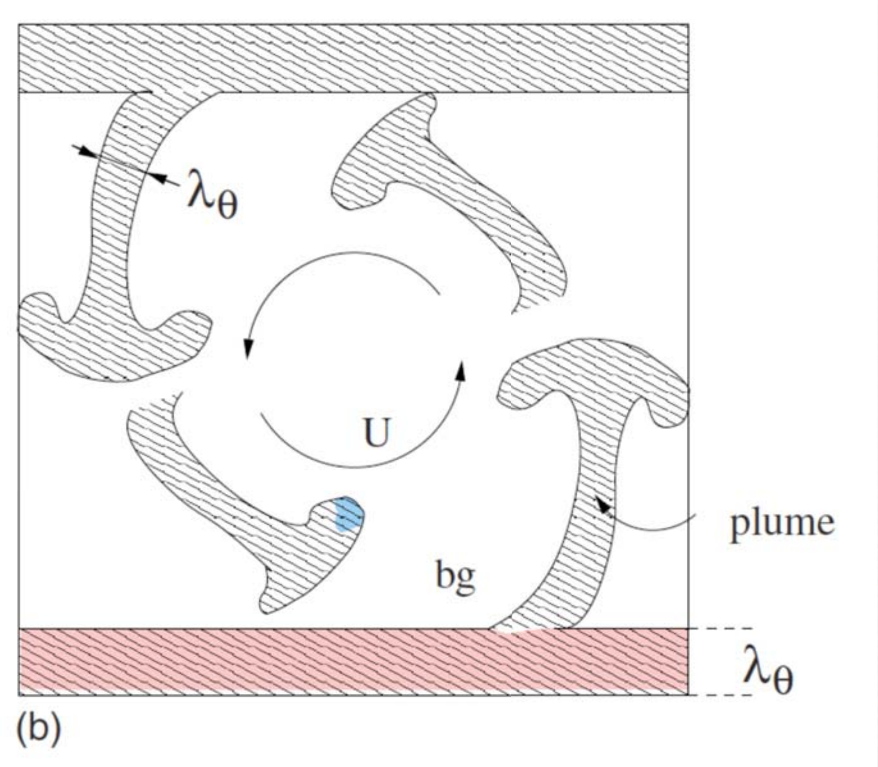
Plumes and Boundary Layers!

What is a boundary layer?

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T = \kappa \nabla^2 T$$

$$\frac{UT}{L} \sim \kappa \frac{T}{\delta_T^2} \qquad \mathrm{Pe} = \frac{UL}{\kappa}$$

$$\delta_T \sim L\,\mathrm{Pe}^{-1/2}$$



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Grossmann & Lohse (2000,2001, 2002, 2004)

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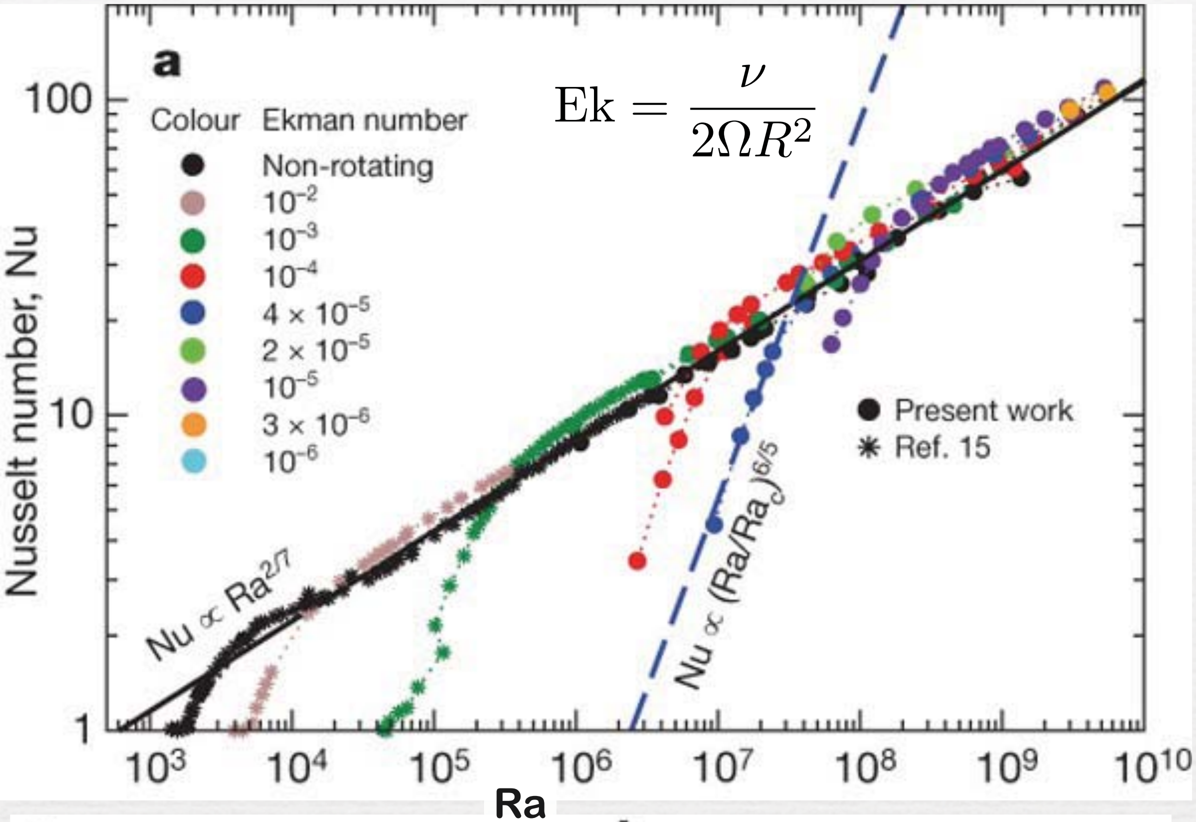
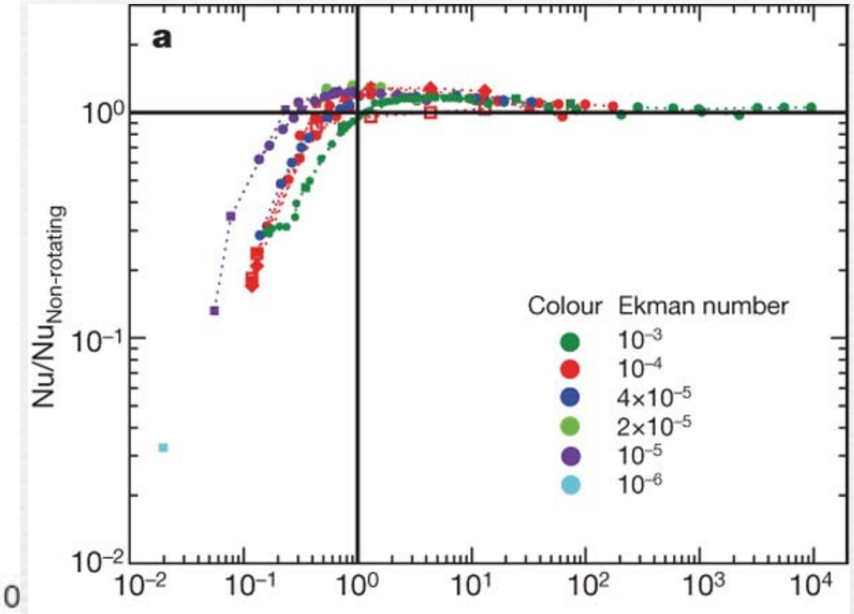
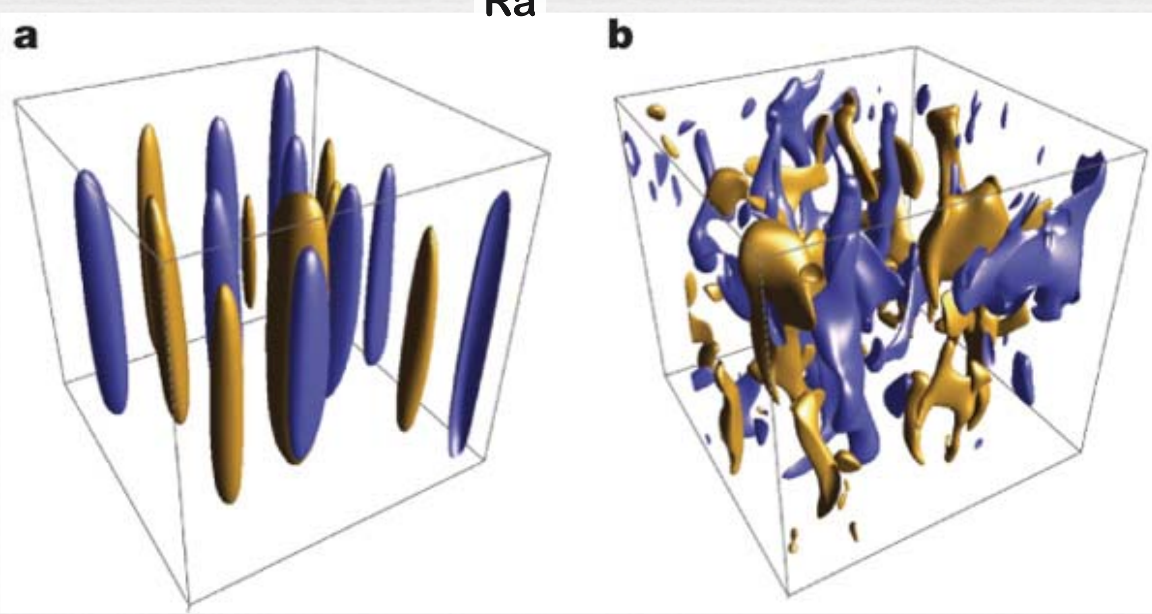
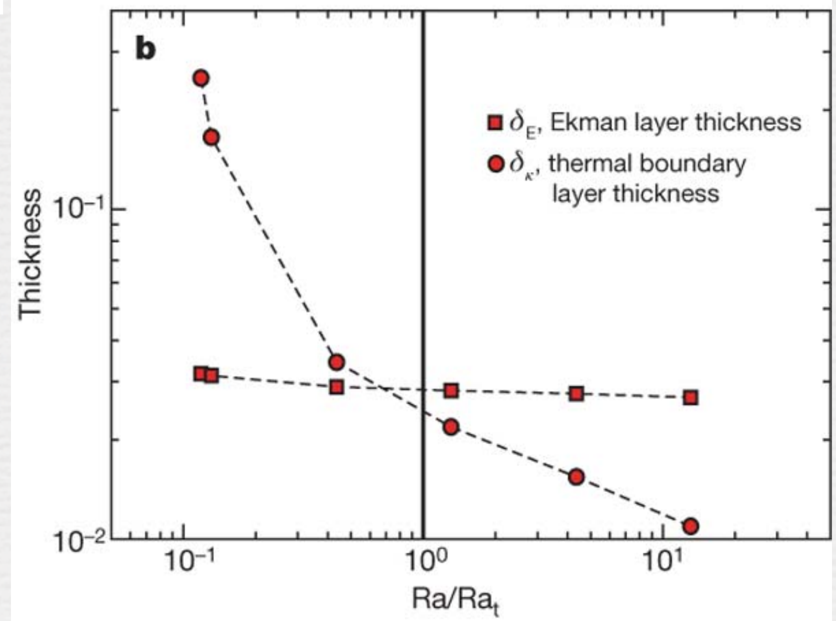
Rotation: ***Helical plumes and Even more Boundary Layers!***



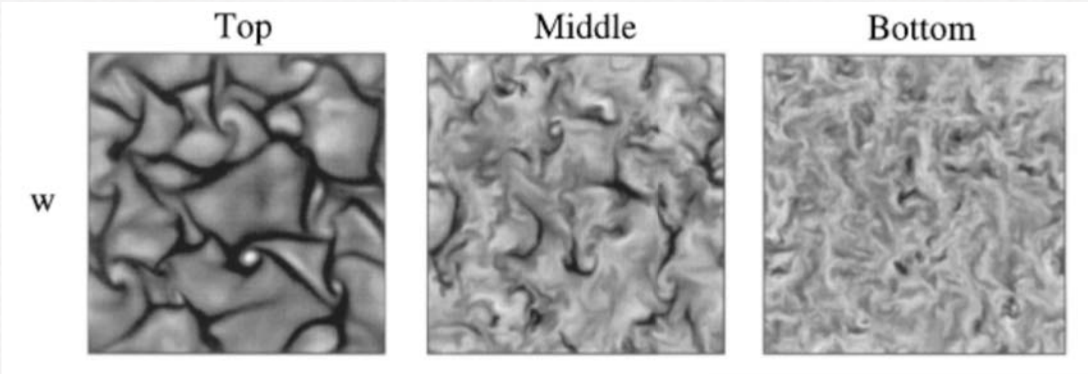
King et al (2009)



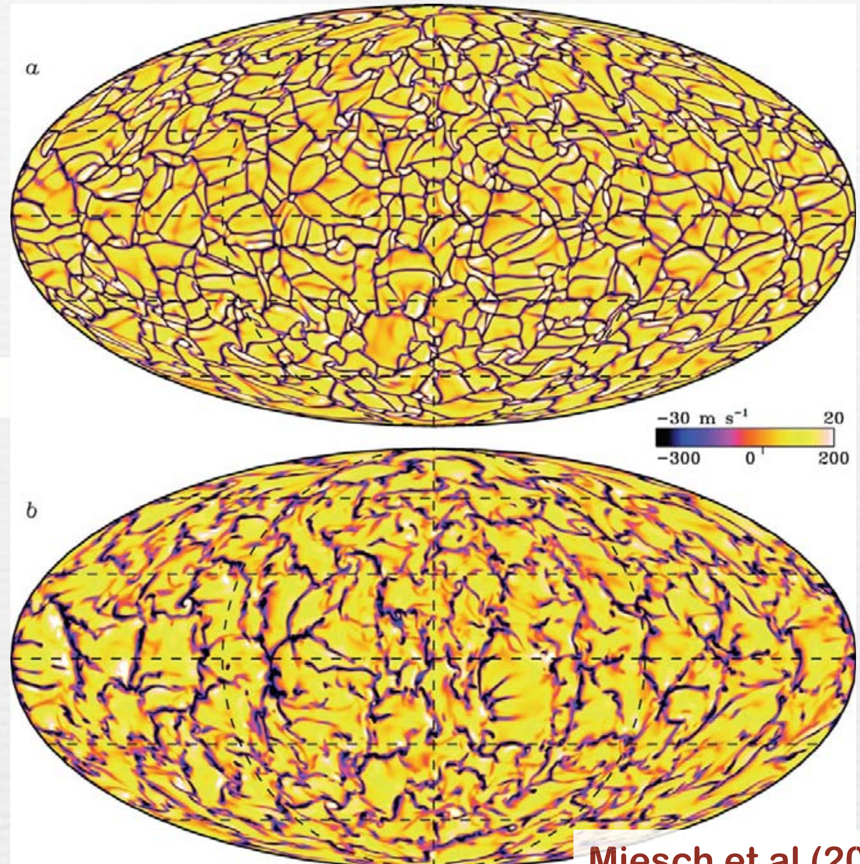




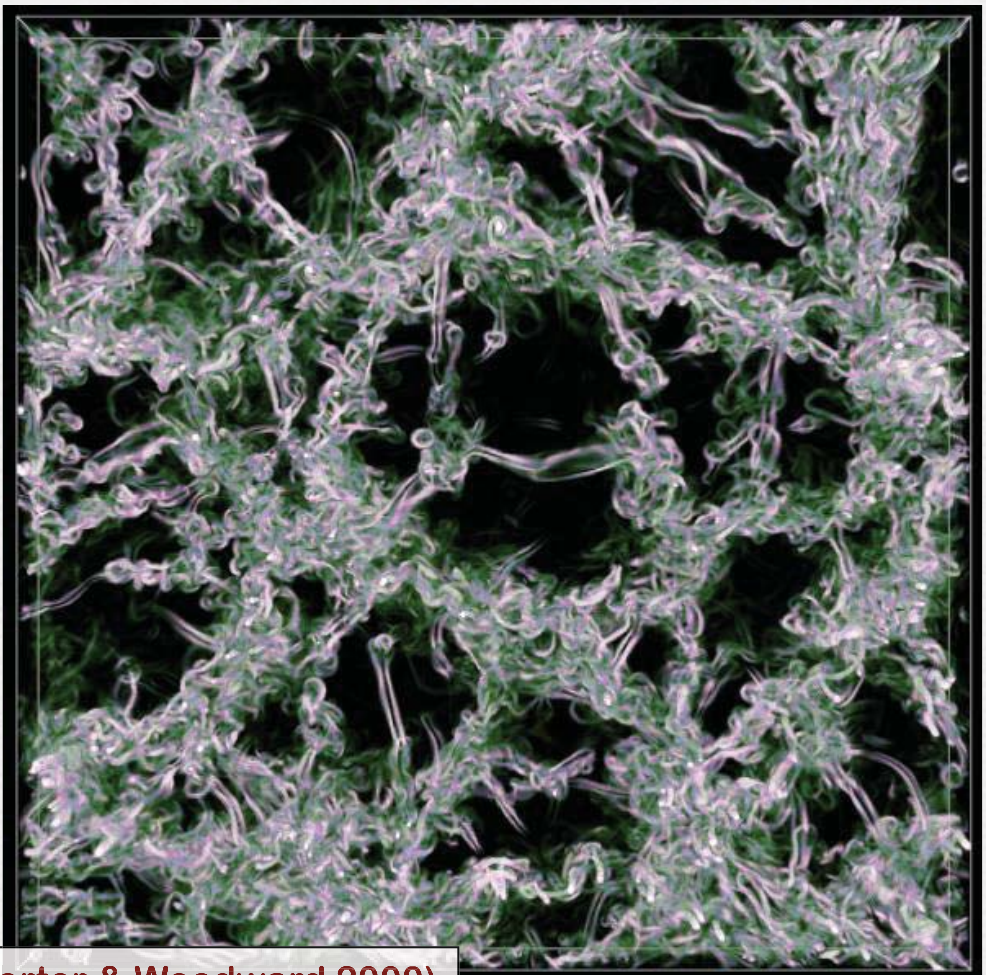
Density Stratification: Downflow lanes and plumes



Brummell et al (1996)



Miesch et al (2008)

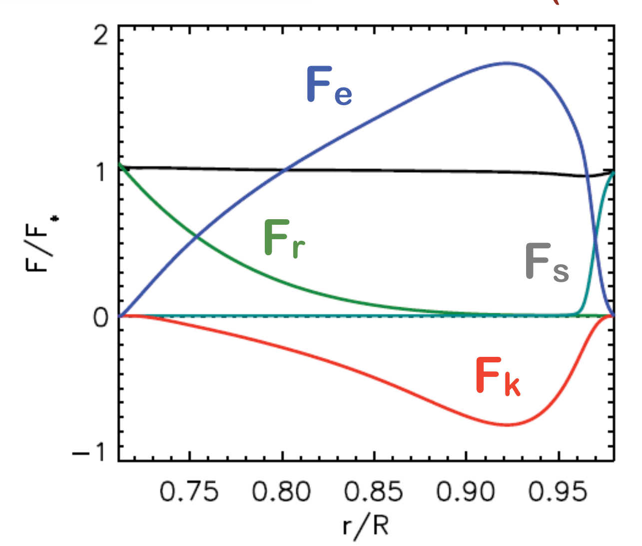


(Porter & Woodward 2000)

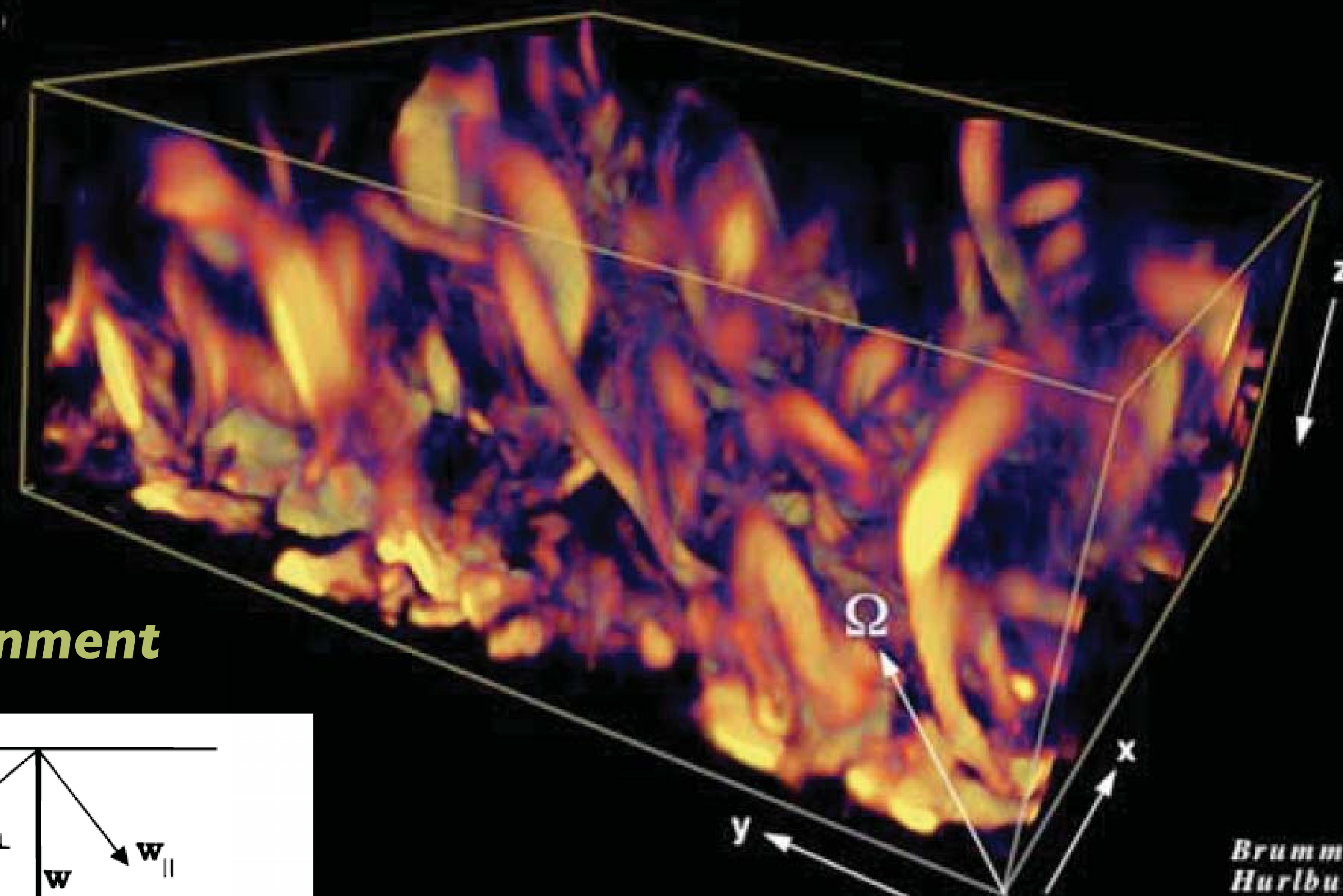
Fast, turbulent downflows

slower, more laminar upflows

Downward KE flux

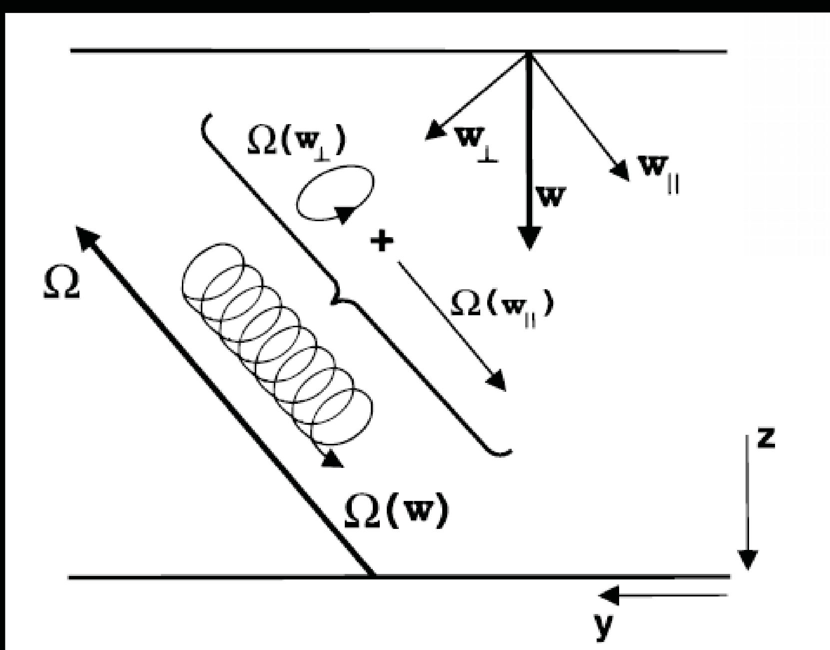


Rotation + Density Stratification: Helical Downflows



Brummell, Hurlburt, Toomre

Turbulent Alignment

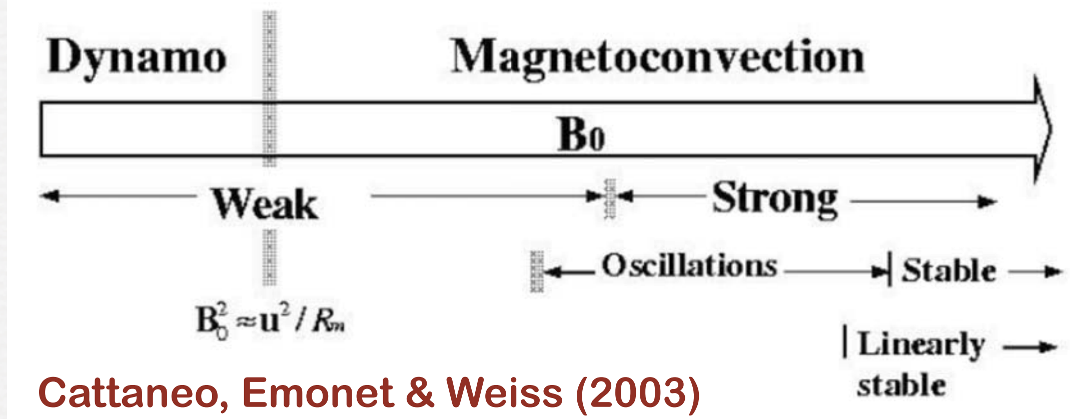


Rossby Number

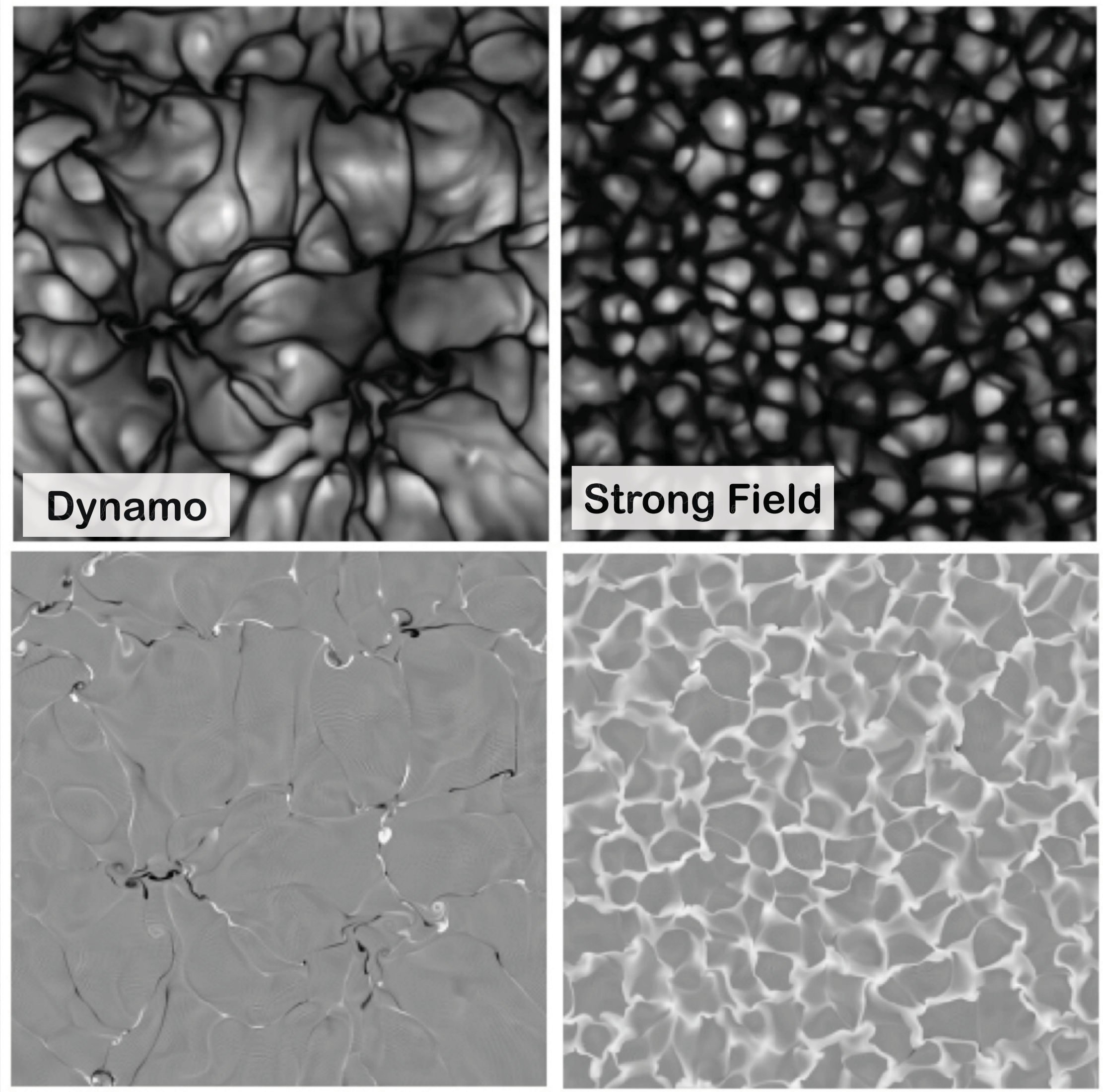
$$\mathrm{Ro} = \frac{\omega_{rms}}{2\Omega}$$

Magnetism:
Field Amplification and Advection

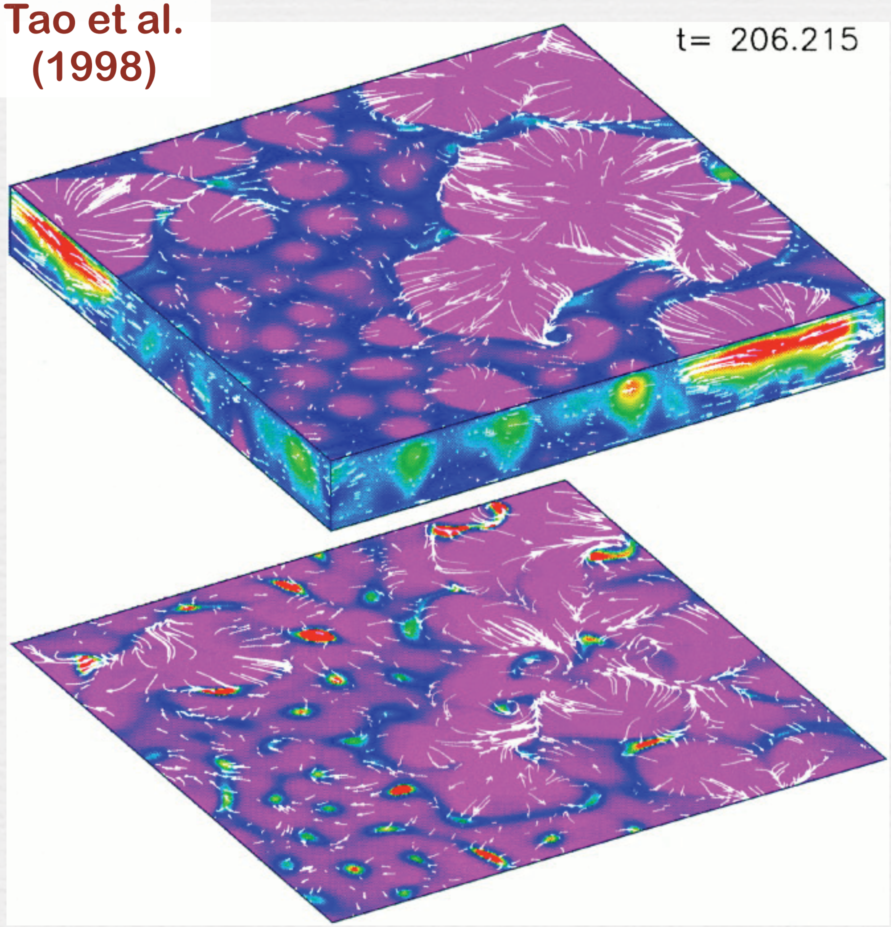
Flux Expulsion
Flux Separation
Turbulent Diamagnetism
Magnetic Pumping
Subcritical instability



