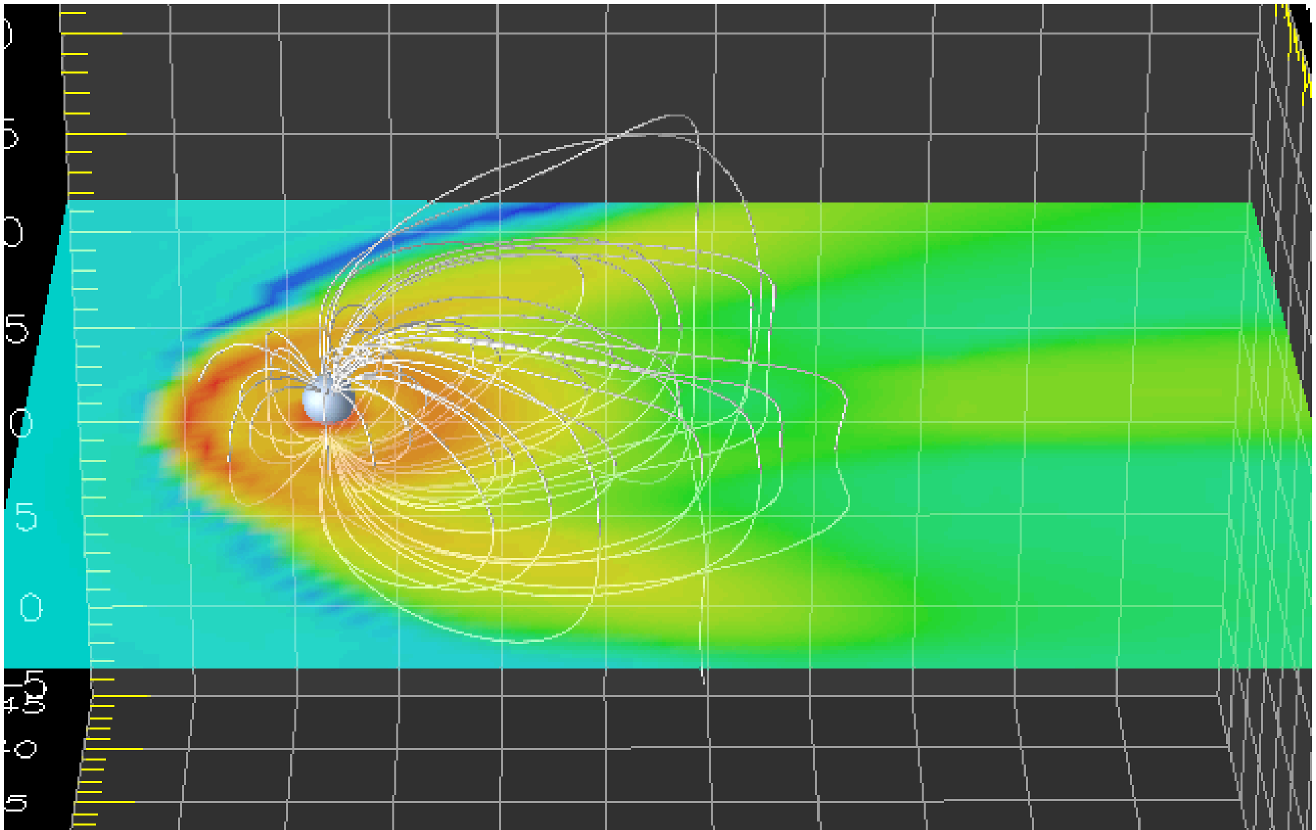
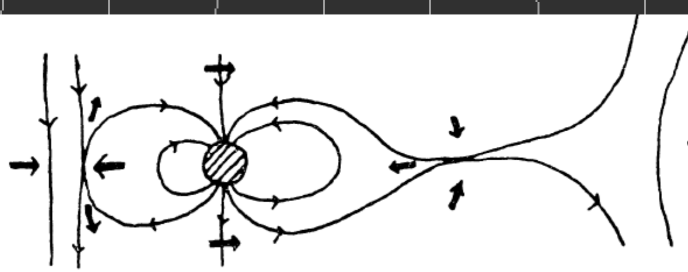
Magnetic field lines and their reconnection