Cattaneo, Emonet & Weiss (2003)



Tao et al. (1998)



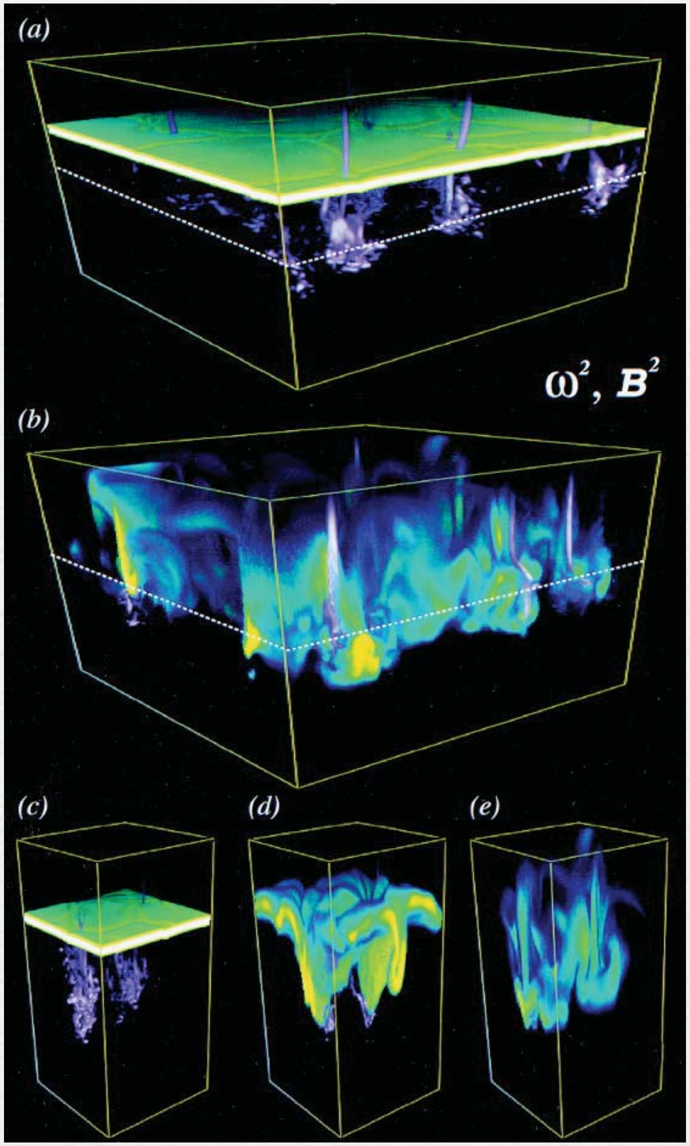
Magnetism: Magnetic Pumping

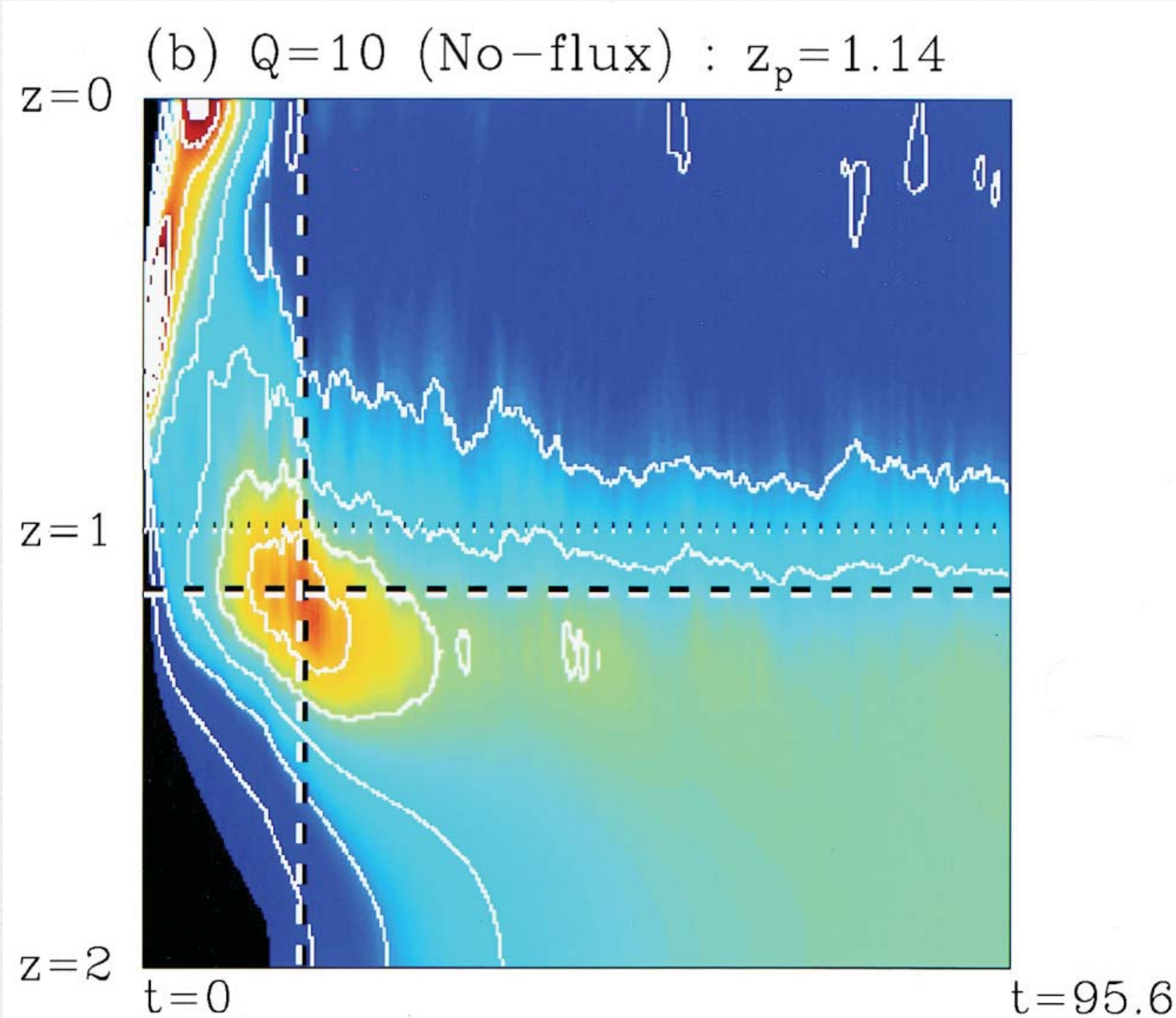
Why downward transport?

Flow asymmetry (downflows are faster)

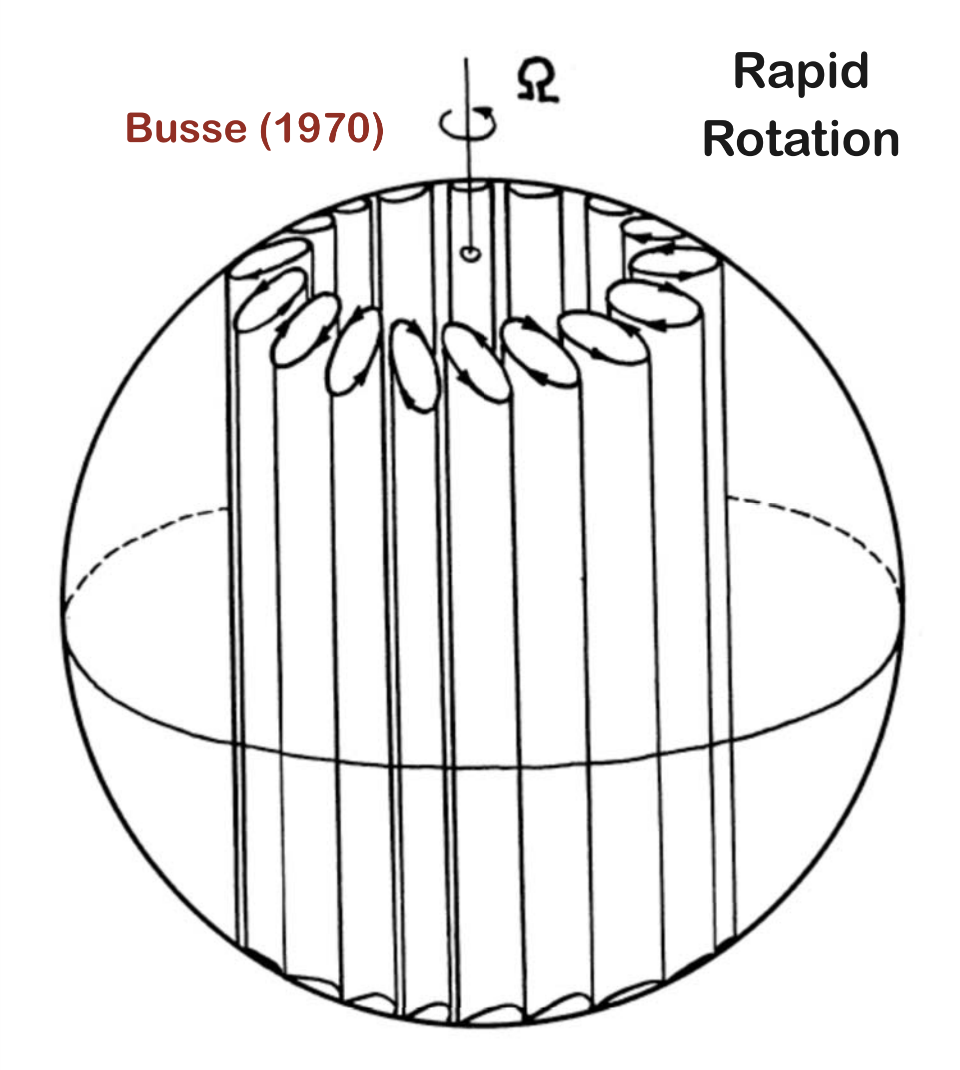
Topological connectivity (Moffatt 1978)

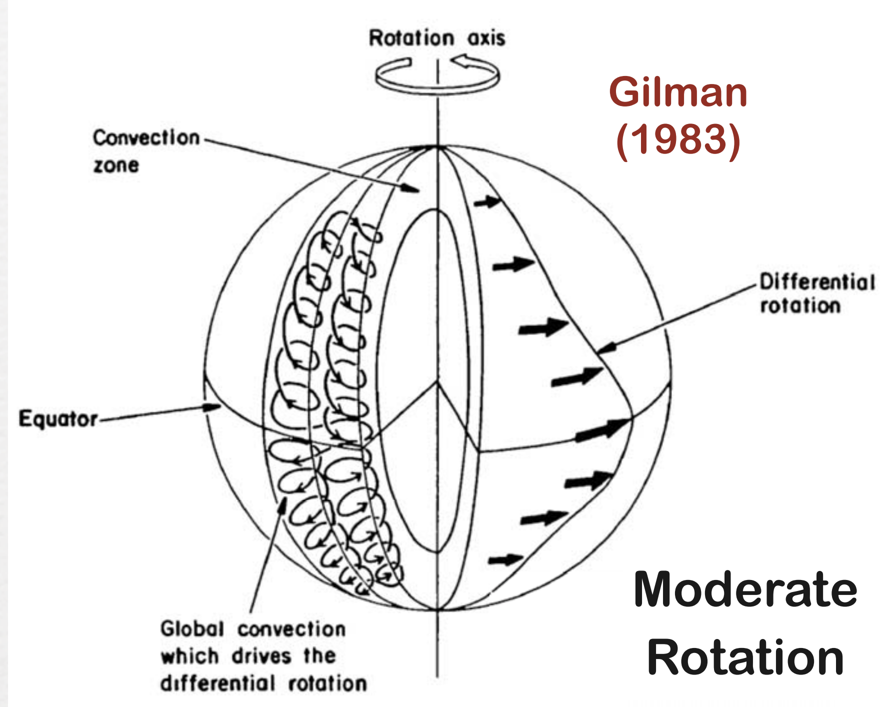
Tobias et al. (2001)





Spherical Geometry: ***Convective Columns, Tangent Cylinder***





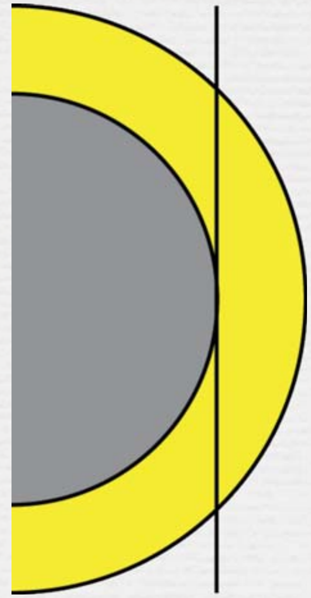
Busse Columns
Banana Cells
Thermal Rossby Waves

In convective shells, columnar convection modes only exist outside the

tangent cylinder

Delineates two distinct convection regimes:

Equatorial modes
Polar Modes



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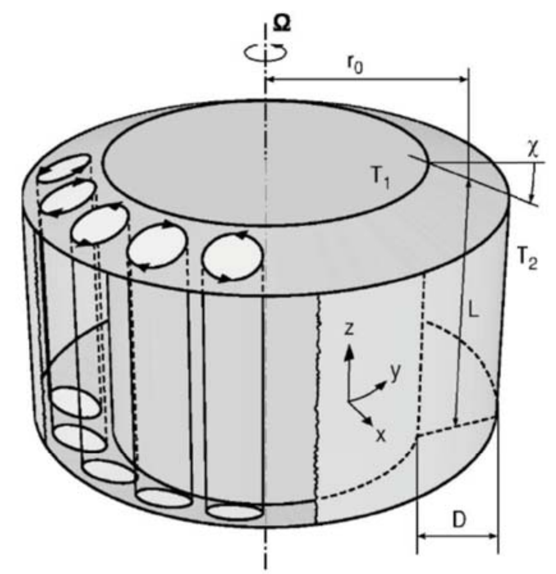
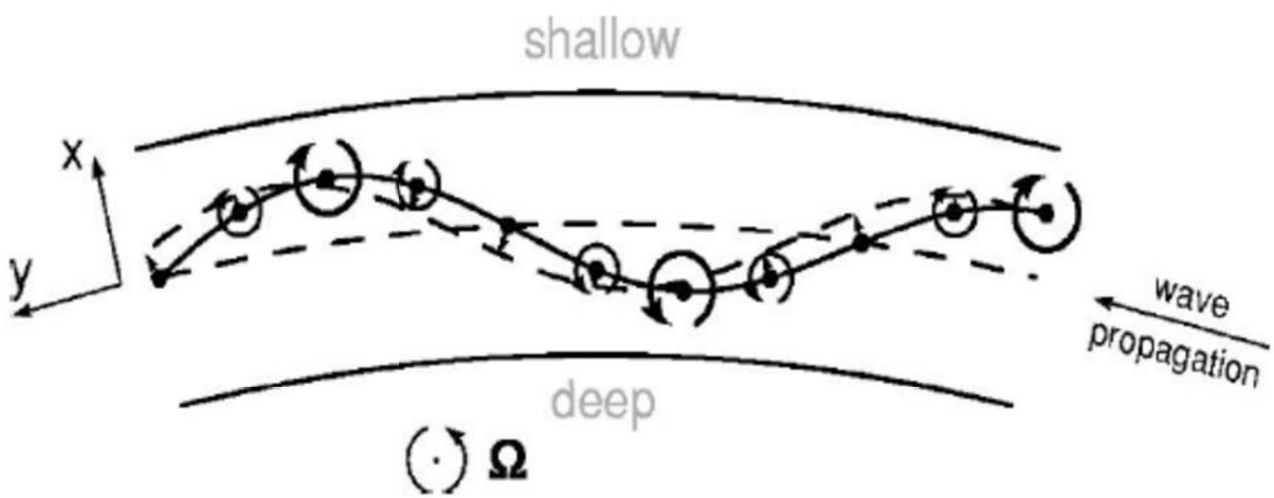


Busse (2002)



Potential Vorticity

$$Q = \frac{\omega_z + 2\Omega}{H\rho}$$

$$\frac{DQ}{Dt} = 0$$

anelastic, adiabatic motions, inviscid, non-magnetic, Ro << 1,

$$\boldsymbol{\Omega} \cdot \boldsymbol{\nabla} \rho = 0$$

(Glatzmaier & Gilman 1981)

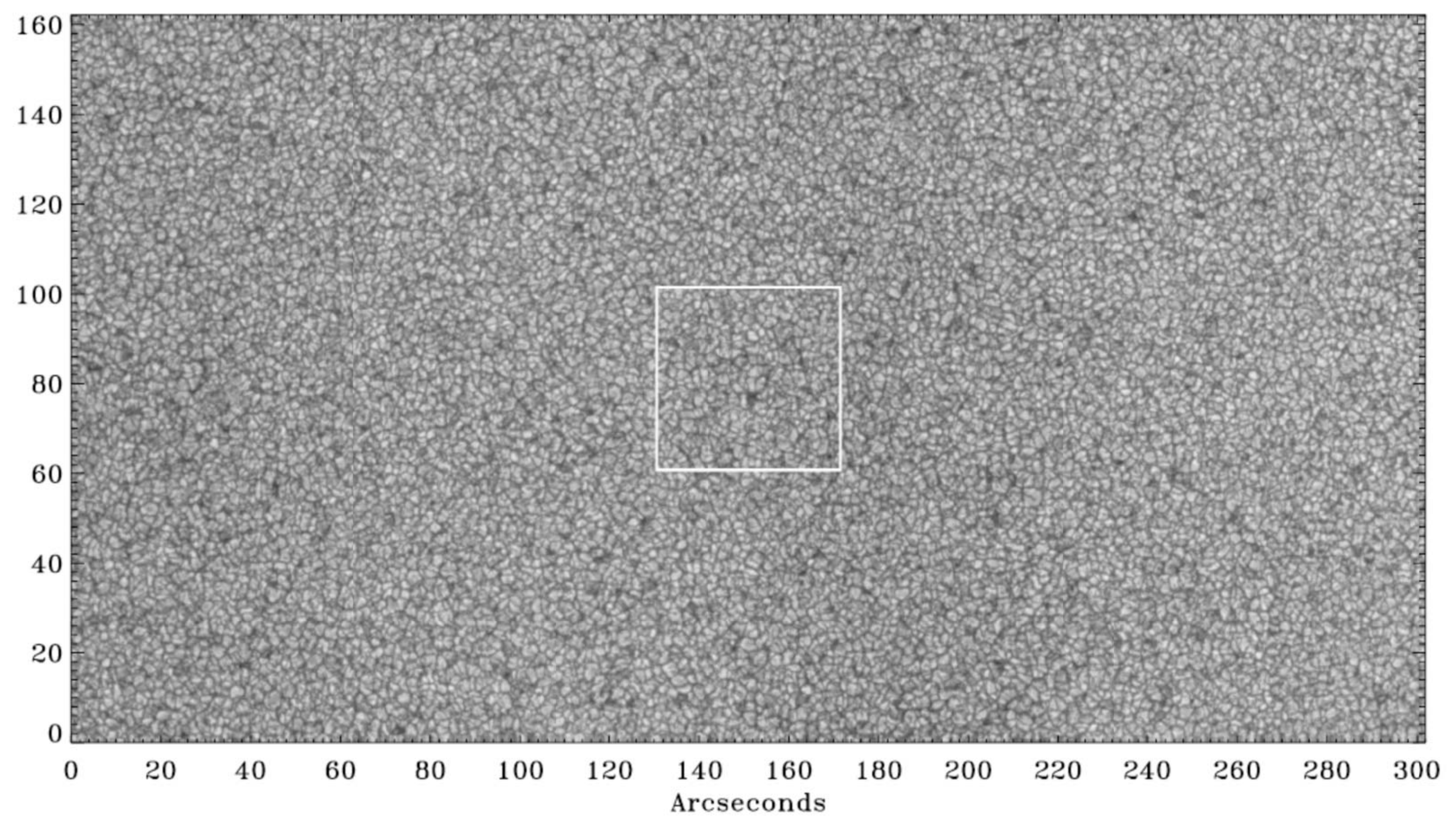
Can be driven either by the spherical curvature of the outer boundary or by the density stratification

Simplest example: Boussinesq fluid, centrifugal gravity, local, linear perturbations, small boundary curvature (Busse 2002)

$$v_p = \frac{4\Omega}{L} \frac{\tan\chi}{(1+P_r)(k_y^2 + k_x^2)}$$

What does all this have to do with the Sun?

Lites et al (2008)



Solar Granulation

$L \sim$ 1-2 Mm
$U \sim 1$ km s^{-1}
$\tau \sim$ 10-15 min

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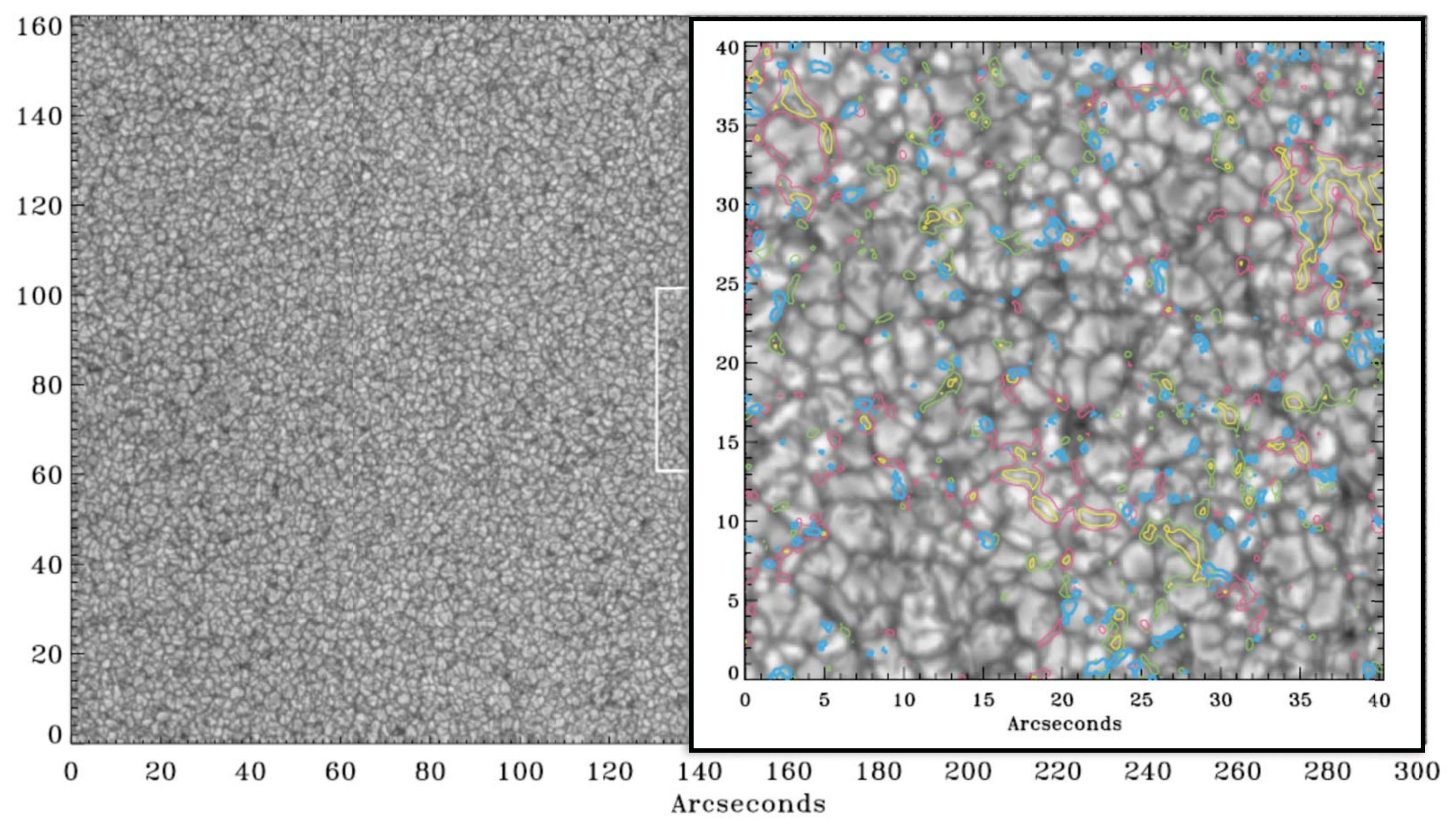


Solar Granulation

$L \sim$ 1-2 Mm
$U \sim 1$ km s^{-1}
$\tau \sim$ 10-15 min

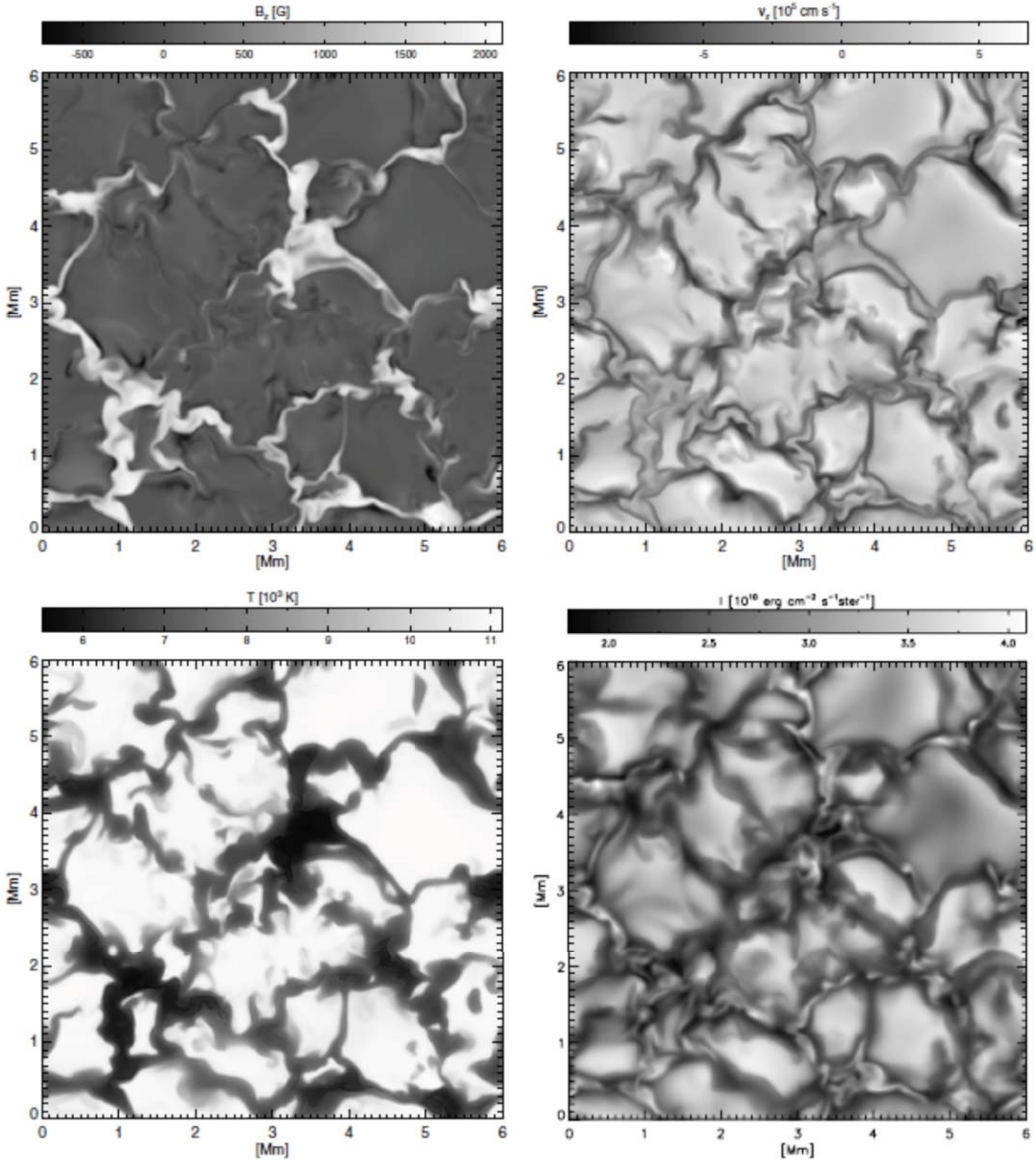
Radiative MHD Simulations of Solar Granulation

Upflows
warm, bright

Downflows
cool, (dark?)