Dana Longcope
Montana State University

Reconnection: the magnetosphere

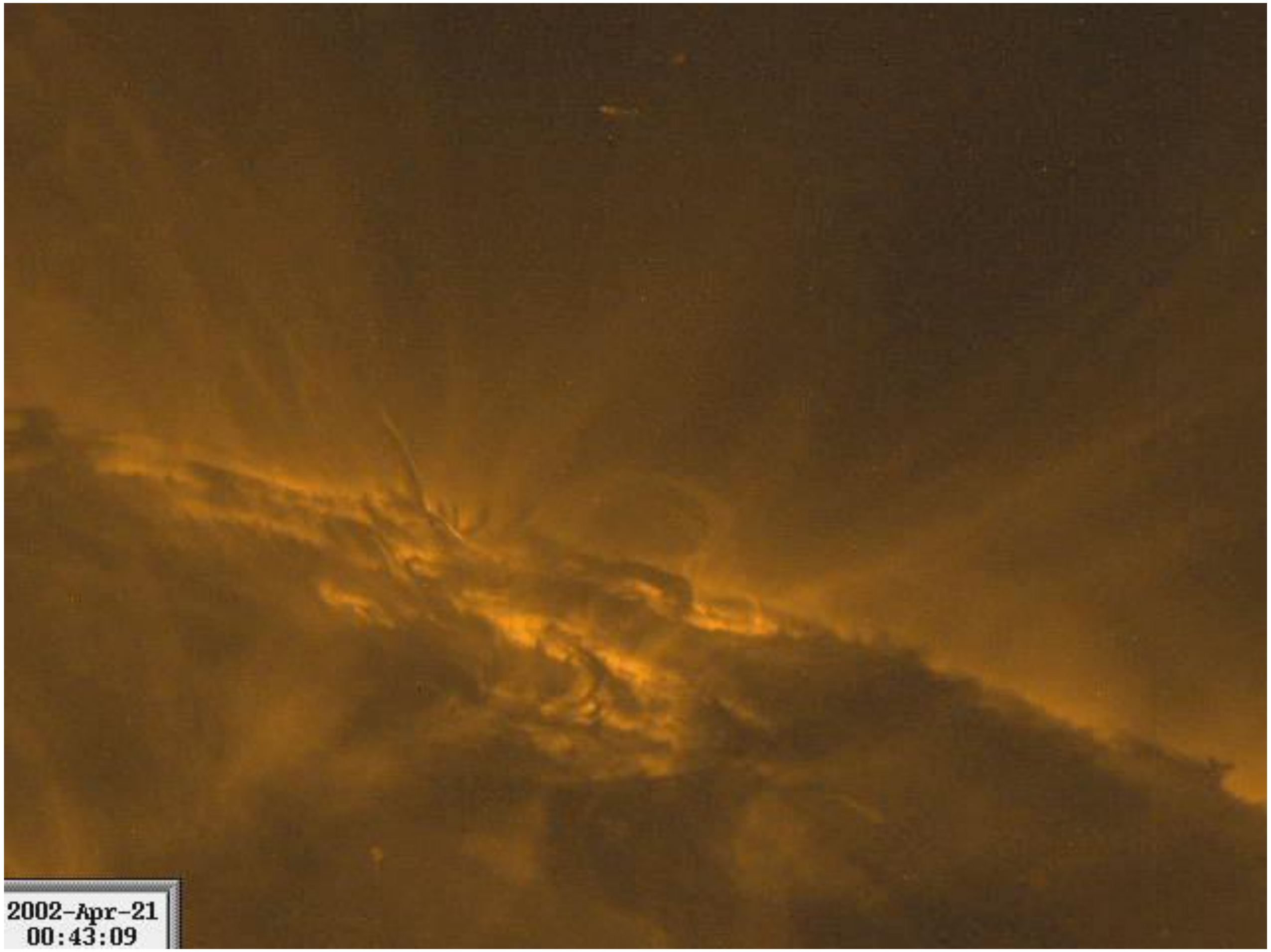
Courtesy R. Winglee

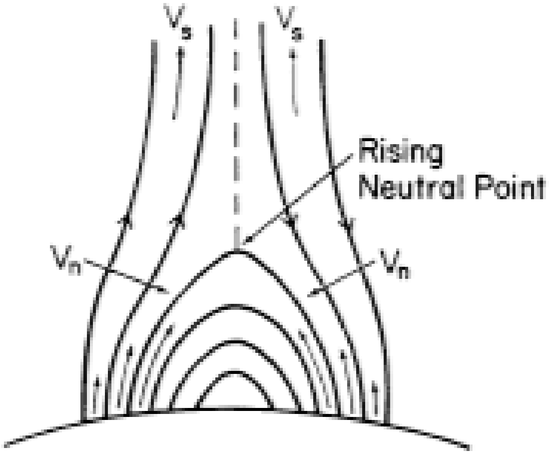




Dungey 1962

Reconnection: solar corona

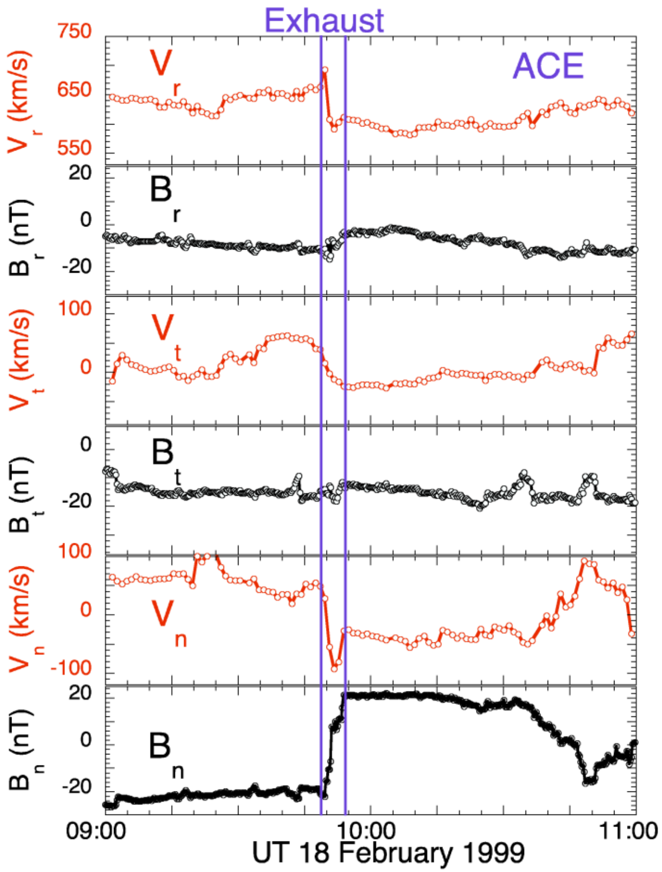


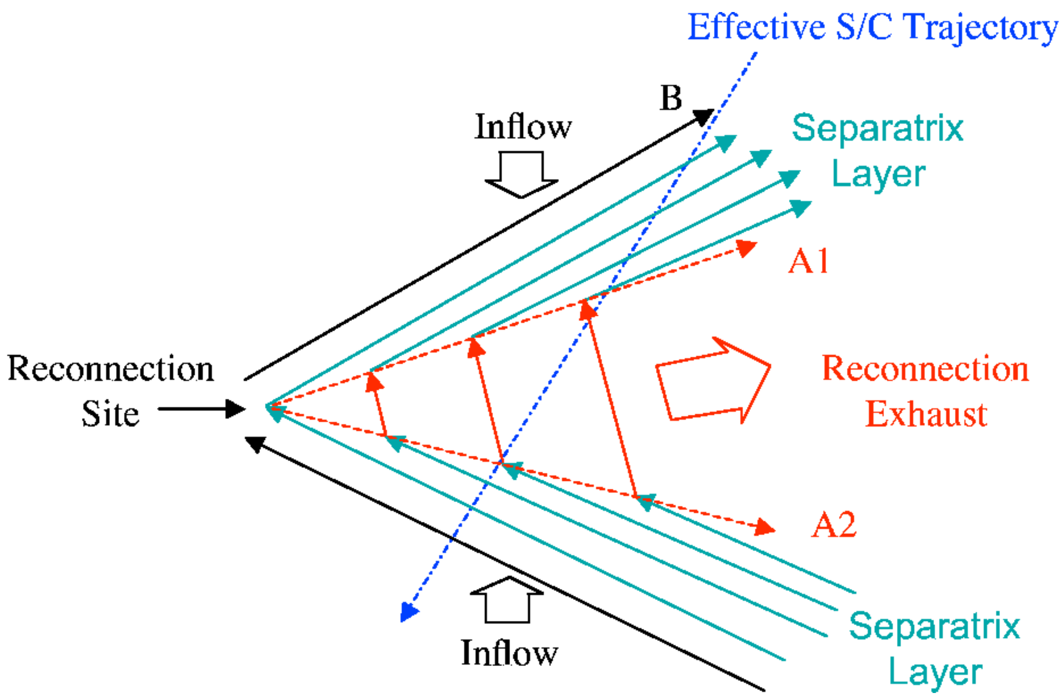


Kopp & Pneman 1976

Courtesy TRACE team

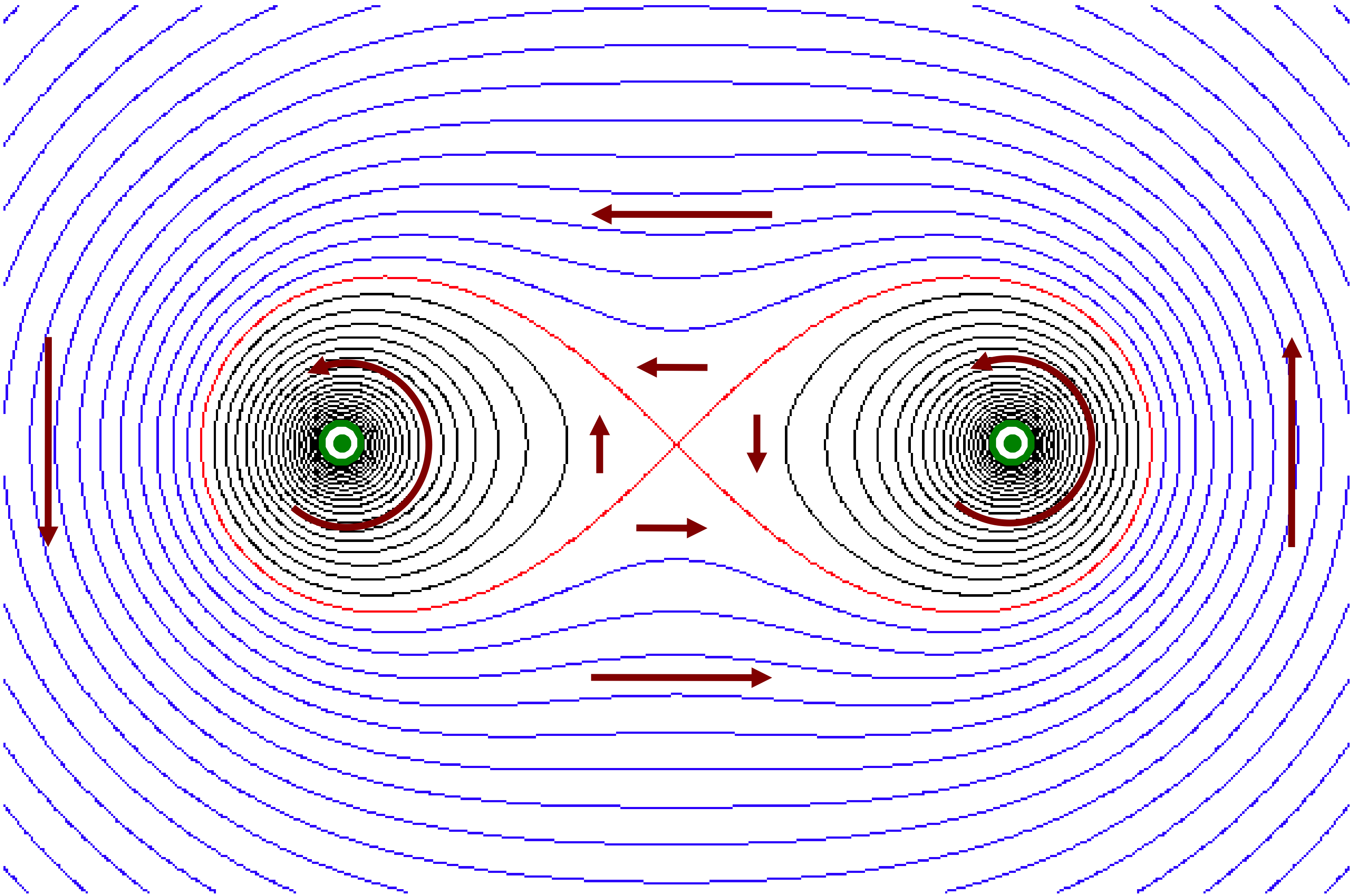
Reconnection: the heliosphere



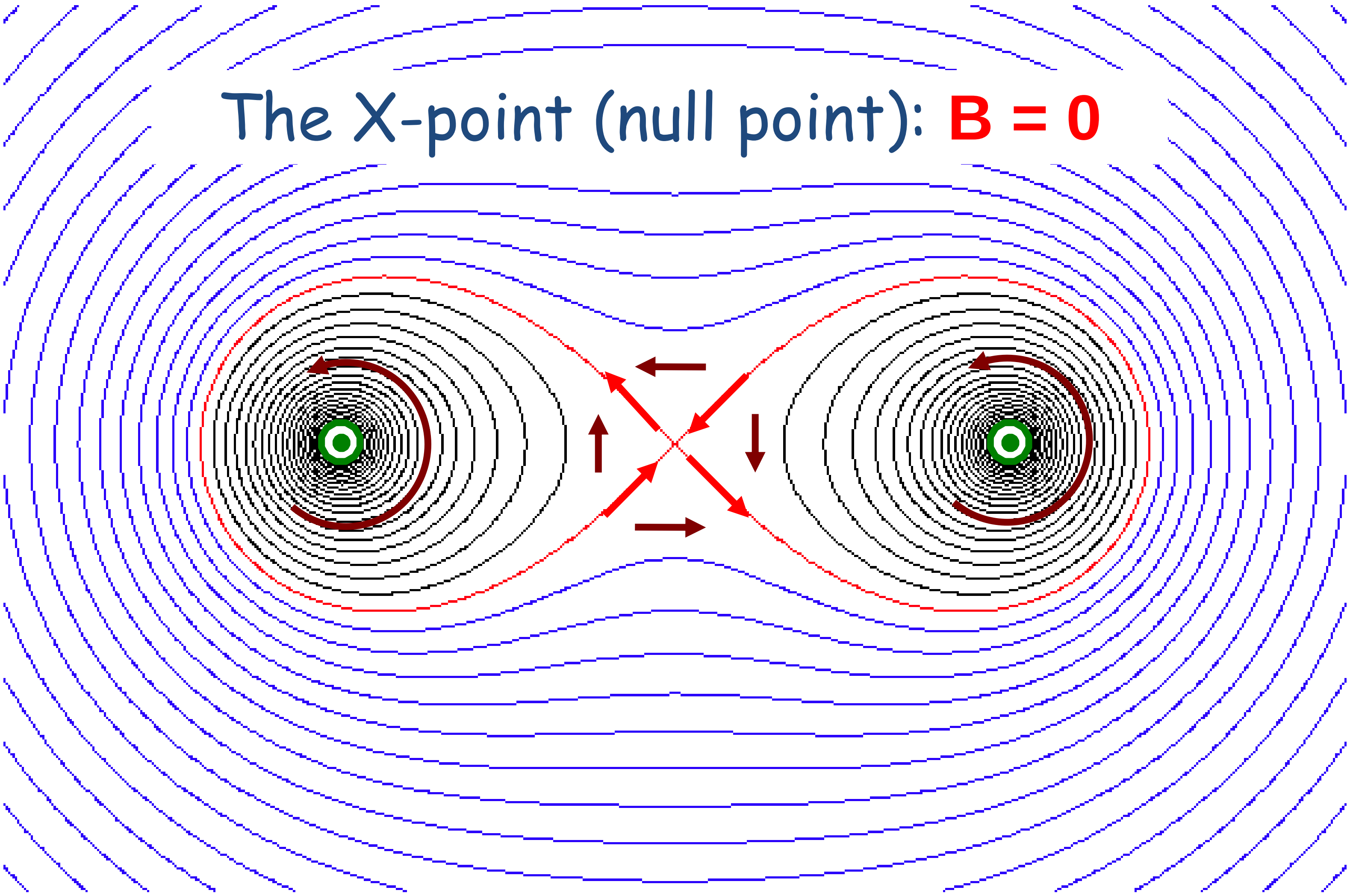


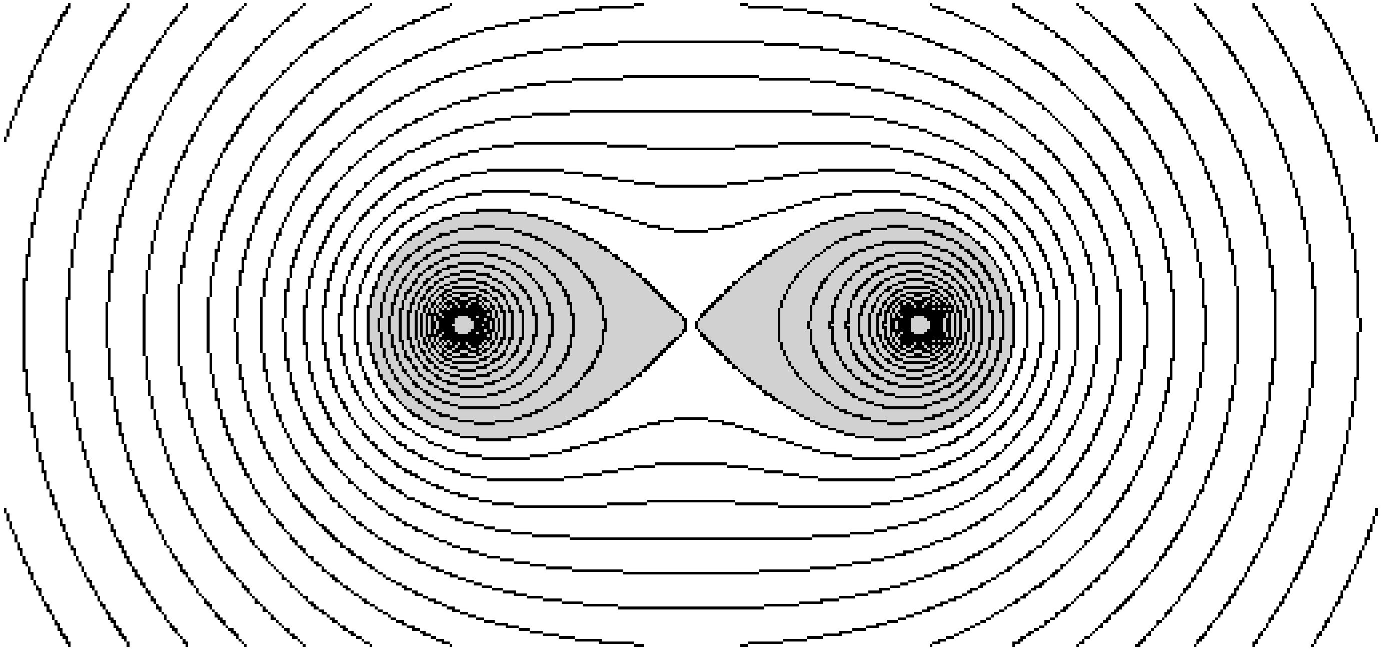
Courtesy J. Gosling

Reconnection: Parallel wires in vacuum

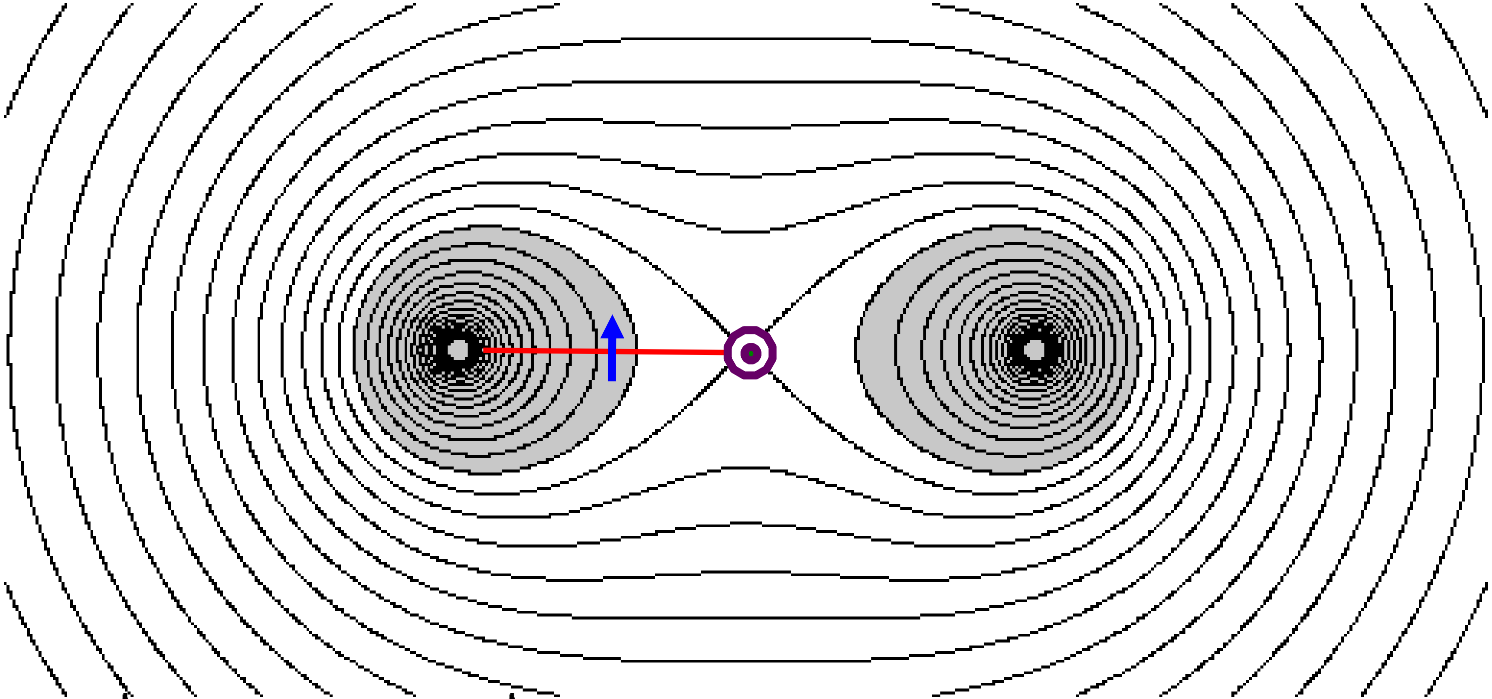


Reconnection: Parallel wires in vacuum





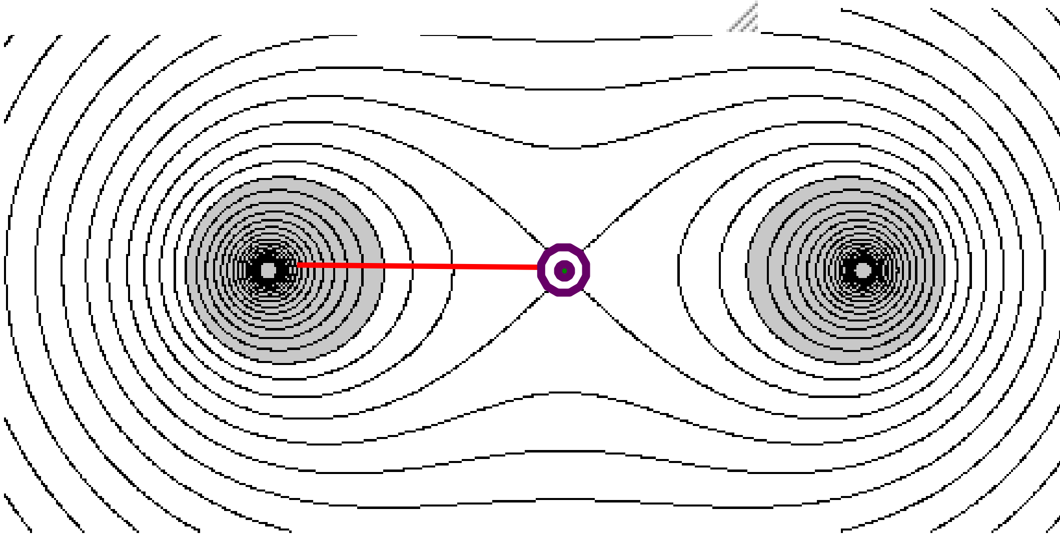
Separate wires **slowly**



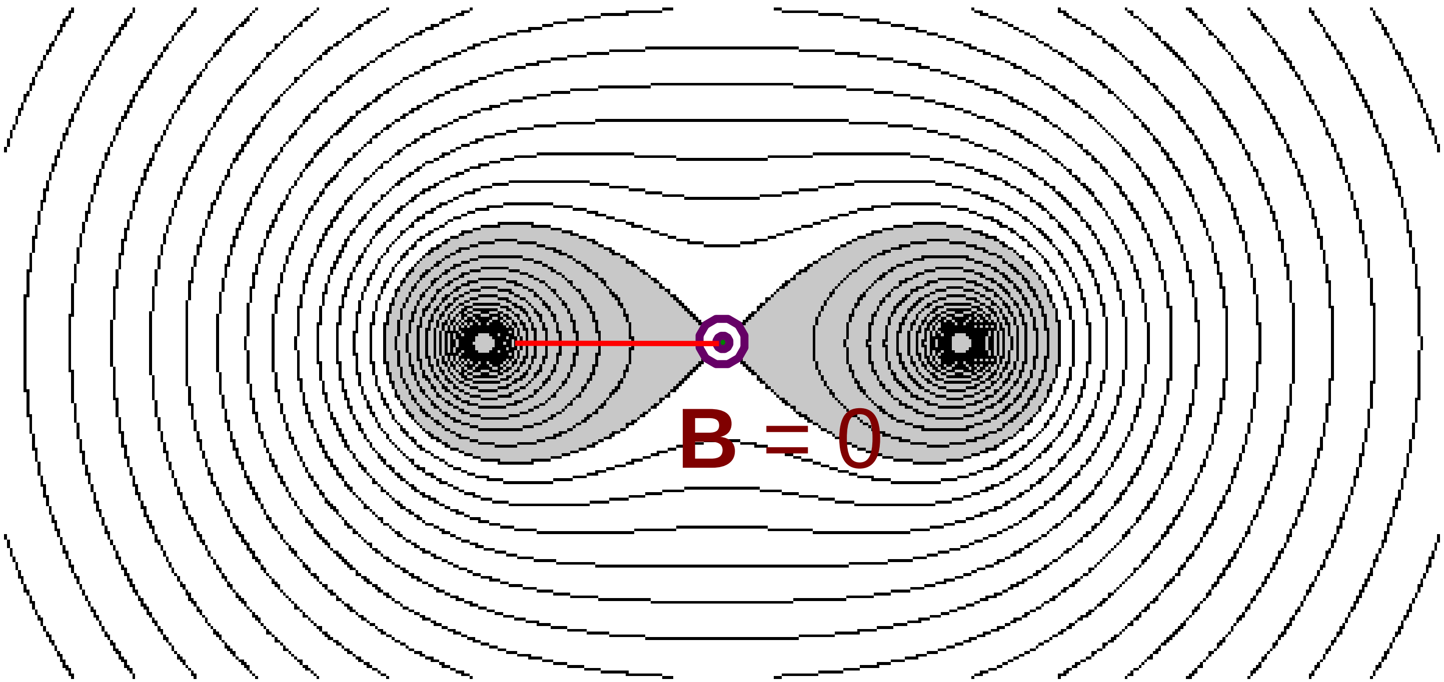
$$\Phi = \int \mathbf{B}\cdot d\mathbf{a} = L_z \underbrace{\int B_y dx}_{\psi}$$

$$\frac{d\Phi}{dt} = -\oint \mathbf{E}\cdot d\mathbf{l} = L_z E_z(0)$$

$E_z = d\psi/dt$

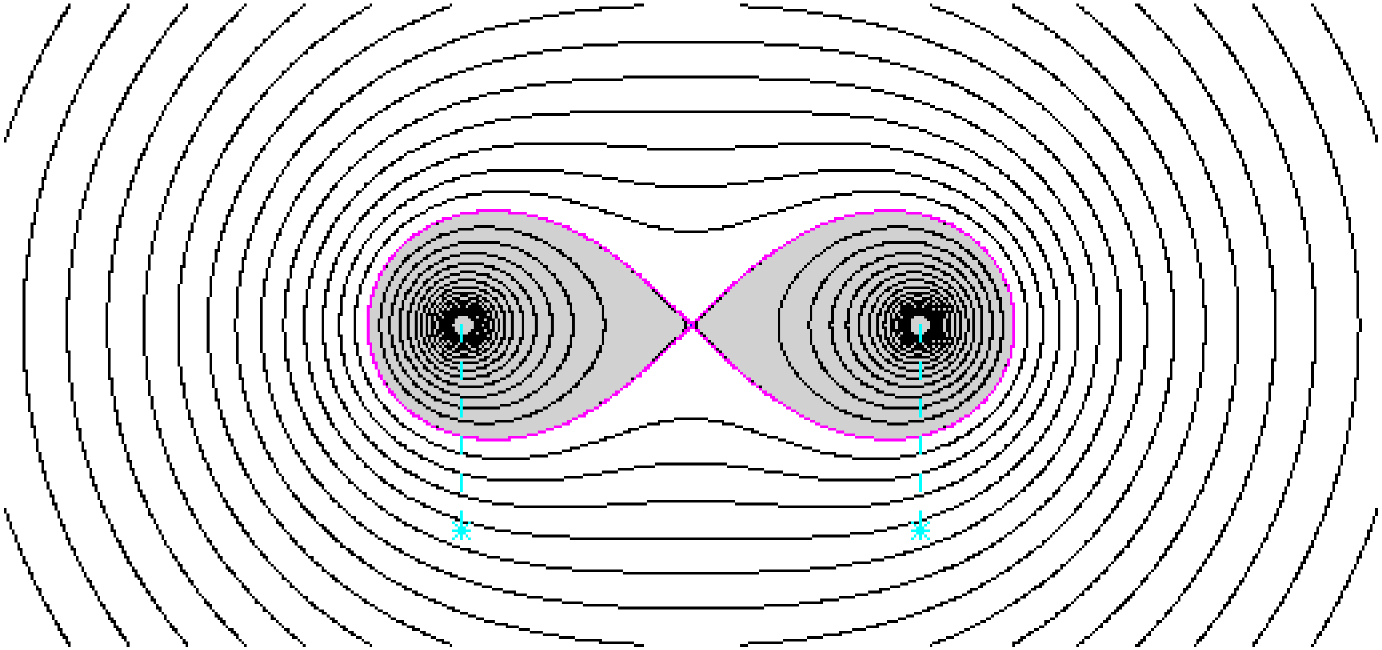


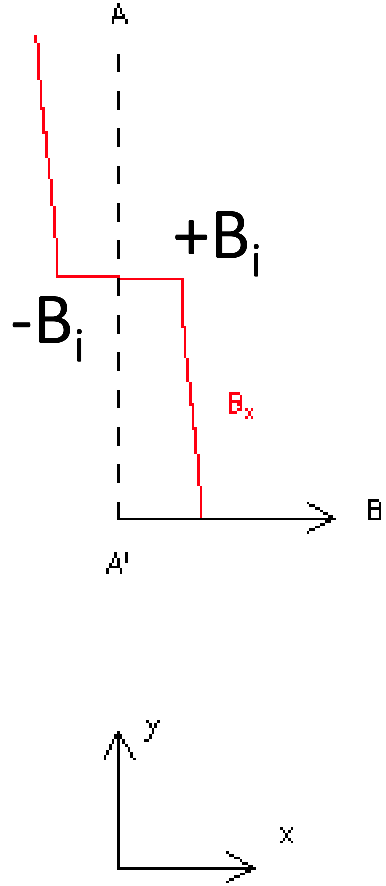
- Good conductor: $\mathbf{E} = 0$
- Good moving conductor: $\mathbf{E} + \mathbf{v} \quad \mathbf{B} = 0$

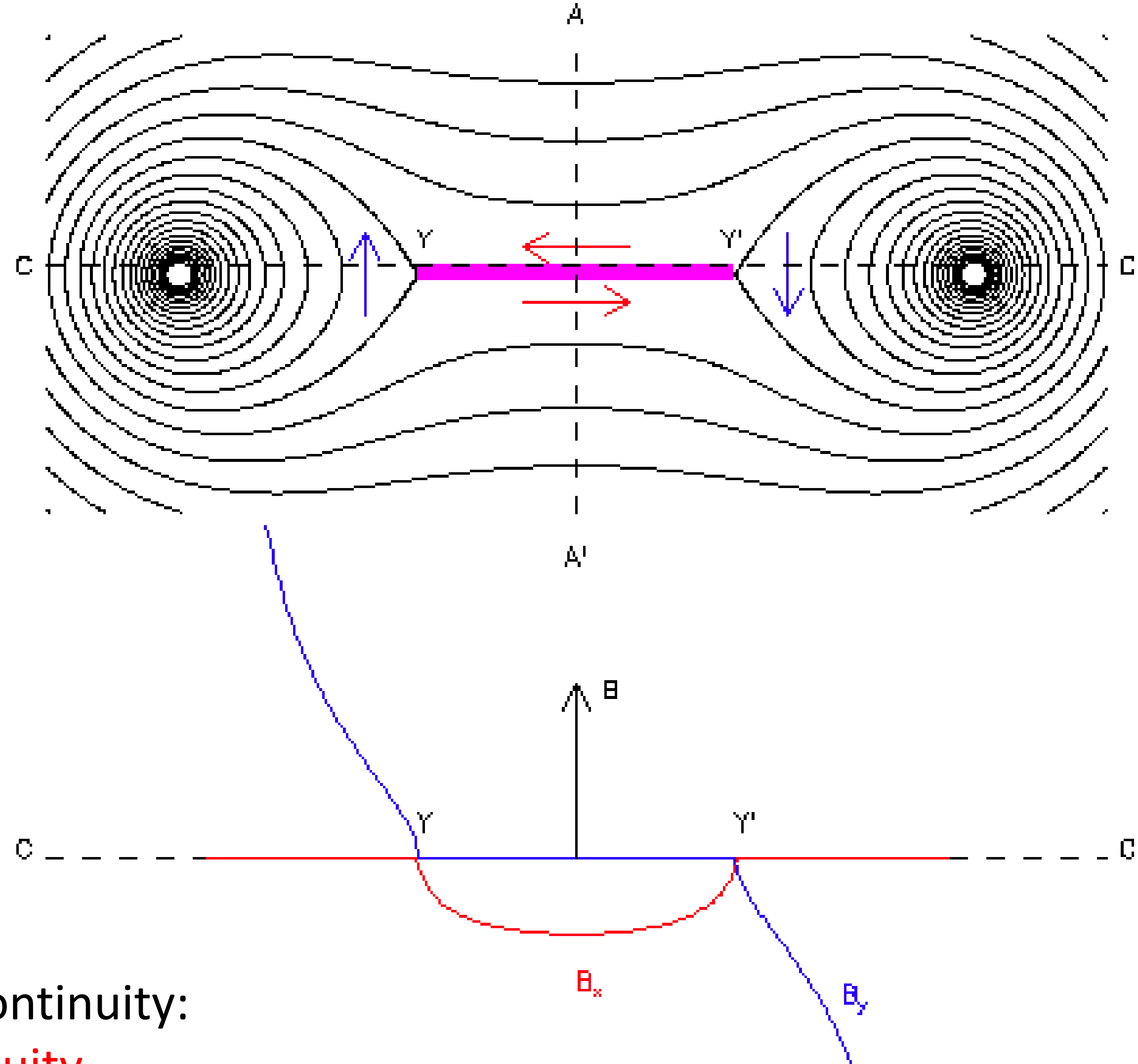


- $E_z + (\mathbf{v} \quad \mathbf{B})_z = 0$ (with $\mathbf{v} \quad \mathbf{B}$ struck through) $\qquad E_z = d\psi/dt$
- ψ = const.