Vertical magnetic fields swept to downflow lanes by converging horizontal flows

Bright spots in downflow lanes attributed to magnetism



Vogler et al. (2005)

Cool doesn't necessarily mean dark

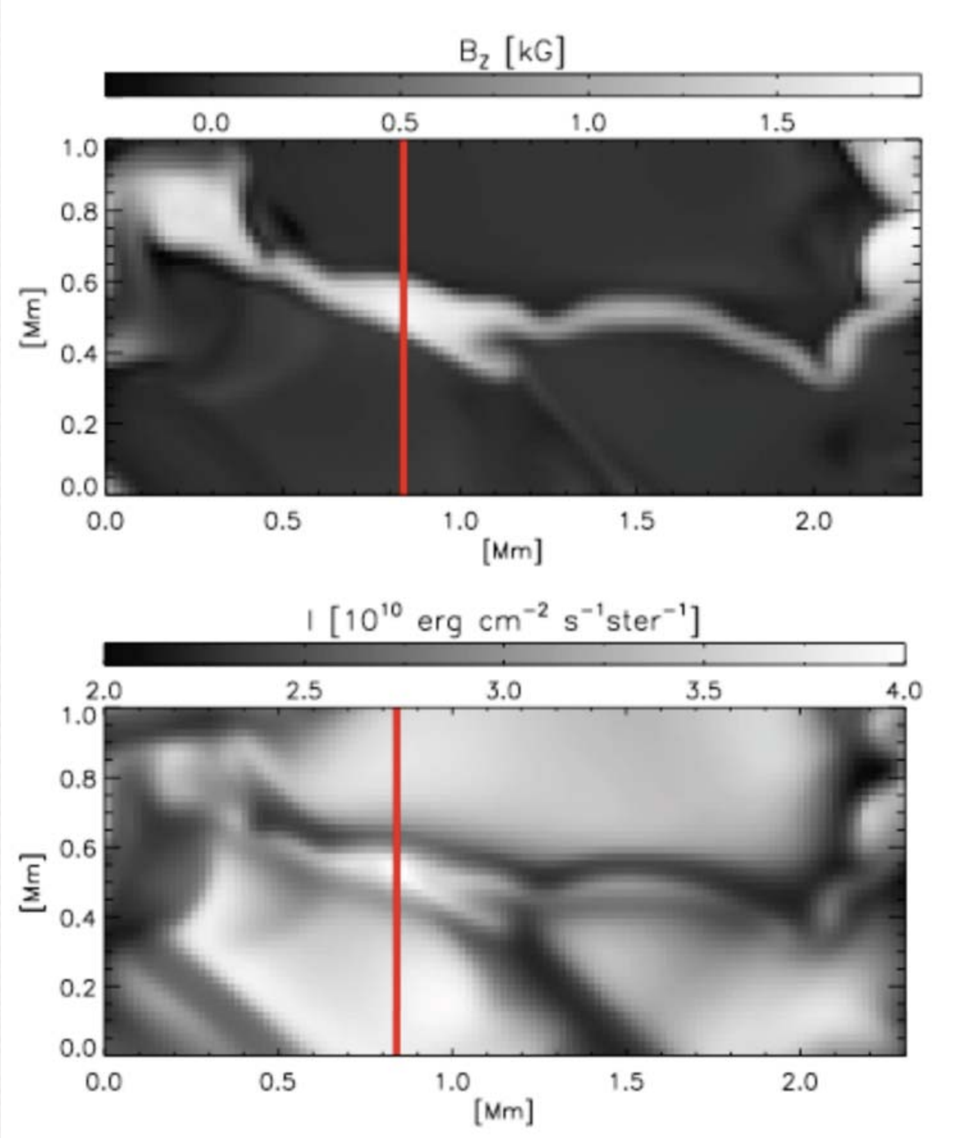
***Channelling of radiation in magnetic flux concentrations* ($B_z > 1$ kG)**

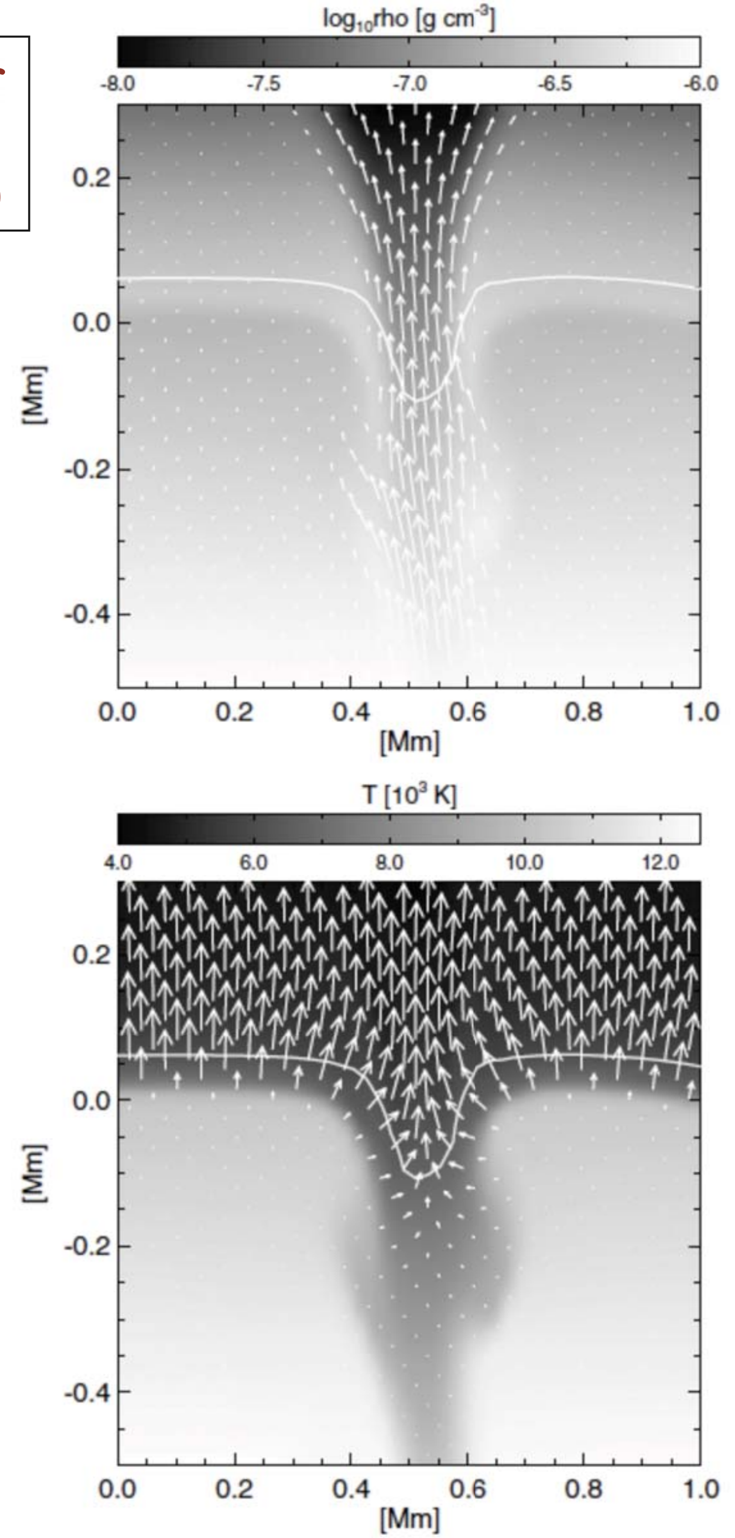
Vogler et al. (2005)

Viewed at an angle they look brighter still

Faculae

Keller et al (2004)





Scale Selection

Granulation is driven by strong radiative cooling in the photosphere

Downflows dominate buoyancy work

Upflows are largely a passive response induced by horizontal pressure gradients; peak velocities occur adjacent to downflows

When granules get too wide, radiative cooling overcomes the convective flux coming up from below, reversing the buoyancy driving in the center of the granule

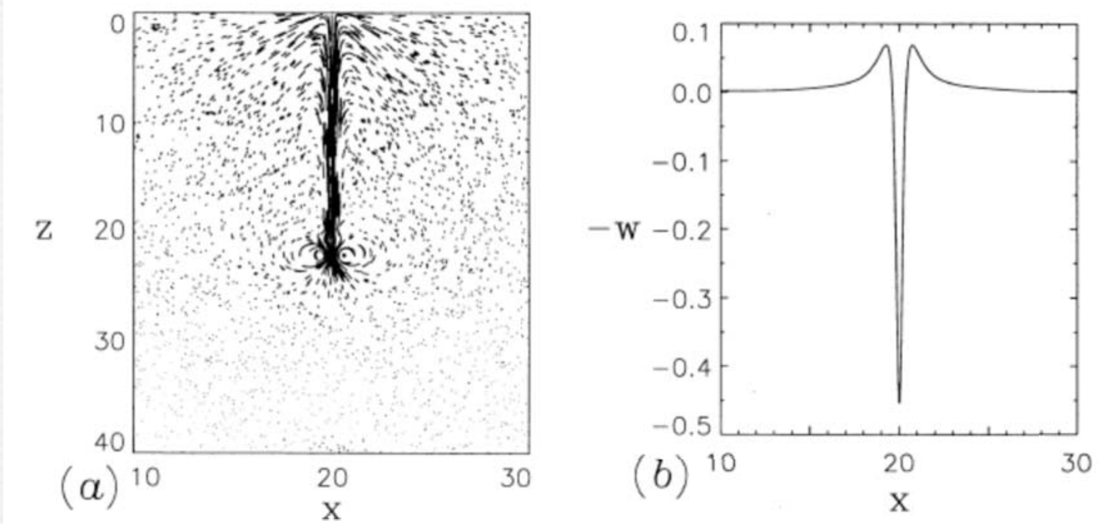
Upflow becomes downflow and the granule bisects (exploding granules)

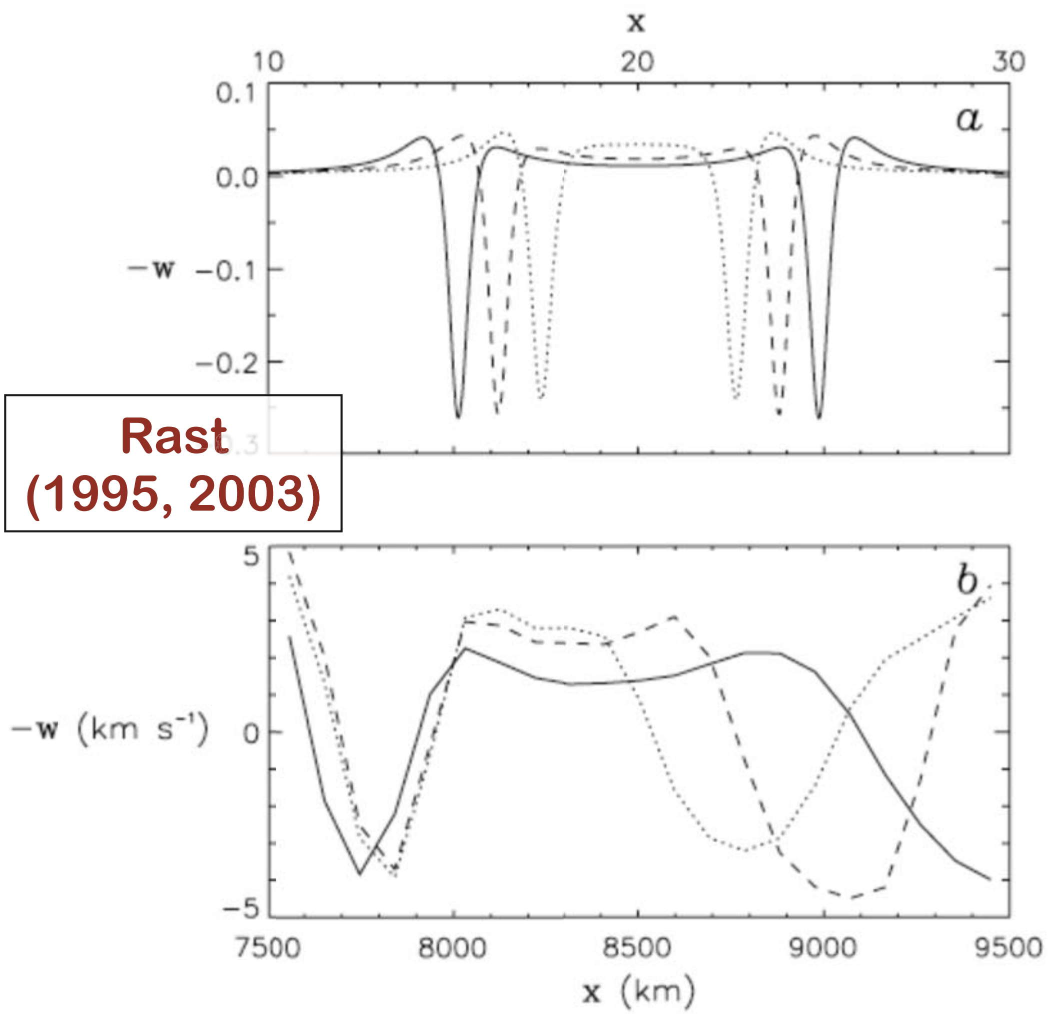
$$\rho v_z y N_A \chi_H \gtrsim \sigma T^4$$

$$L \sim D \frac{v_h}{v_z}$$

$$v_h \lesssim c_s$$

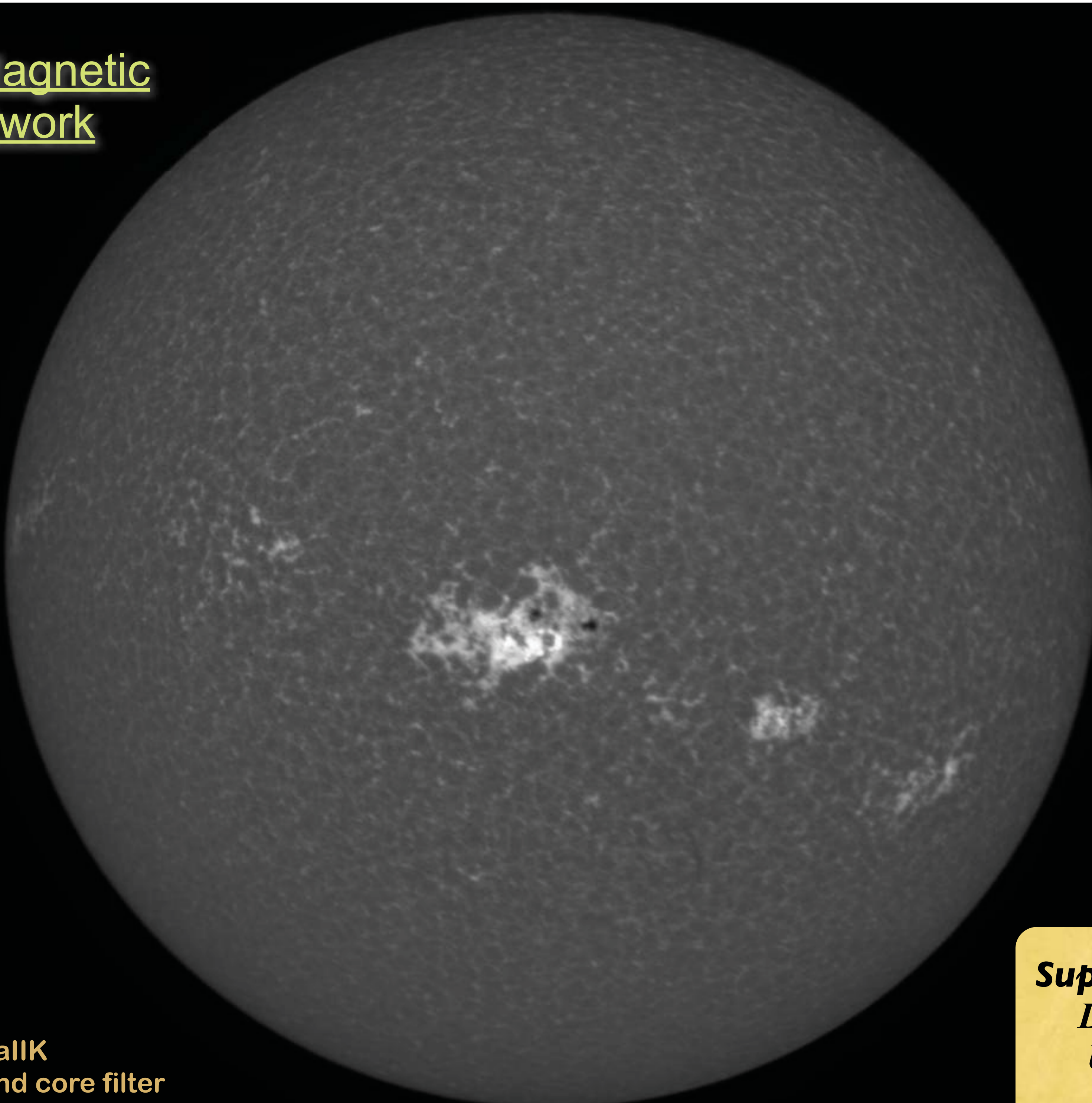
$$D \sim H_\rho$$





Rast (1995, 2003)

The Magnetic Network



CaIIK
narrow-band core filter
PSPT/MLSO

Supergranulation

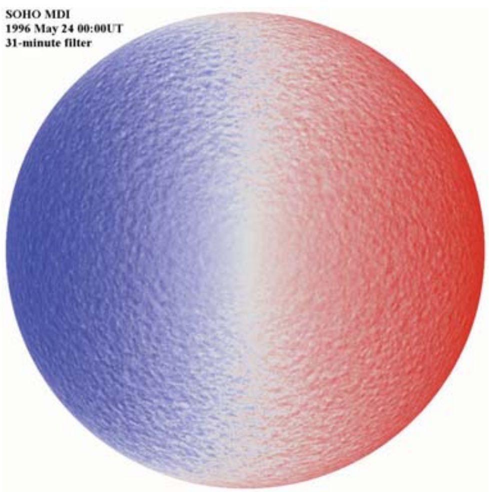
$L \sim 30\text{-}35$ Mm

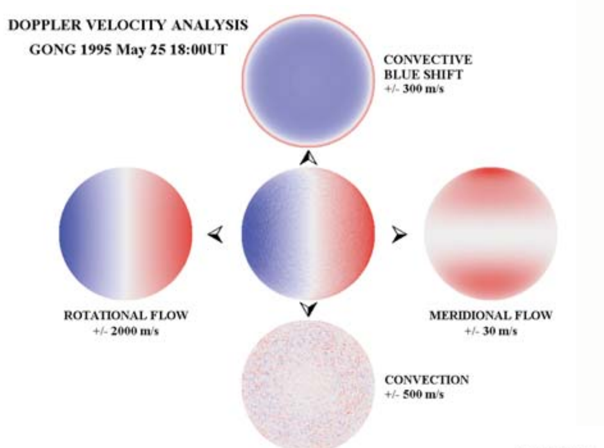
$U \sim 500$ m s^{-1}

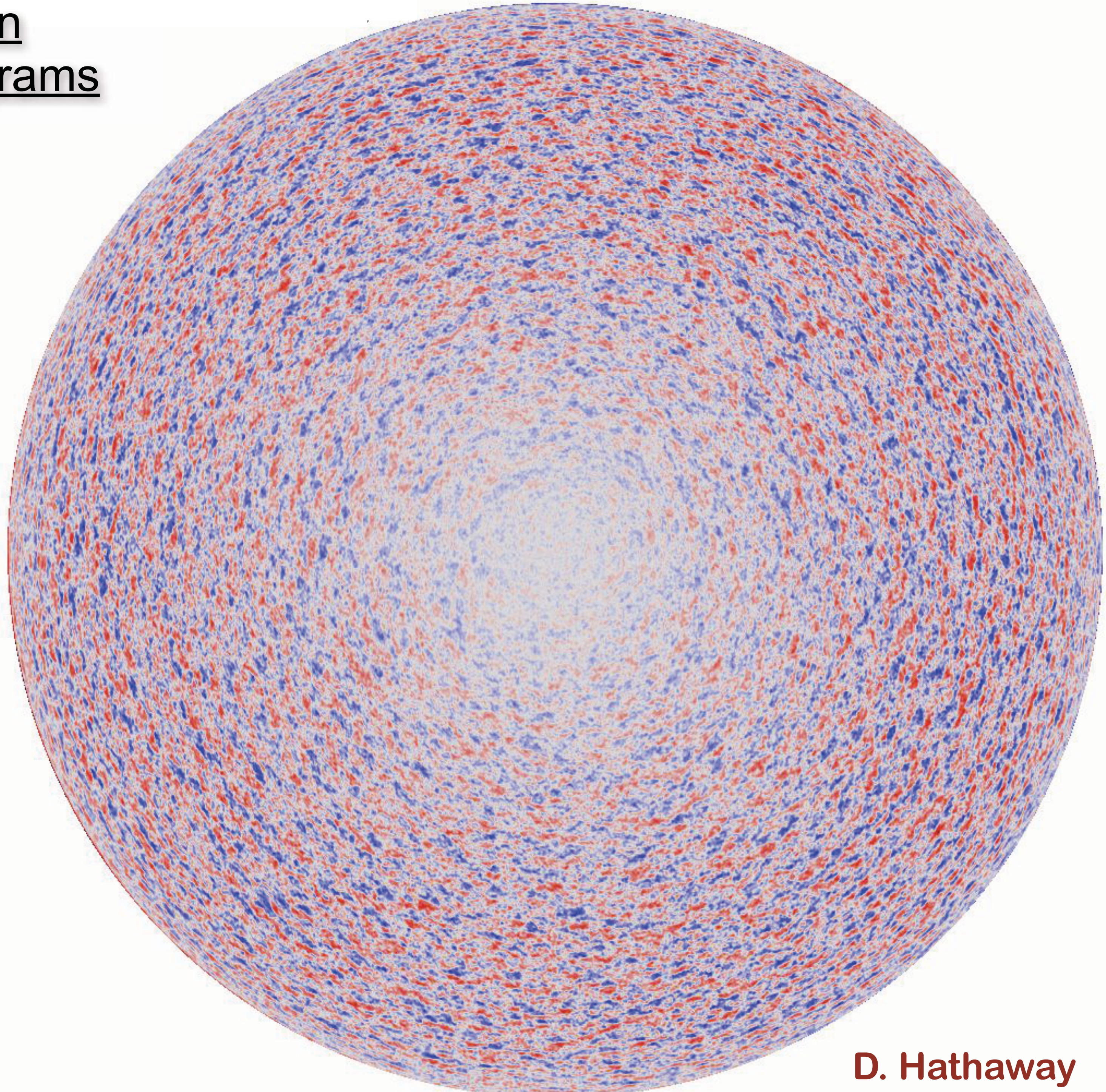
$\tau \sim 20$ hr

Supergranulation in Filtered Dopplergrams

Most prominent in horizontal velocities near the limb

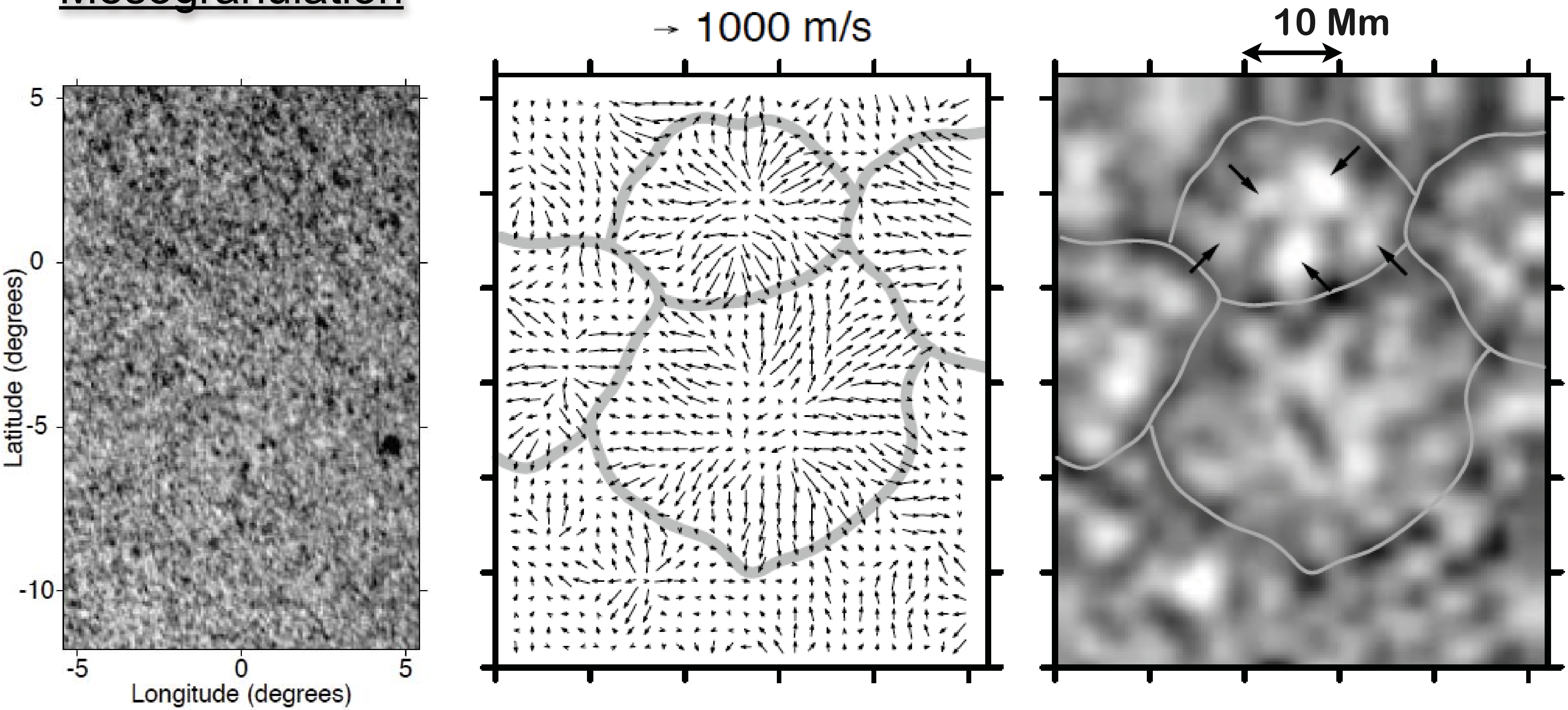






D. Hathaway (NASA MSFC)

Mesogranulation



Shine, Simon & Hurlburt (2000)

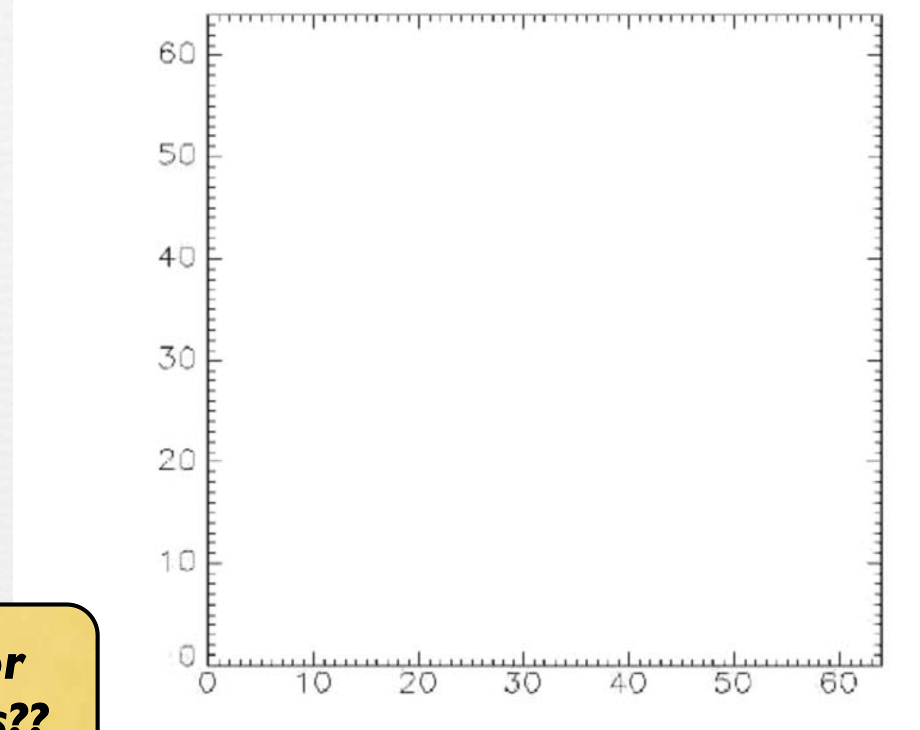
Most readily seen in horizontal velocity divergence maps obtained from local correlation tracking (LCT)

Vertical velocity and temperature signatures of mesogranulation and supergranulation are still elusive
hard to verify that they are convection per se

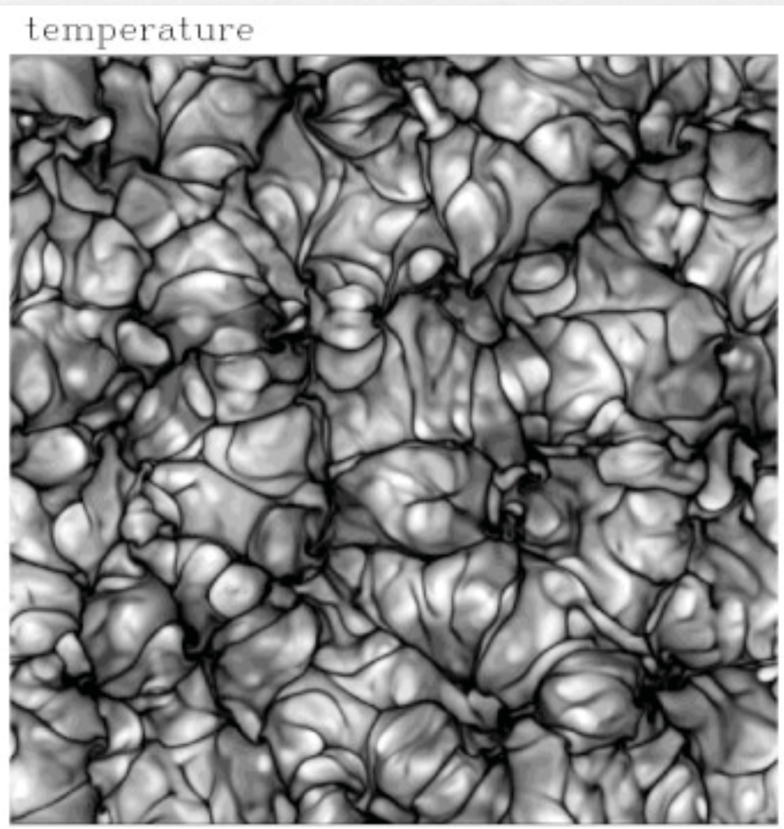
$L \sim 5$ Mm
$\tau \sim 3\text{-}4$ hr

A heirarchy of convective motions

"toy model" Rast (2003)

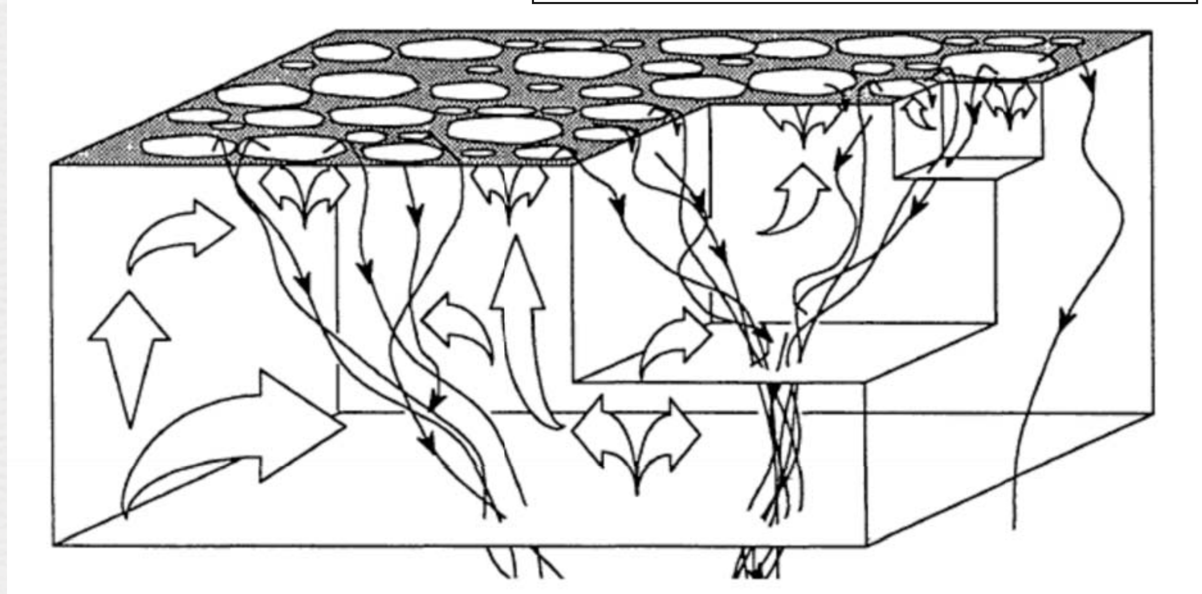


- **Kinematic advection**
  - **Converging flows cause plumes to cluster**
- **Density stratification**
  - **Promotes mergers by squeezing plumes together as they penetrate deeper**



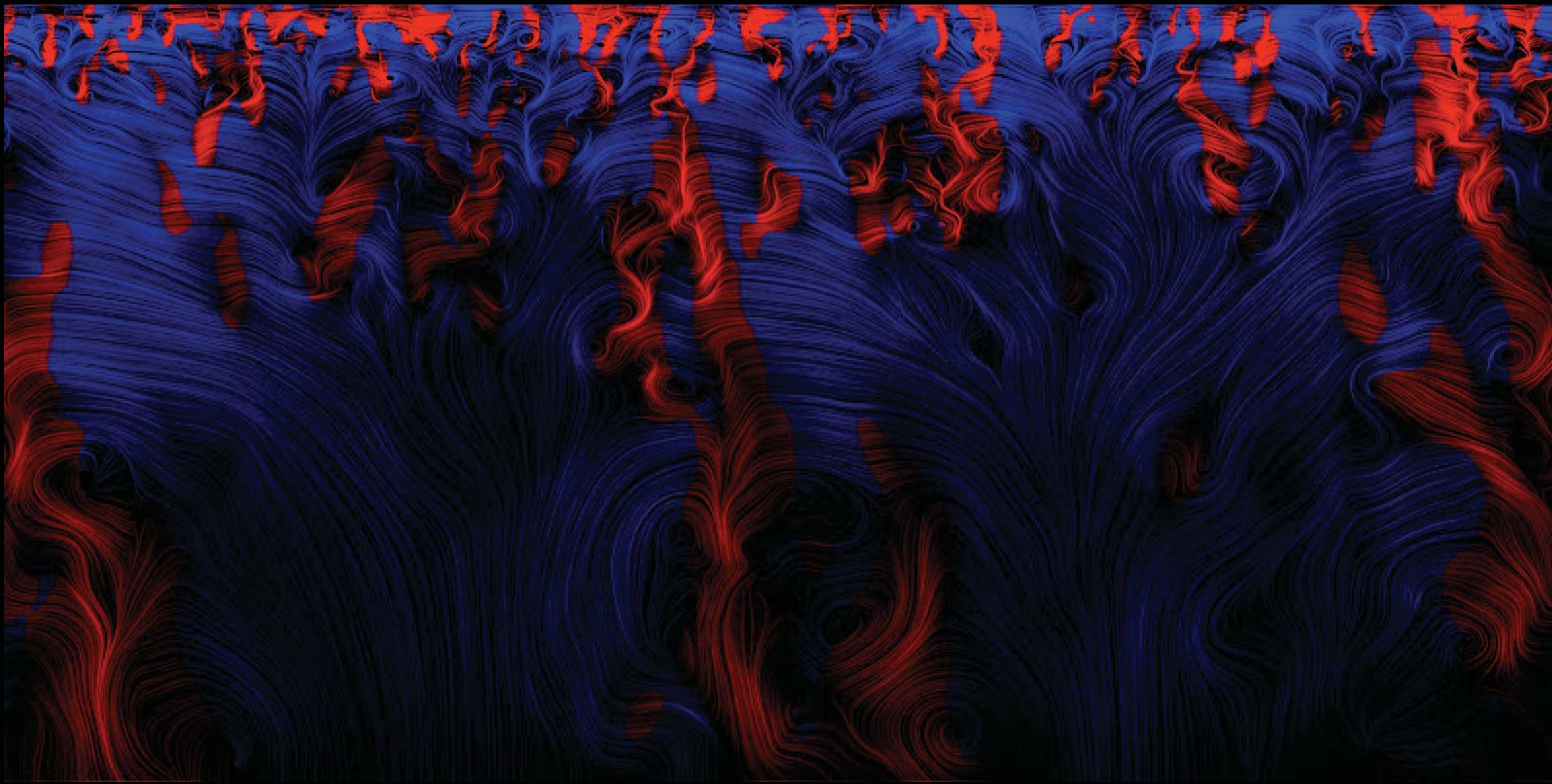
Discrete or Continuous??

Spruit, Nordlund & Title (1990)



Cattaneo, Lenz & Weiss (2001)

simulation by Stein et al (2006), visualization by Henze (2008)



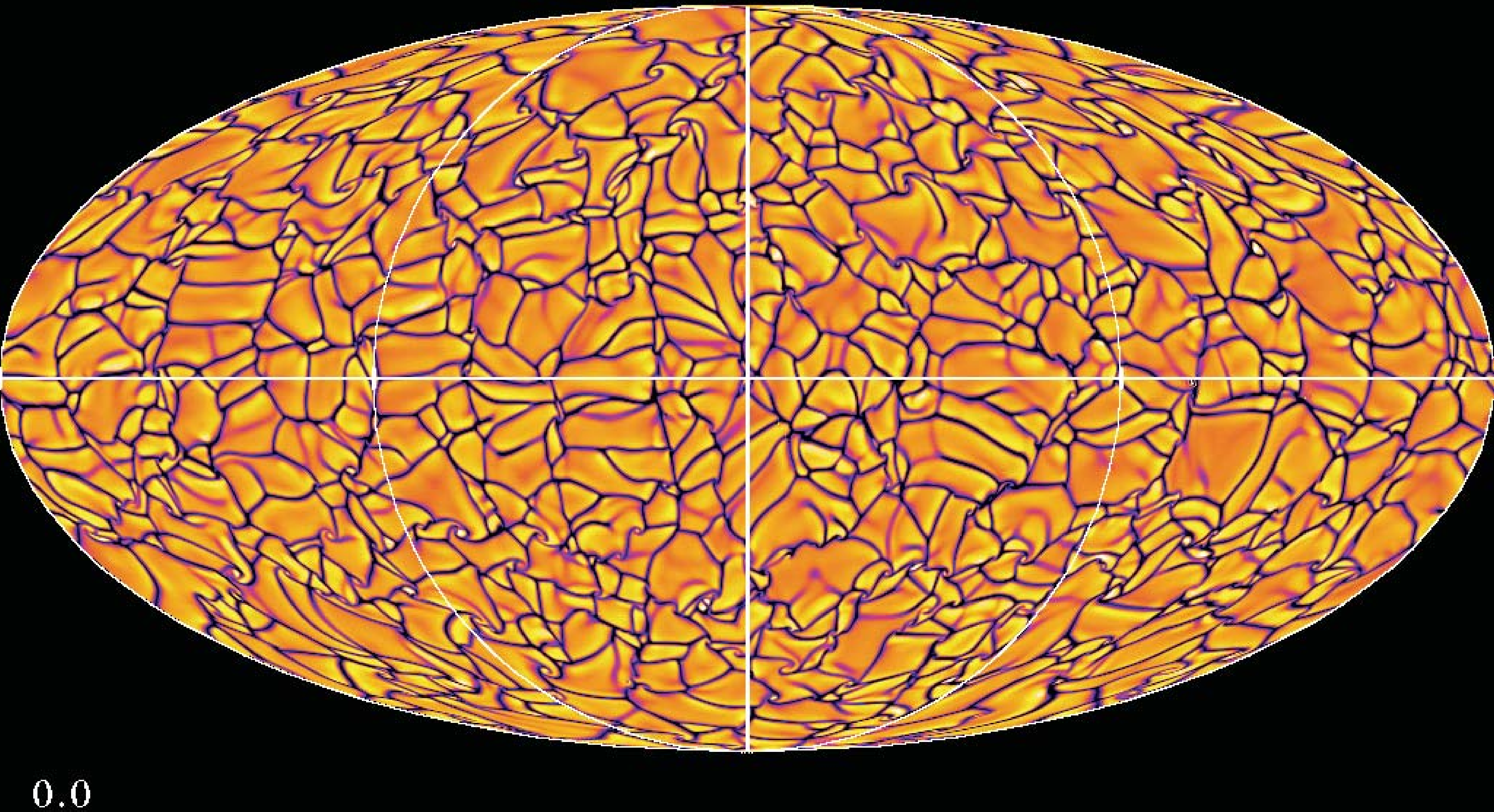
Size, time scales of convection cells increases with depth

Beyond Solar Dermitology
But still stops at 0.97R!
what lies deeper still?

Giant Cells

(Loosely, anything bigger than supergranulation)

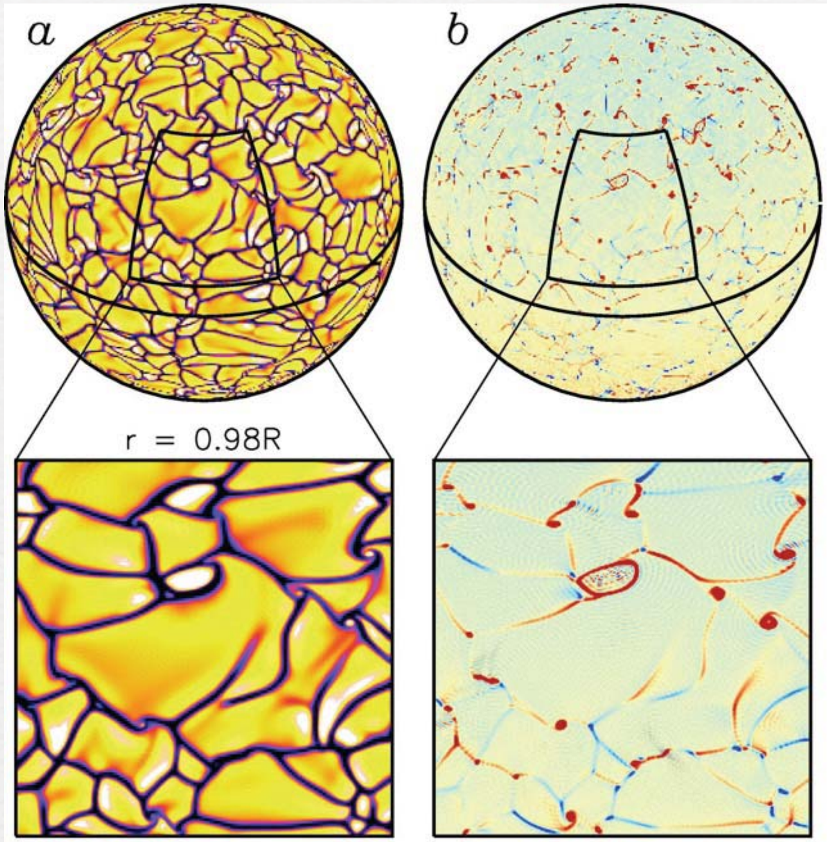
Eventually the heirarchy must culminate in motions large enough to sense the spherical geometry and rotation

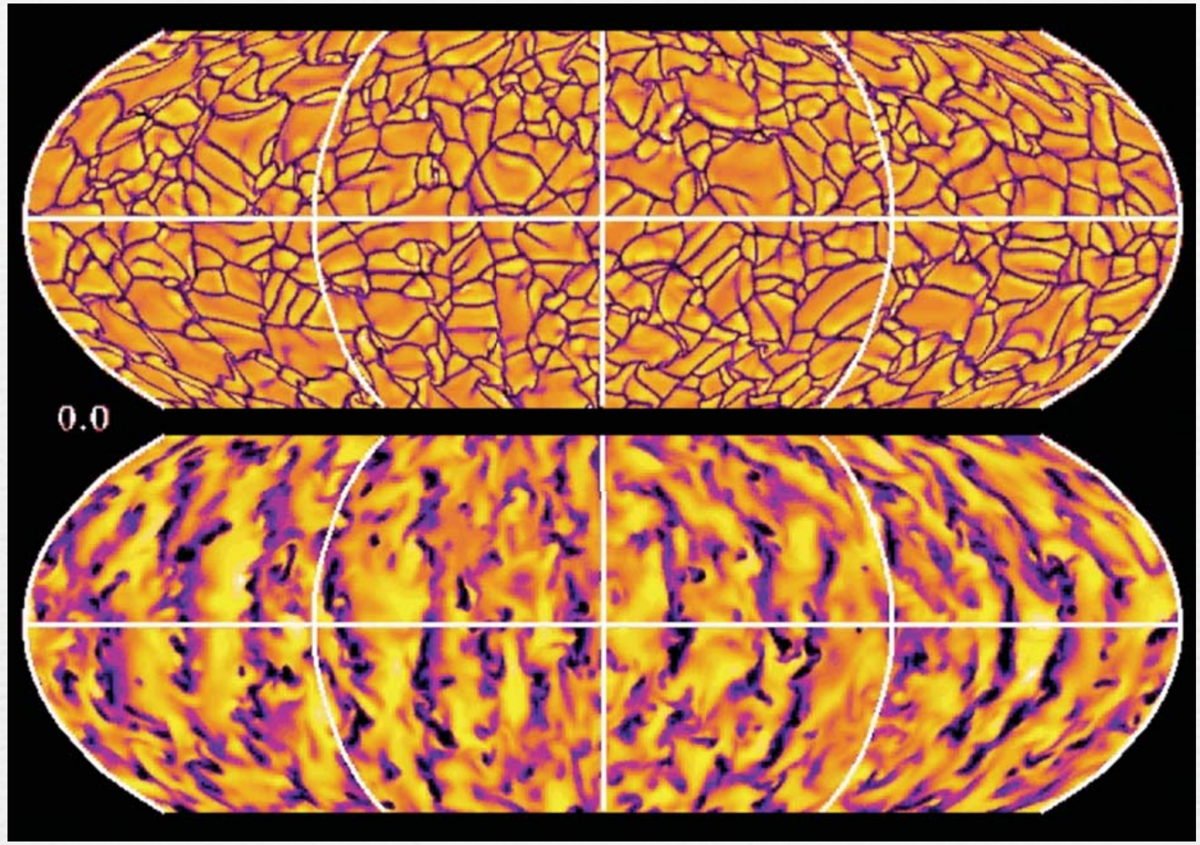


radial velocity, r = 0.98R

Miesch et al (2008)

Structure of Giant Cells





Solar Cyclones at high latitudes
(cool, helical downflows)

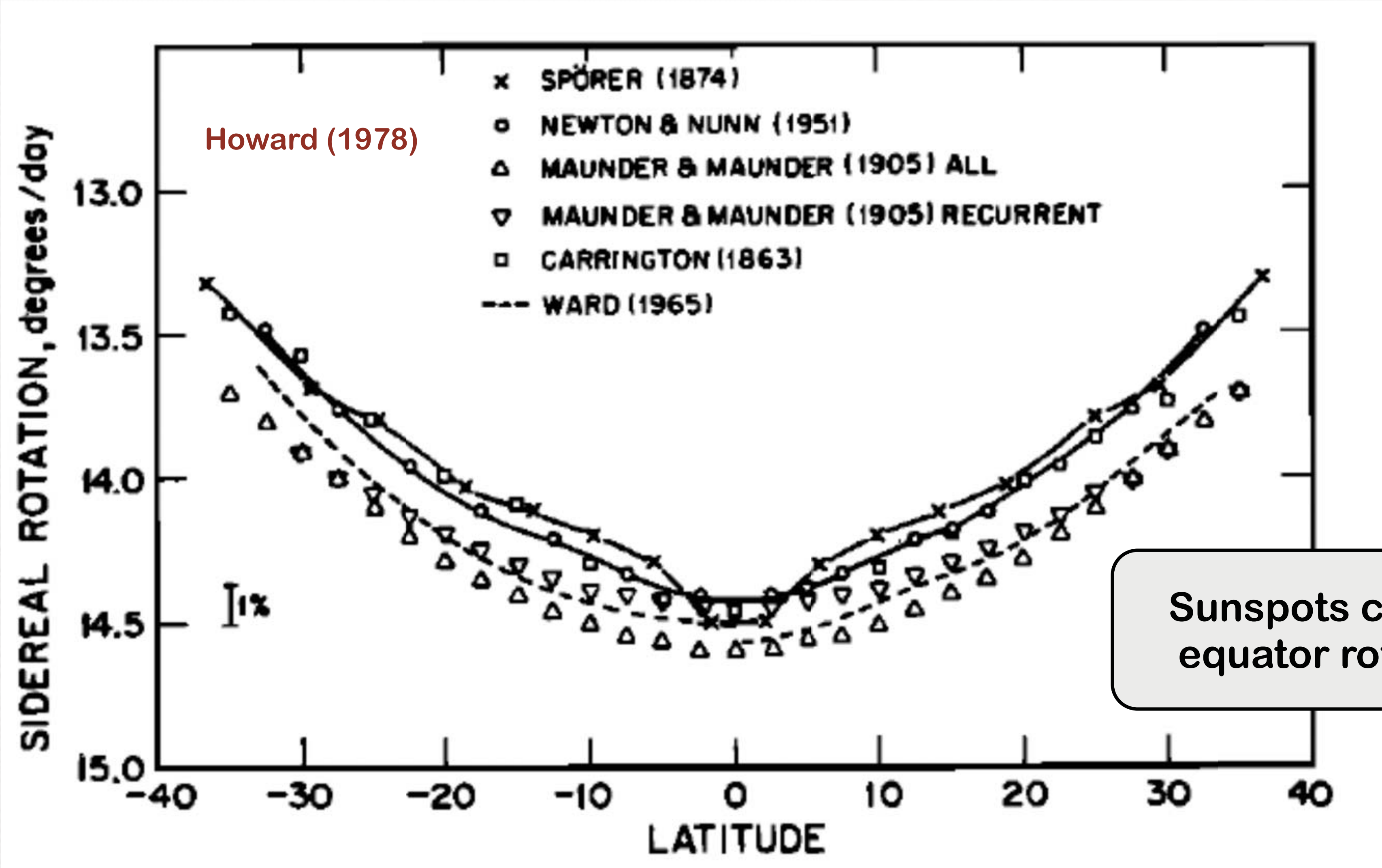
Convective columns at low latitudes
(thermal Rossby waves: prograde propagation)

Summary: Solar Convection

- **Plumes and Boundary Layers**
  - Characteristic feature of turbulent convection (lab, simulations, stars...)
  - Strong influence on dynamics throughout the domain despite their small extent
  - Granulation driven by strong radiative cooling in the photosphere
  - Merging of downflow plumes produces hierarchy of convective motions (granulation, mesogranulation, supergranulation, [giant cells] ~ 1-100+ Mm)
- **Stratification and Rotation**
  - Density stratification introduces asymmetry: downflows stronger
  - Rotation imparts helicity (sign = $\hat{\Omega} \cdot \hat{g}$): solar cyclones
  - Rotation imparts tilt: Turbulent alignment
- **Magnetism**
  - Weak fields amplified by convection: dynamo action
  - Intermediate fields pushed aside by convection: flux separation, magnetic pumping
  - Strong fields suppress convection: sunspots
- **Spherical Geometry**
  - Tangent Cylinder
  - Convective Columns/Thermal Rossby waves
  - *Giant Cells!*

Anisotropy, inhomogeneity induce transport
Mean Flows!

Solar Differential Rotation



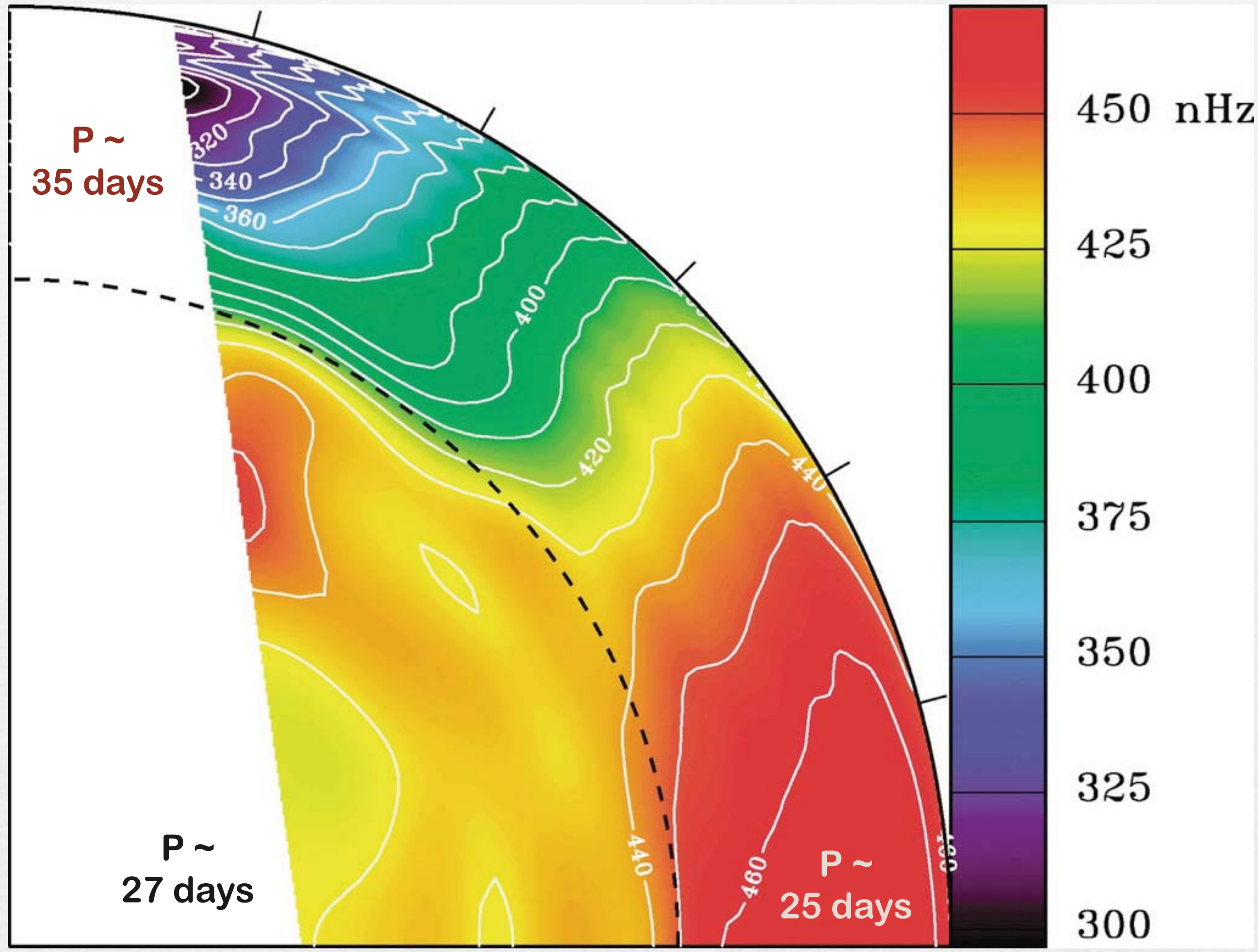
Sunspots closer to the equator rotate faster!

Persistent

The rotation rate determined from spots has not changed by more than a few % since Carrington's measurements spanning 1853-1861 (published in 1863)

The Internal Rotation of the Sun

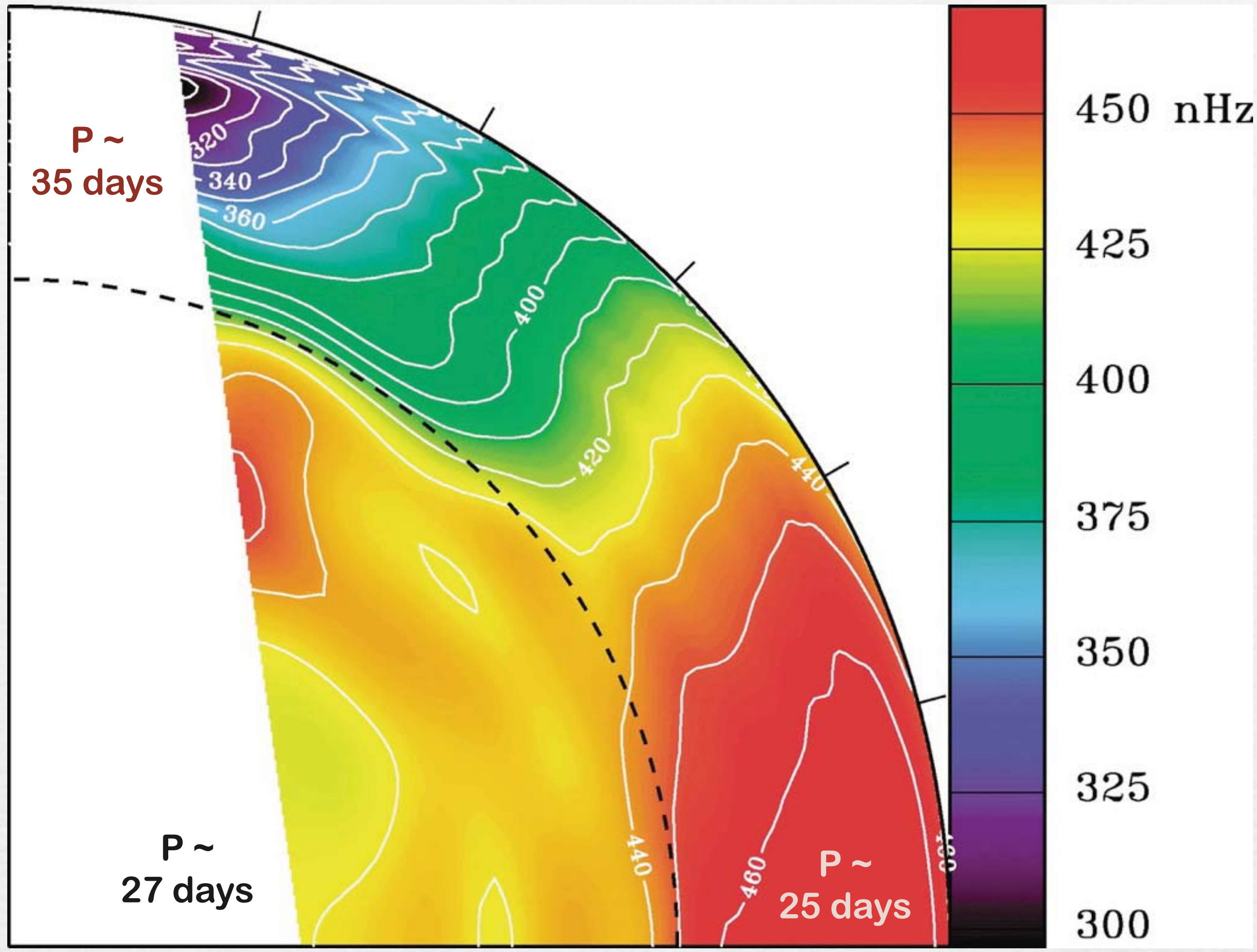
Thompson et al. (2003)



The Internal Rotation of the Sun

Differential Rotation (DR)
Monotonic decrease in Ω of ~ 30% from equator to high latitudes in CZ

Thompson et al. (2003)



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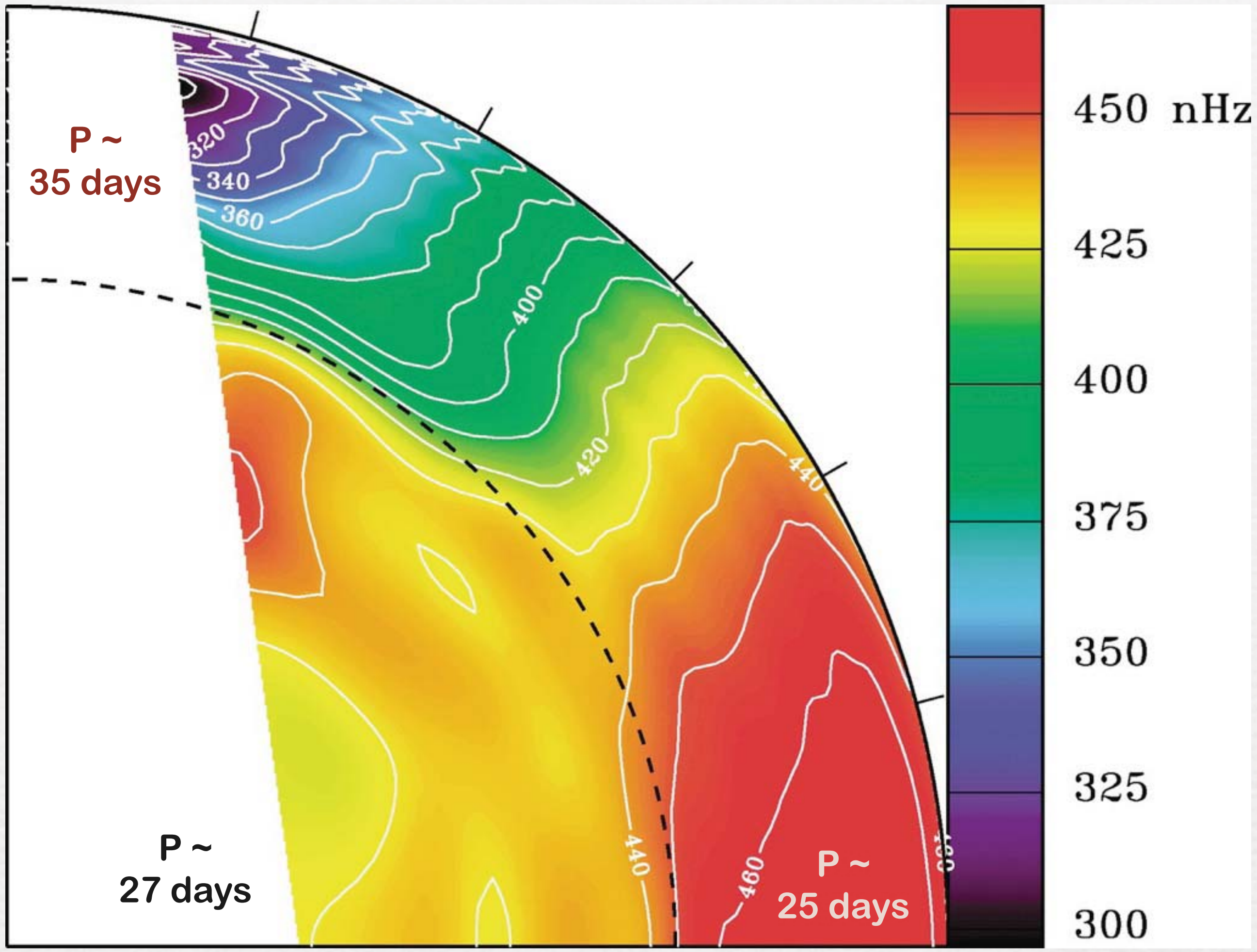

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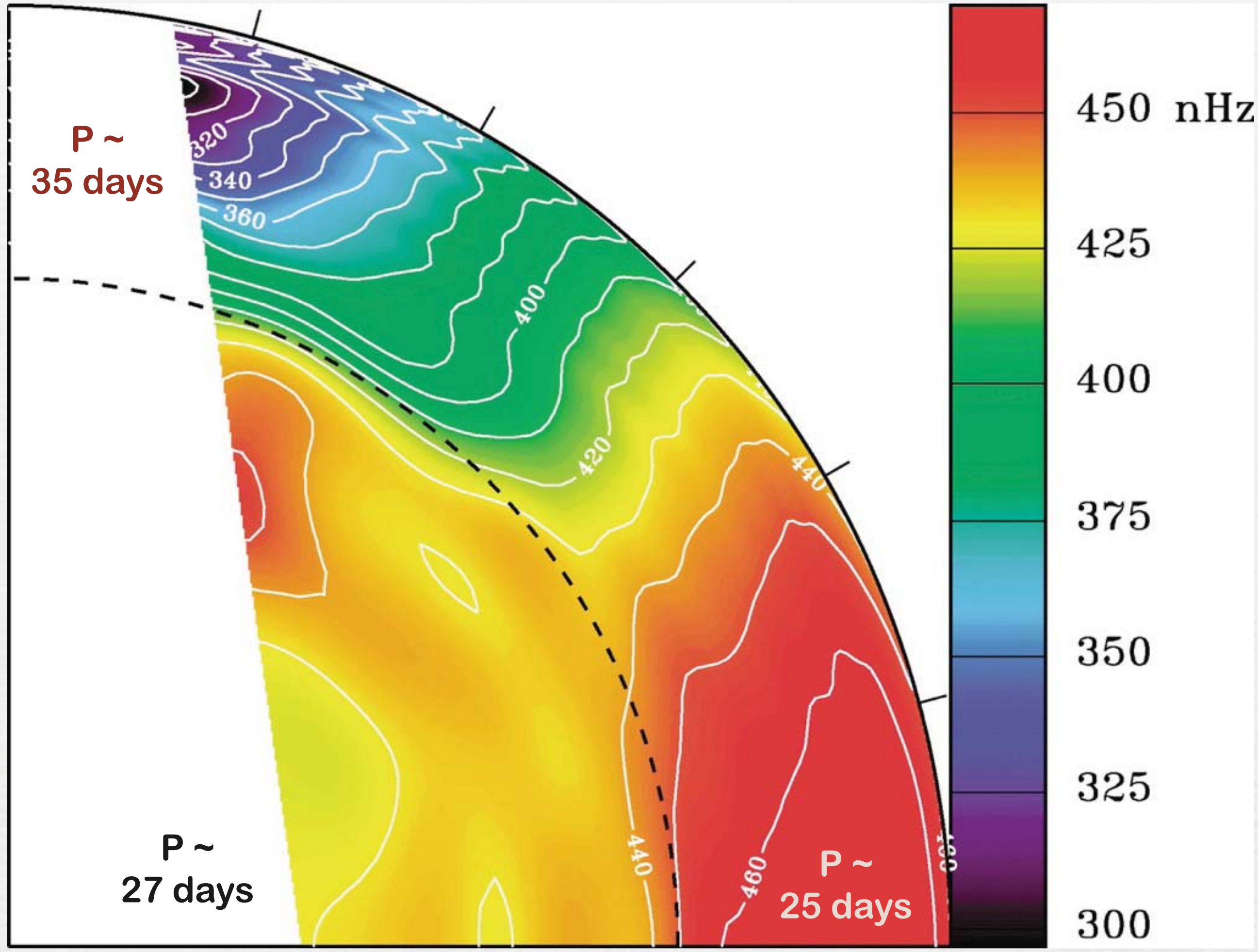