- Mag. Energy >> plasma energy (β<<1)
- Move slowly
- Min. magnetic energy







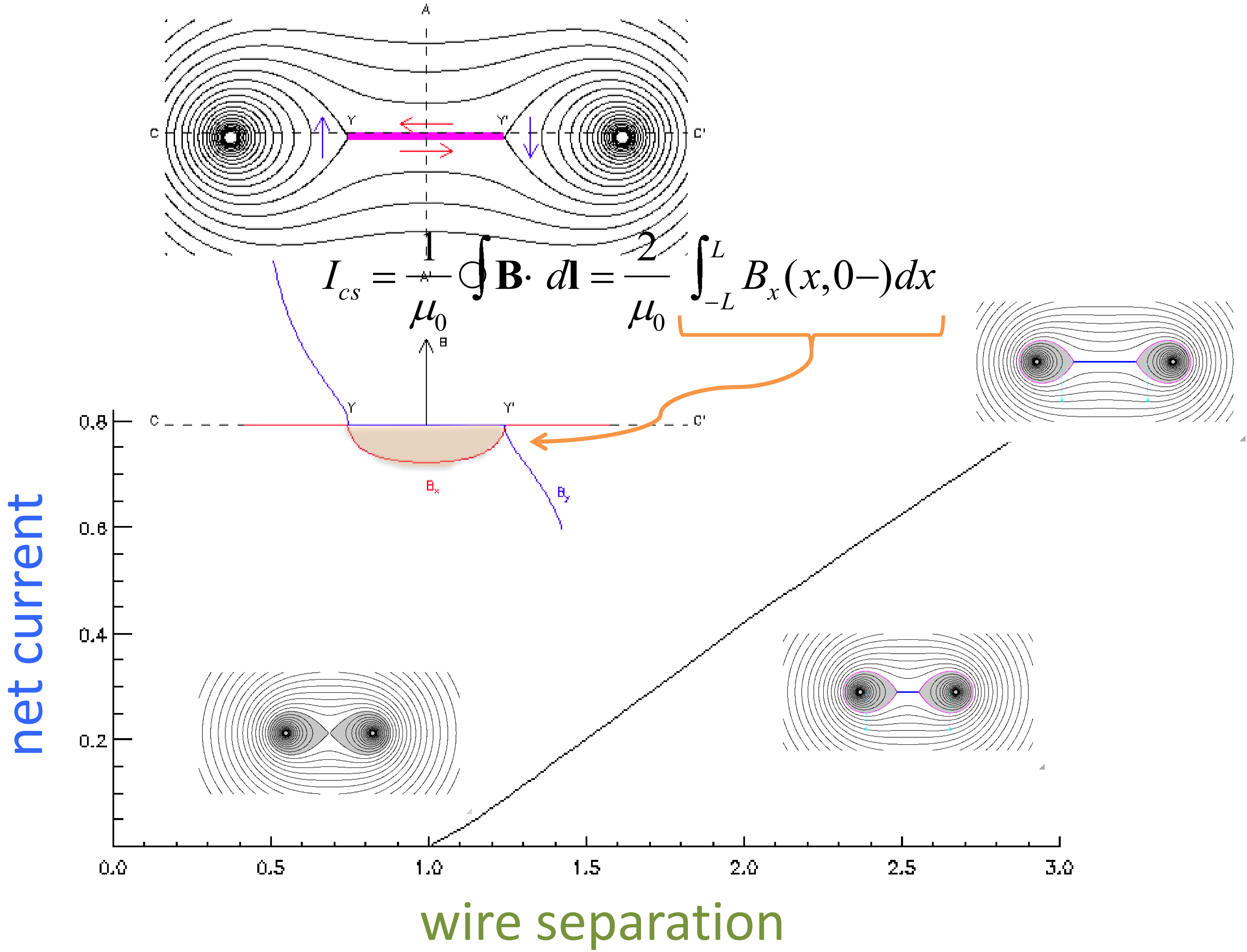
Magnetic field discontinuity:
tangential discontinuity

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B} = 2B_x(x,0-)\delta(y)\,\hat{\mathbf{z}}$$

Magnetic equilibrium

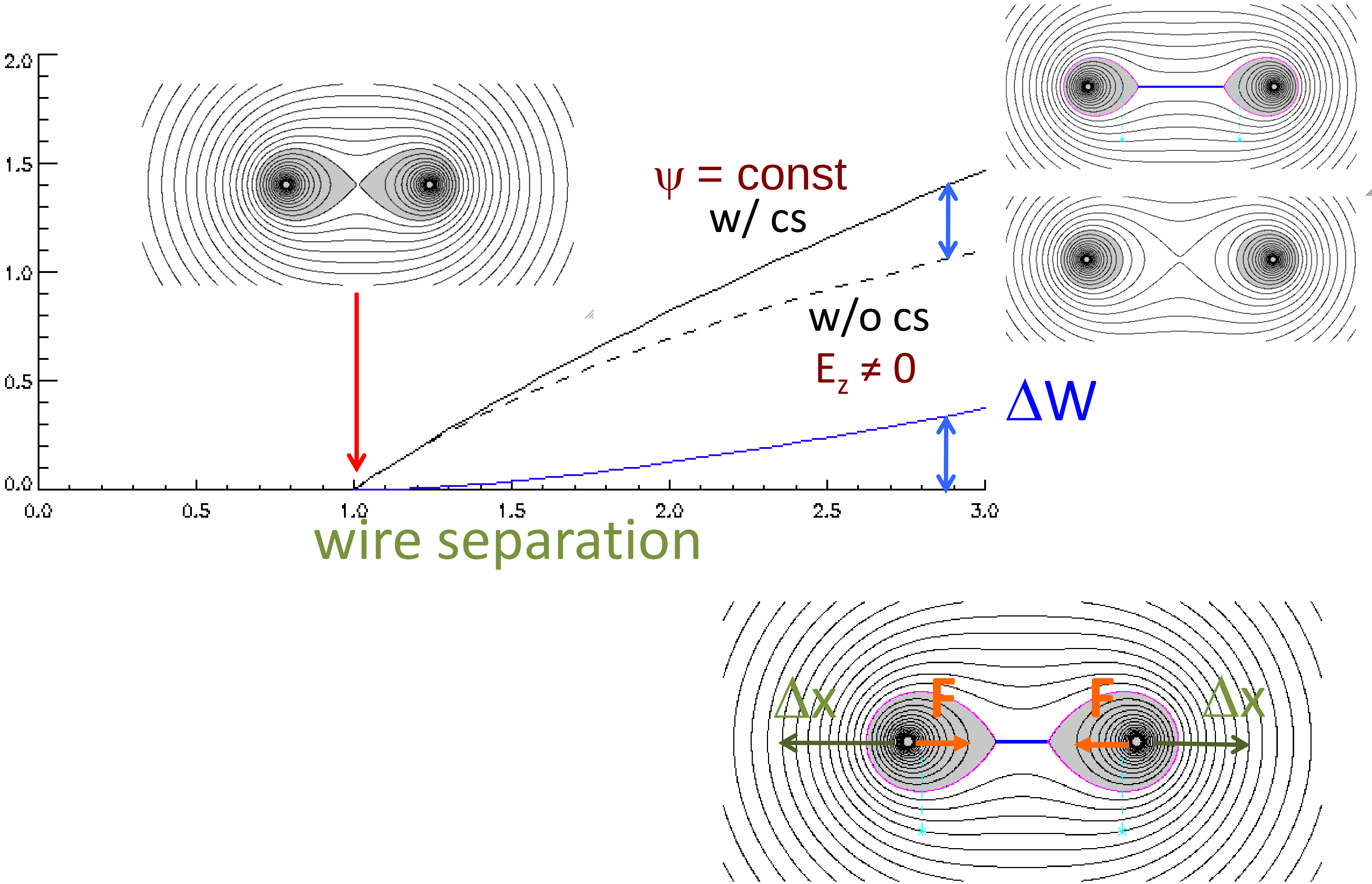
$$\mathbf{J} \times \mathbf{B} \propto (\nabla \times \mathbf{B}) \times \mathbf{B} = \tfrac{1}{2}\hat{\mathbf{y}}\frac{\partial}{\partial y}|\mathbf{B}|^2 = 0$$

Minimum magnetic energy state

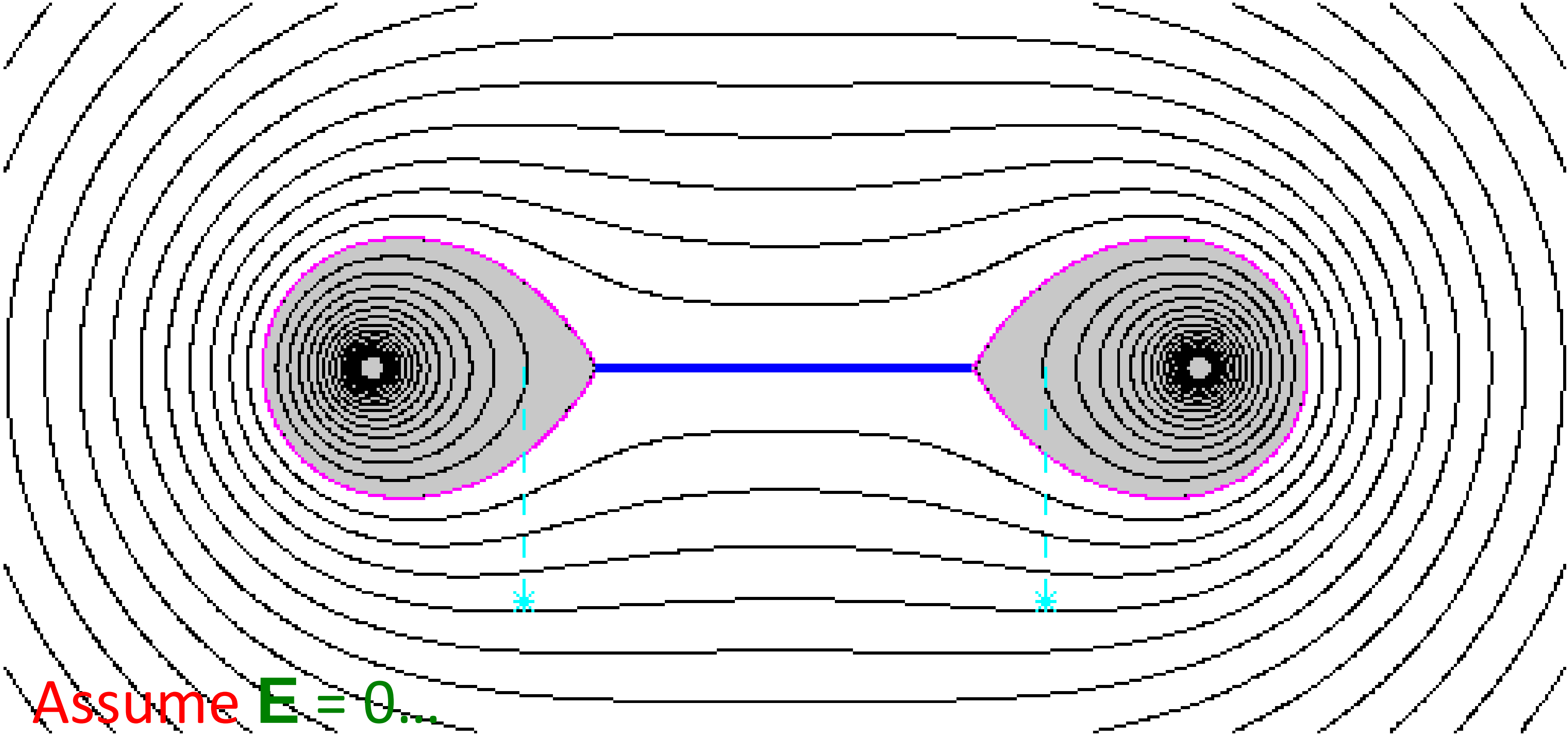


$$I_{cs} = \frac{1}{\mu_0}\oint \mathbf{B}\cdot d\mathbf{l} = \frac{2}{\mu_0}\int_{-L}^{L} B_x(x,0-)\,dx$$
A
C
C'
Y
Y'
B
B_x
B_y
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
net current
wire separation

Work done on wires



Good vs. Fair conductance



Assume **E** = 0...

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B} = 2B_x(x,0-)\,\delta(y)\,\hat{\mathbf{z}}$$

Can **E** = 0 when **J** = ∞?

Ohm's law

- Q: What makes a conductor "good"? (**E**=0)

  (e.g. copper or gold? a plasma?)

- A: electrons move to eliminate **E**
- Q: What might limit "goodness"? (**E**≠0)
- A: electrons cannot respond eff

$$\mathbf{J} = en_e(\mathbf{v}_i - \mathbf{v}_e)$$

momentum eq. of electron fluid

drag

$$m_e n_e \frac{d\mathbf{v}_e}{dt} = -\nabla\cdot \overset{t}{P}_e - en_e\mathbf{E} - en_e\mathbf{v}_e \times \mathbf{B} + m_e n_e \nu_{ei}(\mathbf{v}_i - \mathbf{v}_e)$$

electron inertia

Hall term

$1/\sigma = \eta_e$

$$\mathbf{E} + \mathbf{v}_i \times \mathbf{B} = -\frac{m_e}{e}\frac{d\mathbf{v}_e}{dt} - \frac{1}{en_e}\nabla\cdot \overset{t}{P}_e + \frac{1}{en_e}\mathbf{J}\times\mathbf{B} + \frac{m_e \nu_{ei}}{e^2 n_e}\mathbf{J}$$

Generalized Ohm's law

What's really important?

$$\mathbf{E} = -\mathbf{v}_i \times \mathbf{B} - \frac{m_e}{e}\frac{d\mathbf{v}_e}{dt} - \frac{1}{en_e}\nabla\cdot \overset{t}{P}_e + \frac{1}{en_e}\mathbf{J}\times\mathbf{B} + \eta_e \mathbf{J}$$

(i) (ii) (iii) (iv) (v)

$$\frac{\text{(i)}}{\text{(v)}} = \frac{vB}{\eta_e B/\mu_0 l} = \frac{l v \mu_0}{\eta_e} \equiv Rm$$

$$\frac{\text{(i)}}{\text{(ii)}} = \frac{vB}{m_e vB/\mu_0 e^2 n_e l^2} = \frac{l^2}{m_e/\mu_0 e^2 n_e} = \left(\frac{l}{c/\omega_{pe}}\right)^2$$

$$\frac{\text{(i)}}{\text{(iv)}} = \frac{vB}{B^2/\mu_0 e n_e l} = \frac{vl}{B/\mu_0 e n_e} = \left(\frac{v}{v_A}\right)\left(\frac{l}{c/\omega_{pi}}\right)$$

$$\frac{\text{(i)}}{\text{(iii)}} = \frac{vB}{p_e/en_e l} = \frac{vl}{k_B T/eB} = \left(\frac{v}{c_s}\right)\left(\frac{l}{\rho_i}\right)$$

Importance of term depends on length scale of solution

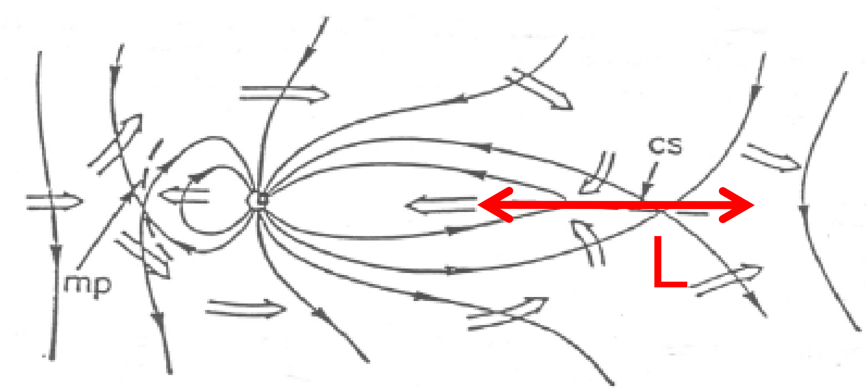
On large scales plasma is ideal conductor*

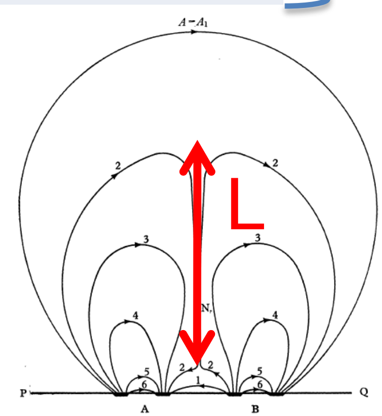
* except where **B**=0

How large is "large"?

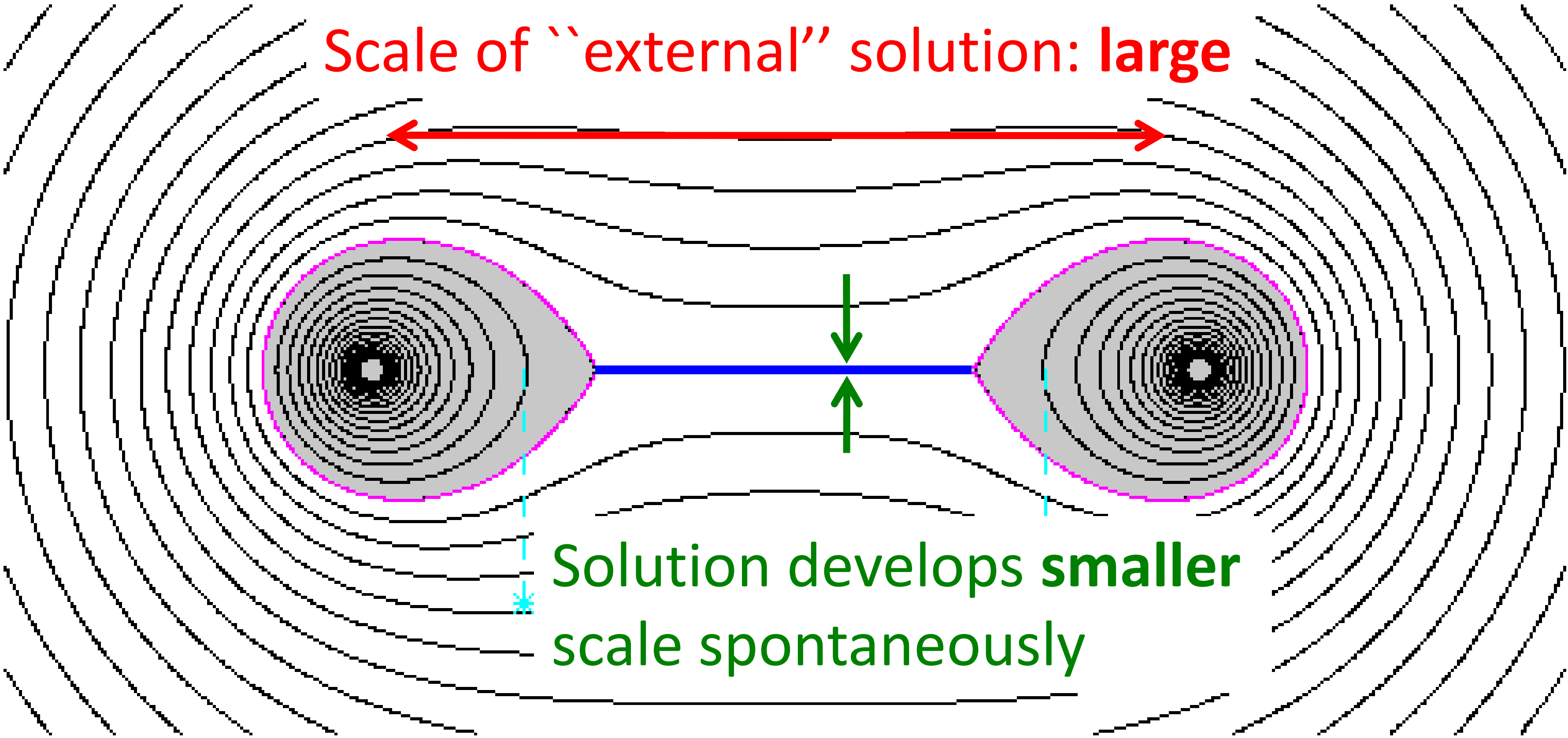
		Earth's magnetotail	Solar corona
External scale	L	10^8 m	10^8 m
Collisionless skin depth	c/ω_{pe}	10^4 m	0.1 m
ion skin depth	c/ω_{pi}	10^6 m	10 m
Ion gyro-radius	ρ_i	10^5 m	0.1 m