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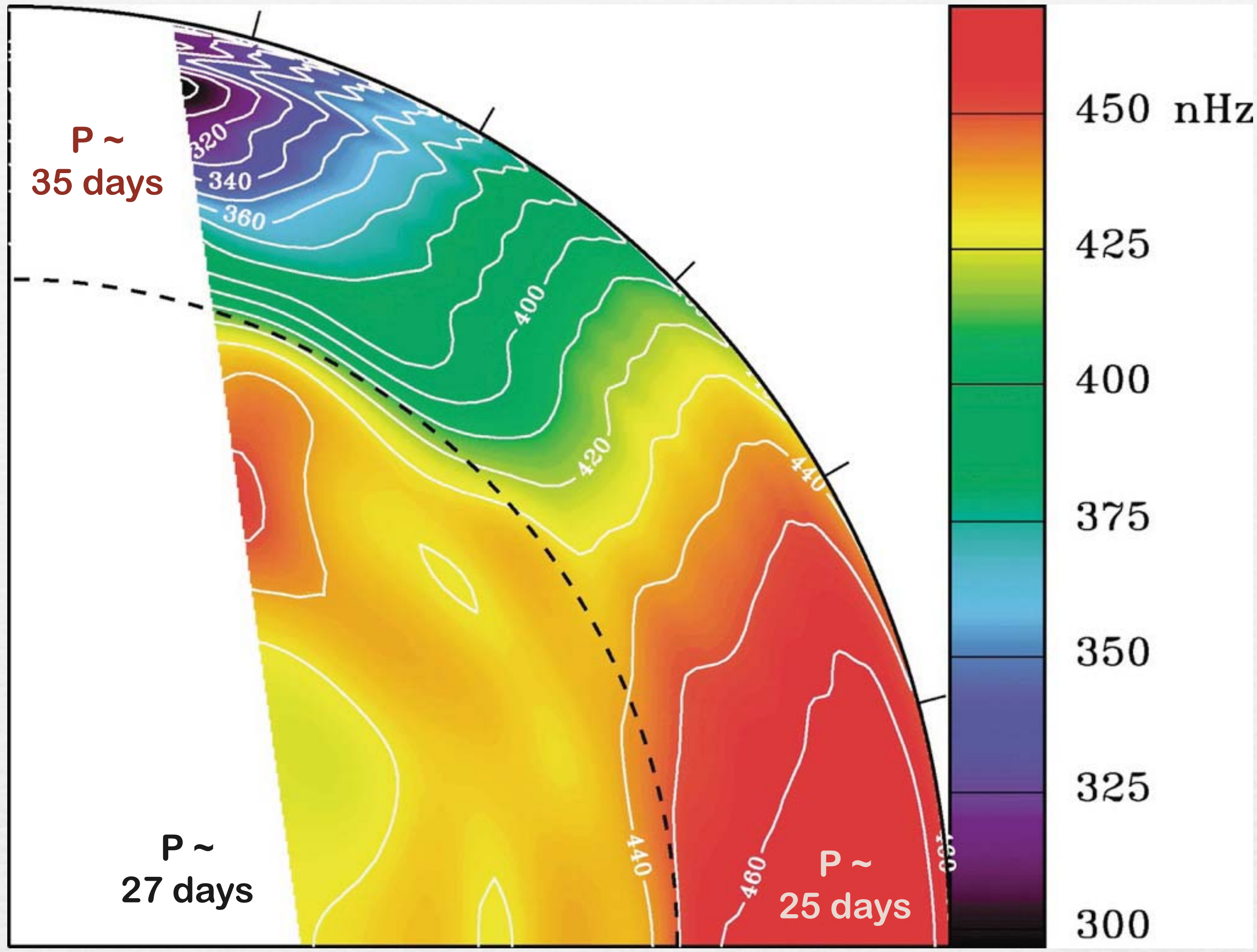

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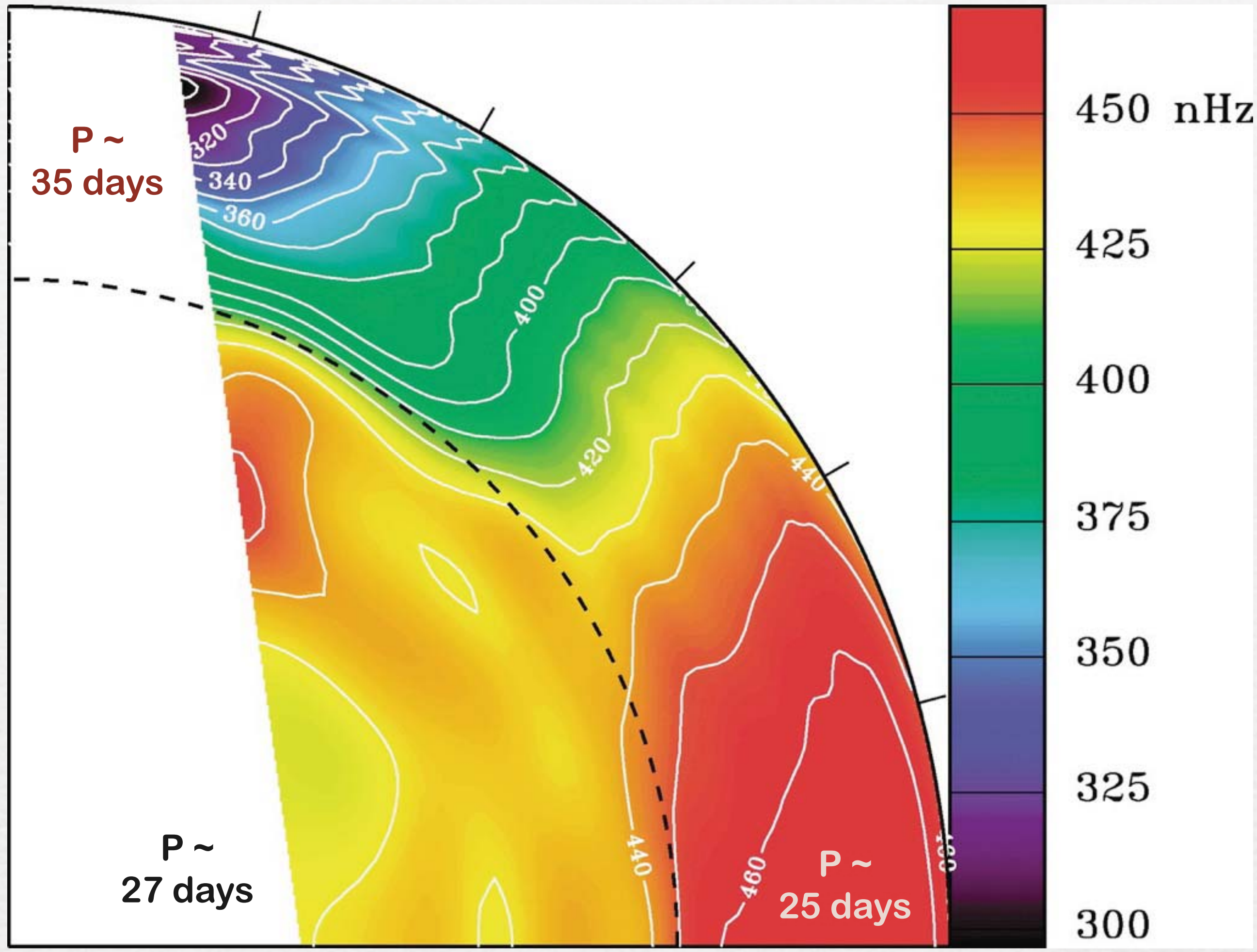
Conical isosurfaces at mid-latitudes

Thompson et al. (2003)



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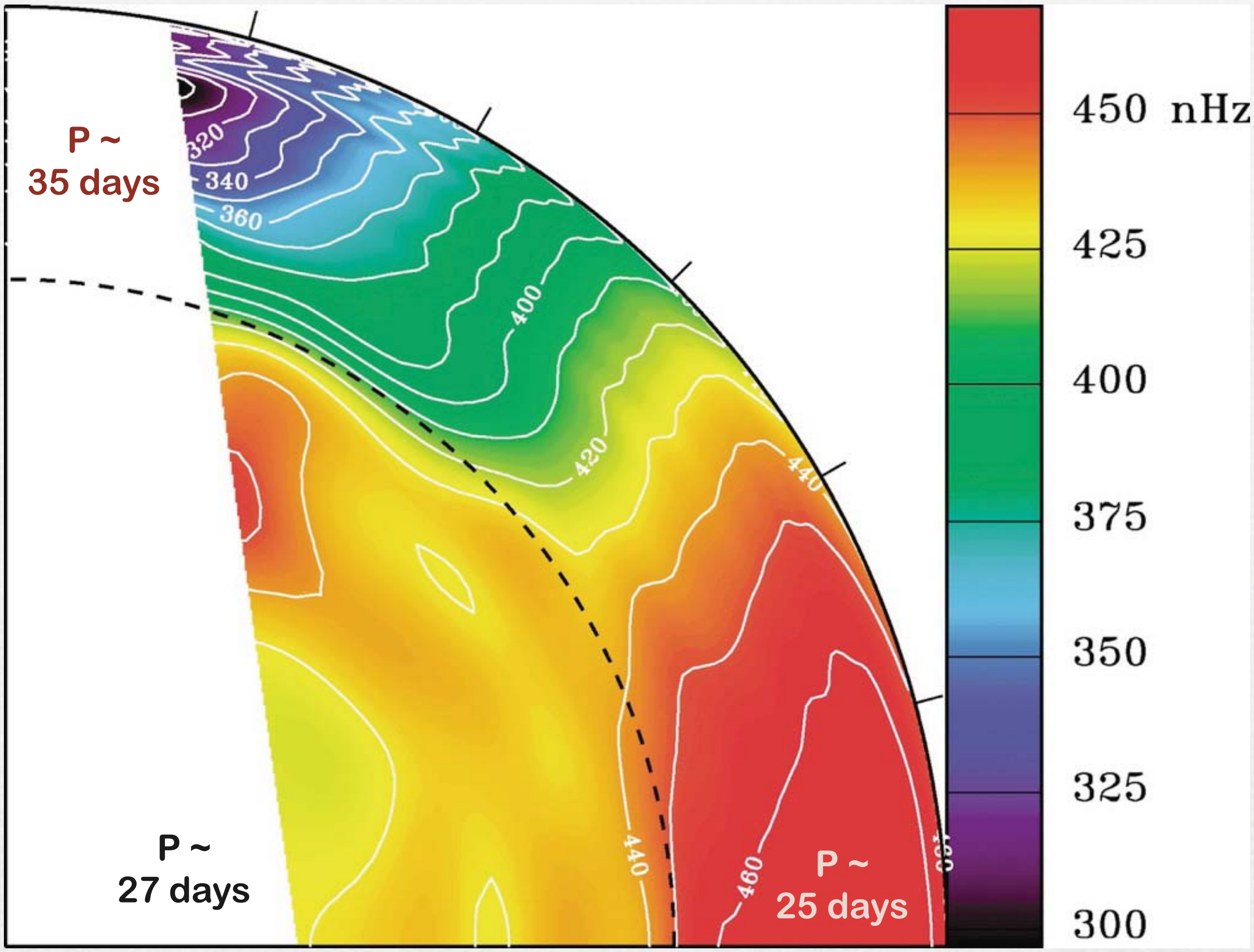

Conical isosurfaces at mid-latitudes

Near-surface shear layer $(0.95R < r < R)$

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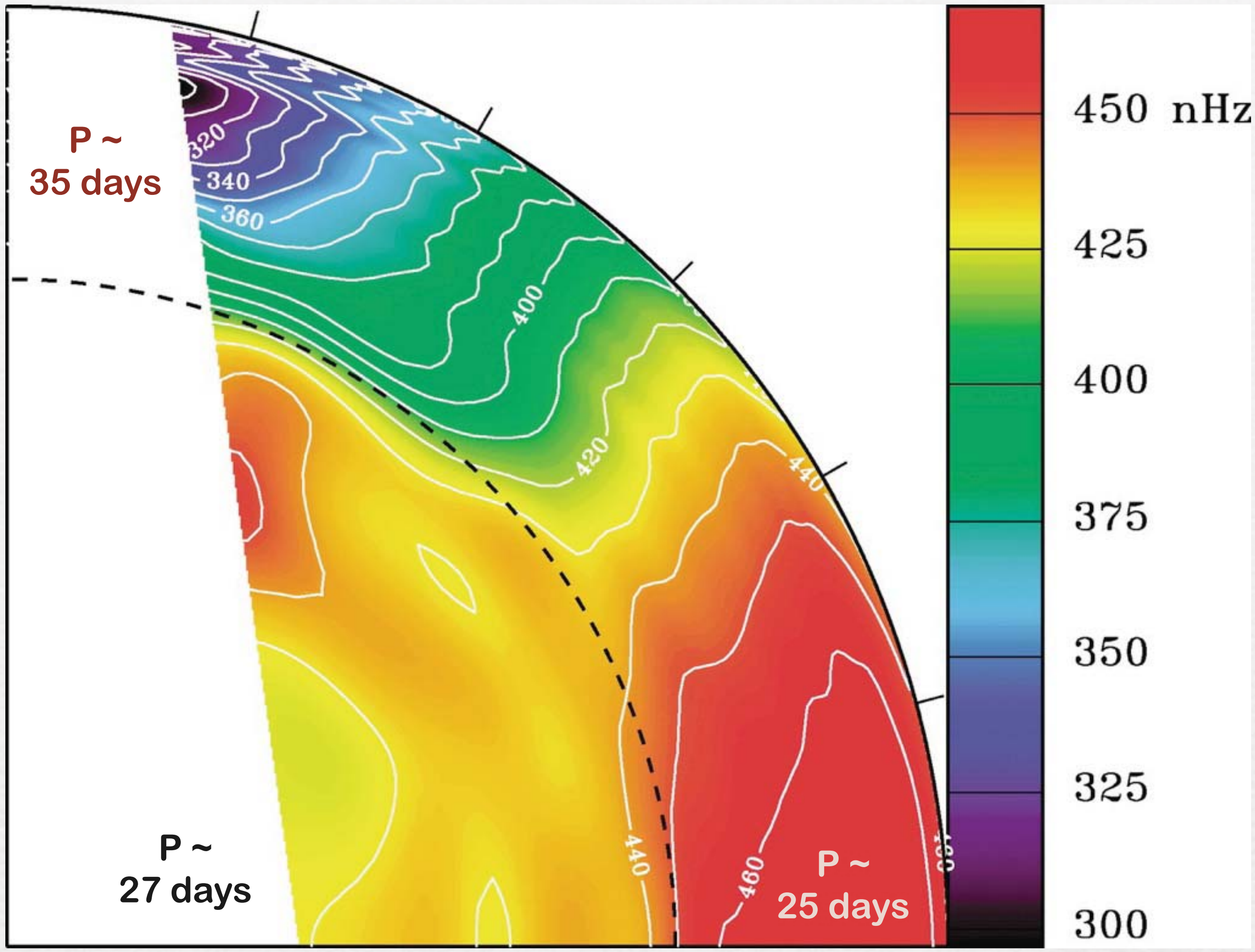


Conical isosurfaces at mid-latitudes

Near-surface shear layer $(0.95R < r < R)$

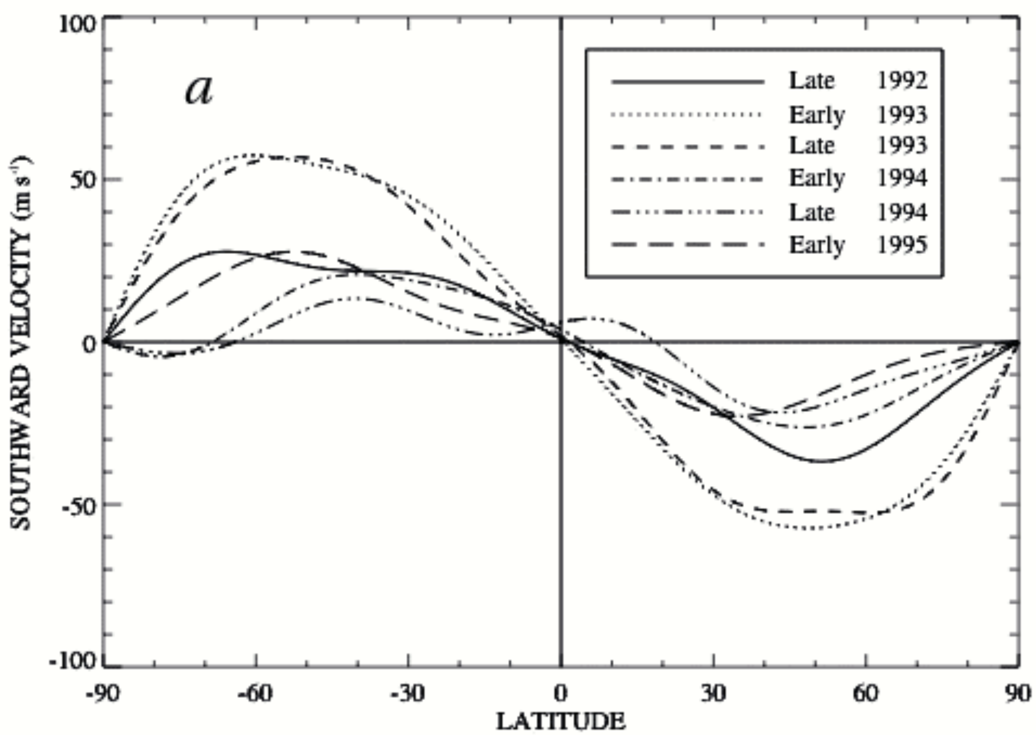
Tachocline $(0.69R < r < 0.72R$; CZ base $= 0.713R$ +/- $0.003)$

- ***Toroidal field generation by rotational shear (critical for global dynamo)***
- ***Penetrative convection, internal gravity waves***
- ***Instabilities (magnetic buoyancy, magneto-shear)***
- ***Confinement***

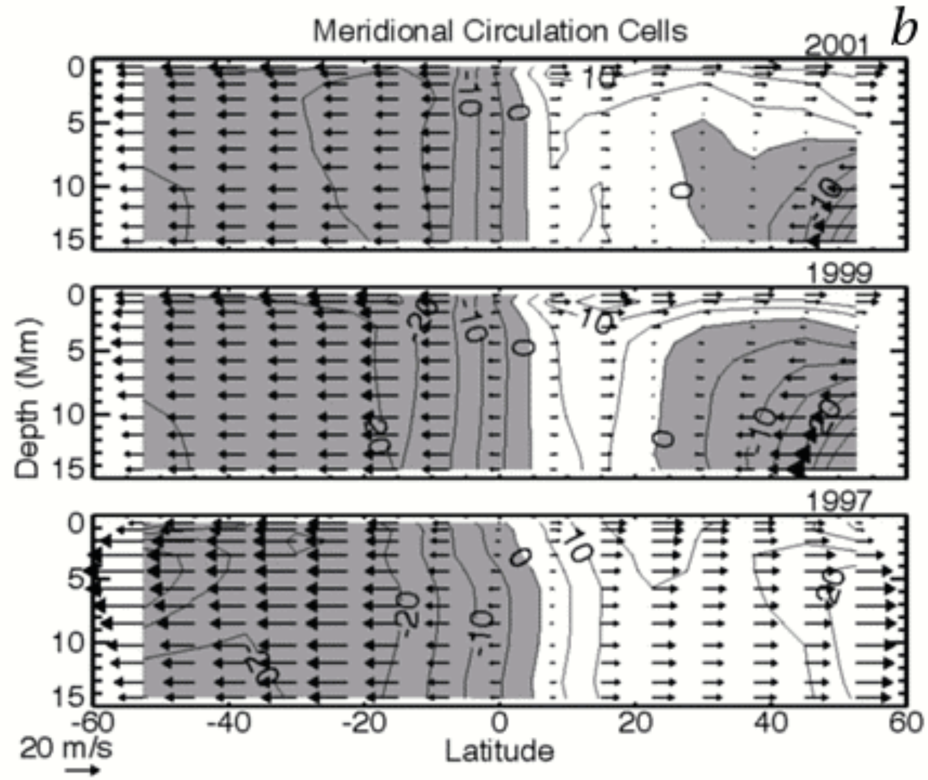
See "The Solar Tachocline", ed. D.W. Hughes, R. Rosner, N.O. Weiss, Cambridge Univ. Press (2007)

Meridional Flow

Photospheric Doppler measurements



Local Helioseismology



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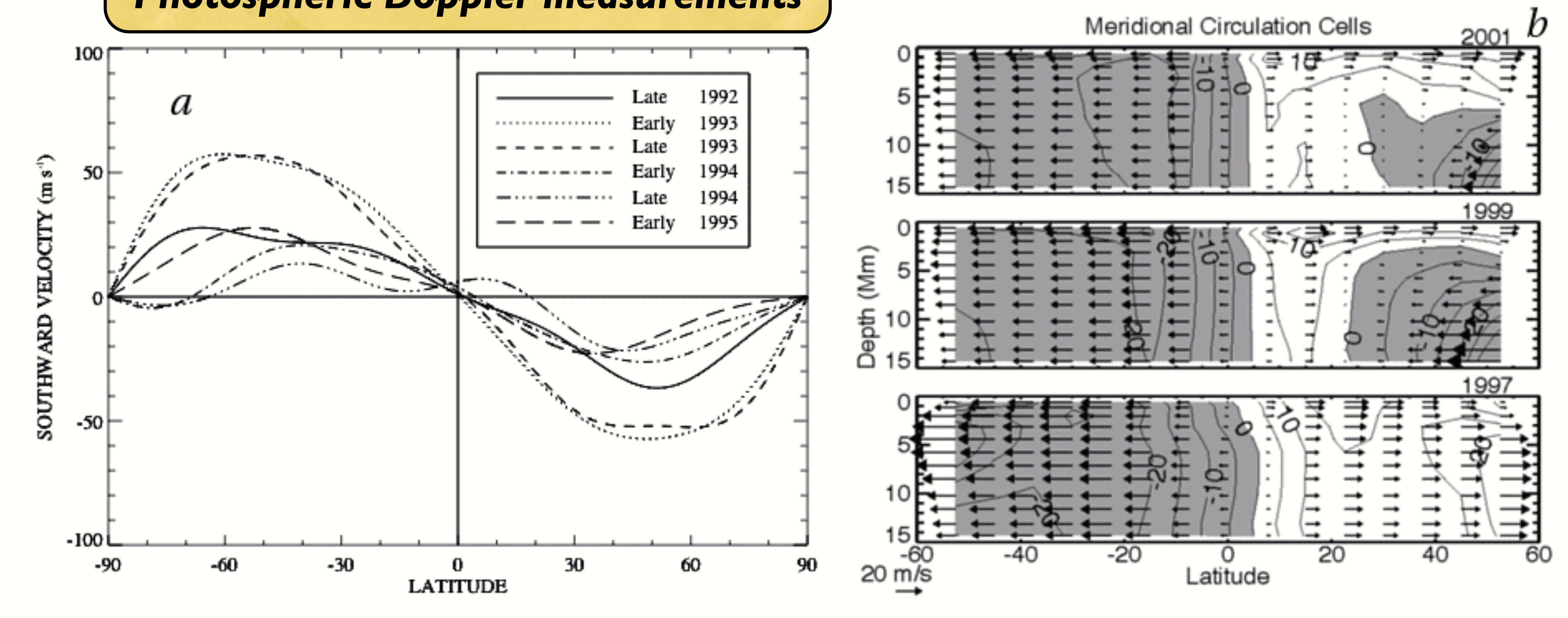
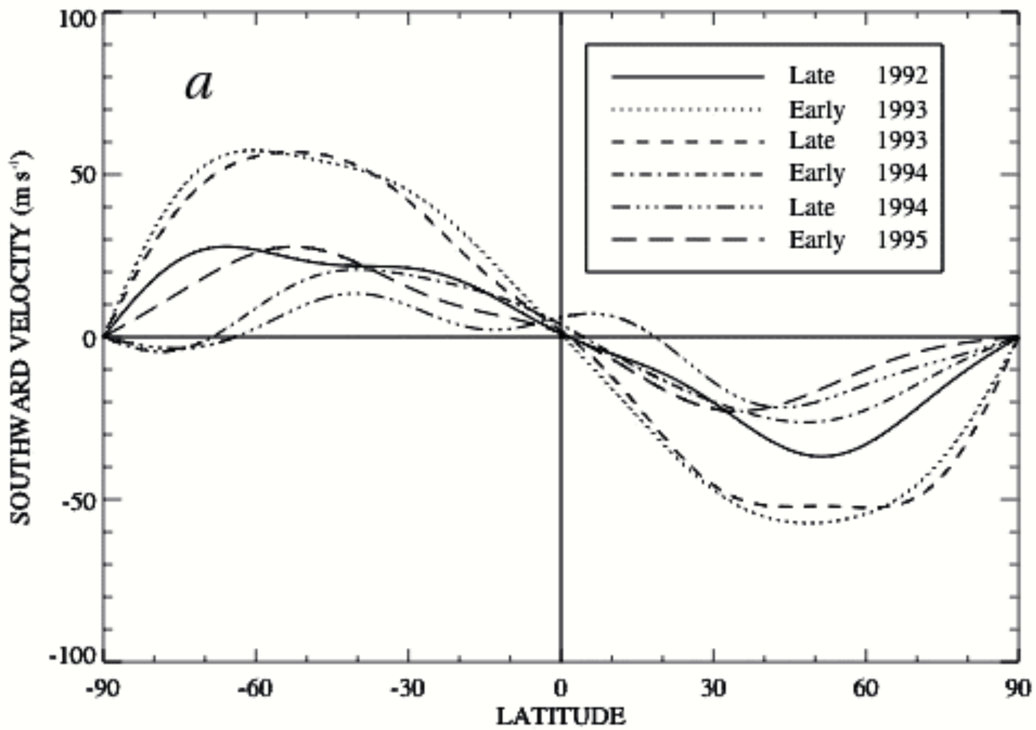


Poleward near surface (r > 0.97R) at latitudes < 60º (unknown elsewhere)

Meridional Flow

Photospheric Doppler measurements



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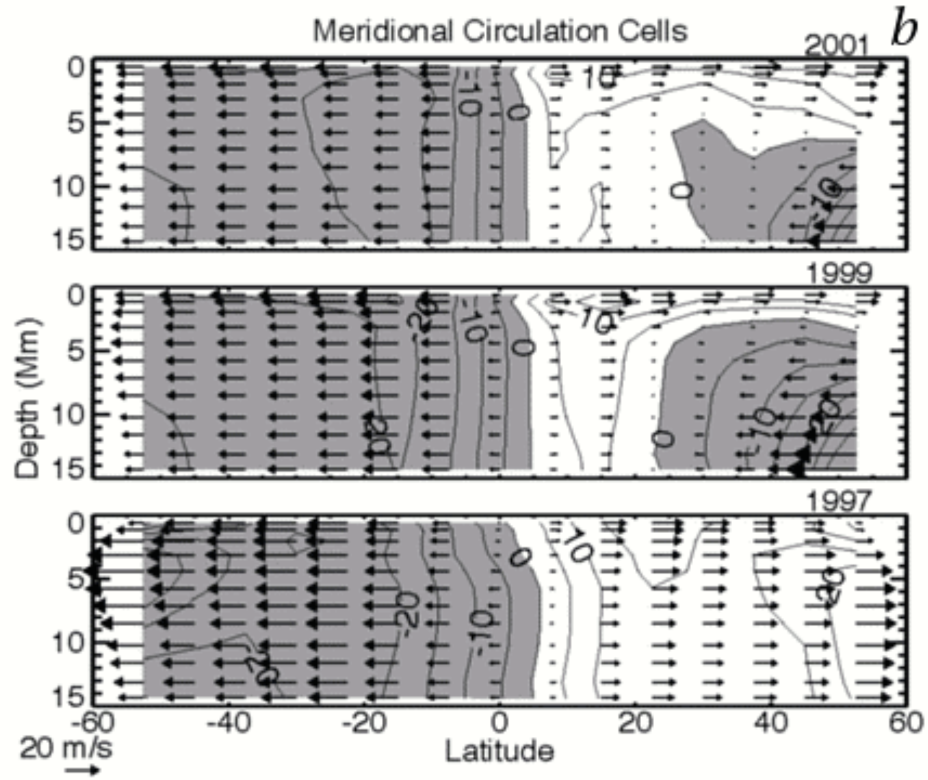

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Amplitude ~ 10-20 m s^{-1} but highly variable (much weaker than DR)

Meridional Flow

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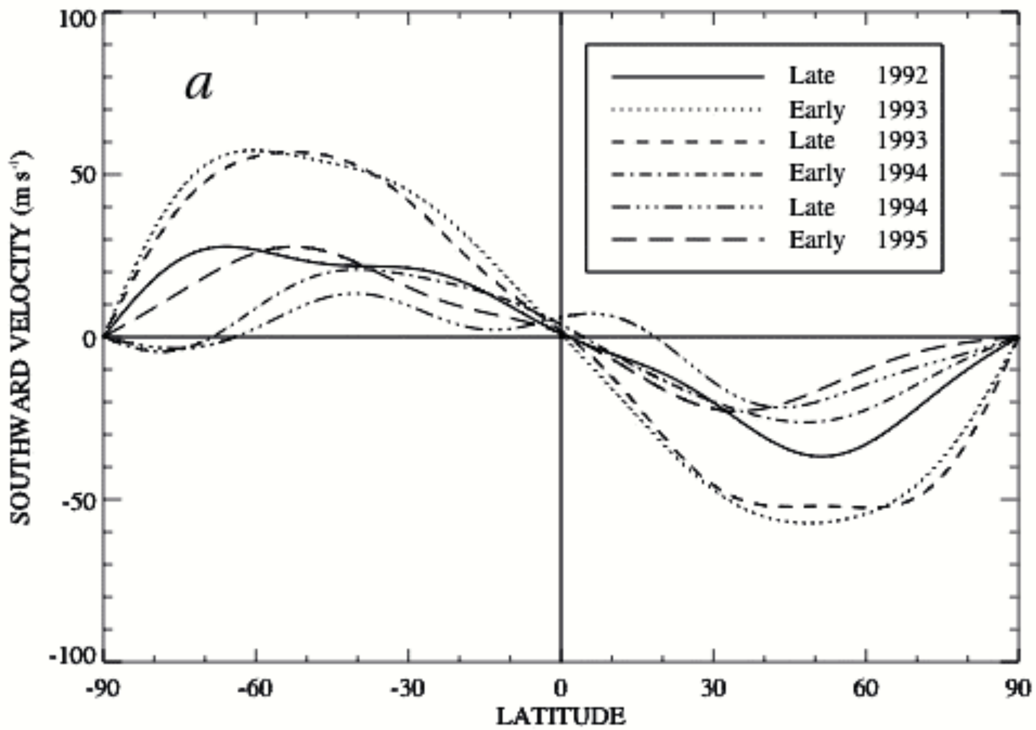