All small :
E+v B=0

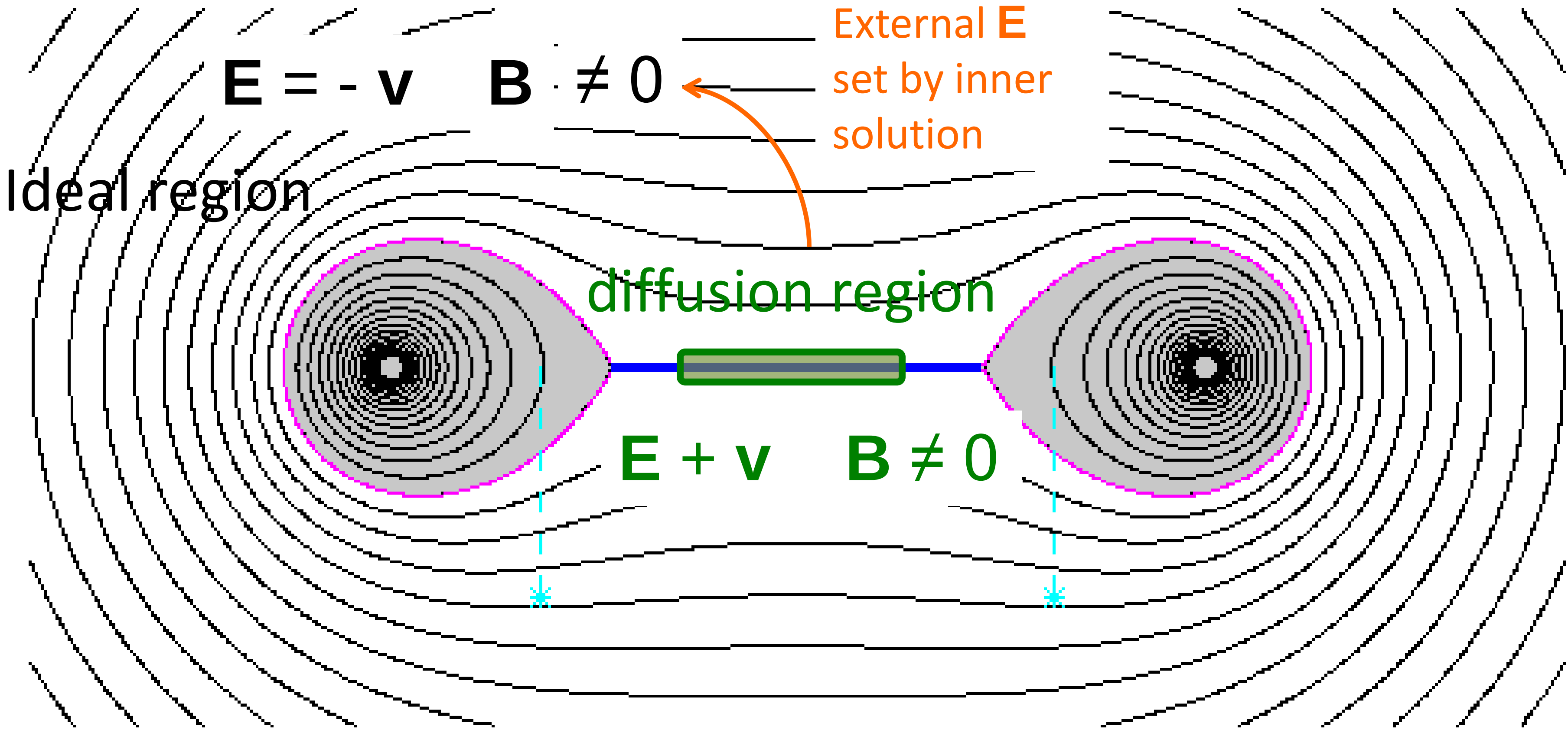




Good vs. Fair conductance

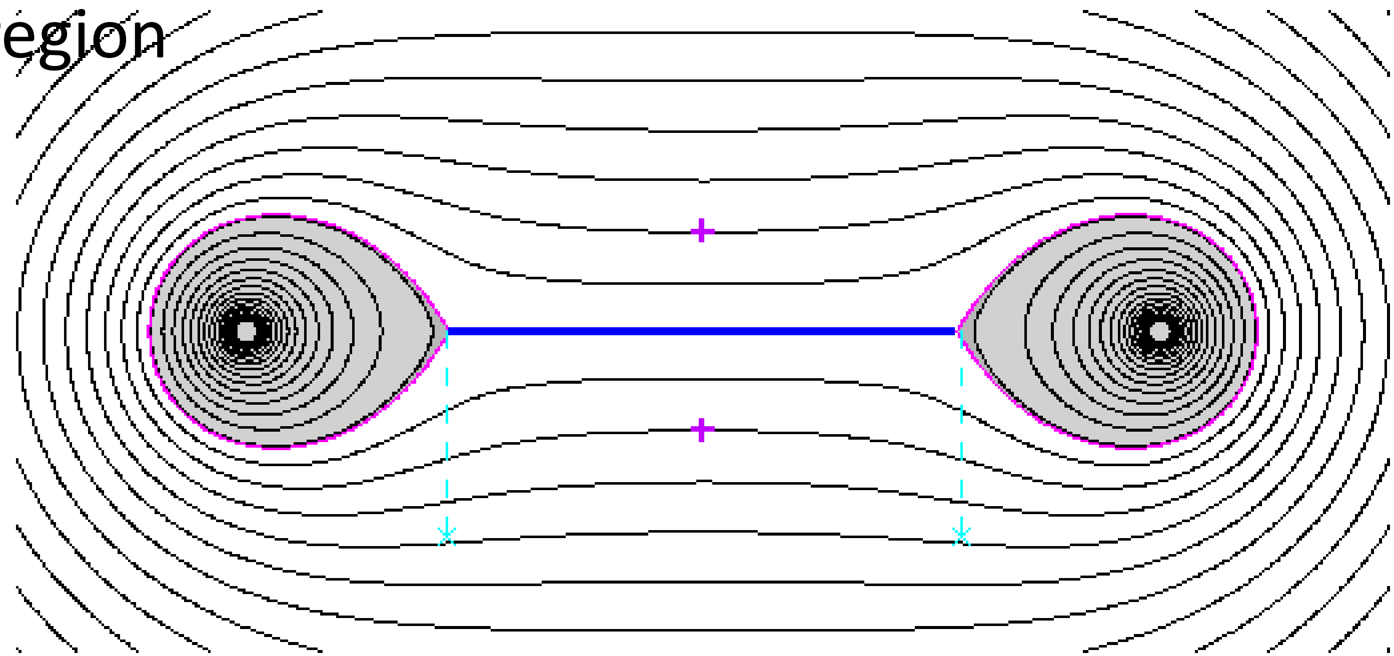


Good vs. Fair conductance

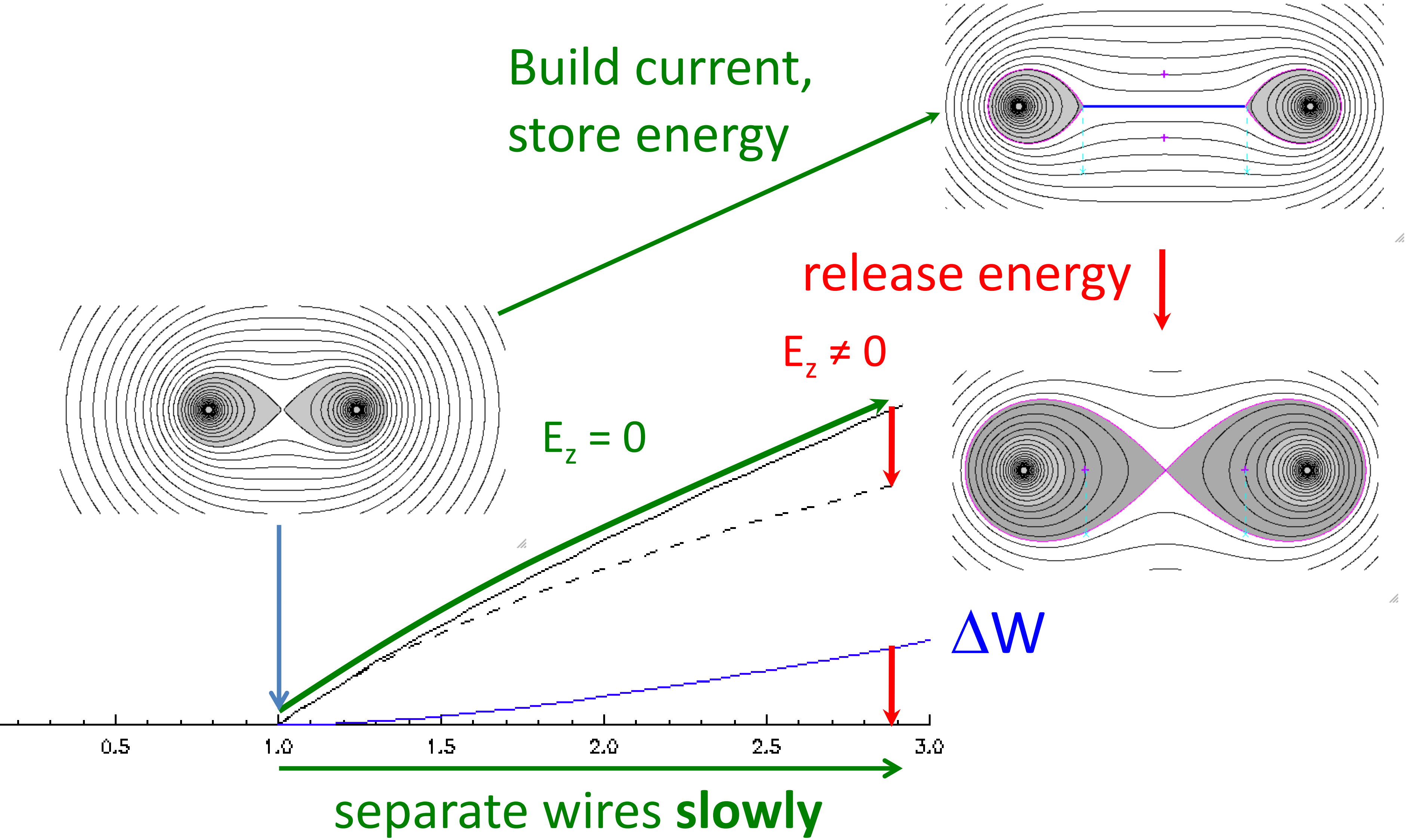


$\mathbf{E} = -\,\mathbf{v} \quad \mathbf{B} \neq 0$

Ideal region



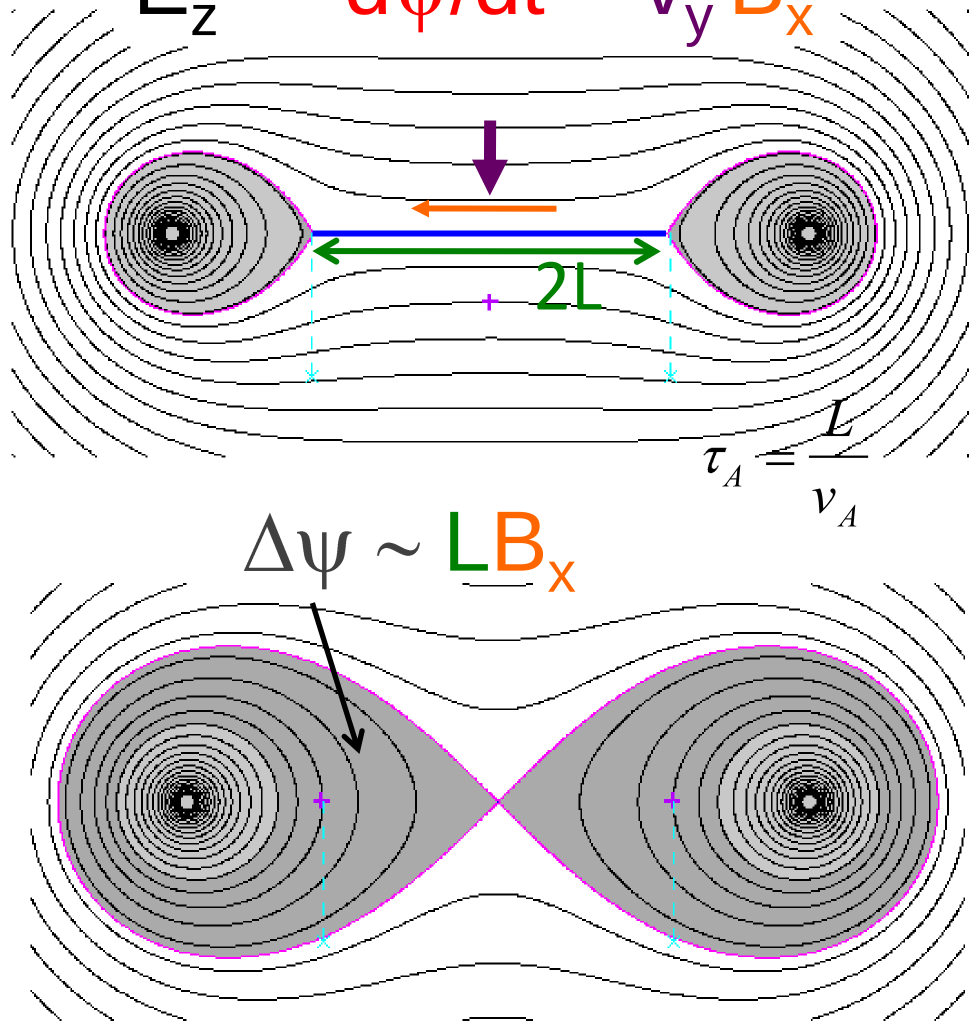
Work done on wires



$$\Delta W = \int I_{cs} E \, dt = -\int I_{cs} \frac{d\psi}{dt} dt = -\int I_{cs} d\psi$$

Spontaneous & irreversible

$$E_z = -d\psi/dt = v_y B_x$$



Total reconnection time

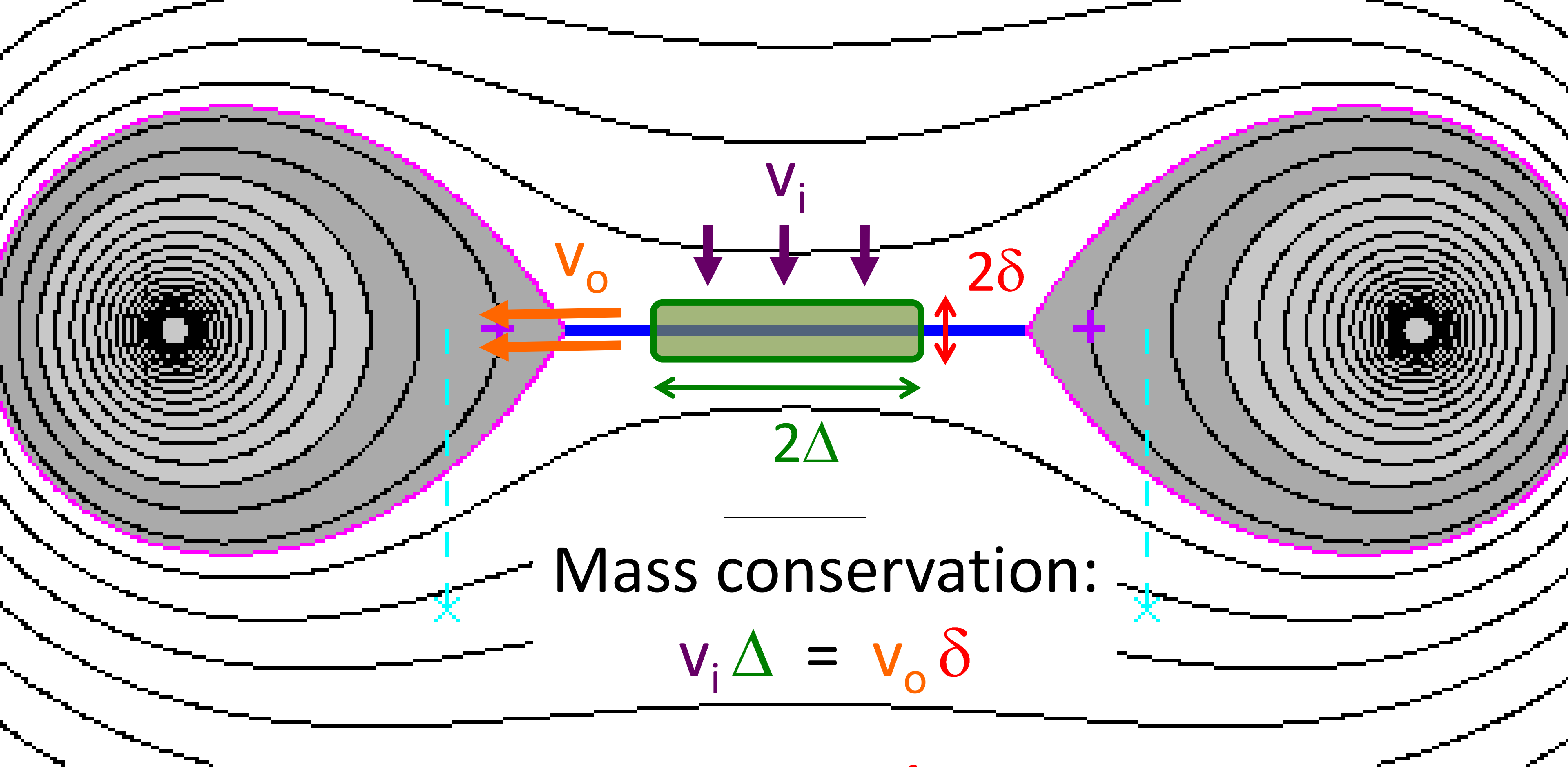
$$\tau_{\text{rx}} = \frac{\Delta\psi}{d\psi/dt} = \frac{LB_x}{v_y B_x}$$

$$\tau_{\text{rx}} = \frac{L}{v_i} = \frac{v_A}{v_i}\,\tau_A = \frac{\tau_A}{M_{Ai}}$$

$$M_{Ai} = \frac{v_i}{v_A}$$

$M_{Ai} << 1$: Slow reconnection

$M_{Ai} \sim 1$: Fast reconnection



Mass conservation:

$$v_i \Delta = v_o \delta$$

$$M_{Ai} = \frac{v_i}{v_A} = \frac{v_o}{v_A}\frac{\delta}{\Delta} \approx \frac{\delta}{\Delta}$$

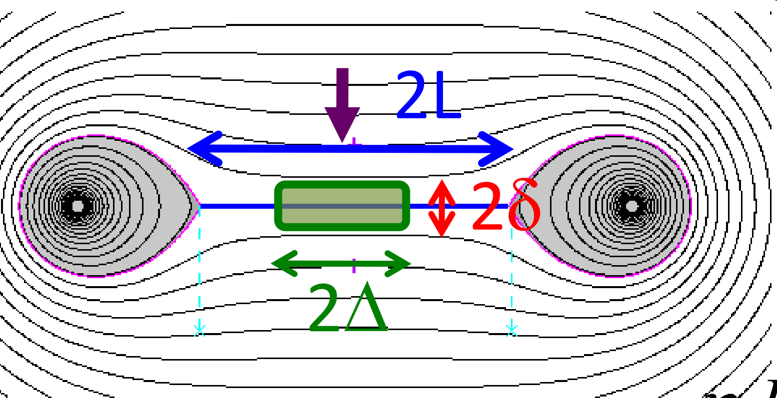
~1 (the $\frac{v_o}{v_A}$ term is struck through)

Aspect ratio of diffusion region

Classic Ohm's law

$$\mathbf{E}+\mathbf{v}_i\times\mathbf{B}=-\cancel{m_e\frac{d\mathbf{v}_e}{dt}}-\cancel{\frac{1}{en_e}\nabla\cdot\mathbf{P}_e}+\cancel{\frac{1}{en_e}\mathbf{J}\times\mathbf{B}}+\eta_e\mathbf{J}$$

Ideal region $E_z \approx v_i B_i$



diffusion region $E_z \approx \eta_e J_z \sim \dfrac{\eta_e B_i}{\mu_0 \delta}$

$$v_i \sim \frac{\eta_e}{\mu_0\delta}$$

advection balances diffusion $\quad Rm=\dfrac{v_i\mu_0\delta}{\eta_e}\sim 1$

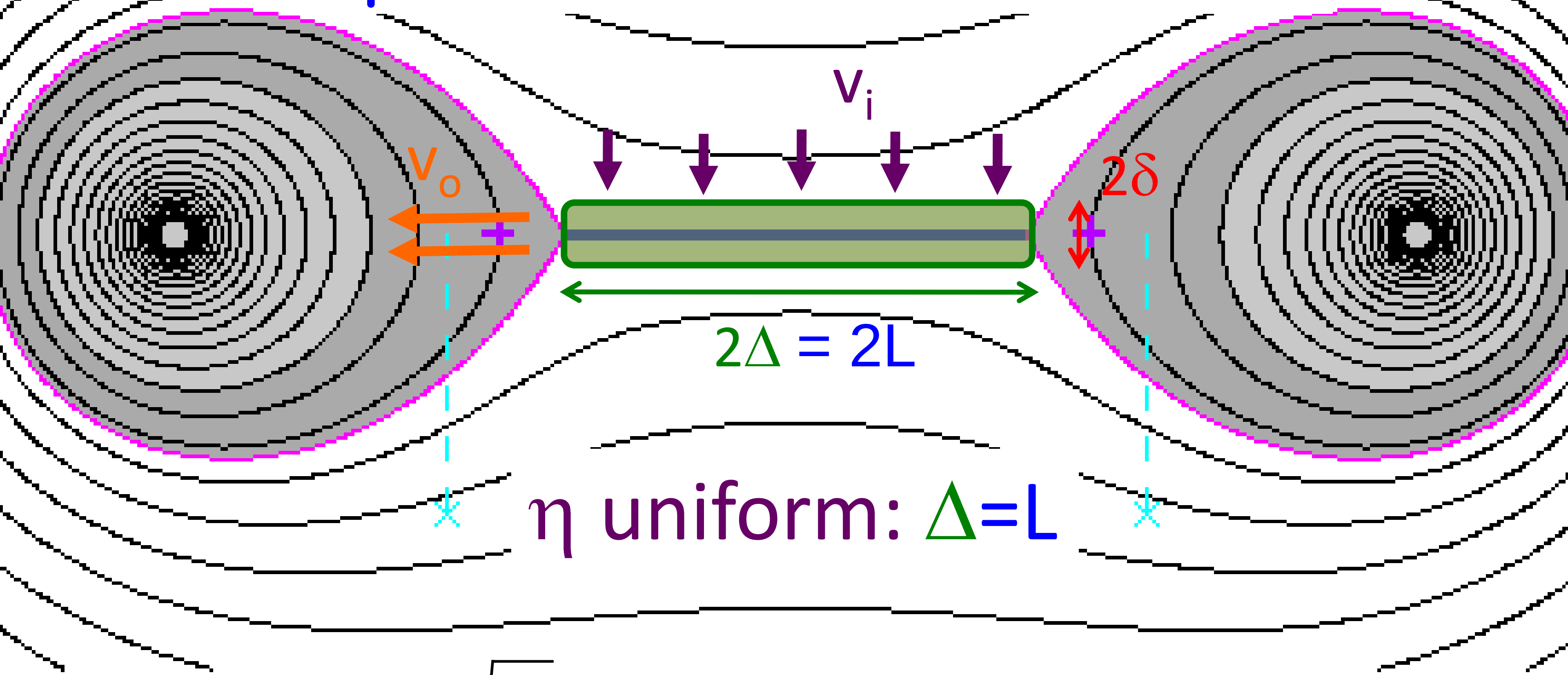
$$M_{Ai}=\frac{v_i}{v_A}=\frac{\eta_e}{v_A\mu_0\delta}=\frac{\eta_e}{\mu_0 L v_A}\frac{L}{\delta}=Lu^{-1}\frac{L}{\delta}$$

$$M_{Ai}=\frac{\delta}{\Delta}$$

from mass conservation

$$M_{Ai}=Lu^{-1/2}\sqrt{\frac{L}{\Delta}} \quad \& \quad \delta=Lu^{-1/2}\sqrt{L\Delta}$$