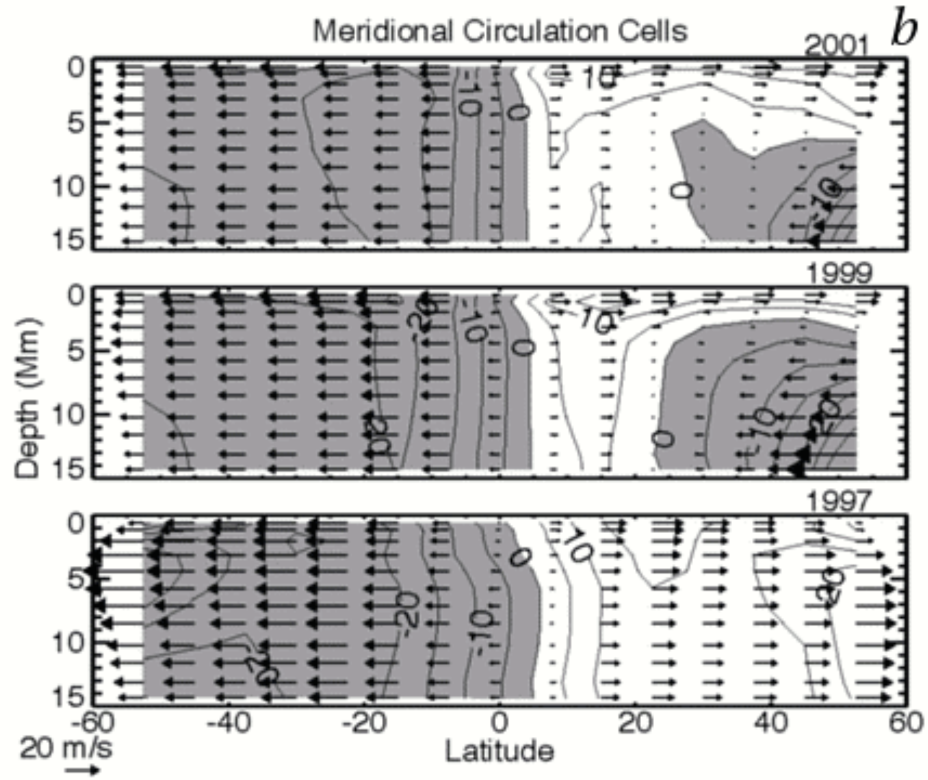

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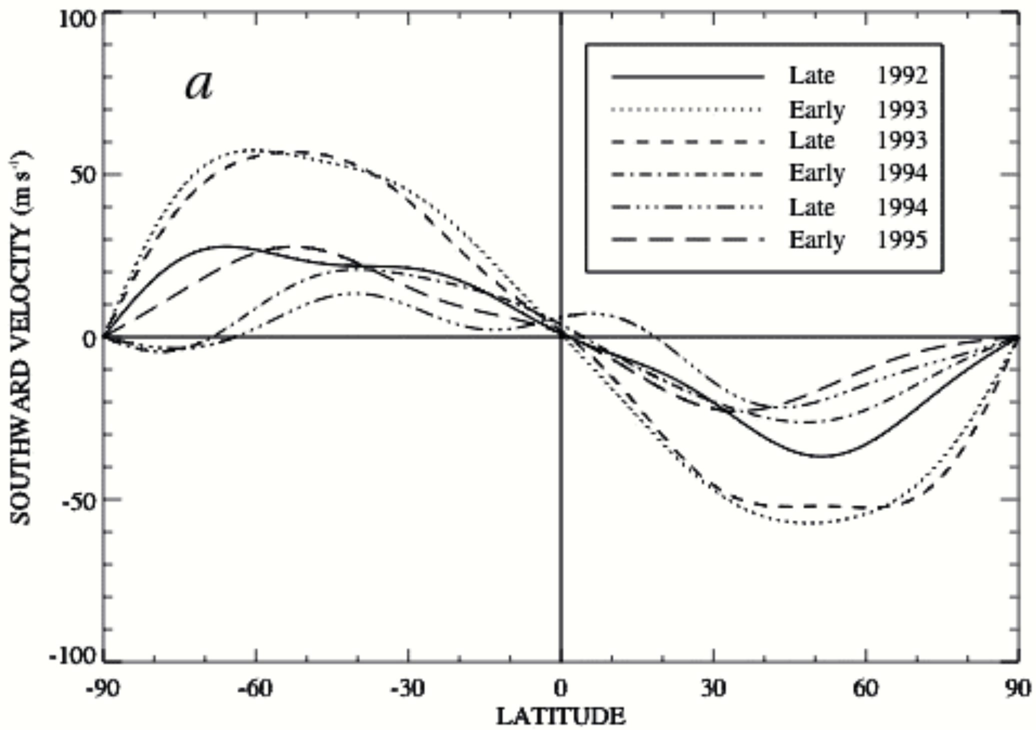
Possible evidence for multiple cells at high latitudes, deeper levels

Meridional Flow

Photospheric Doppler measurements



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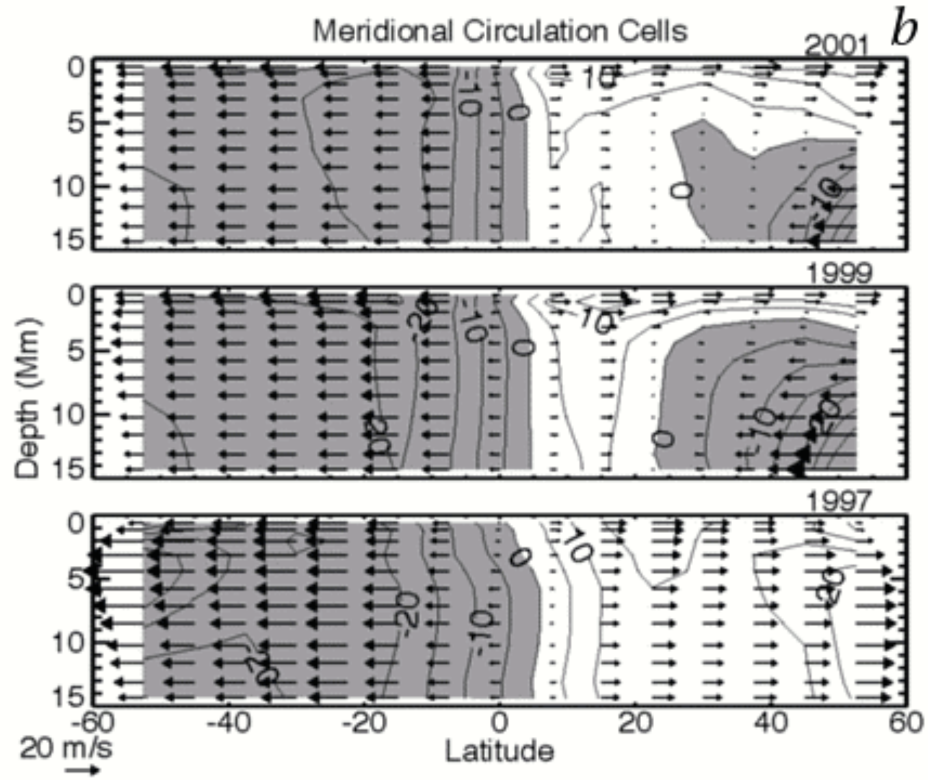

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Amplitude ~ 10-20 m s^{-1} but highly variable (much weaker than DR)

Possible evidence for multiple cells at high latitudes, deeper levels

Solar cycle variations; convergence into activity bands (near surface)

Dynamical Balances

conservation of momentum in a rotating fluid

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \left(\mathbf{v} \cdot \boldsymbol{\nabla} \right) \mathbf{v} = - \boldsymbol{\nabla} P - \rho \mathbf{g}$$

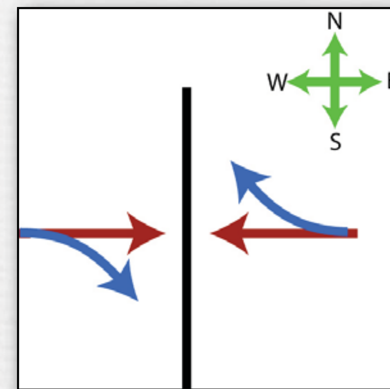
(1) Meridional Circulation = Reynolds stress

$$\left\langle \rho \mathbf{v}_m \right\rangle \cdot \boldsymbol{\nabla} \mathcal{L} = - \boldsymbol{\nabla} \cdot \left[\rho \lambda \left\langle v'_\phi \mathbf{v}'_m \right\rangle \right]$$

$$\mathcal{L} = \lambda^2 \Omega$$

$$\lambda = r \sin \theta$$

Coriolis-induced tilting of convective structures



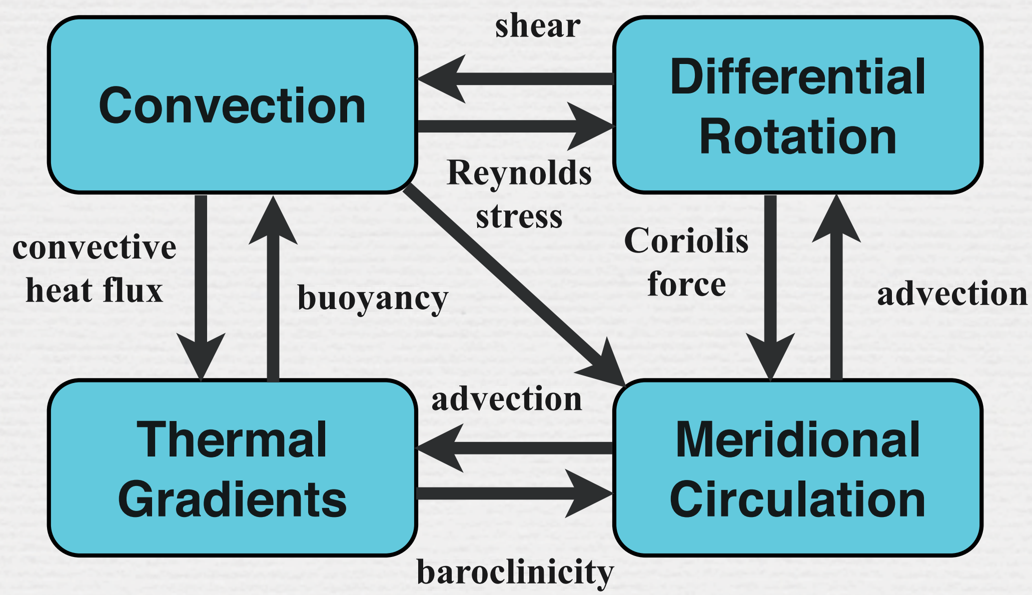
(2) Thermal Wind Balance *(Taylor-Proudman theorem)*

$$\Omega = \frac{\left\langle v_\phi \right\rangle}{\lambda}$$

$$\frac{\partial \Omega^2}{\partial z} = \frac{g}{r \lambda C_P} \frac{\partial \left\langle S \right\rangle}{\partial \theta}$$

- Steady State
- Neglect LF, VD
- Rapid Rotation RS << CF
- ideal gas
- hydrostatic, adiabatic background

See Homework Problem I

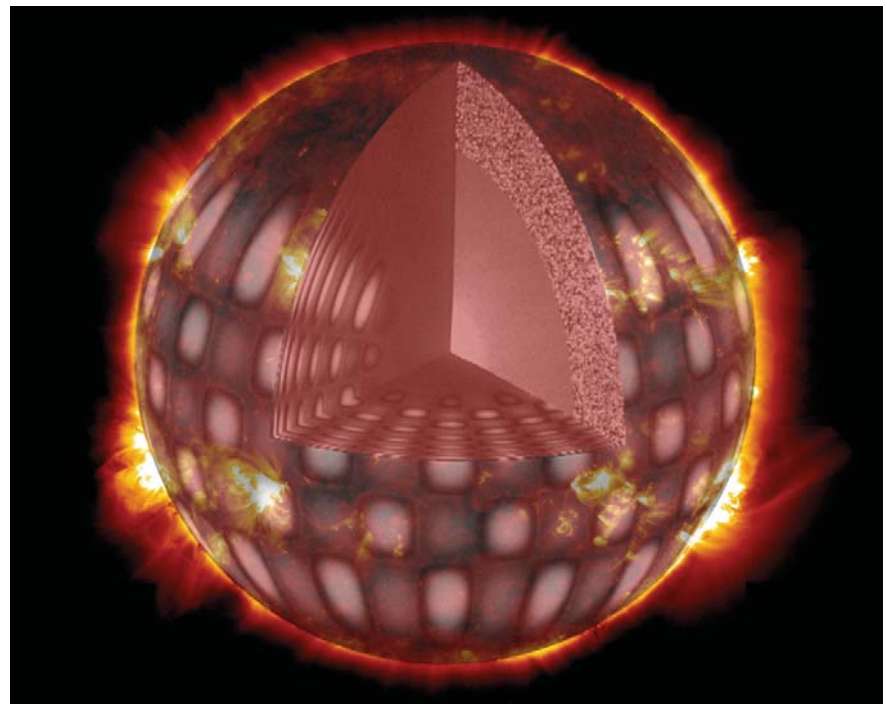


Summary: Mean Flows

***Mean* = averaged over longitude and time**

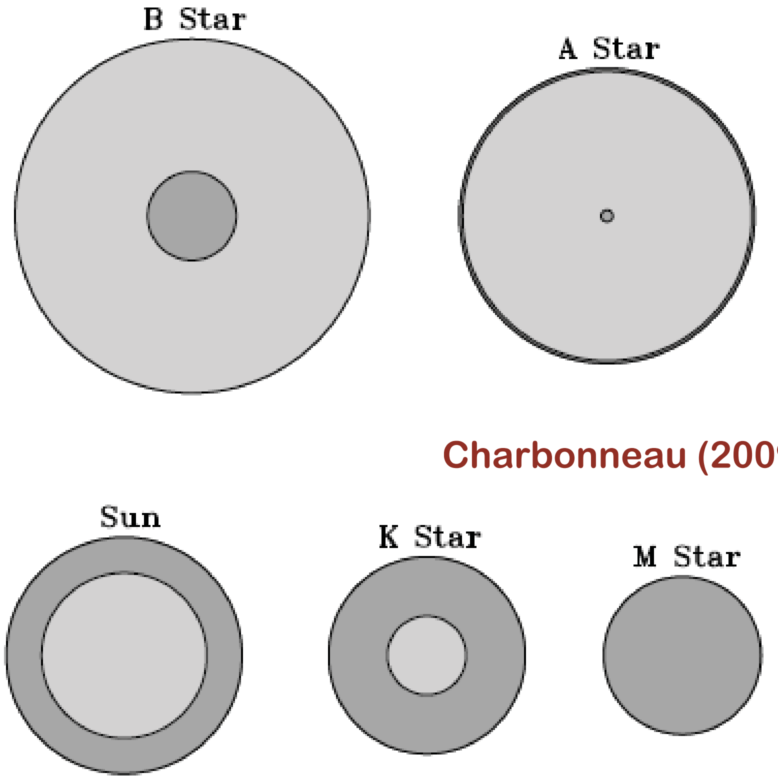
Inferred from Surface observations, Helioseismology

- **Differential Rotation**
  - zonal (east-west) Mean Flow
  - Known throughout most of the convection zone
  - fast equator, slower poles
- **Meridional Circulation**
  - Mean Flow in the radius-latitude plane
  - Only known above about 0.97R, low-mid latitudes (poleward)

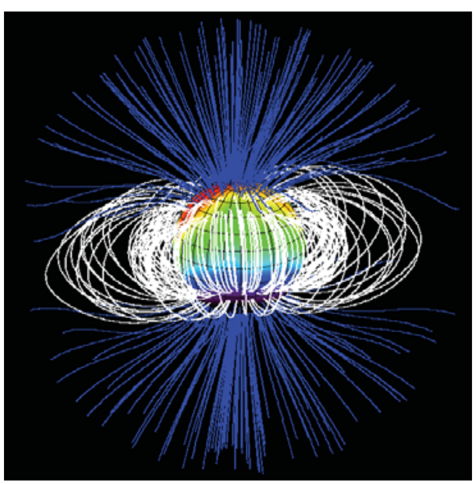


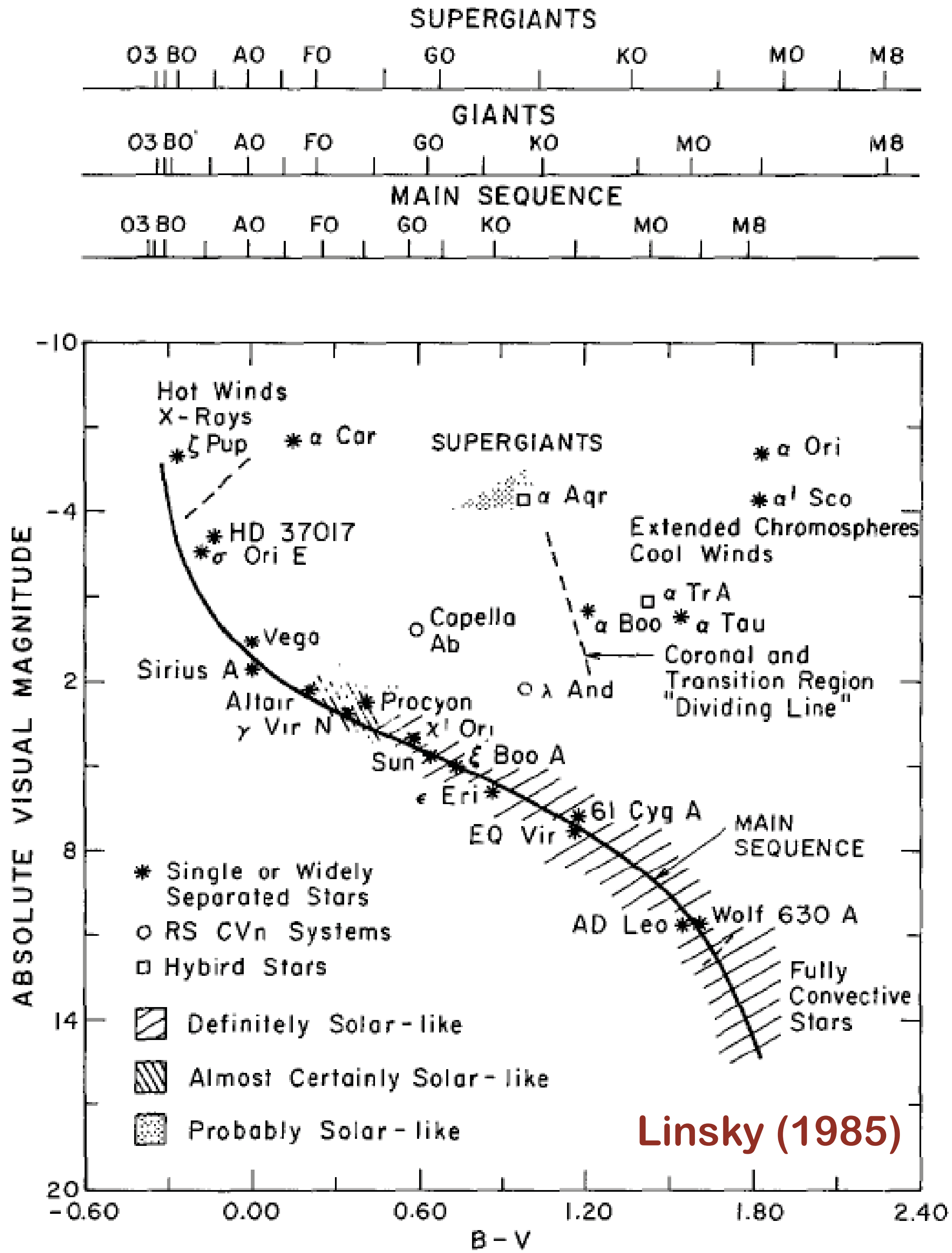
Maintained via momentum and energy transport by Giant Cells

Convection Breeds Magnetism



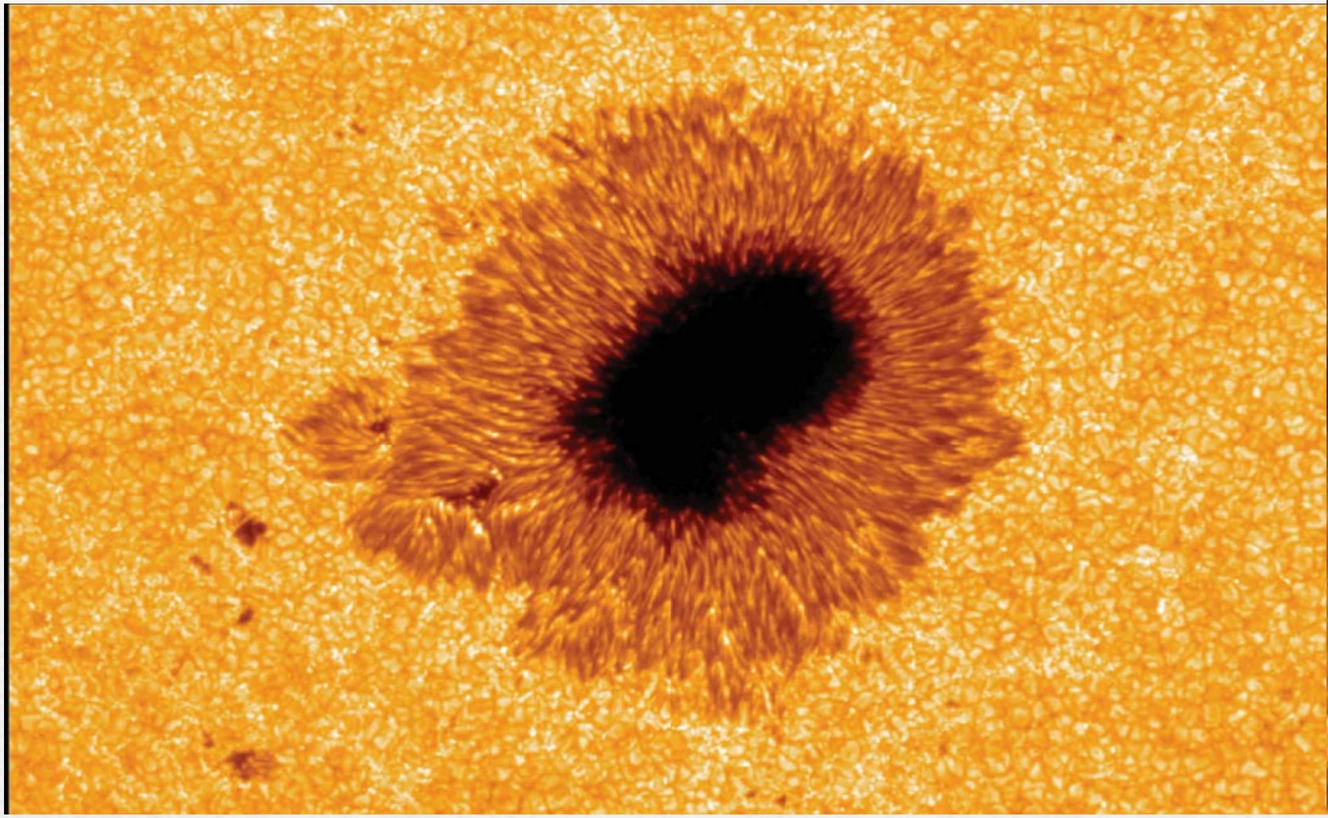
Charbonneau (2009)

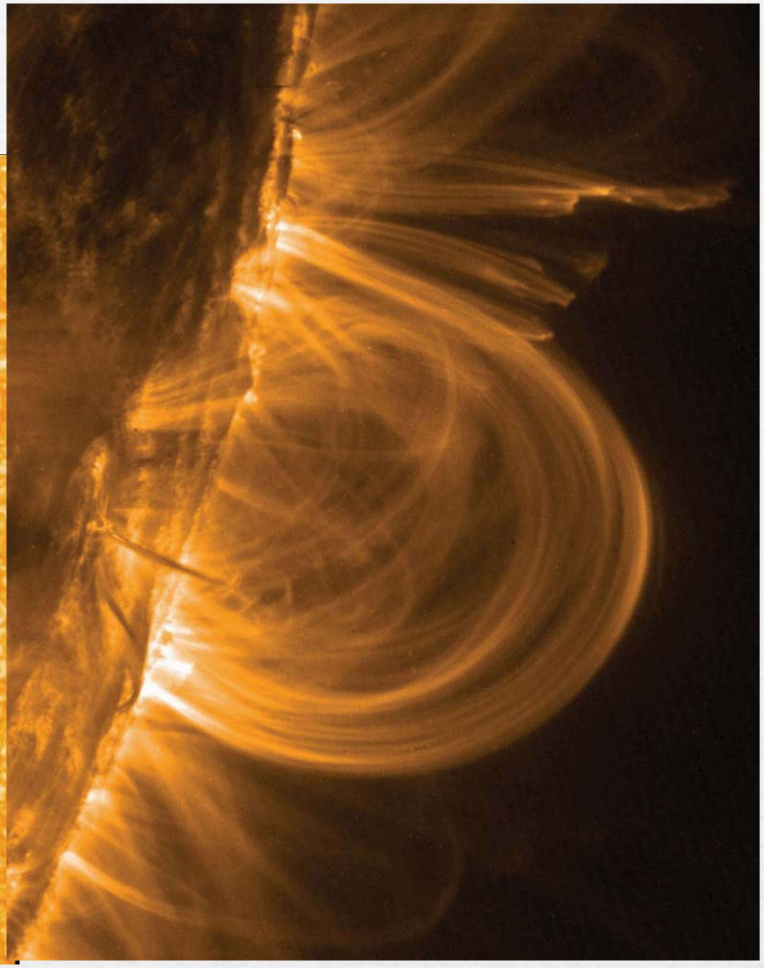

Donati et al (2006)

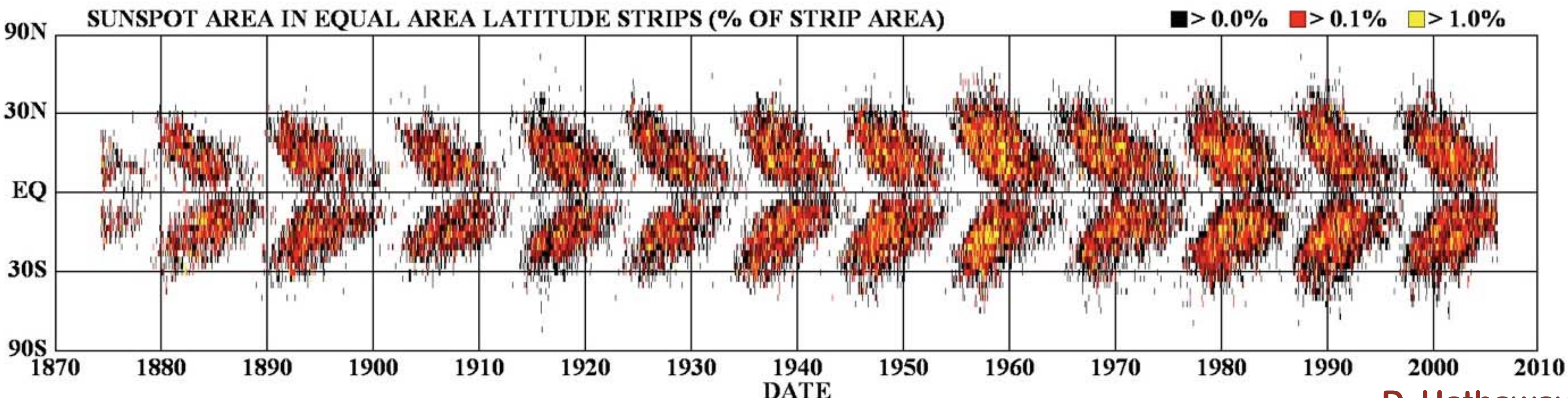


Linsky (1985)

The Global Solar Dynamo







D. Hathaway

Generation of Magnetic Fields: The MHD Magnetic Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times \left(\mathbf{v} \times \mathbf{B} - \eta \boldsymbol{\nabla} \times \mathbf{B} \right)$$

Follows from Faraday's Law of Induction

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

And Ohm's Law (with a Galilean transformation)

$$\mathbf{J} = \sigma \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right)$$

With the usual MHD assumptions

Highly ionized, Quasi-neutral

High collision frequency/ short mean-free paths (high density, temperature)

sub-relativistic bulk velocity

Lagrangian Chaos

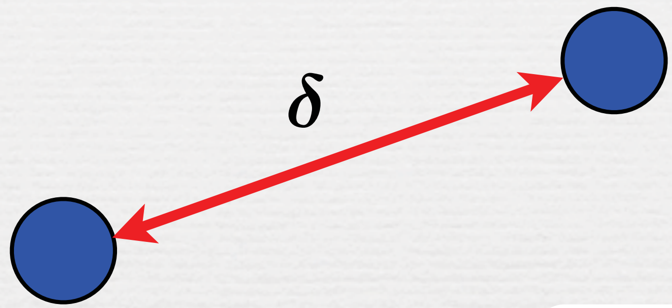
Chaotic fluid trajectories amplify magnetic fields

(provided that chaotic stretching wins the battle against ohmic diffusion)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

$$\frac{D\mathbf{B}}{Dt} = \frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{B} = (\mathbf{B} \cdot \nabla)\,\mathbf{v} - \mathbf{B}\,(\nabla \cdot \mathbf{v}) - \nabla \times (\eta \nabla \times \mathbf{B})$$

If $\nabla \cdot \mathbf{v} = \eta = 0$ **then**

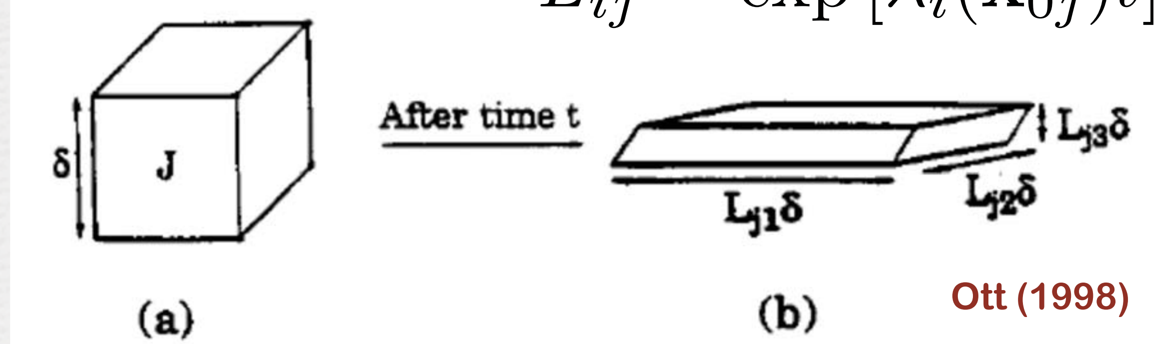


λ = Local Lyapunov exponents

$$\frac{D\mathbf{B}}{Dt} = (\mathbf{B} \cdot \nabla)\,\mathbf{v}$$

$$\frac{d\boldsymbol{\delta}}{dt} = (\boldsymbol{\delta} \cdot \nabla)\,\mathbf{v}$$

$$L_{ij} = \exp\left[\lambda_i(\mathbf{x}_{0j})t\right]$$



Ott (1998)

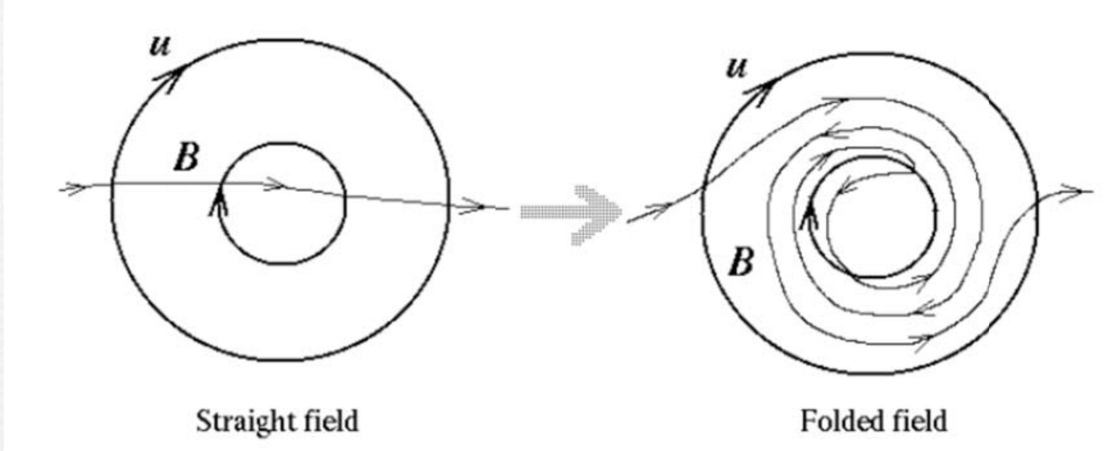
$$\frac{d\delta_i(\mathbf{x}_0, t)}{dt} = \mathcal{J}_{ij}(\mathbf{x}_0, t)\;\delta_j\,(\mathbf{x}_0, t)$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 0$$

Spatially smooth, temporally chaotic flows work best

$$R_m = \frac{UL}{\eta}$$

$$P_m = \frac{\nu}{\eta}$$

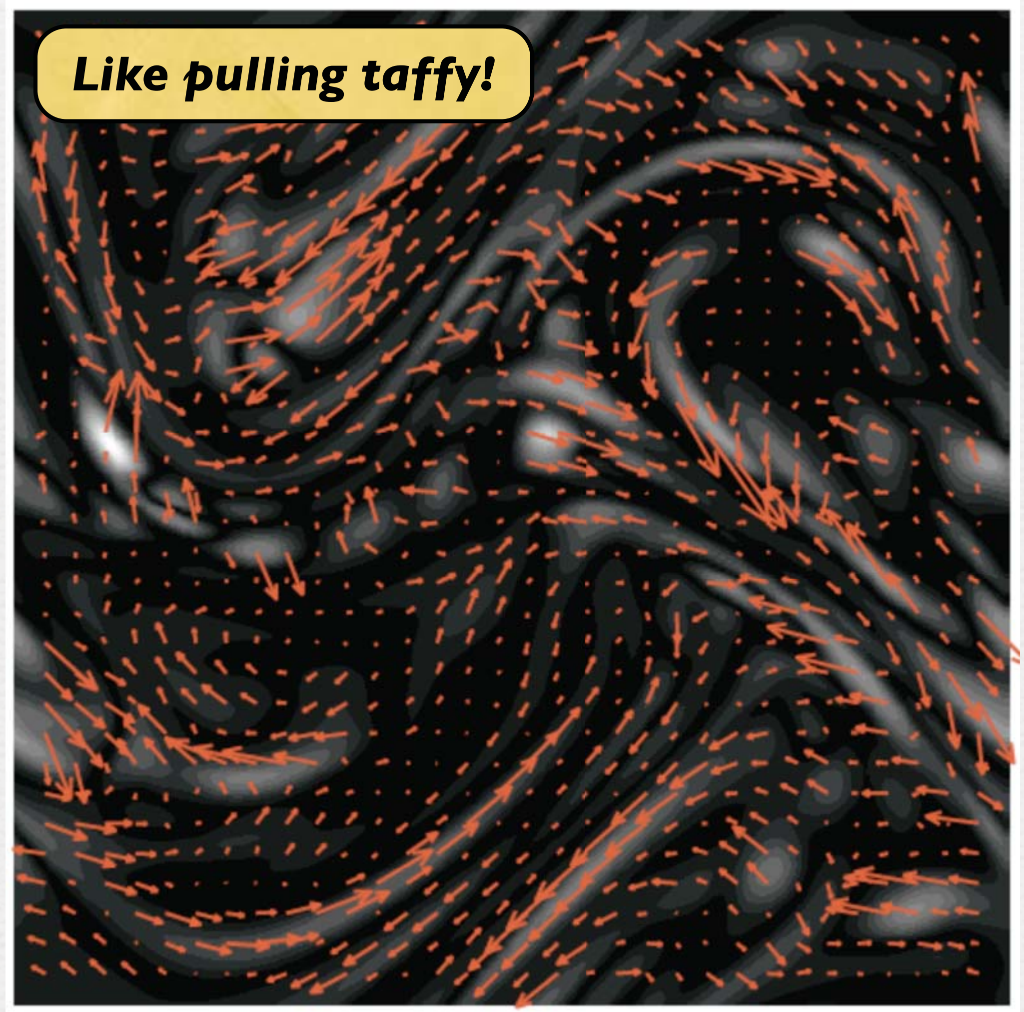


Schekochihin et al (2004)

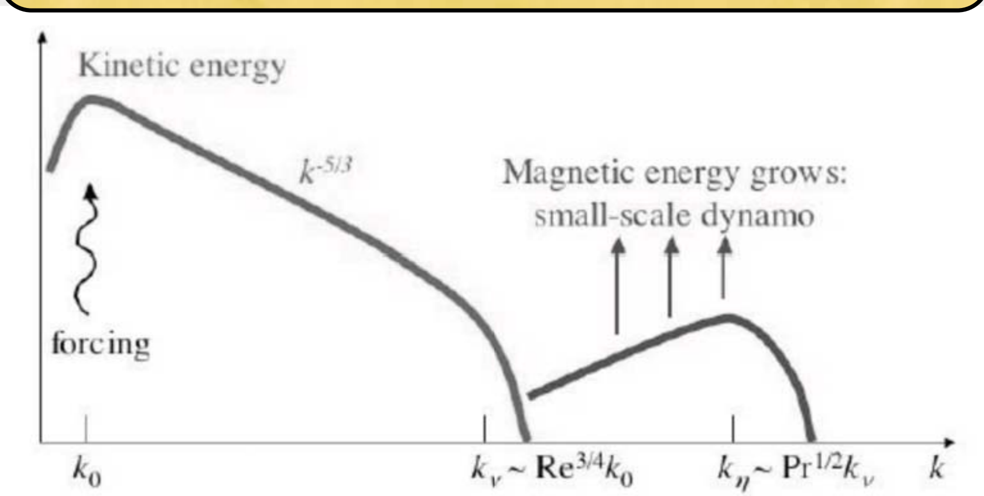
If $P_m > 1$ then turbulent dynamos build fields on sub-viscous scales (near resistive scale)

Folded field topologies sheets and filaments

Like pulling taffy!



Turbulent flows beget turbulent fields!



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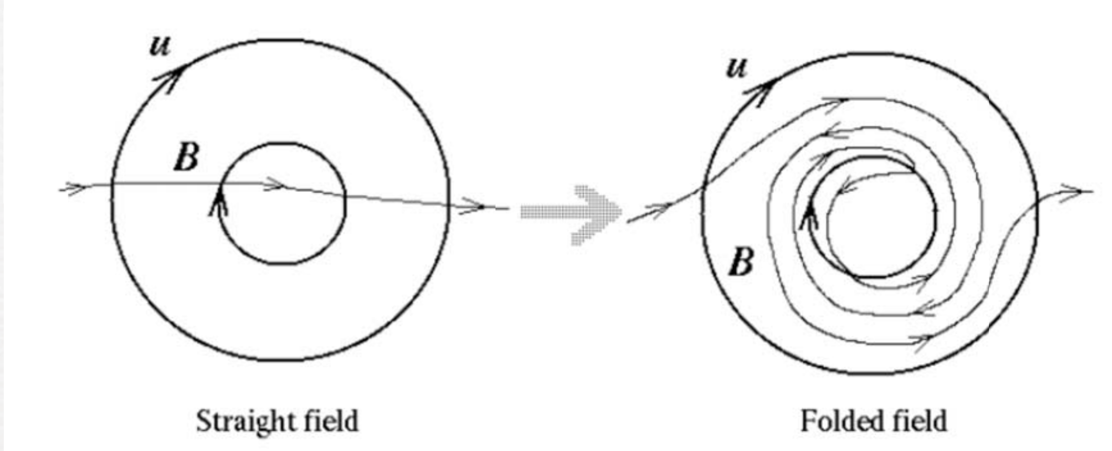

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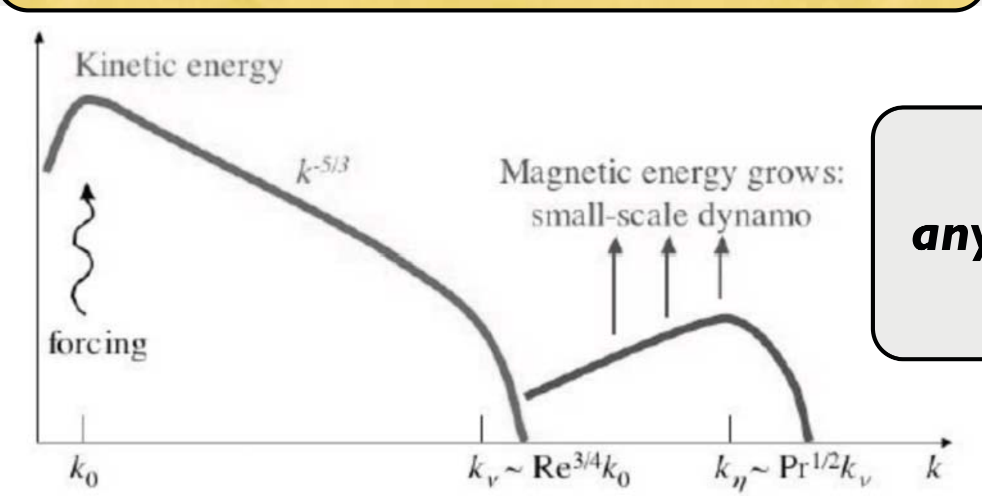
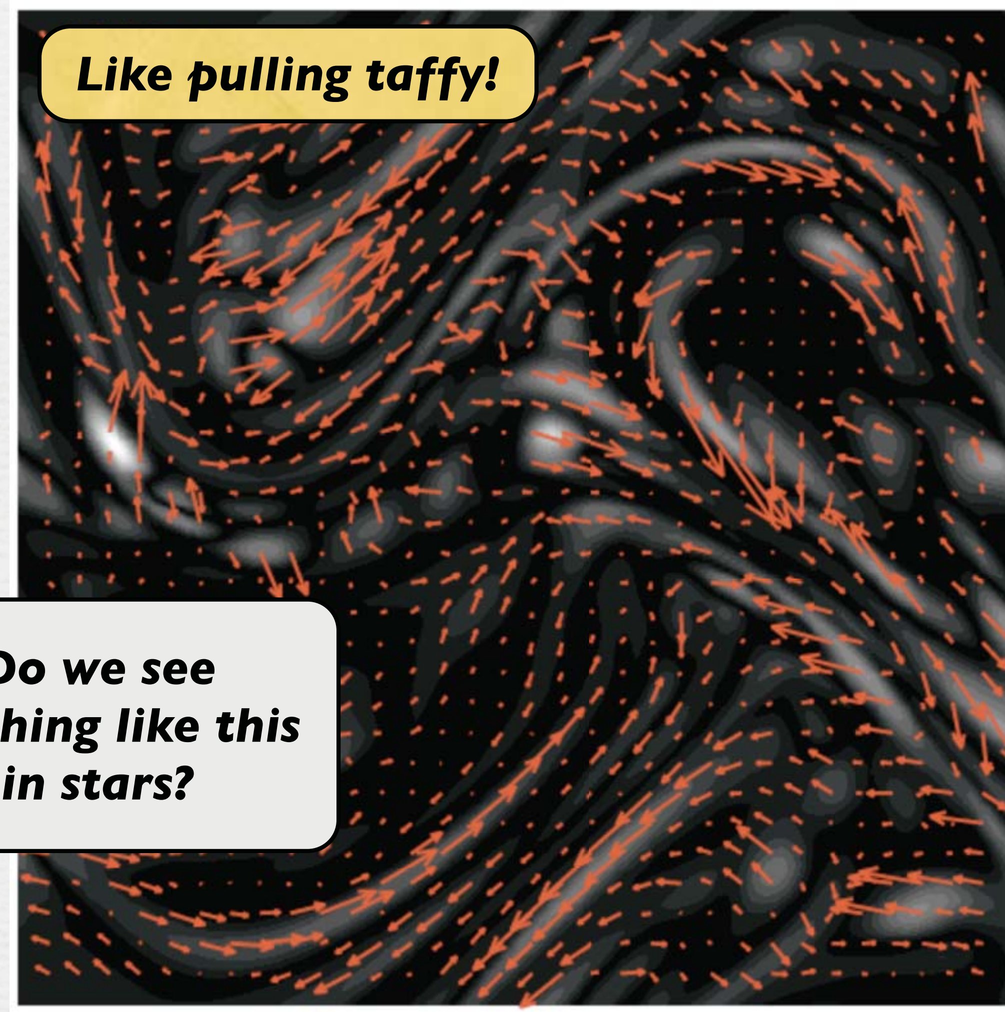
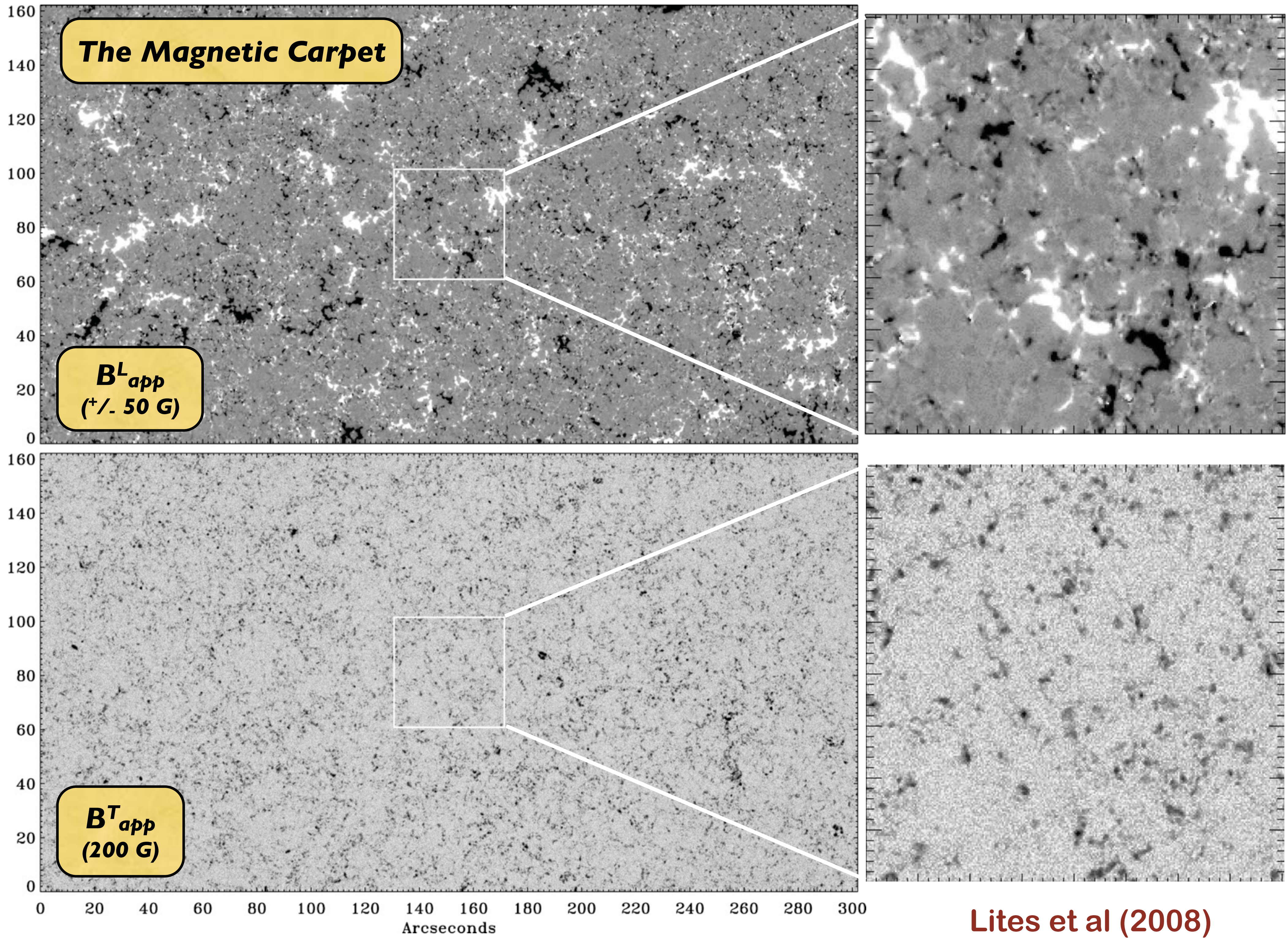




Do we see anything like this in stars?



Lites et al (2008)

Types of Dynamos

define
Small-scale dynamo

Generates magnetic fields on scales smaller than the velocity field

$$\ell_B \leq \ell_v$$

define
Large-scale dynamo

Generates magnetic fields on scales larger than the velocity field

$$\ell_B >> \ell_v$$

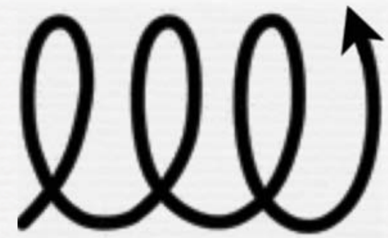
Are local solar/stellar dynamos small-scale dynamos?

Probably - but intimately coupled to deep CZ

Are global solar/stellar dynamos large-scale dynamos?

Probably - but v-B correlations induced by large-scale convective modes or instabilities may contribute to global field generation

Recipe for a Large-Scale Dynamo



Lagrangian Chaos

- Builds magnetic energy

Rotational Shear

- Builds large-scale toroidal flux (Ω-effect)
- Enhances dissipation of small-scale fields
- Promotes magnetic helicity flux

Helicity

- Rotation and stratification generate kinetic helicity
- Kinetic helicity generates magnetic helicity
- Upscale spectral transfer of magnetic helicity generates large-scale fields
  - Local transfer: inverse cascade of magnetic helicity
  - Nonlocal transfer: α-effect

$$H_k = \langle \boldsymbol{\omega} \cdot \boldsymbol{v} \rangle$$

$$H_m = \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle$$

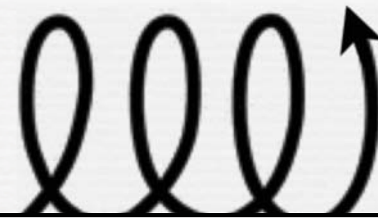
$$H_c = \langle \boldsymbol{J} \cdot \boldsymbol{B} \rangle$$

$$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$$

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

$$\boldsymbol{J} = \frac{c}{4\pi} \boldsymbol{\nabla} \times \boldsymbol{B}$$

Recipe for a Large-Scale Dynamo



- Lag
  - Builds
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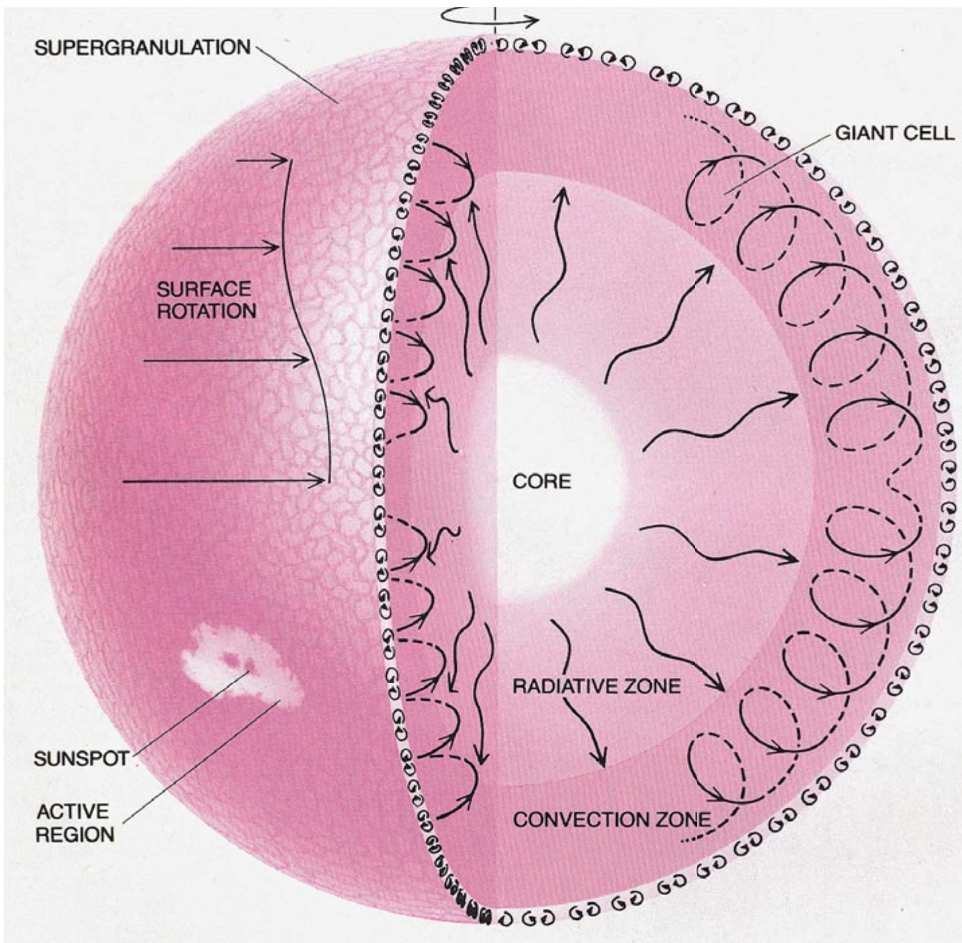
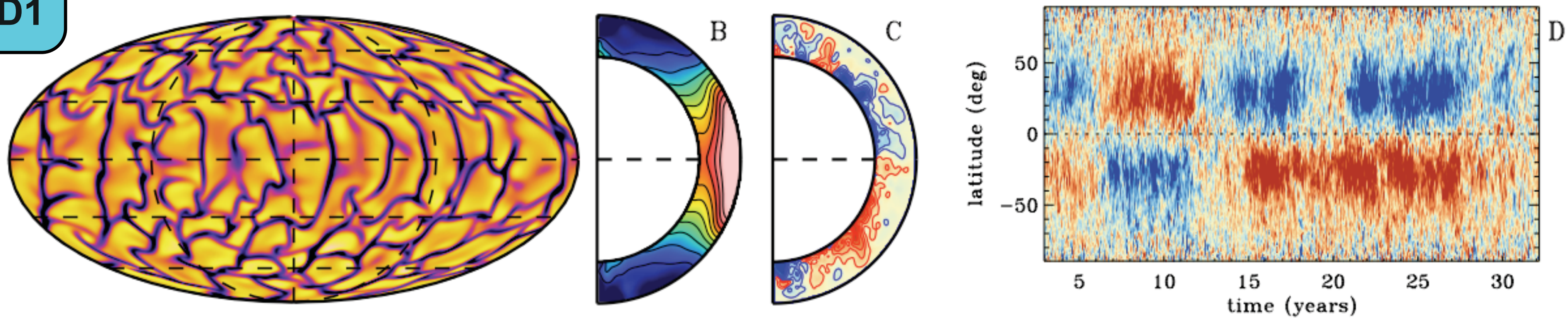


A local, small-scale dynamo may be churning away in the surface layers (growth rate ~ 5 min) while the global dynamo plods along deeper down (activity cycle ~ 22 years)

Building Mean Fields: Rotation Helps!

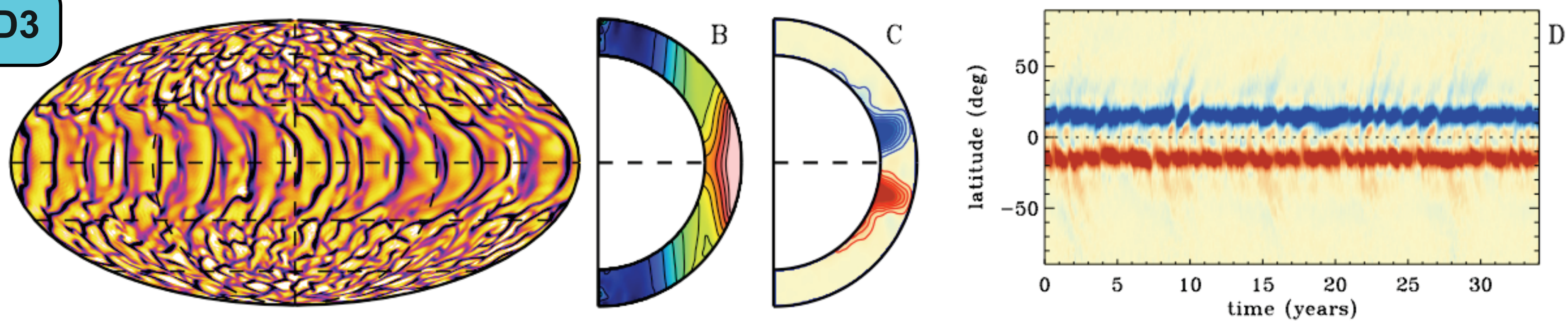
P = 28 days, 9.3 days, 5.6 days

D1



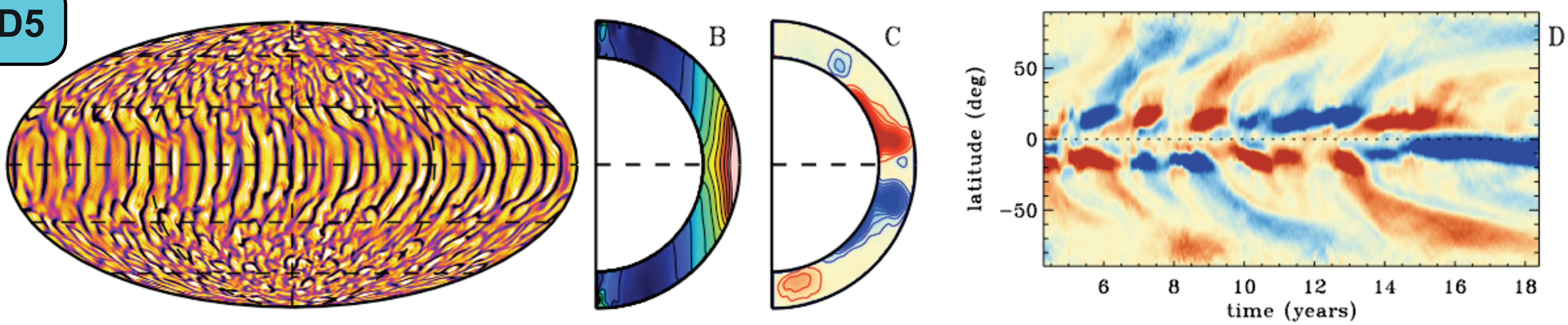
Miesch et al (2010)

D3



Brown et al (2010, 2011)

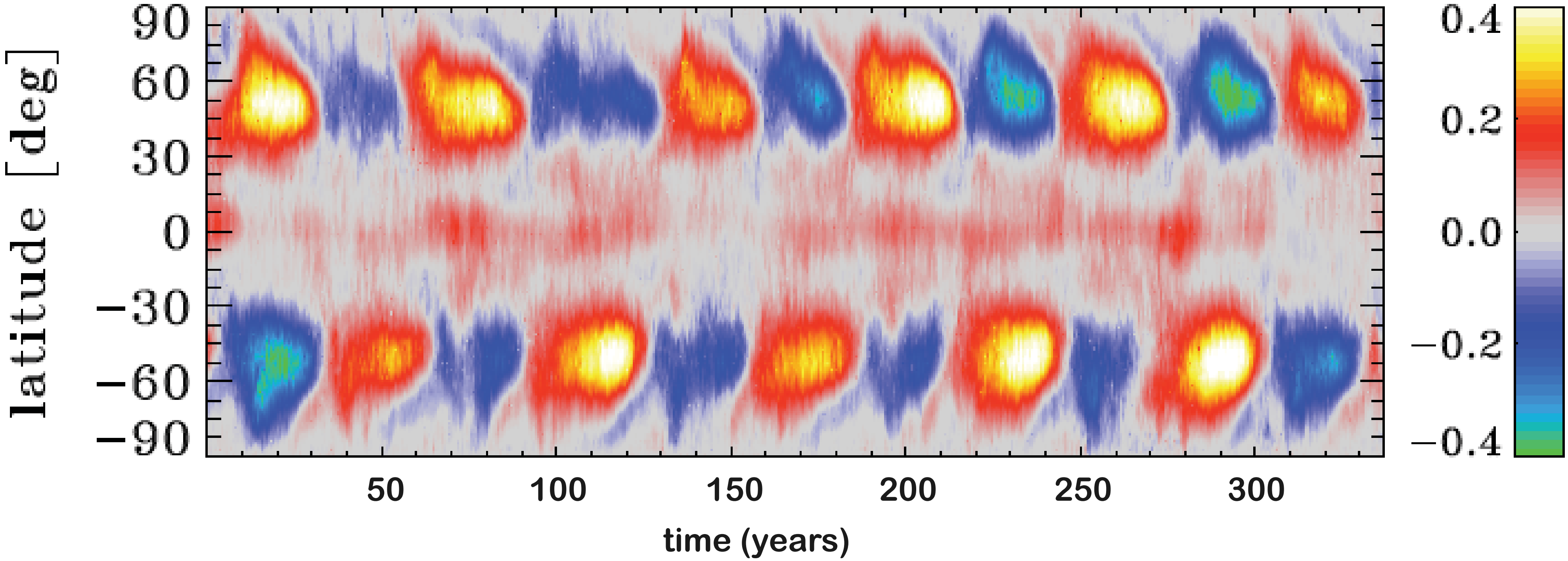
D5



Magnetic Cycles in Convective Dynamos

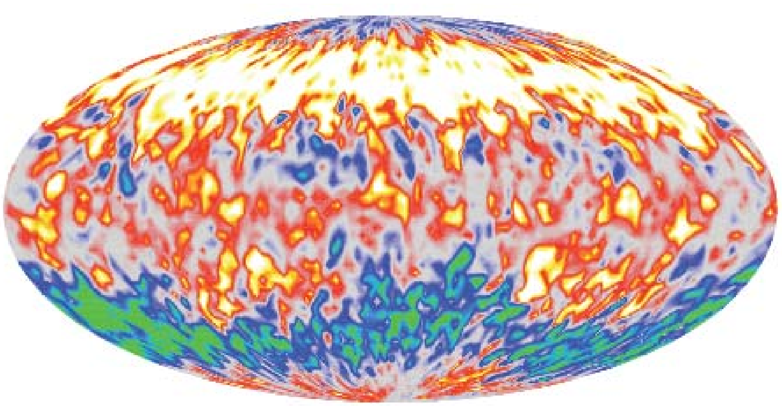
Racine et al (2011)

cf. “Butterfly Diagram”

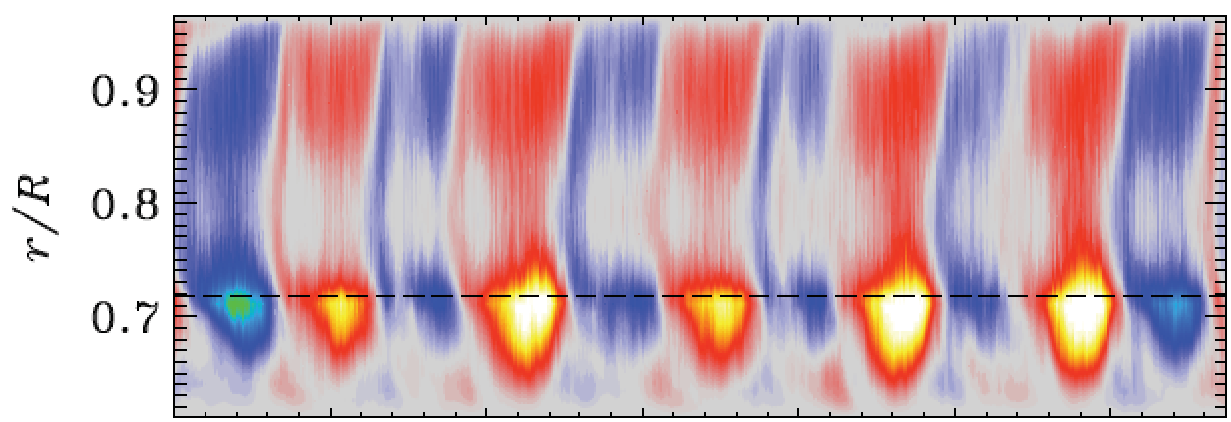


Coexistence of turbulent and mean fields

$\langle B_\phi \rangle$

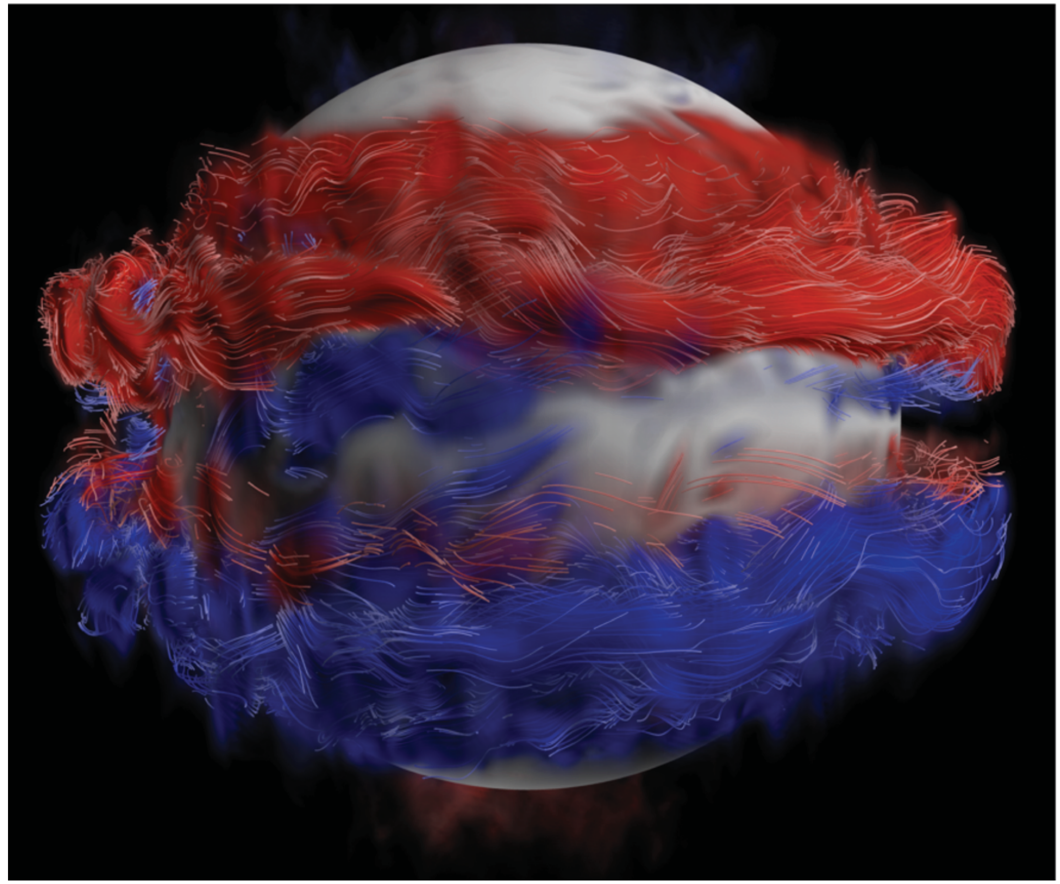


Toroidal field generated near base of the convection zone



Summary: Convective Dynamos

- **Local Dynamo**
  - Lagrangian Chaos
  - Small-scale fields
  - Magnetic carpet
- **Global Dynamo**
  - Rotational Shear
  - Helicity
  - Spherical Geometry
  - Meridional Circulation
  - Boundary Layers
  - MHD Instabilities
  - Activity cycle



***Solar Activity Cycle* still the most pressing and formidable challenge**
Most solar cycle models still employ Mean-Field Dynamo Theory

Mean-Field Dynamo Theory: Reynolds Decomposition

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \boldsymbol{B} - \eta \boldsymbol{\nabla} \times \boldsymbol{B})$$

$$\boldsymbol{B} = \overline{\boldsymbol{B}} + \mathbf{b}$$

$$\mathbf{v} = \overline{\mathbf{v}} + \mathbf{u}$$

Define

Turbulent emf

$$\mathcal{E} = \overline{\mathbf{u} \times \mathbf{b}}$$

G-current

$$\mathcal{G} = \mathbf{u} \times \mathbf{b} - \overline{\mathbf{u} \times \mathbf{b}}$$

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times (\overline{\mathbf{v}} \times \overline{\boldsymbol{B}} + \mathcal{E} - \eta \boldsymbol{\nabla} \times \overline{\boldsymbol{B}})$$

$$\frac{\partial \mathbf{b}}{\partial t} = \boldsymbol{\nabla} \times (\overline{\mathbf{v}} \times \mathbf{b} + \mathbf{u} \times \overline{\boldsymbol{B}} + \mathcal{G} - \eta \boldsymbol{\nabla} \times \mathbf{b})$$

No assumptions so far...