Special case: Sweet-Parker

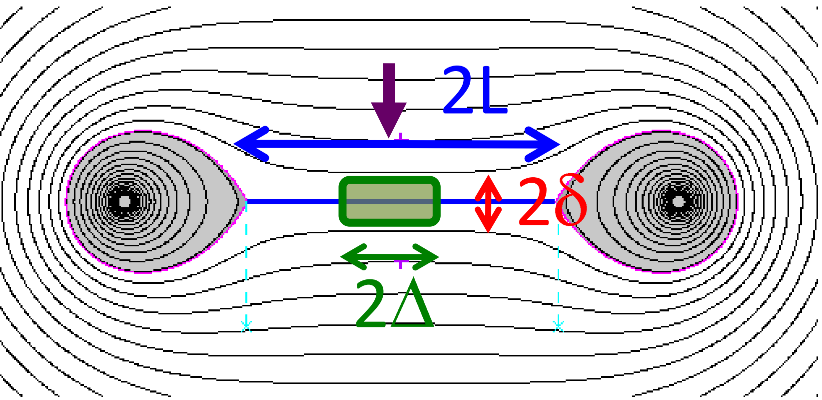


$$M_{Ai} = Lu^{-1/2}\sqrt{\frac{L}{\Delta}} = Lu^{-1/2} << 1$$ *v. slow reconnection*

$$\frac{L}{\delta} = Lu^{1/2}\sqrt{\frac{L}{\Delta}} = Lu^{1/2} >> 1$$ *v. great aspect ratio*

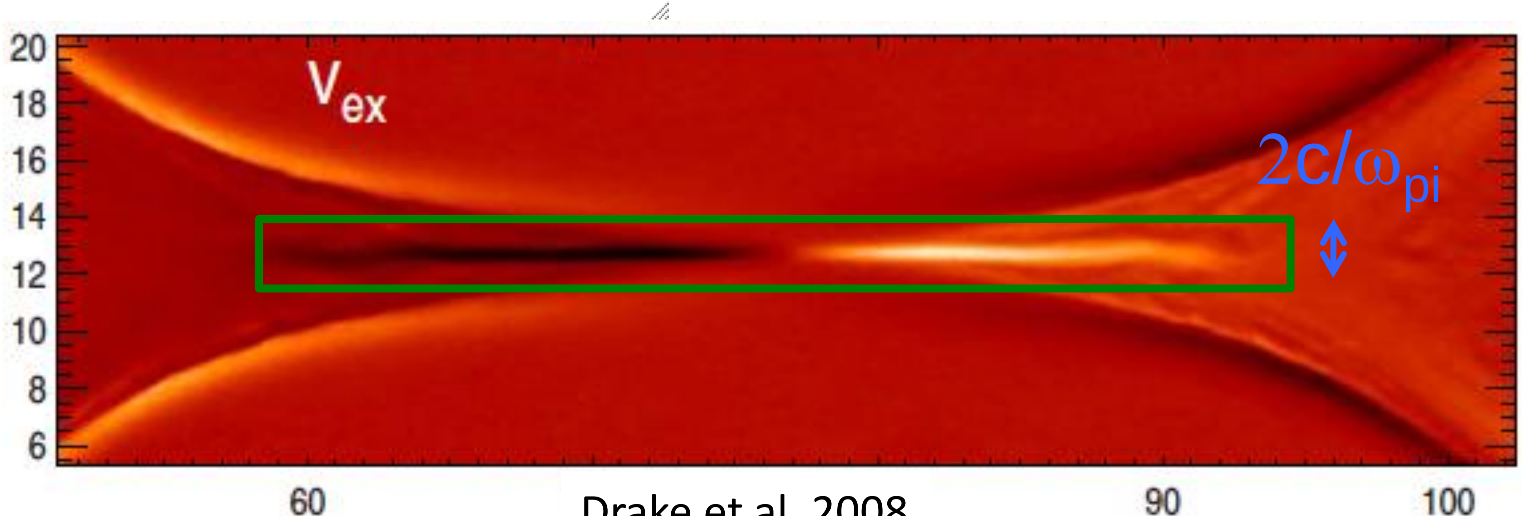
Collisionless reconnection

$$\mathbf{E} + \mathbf{v}_i \times \mathbf{B} = -m_e \frac{d\mathbf{v}_e}{dt} - \cancel{\frac{1}{en_e}\nabla \cdot \overleftrightarrow{P}_e} + \frac{1}{en_e}\mathbf{J} \times \mathbf{B} + \cancel{\eta_e \mathbf{J}}$$

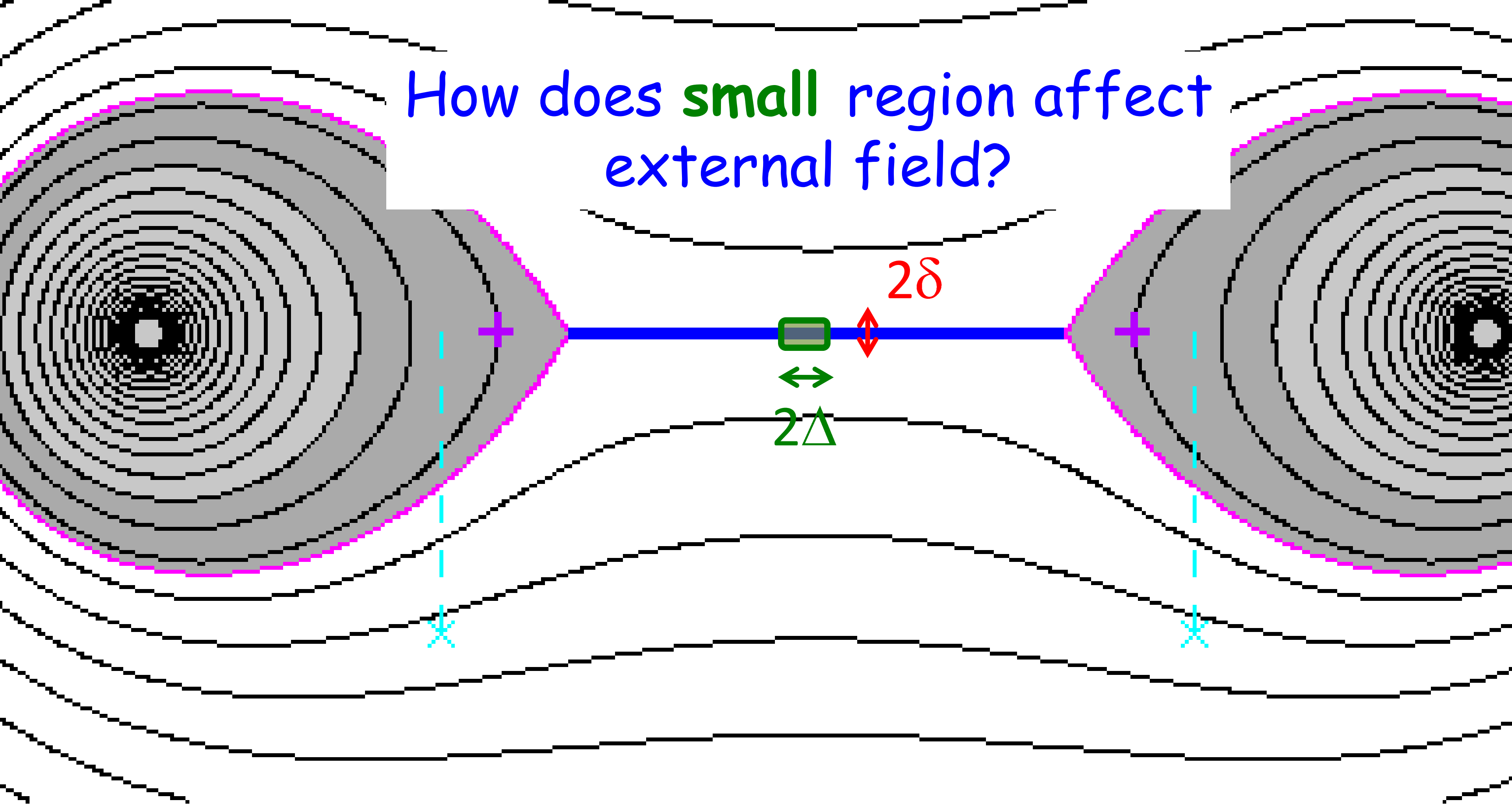


$\Delta \sim \delta \sim c/\omega_{pi}$ from Hall term

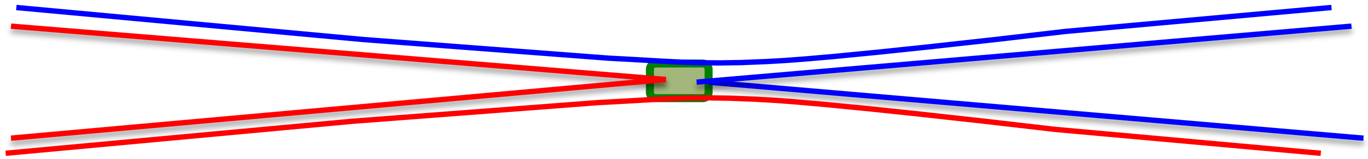
$$M_{Ai} = \frac{\delta}{\Delta} \sim 1$$ from mass conservation



Drake et al. 2008



It creates bent field lines…

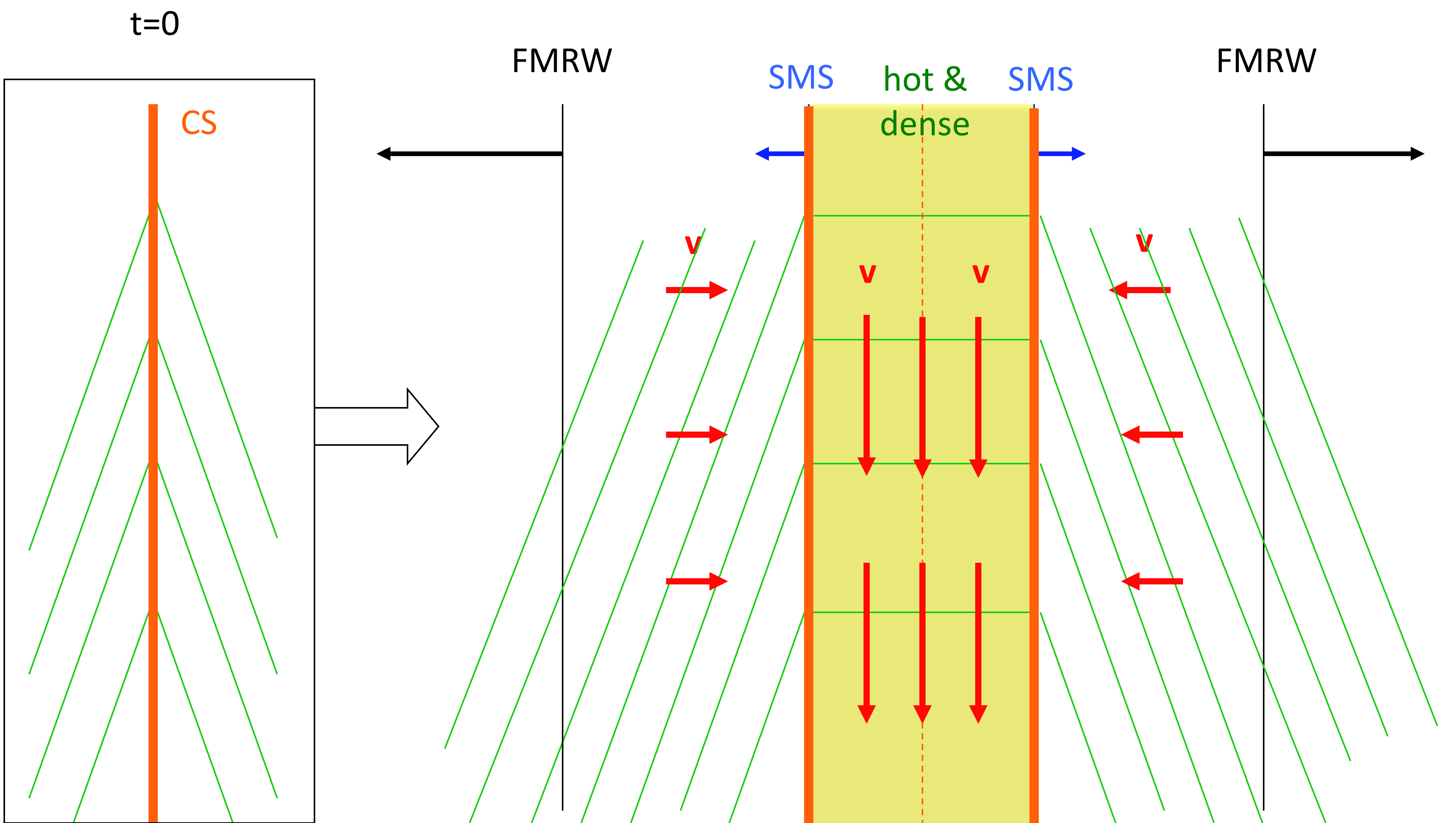


… what next?

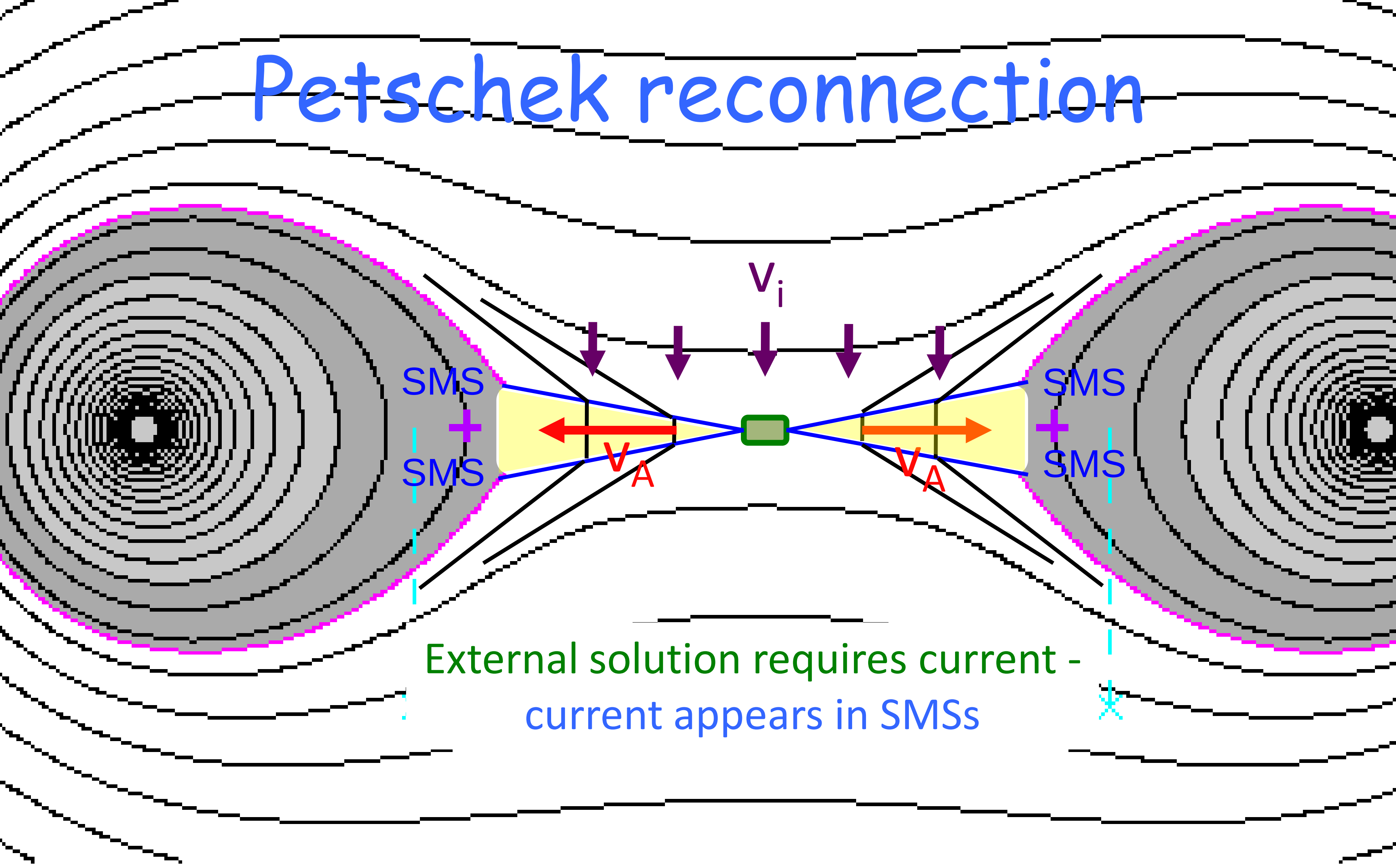
Response to bend

Lin & Lee 1994

Riemann problem for 1D current sheet (CS)



Petschek reconnection

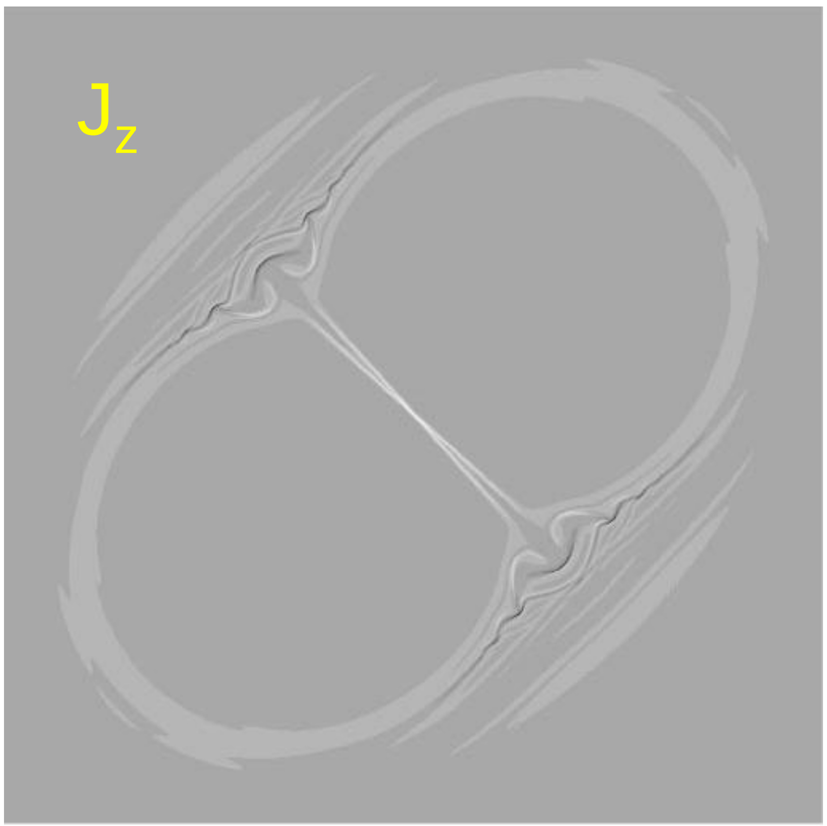


External solution requires current - current appears in SMSs

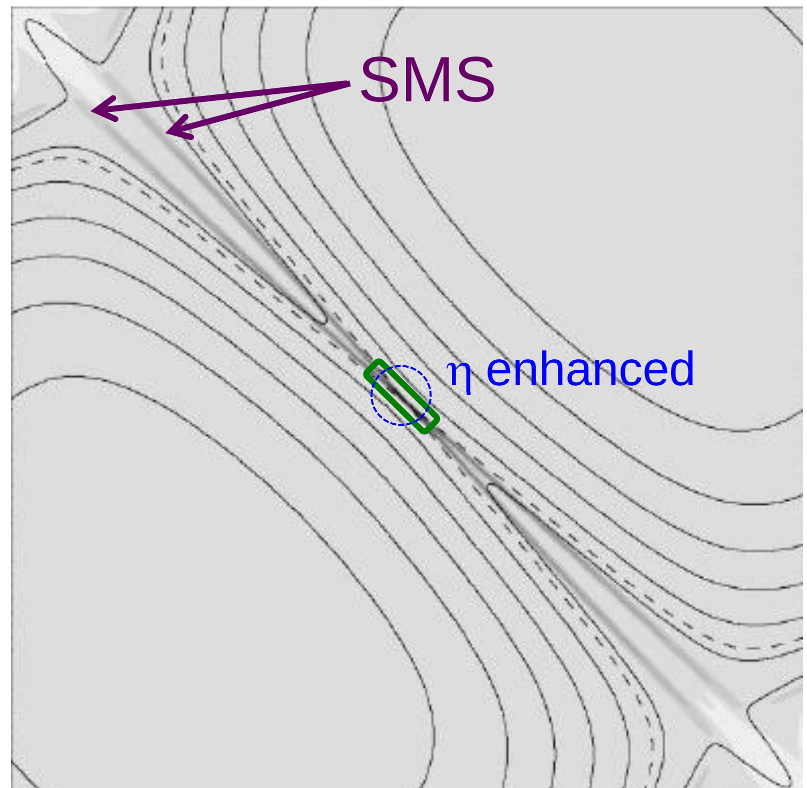
Q: Will resistivity always result in slow (Sweet-Parker) reconnection?