Mean-Field Dynamo Theory: Underlying Assumptions

Now... The two fundamental assumptions central to (traditional) MFT

(I) Kinematic

Velocity field v is independent of B

Cannot be true in any saturated dynamo!

(II) Locality

Spatial scale on which fluctuations operate is small relative to that of mean field (scale separation)
$(\ell << L)$

Unlikely to be true in a turbulent dynamo!

$$\mathcal{E}_i = \alpha_{ij}\overline{B}_j + \beta_{ijk}\frac{\partial \overline{B}_j}{\partial x_k} + \epsilon_{ijkl}\frac{\partial^2 \overline{B}_j}{\partial x_k \partial x_l} + \ldots$$

Moffatt (1978), Krause & Radler (1980), Ossendrijver (2003), Rudiger & Hollerbach (2004), Rempel (2009)

Mean-Field Dynamo Theory: The Mean-Field Equation

Now write the mean velocity field as (spherical coordinates)

$$\overline{\mathbf{v}} = \overline{\mathbf{v}}_m + r \sin\theta \; \Omega \; \hat{\phi}$$

meridional circulation — differential rotation

And (finally!) obtain the mean-field dynamo equation as it it typically solved

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \lambda \left(\overline{\boldsymbol{B}}_p \cdot \boldsymbol{\nabla} \Omega \right) \hat{\phi} + \boldsymbol{\nabla} \times \left(\alpha \overline{\boldsymbol{B}} + \left(\overline{\mathbf{v}}_m + \boldsymbol{\gamma} \right) \times \overline{\boldsymbol{B}} + \left(\eta + \beta \right) \overline{\boldsymbol{J}} \right)$$

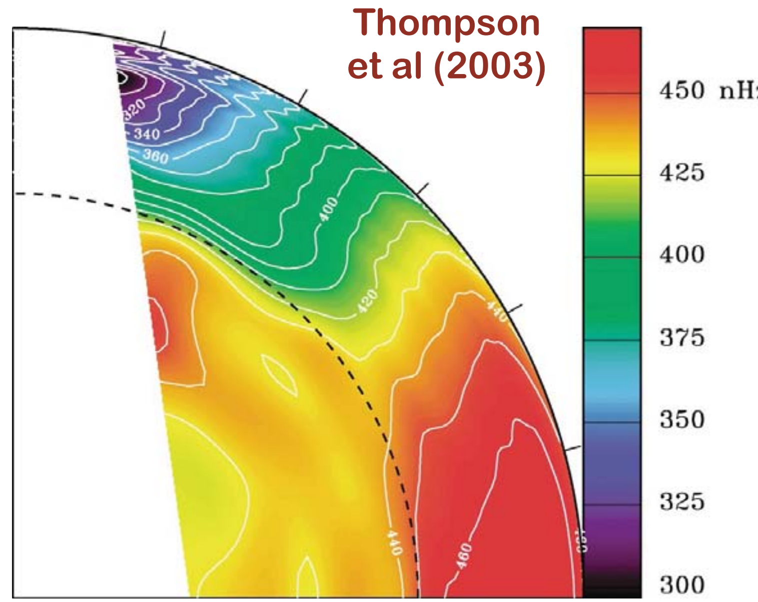
- Ω-effect
- α-effect
- transport by magnetic pumping (often neglected)
- turbulent diffusion
- transport by the meridional circulation
- molecular diffusion (often neglected)

This is the basis for virtually all current models of the solar activity cycle!

α-Ω Models for the Solar Cycle

In spherical geometry, dynamo waves can produce activity cycles!

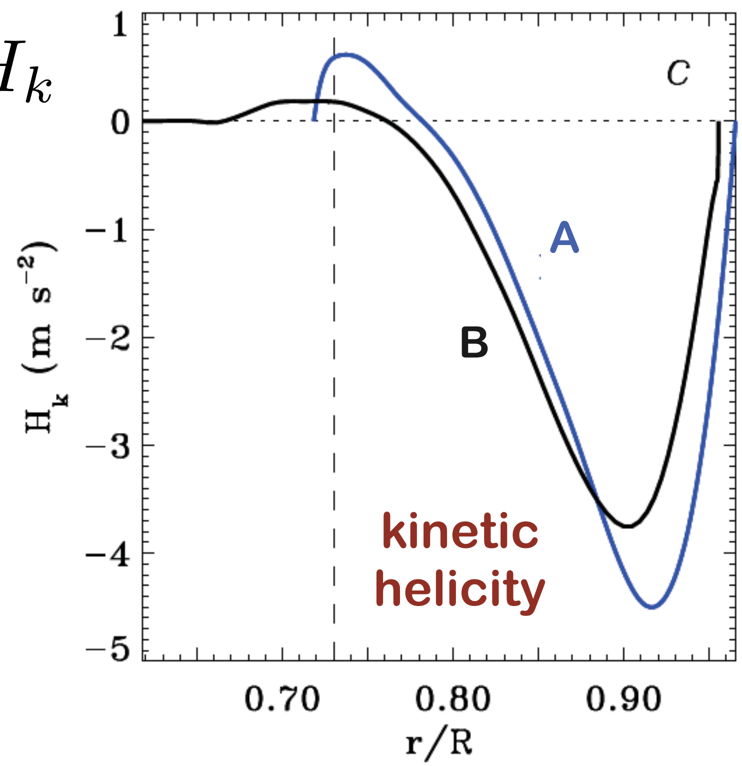
See Homework Problem II



Dynamo Waves: propagating solutions to the mean-field dynamo equations

Growth rate, propagation speed, and propagation direction determined by the product of α and ∇Ω

$$\alpha \propto -H_k$$



Parker-Yoshimura sign rule

$$\alpha \frac{\partial \Omega}{\partial r} < 0$$

in NH gives equatorward propagation

Expect equatorward propagation at low latitudes near the base of the convection zone

For an assessment of how these and other models do, see Ossendrijver (2003), Charbonneau (2010)

Mean-Field Dynamo Theory: Interface Dynamos

Convection Zone	***Large turbulent diffusion alpha effect weak radial shear***
Overshoot Region/ Tachocline	***Small turbulent diffusion no alpha effect strong radial shear***

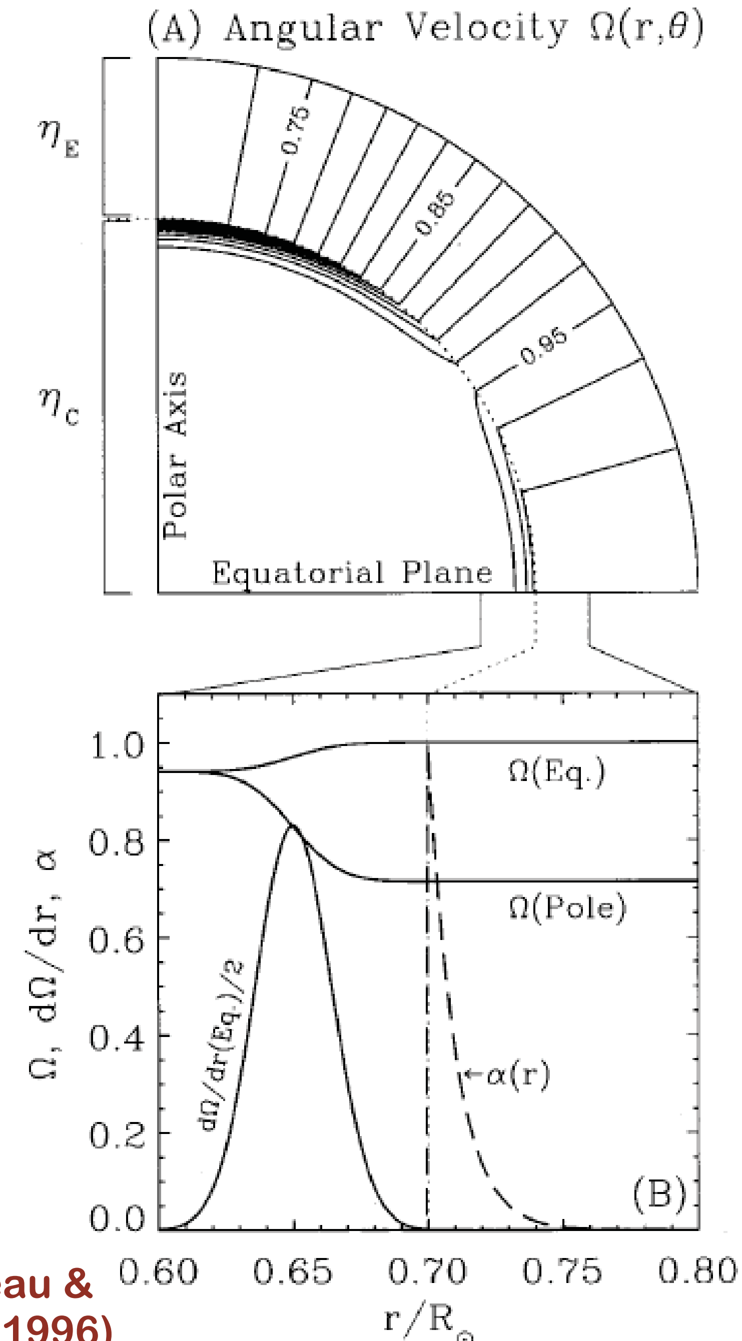
Proposed by Parker (1993)

(see also Charbonneau & Macgregor 1996, 1997: Tobias 1996, 1998)

Still follow Parker-Yoshimura rule for dynamo wave propagation but now the cycle period, dynamo growth rate also depend on transport via β

$$\frac{B_{or}}{B_{cz}} \sim \frac{\eta_{cz}}{\eta_{or}}$$

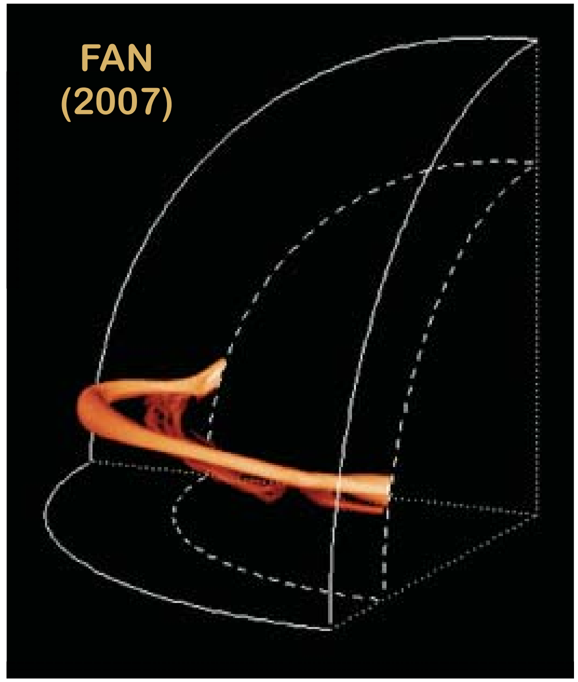
Can help alleviate α-quenching & promote storage of toroidal flux

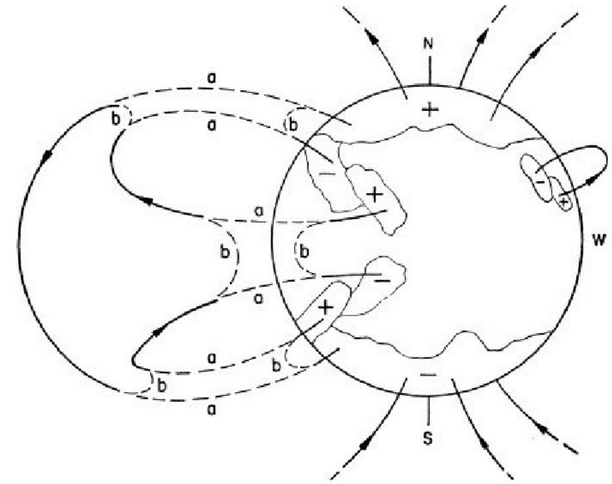


Charbonneau & Macgregor (1996)

The Babcock-Leighton Mechanism

Arises from Coriolis-induced tilts in emerging flux tubes followed by dispersal of poloidal flux in surface layers by turbulent diffusion, meridional flow





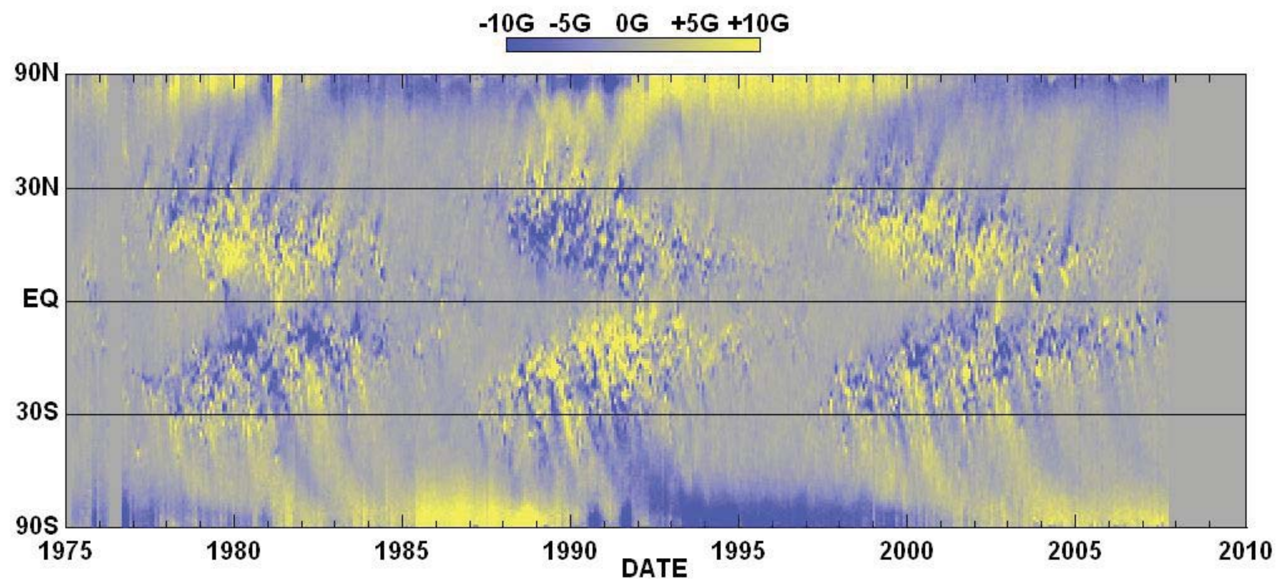
Babcock (1961), Leighton (1964)

Often implemented as a non-local α-effect

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(S \hat{\phi} \right) + \ldots$$

$$S(r, \theta) = \alpha f(r) g(\theta) B_\phi^{bcz}(\theta)$$

with f(r) confined to surface layers



NASA/MSFC/NSSTC/Hathaway 2007/10

One of several alternatives to the conventional turbulent α-effect (Charbonneau 2010)

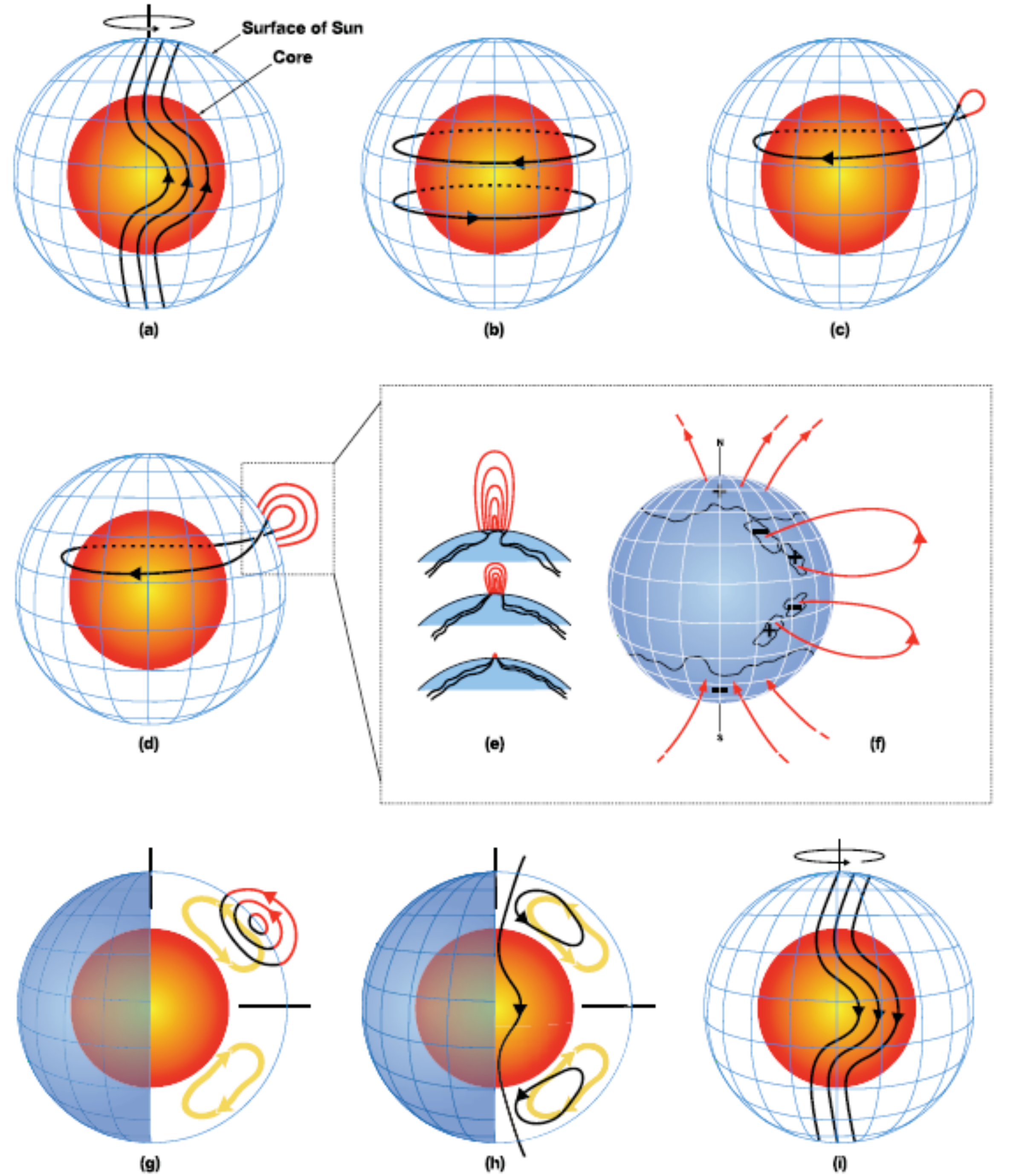
Flux-Transport Dynamo Models

Equatorward meridional flow near the base of the convection zone largely responsible for equatorward migration of active bands (butterfly diagram)

Most current Flux-Transport Models are also Babcock-Leighton Models

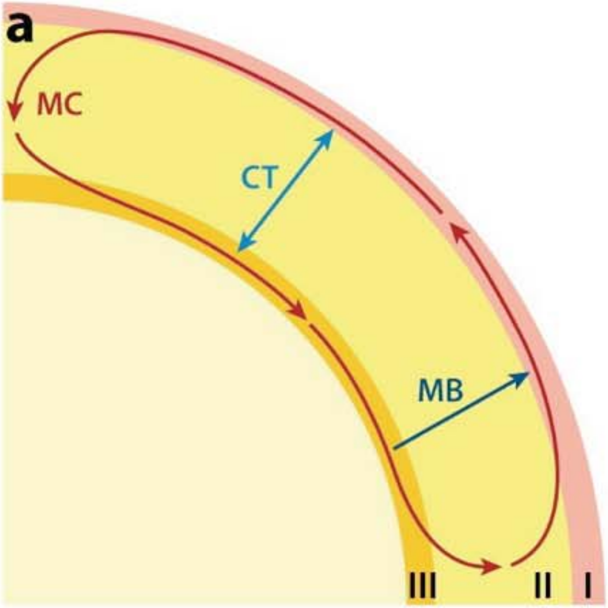
BLFT Models

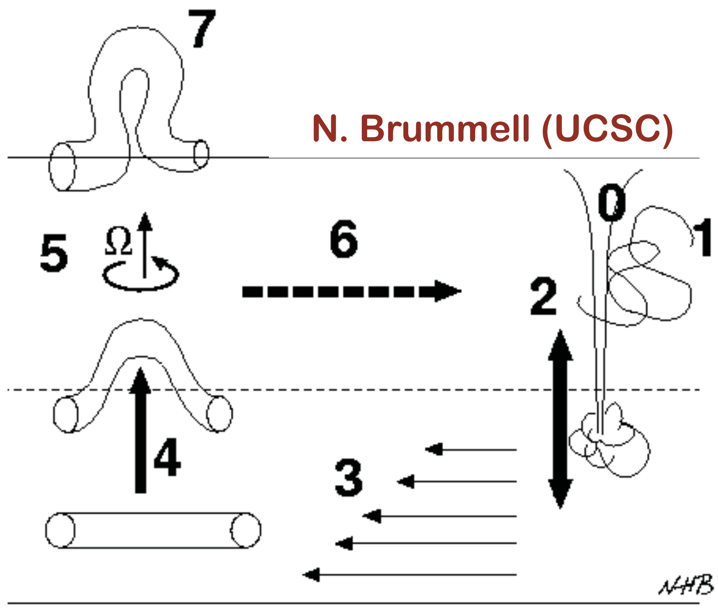
May operate in the advection-dominated or diffusion/pumping-dominated regime



Dikpati & Gilman (2006)

The (Global) Solar Dynamo:





c Miesch & Toomre (2009)

	Toroidal field generation	Poloidal field generation	Principal coupling mechanisms	Cycle period determined by
BLFT models	Region III	Region I	MC, MB	Meridional flow
Interface models	Region III	Region II	CT	Dynamo waves[a]

a. Dispersion relation involving α, $\Delta\Omega$, and η_t.

Boundary Layers

Makes numerical modeling more challenging

Time Delays

Promotes chaotic modulation of cycle periods/amplitudes

Mean-field Models:Current Challenges

General Issues

- Turbulent transport/diffusion not well understood
- Lorentz force back-reactions not well understood
- Meridional flow not well known
- Parity selection (dipole/quadropole)
- What is the dominant source of poloidal field?
- Where do active regions originate?

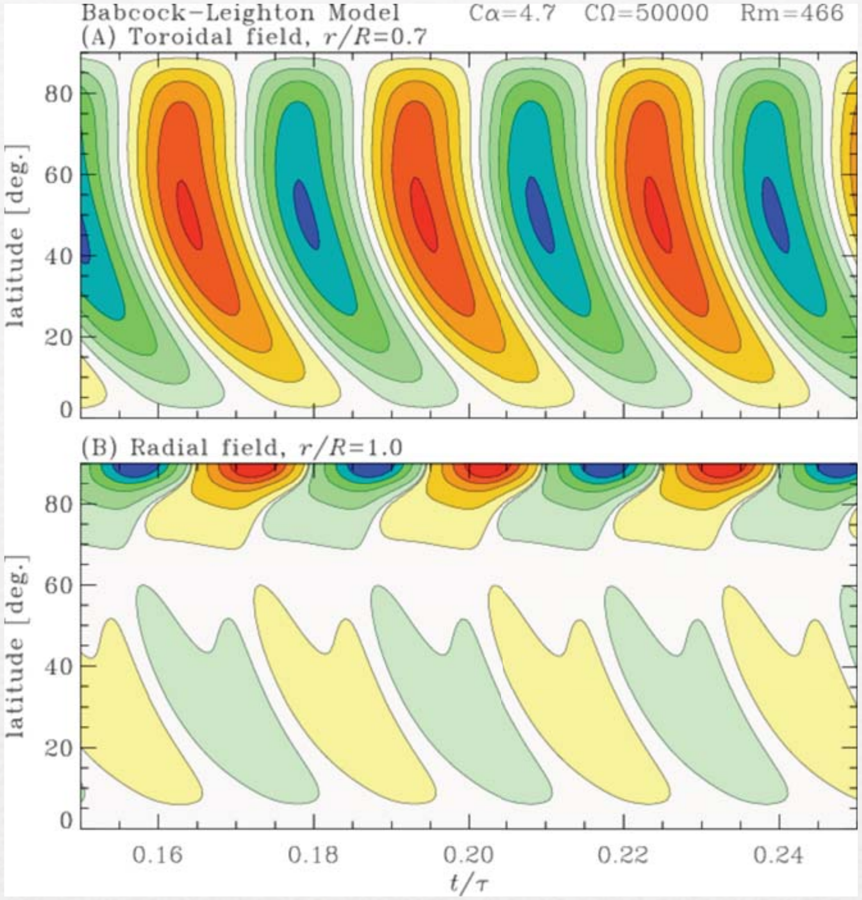
BLFT Dynamos

- Advection-dominated regime not well justified
- Flux emergence not well understood (links toroidal field at base to poloidal source)
- Strong polar fields, self-excitation, etc

Interface/Distributed Dynamos

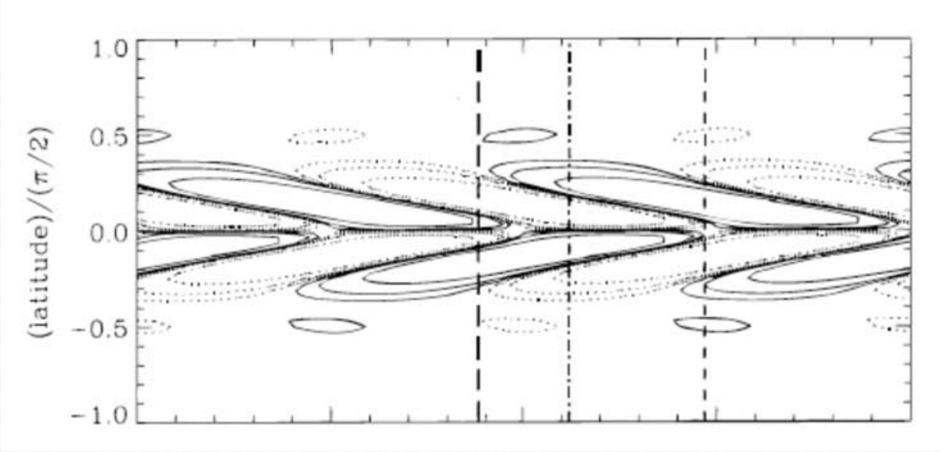
- Turbulent α-effect not well justified/understood
- Tend to produce overlapping cycles with small latitudinal extents

For much more on this see
Ossendrijver (2003), Charbonneau (2010)



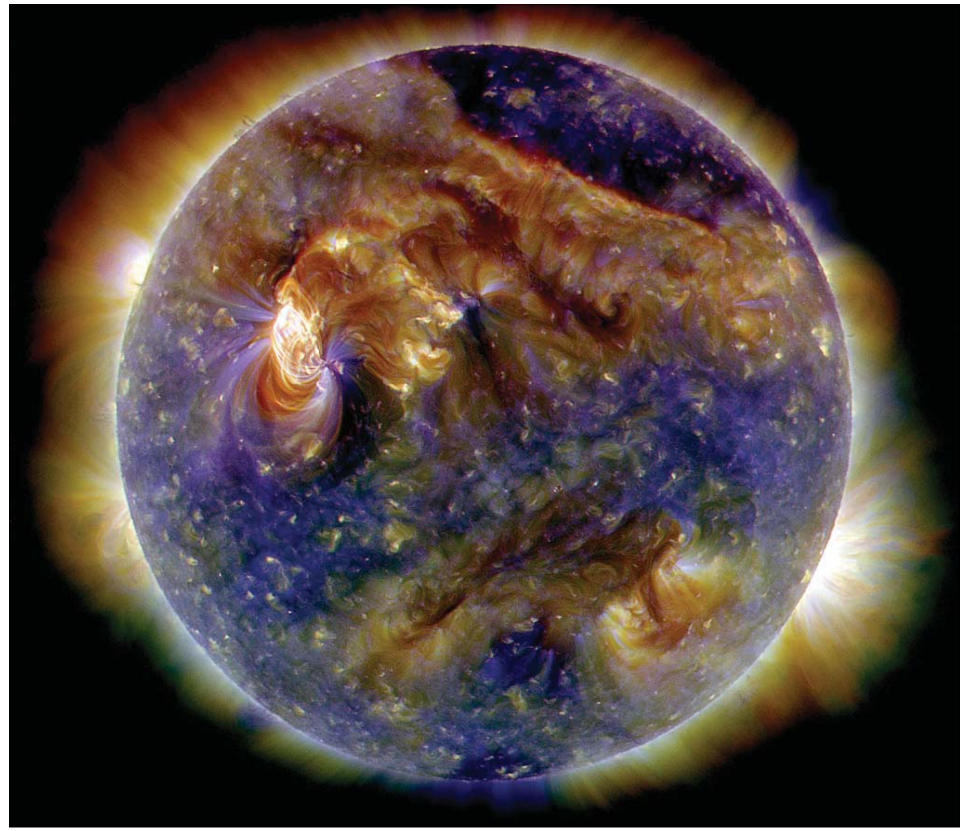
Charbonneau (2010)

Charbonneau & MacGregor (1997)



Summary: The Solar Dynamo

- Dynamos are complex!
- Solar magnetism
  - Multiple scales
    (seconds to centuries, km to Gm)
  - Magnetic Carpet
  - Solar Cycle
    - Convection
    - Differential Rotation
    - Meridional Circulation
    - MHD Instabilities



- Tools of the Trade
  - Solar Observations
  - Stellar Observations
  - Numerical Models
  - Theoretical Insights