A: Yes, if η is uniform in space…

But not, when $\eta(\mathbf{x})$ is locally enhanced*



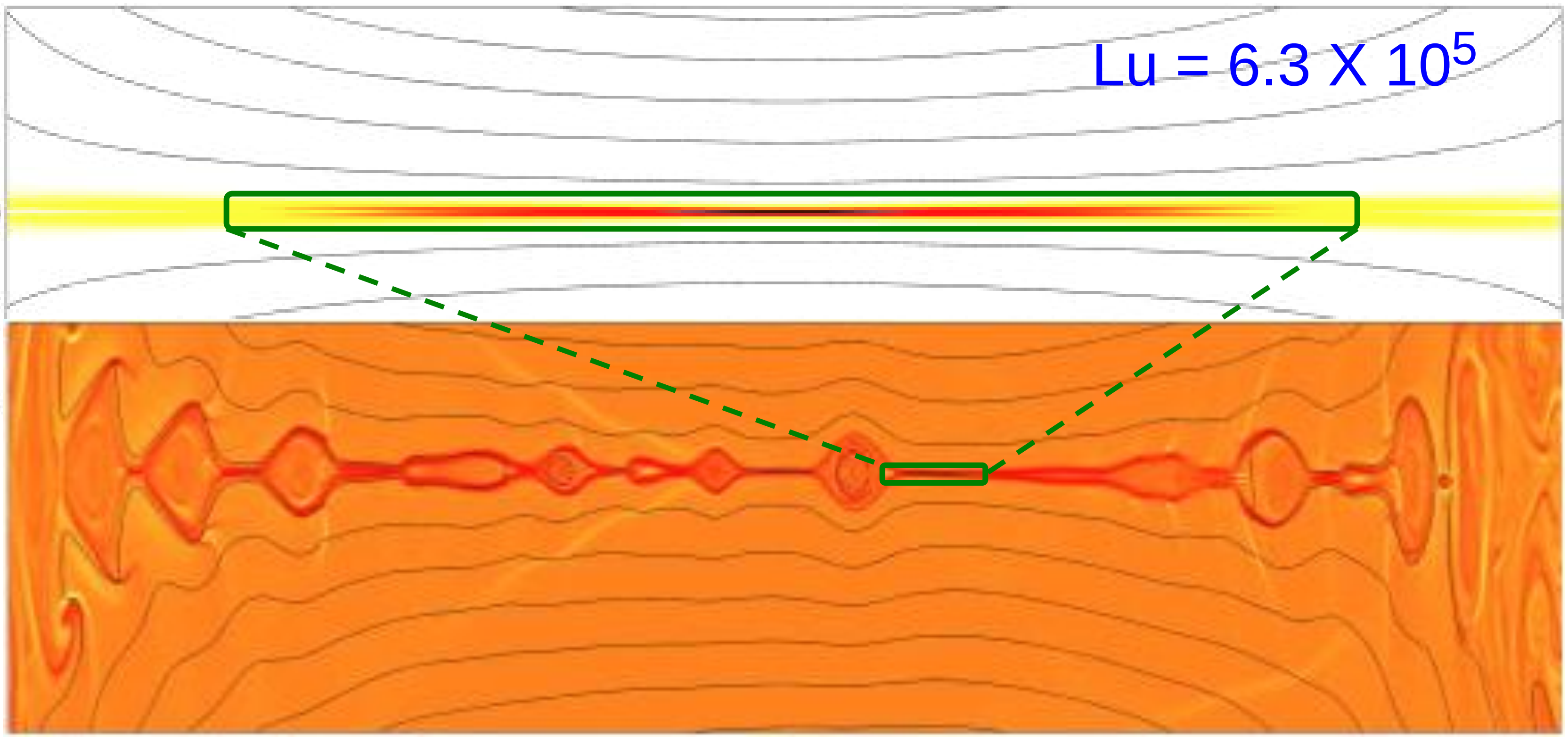
Biskamp & Scwartz 2001



*as by micro-instability

Q: Is slowly reconnecting sheet stable?

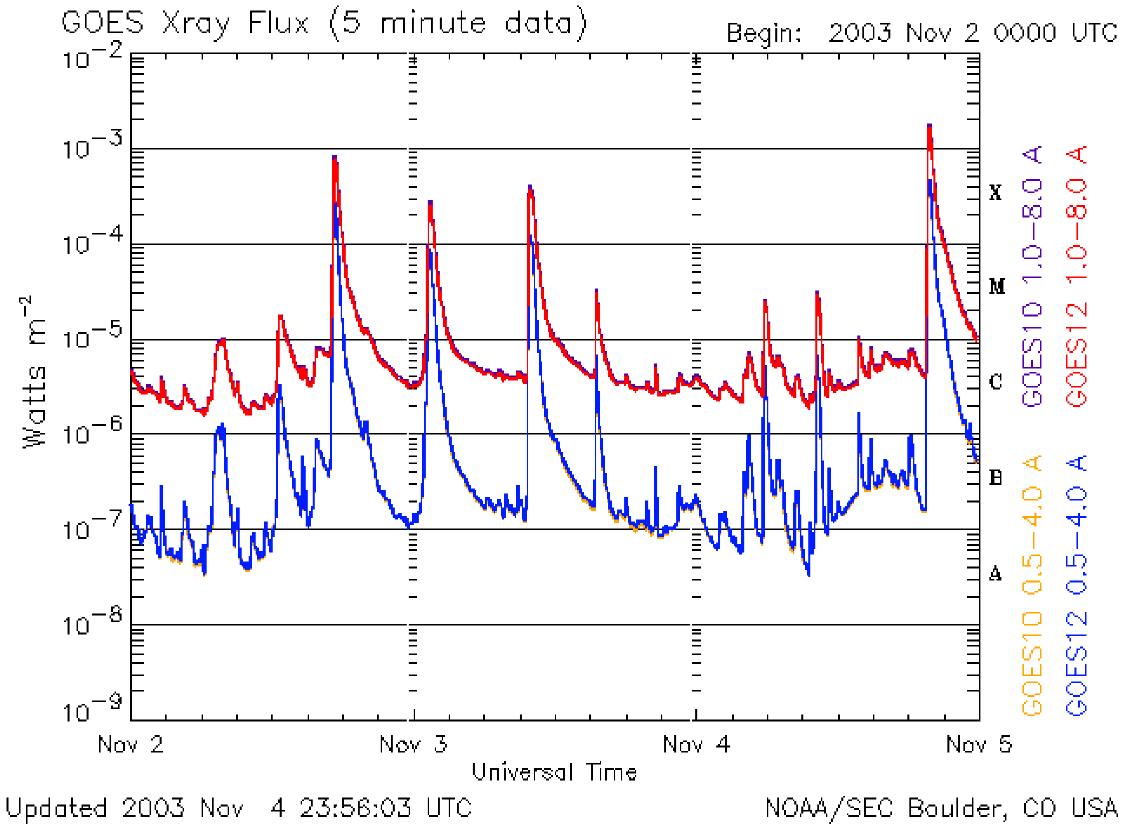
A: No. Subject to resistive instability: tearing mode

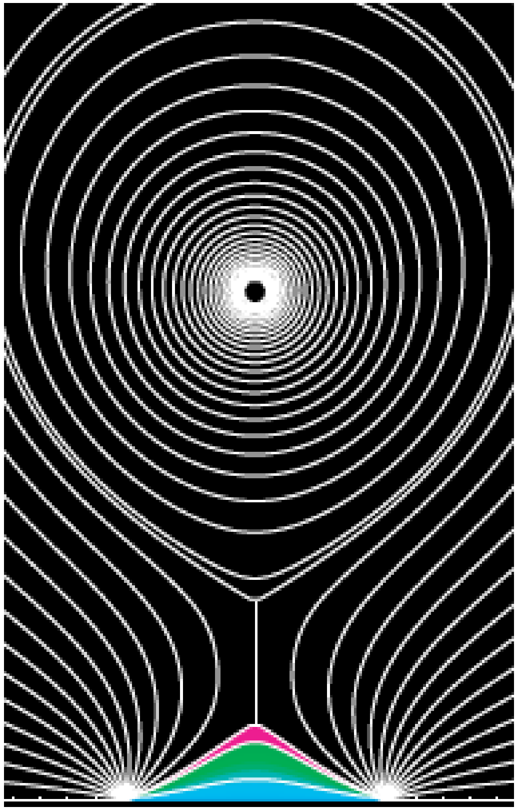


Bhattacharjee *et al.* 2009 (vertical scale is expanded)

- Solution becomes time-dependant
- Smaller diffusion region(s) develop w/ larger $\delta/\Delta \sim M_{Ai}$

Q: what triggers reconnection in the CS?

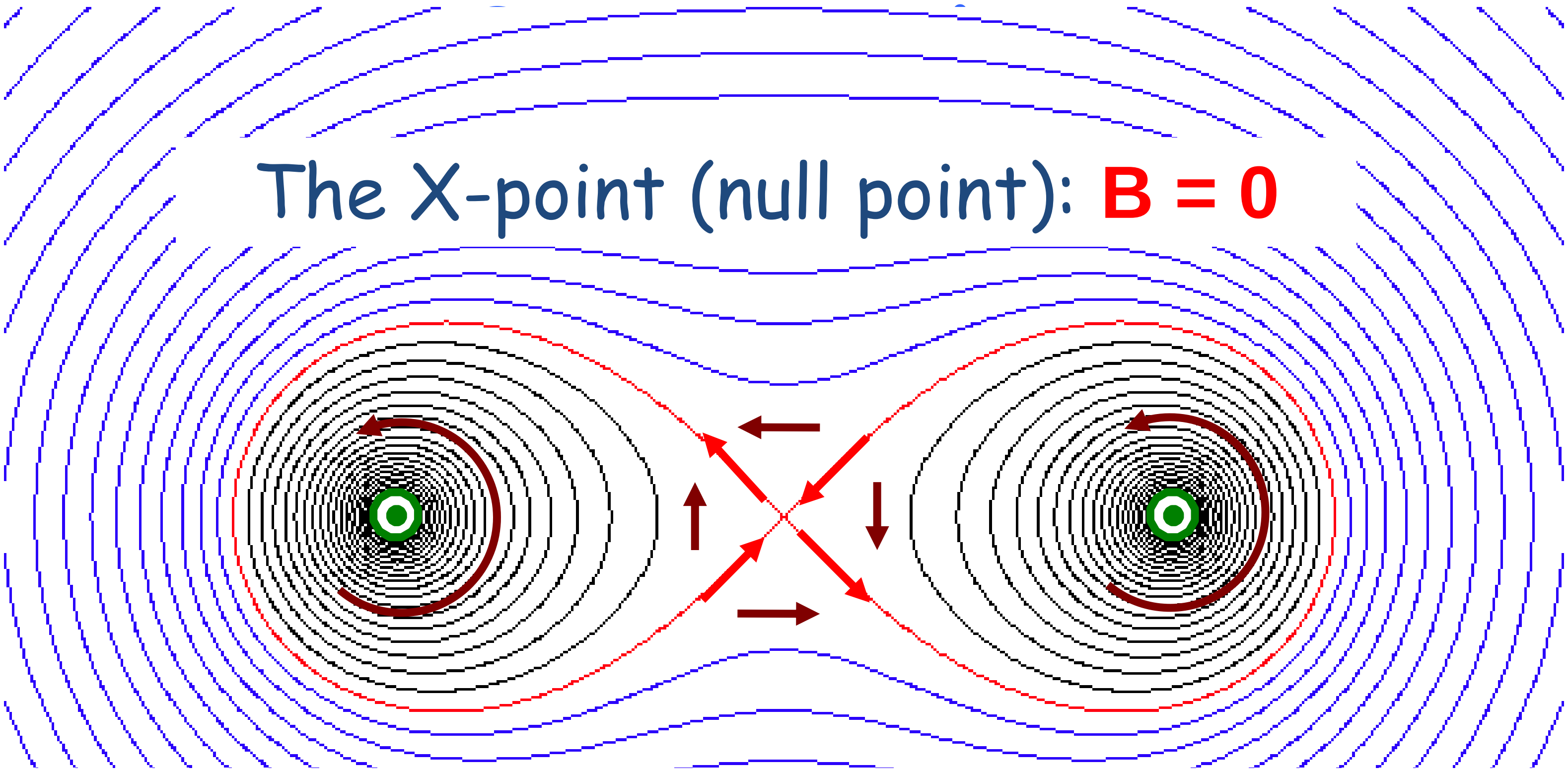




Reeves & Forbes 2005

?

The X-point (null point): B = 0



$$\mathbf{B}(x,y) = \begin{bmatrix} 0 & -B' \\ -B' & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \mathrm{L} \;\; = \frac{d}{ds}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x(s) \\ y(s) \end{bmatrix} = \varepsilon \begin{bmatrix} 1 \\ \pm 1 \end{bmatrix} e^{\mathrm{m}B's}$$

e-values = B’

approaches null as s ➔ ±∞

Bold new world: 3d

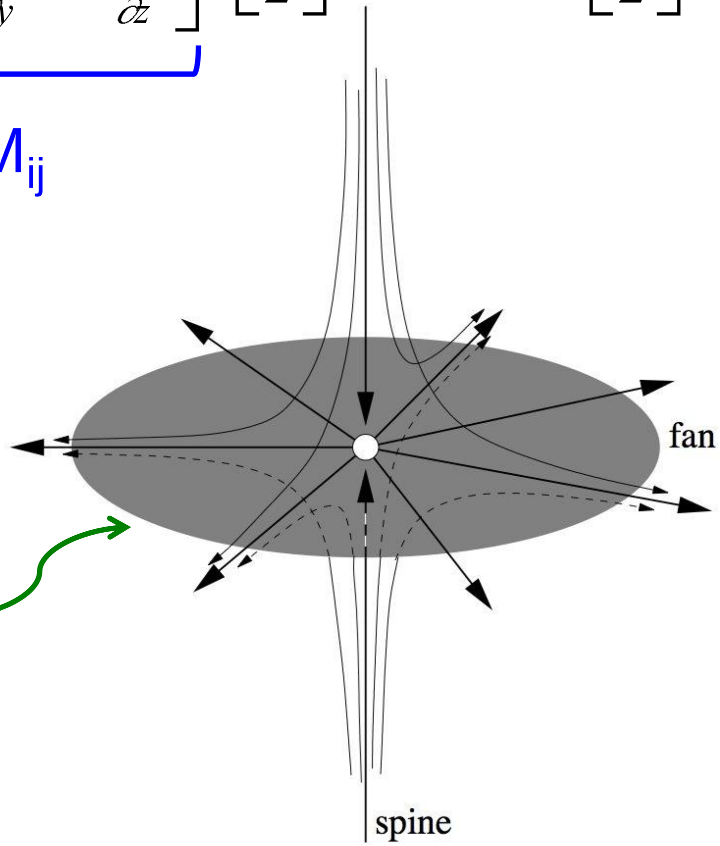
Null point (@ origin)

$$\mathbf{B}(0,0,0)=\mathbf{0} \quad \Rightarrow \quad \mathbf{B}(x,y,z)=\begin{bmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \mathrm{L} \quad = \frac{d}{ds}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

M_{ij}

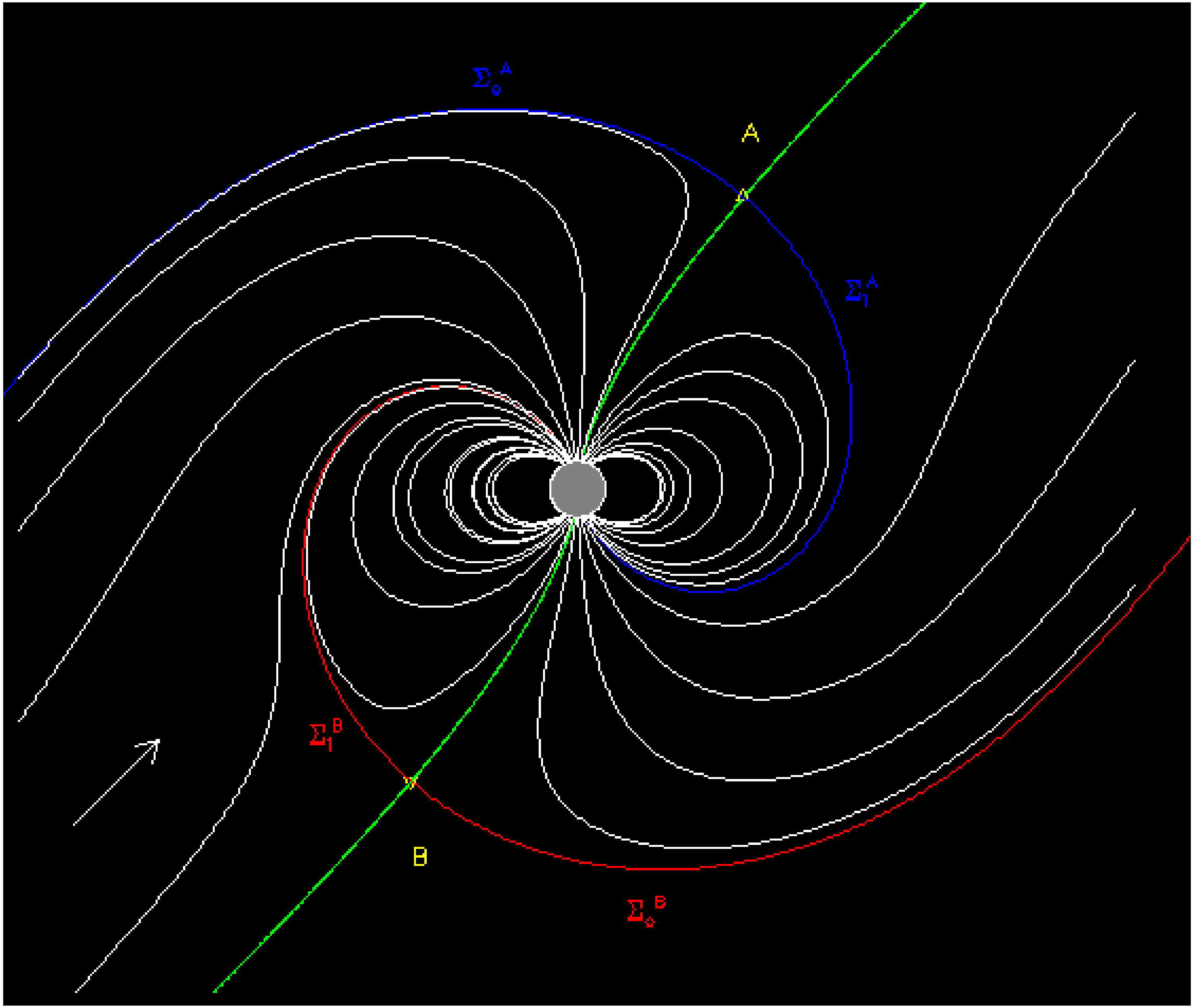
$$\nabla \cdot \mathbf{B} = 0 = \mathrm{Tr}(M_{ij}) = \sum \lambda_i$$

det(M) < 0: **"positive"** null point
2 pos. e-values (fan surface)
1 neg. e-value (spines)

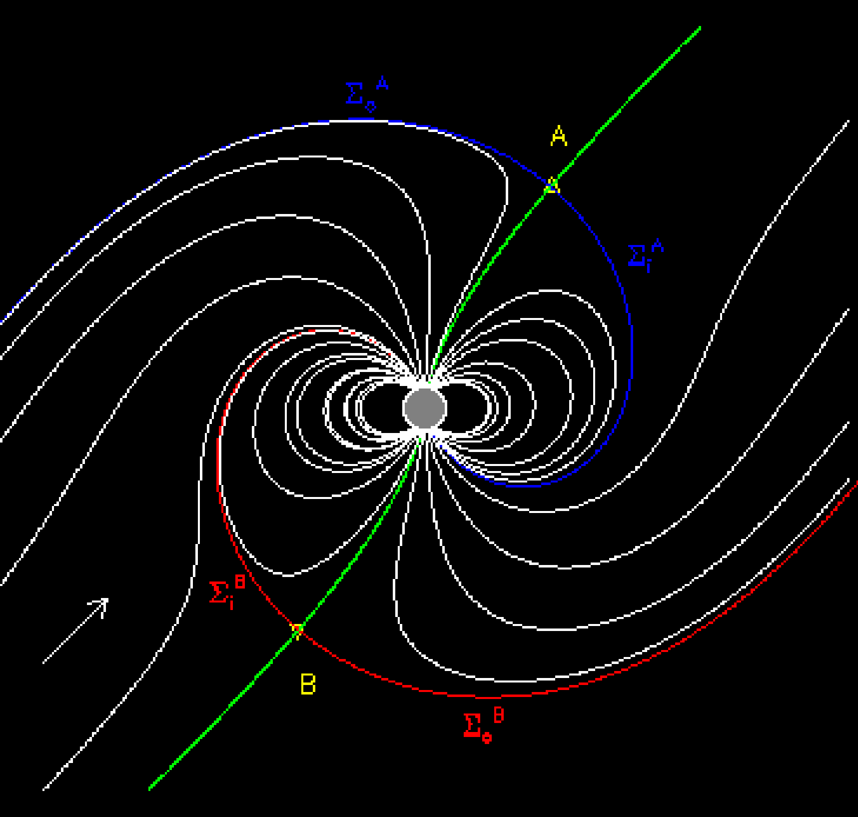


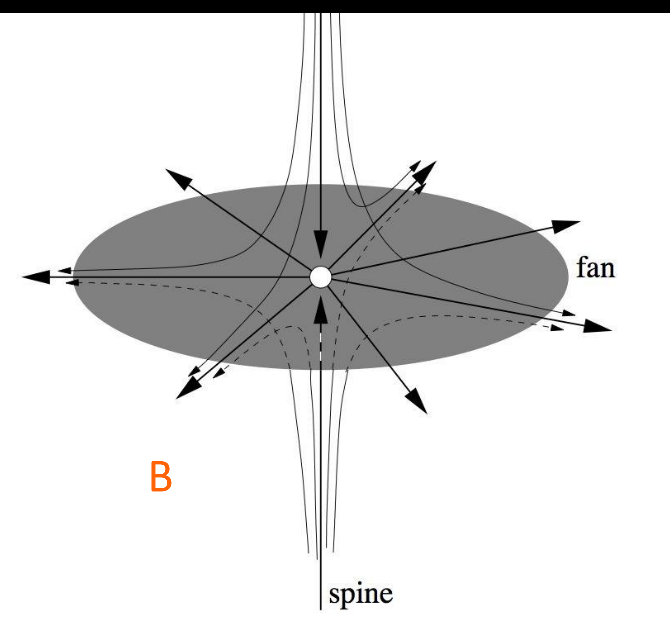
fan = separatrix

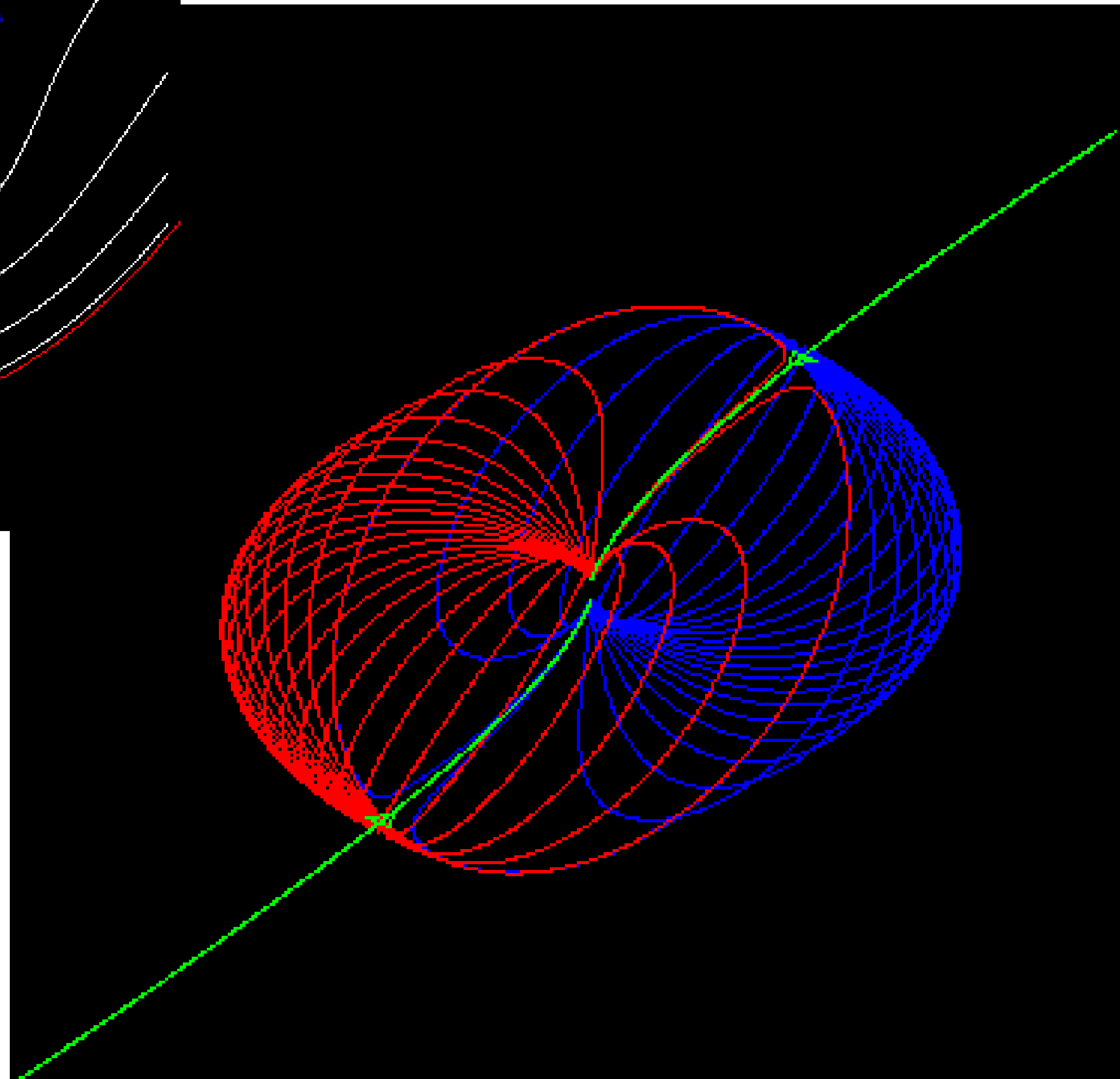
det(M) > 0: **"negative"** null point
(reverse all arrows)

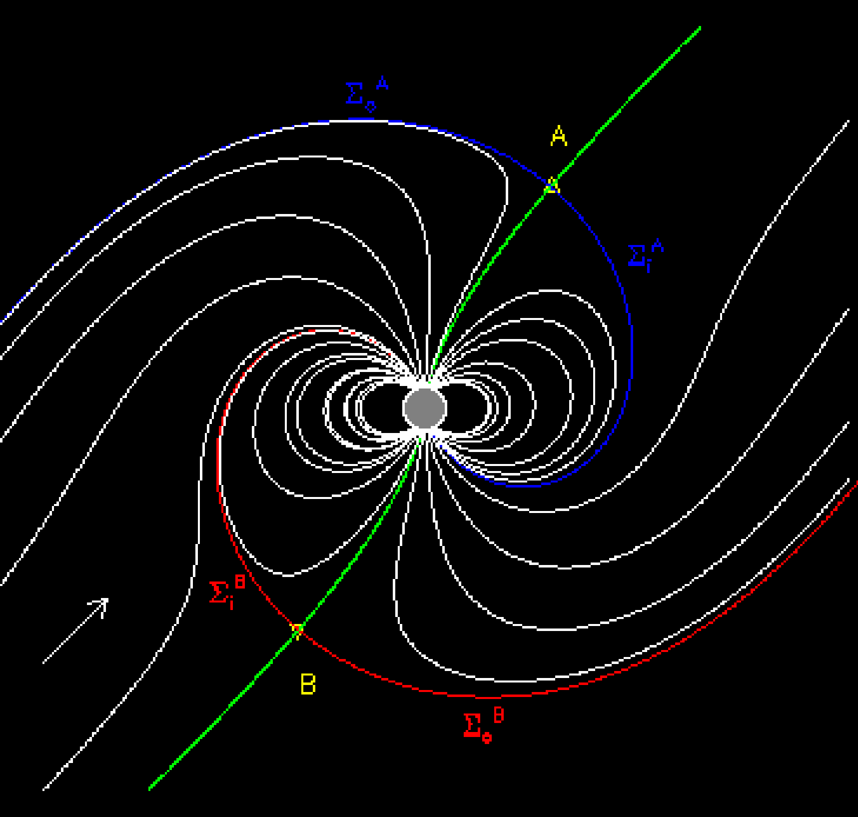


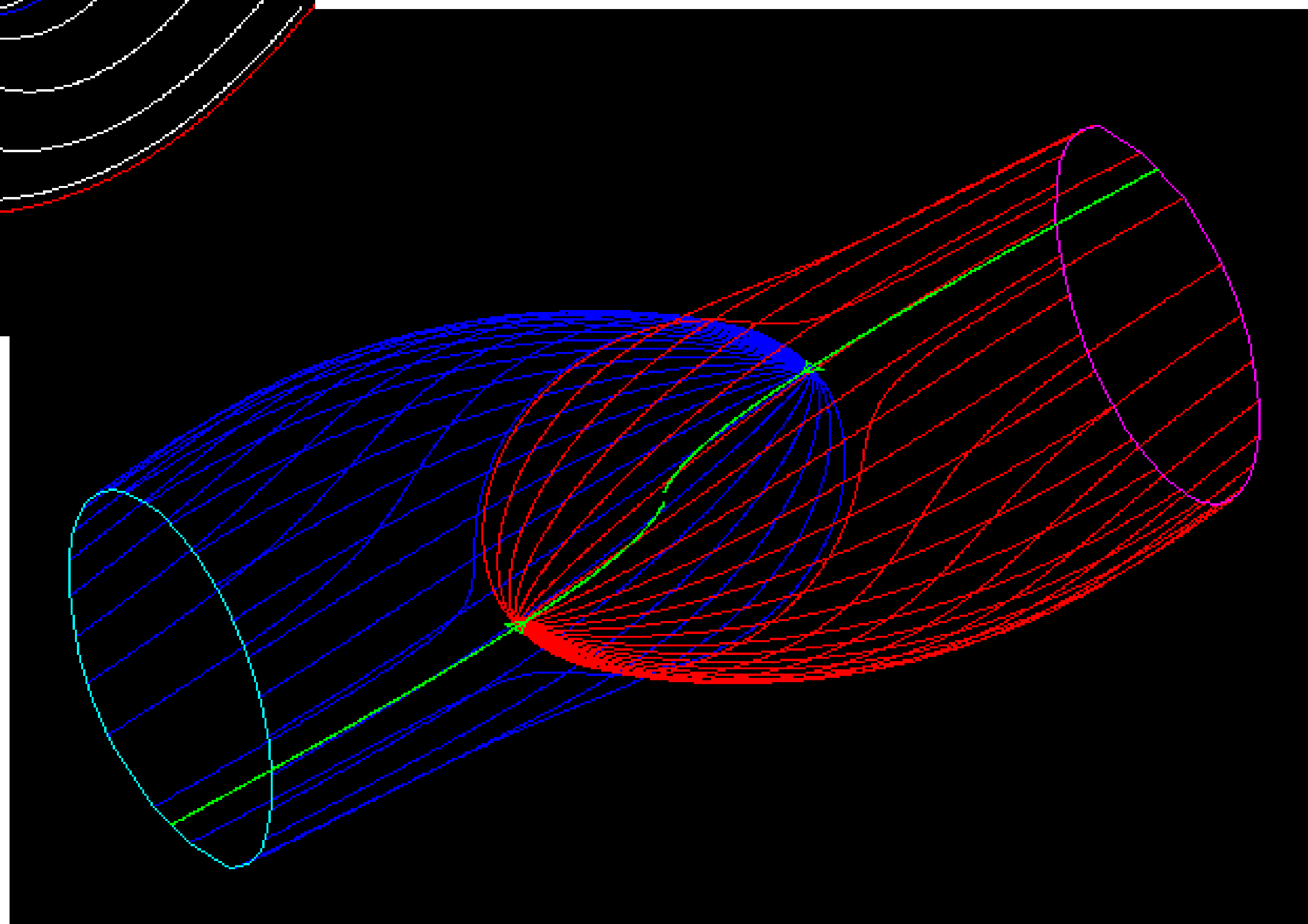
Σ_0^A
A
Σ_1^A
Σ_1^B
B
Σ_0^B


Σ_o^A
A
Σ_i^A
Σ_i^B
B
Σ_o^B

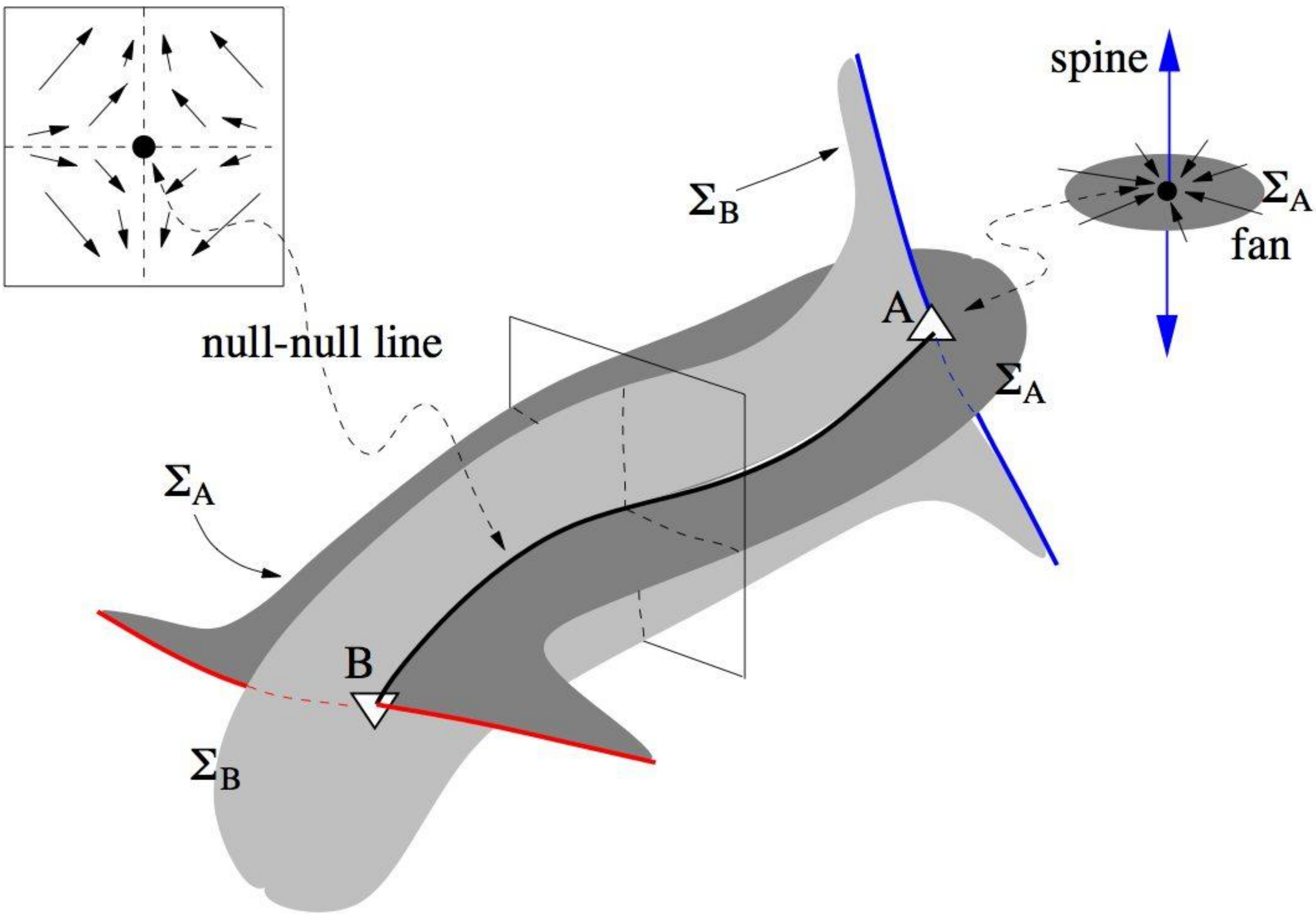

fan
B
spine

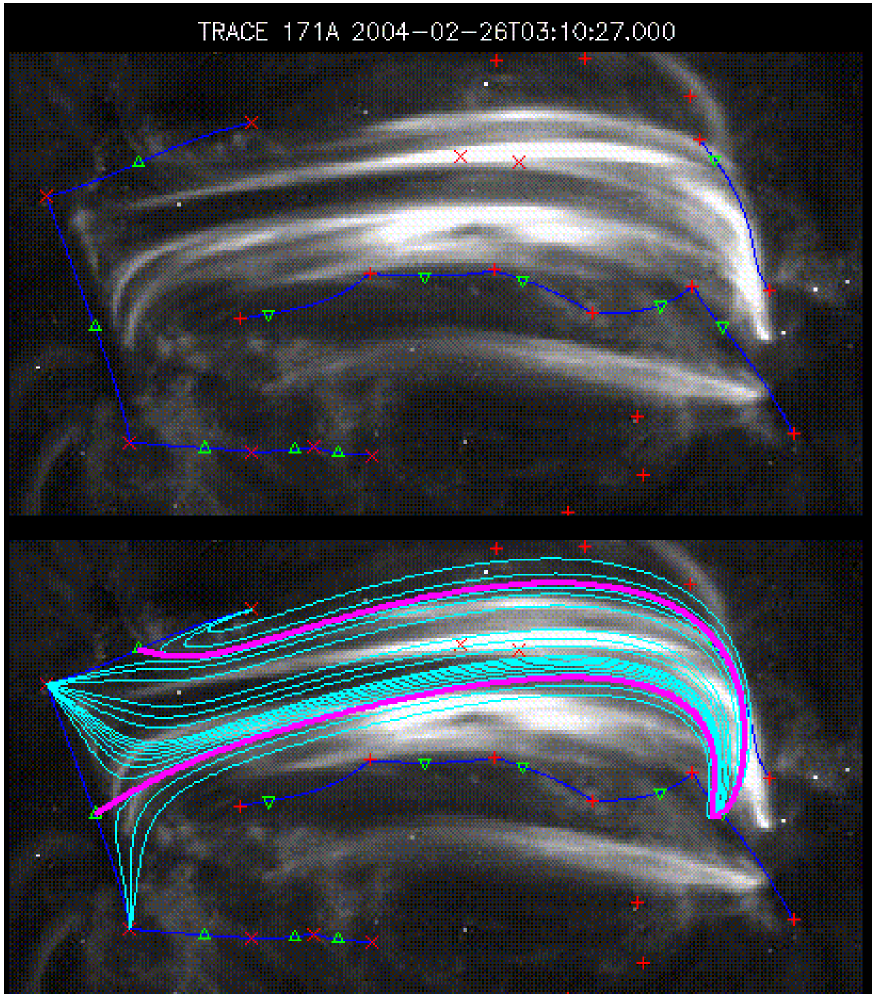


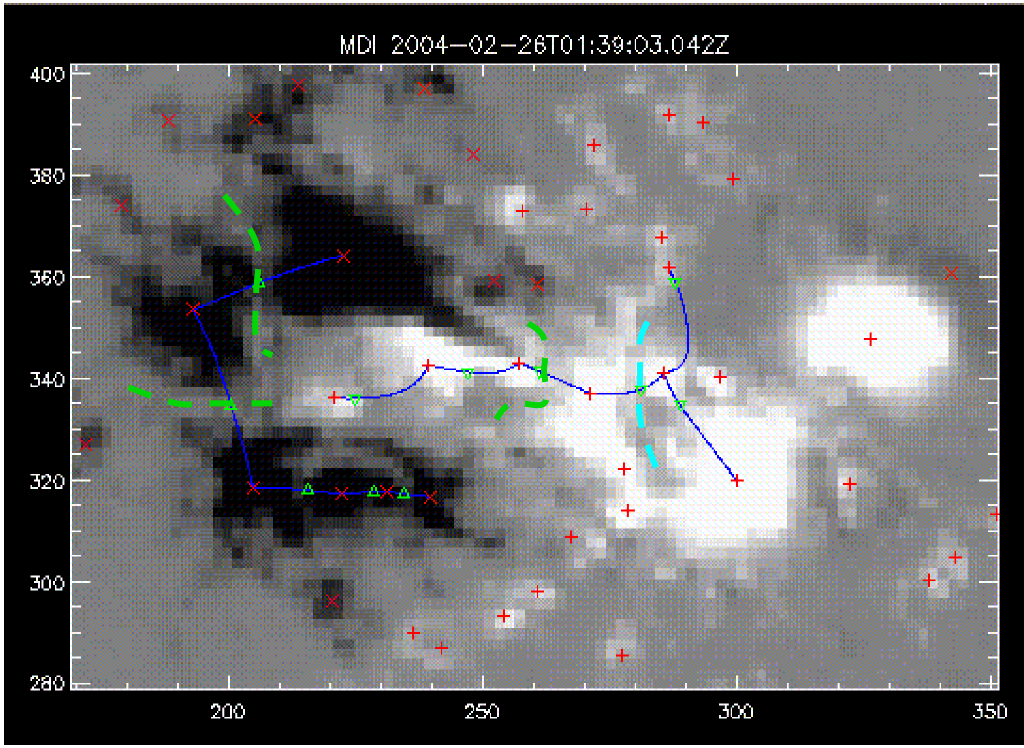

Σ_o^A
A
Σ_i^A
Σ_i^B
B
Σ_o^B

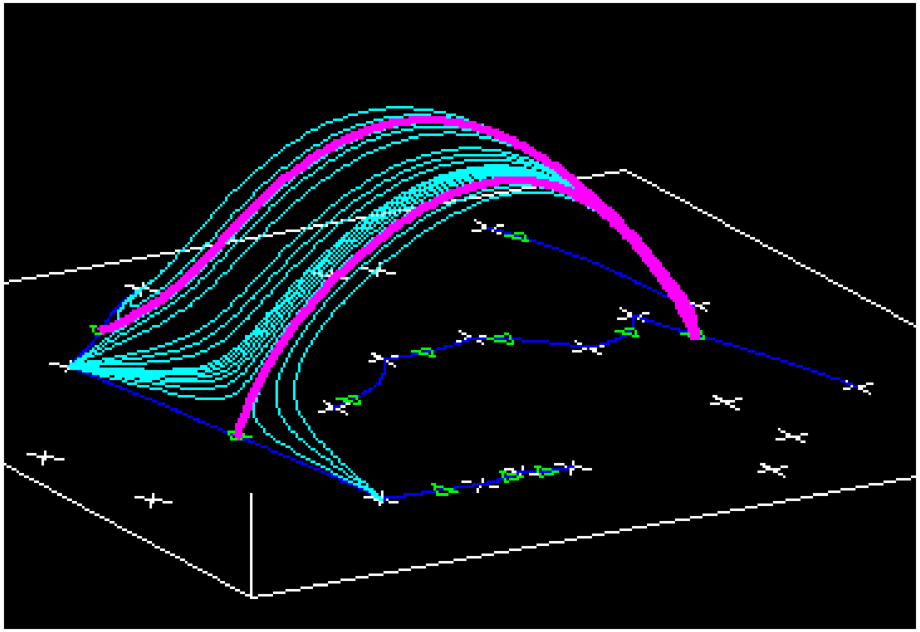


Fans & Null-Null Lines

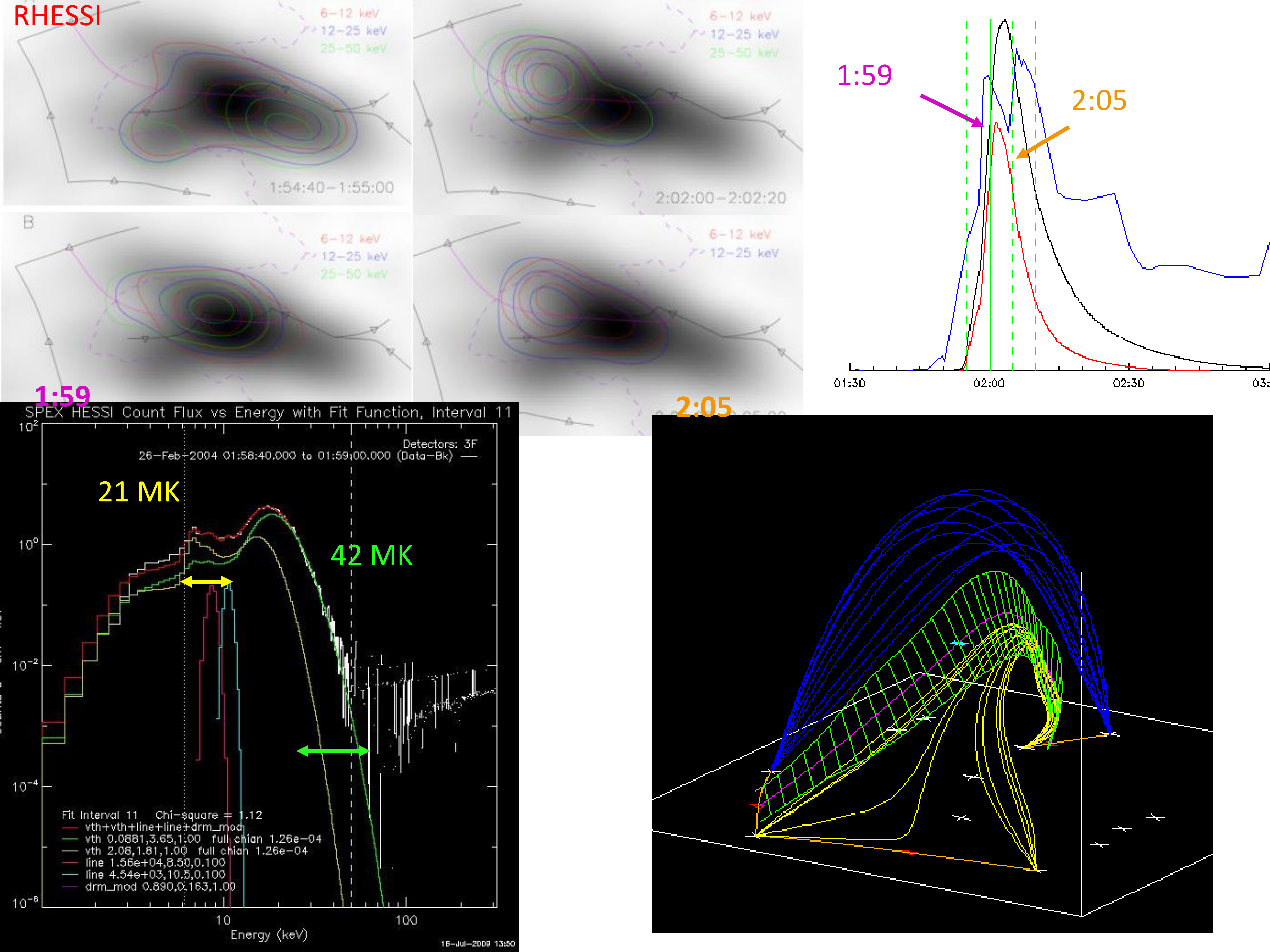




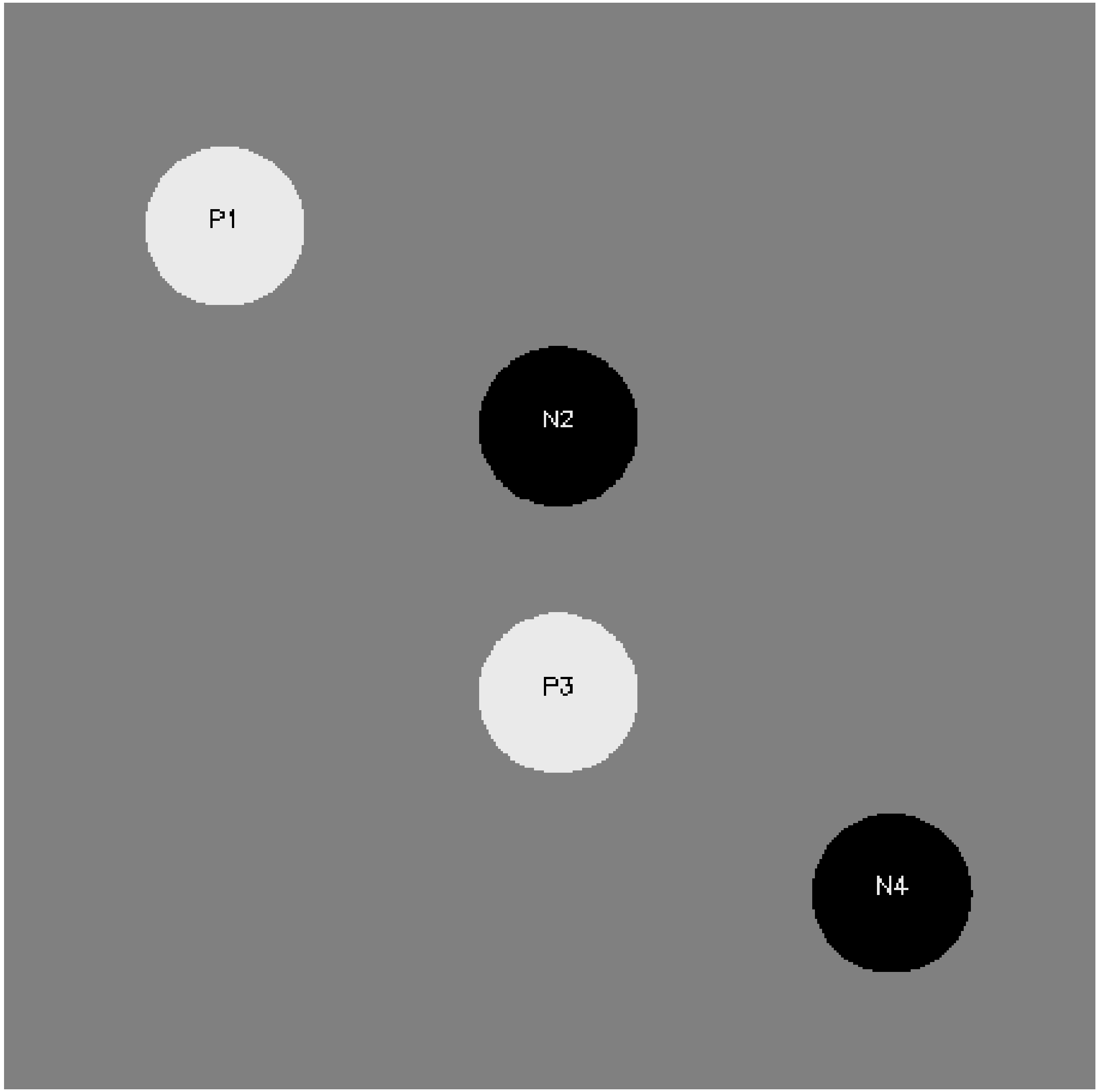


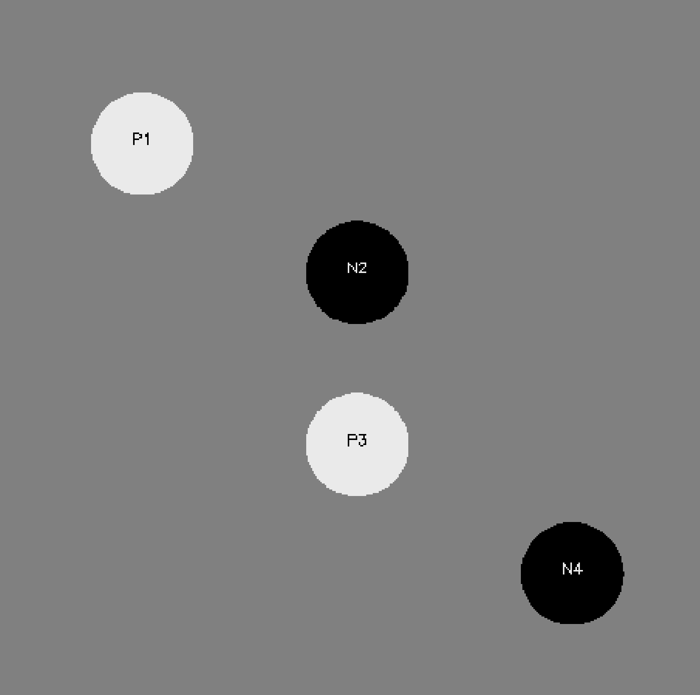


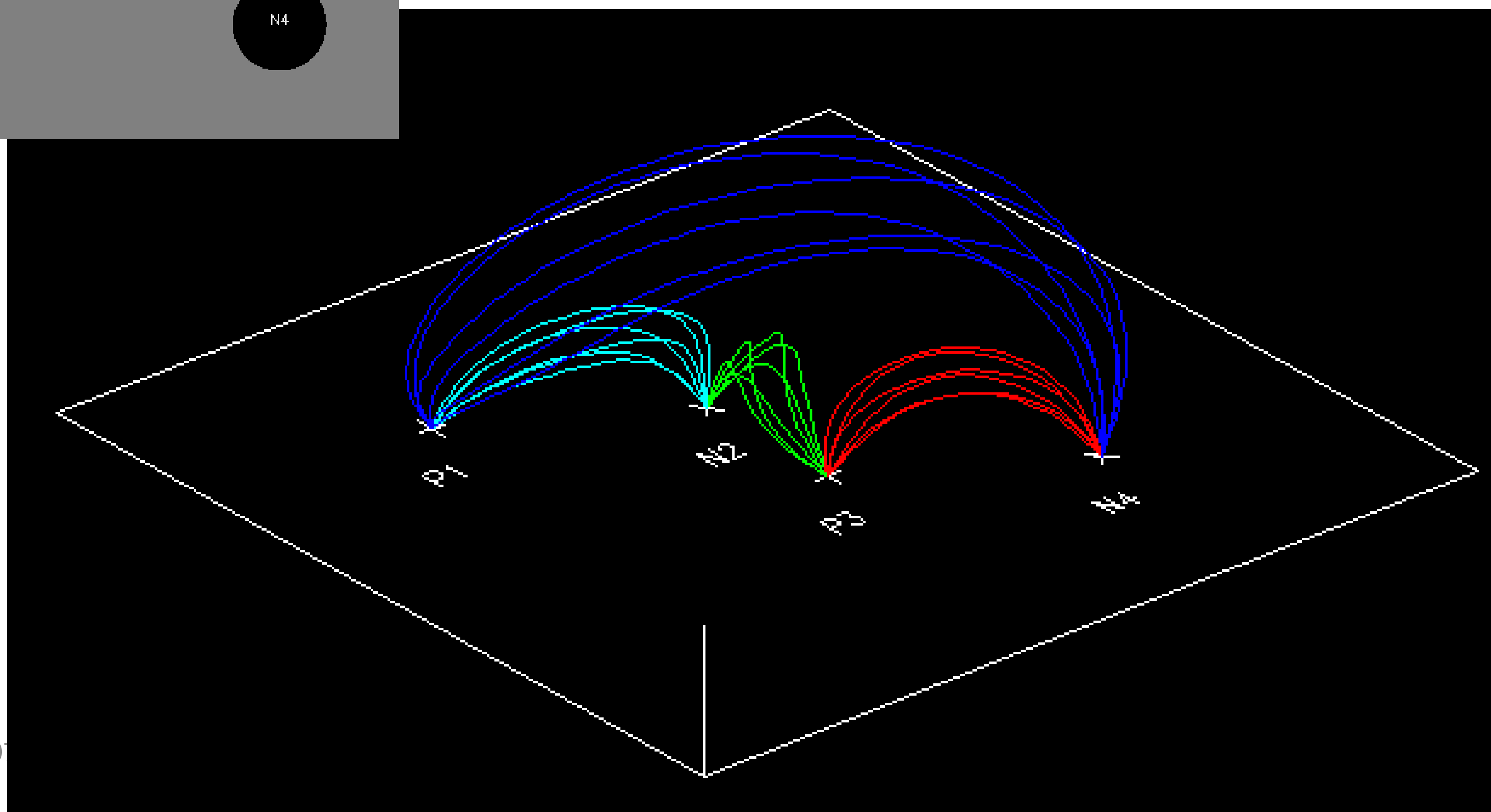
separatrix between new & old positive flux

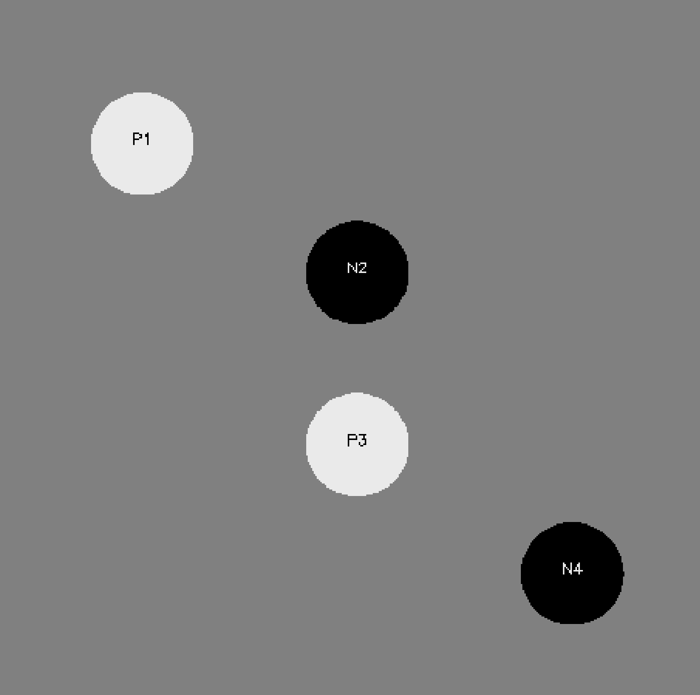


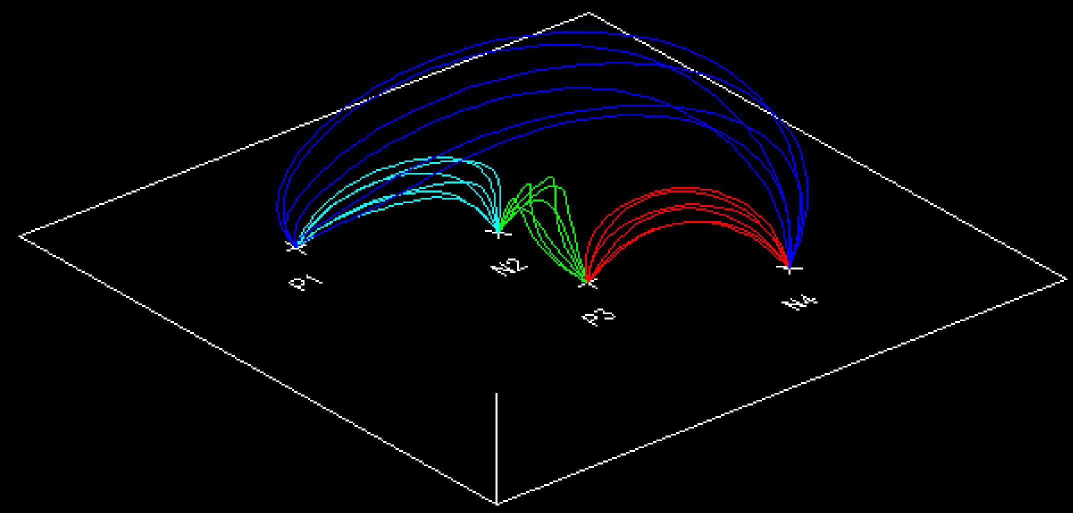
RHESSI
6–12 keV
12–25 keV
25–50 keV
1:54:40–1:55:00
2:02:00–2:02:20
B
1:59
2:05
01:30
02:00
02:30
SPEX HESSI Count Flux vs Energy with Fit Function, Interval 11
Detectors: 3F
26-Feb-2004 01:58:40.000 to 01:59:00.000 (Data-Bk)
21 MK
42 MK
Fit Interval 11 Chi-square = 1.12
vth+vth+line+line+drm_mod
vth 0.0881,3.65,1.00 full chian 1.26e-04
vth 2.08,1.81,1.00 full chian 1.26e-04
line 1.56e+04,8.50,0.100
line 4.54e+03,10.5,0.100
drm_mod 0.890,0.163,1.00
Energy (keV)
15-Jul-2009 13:50

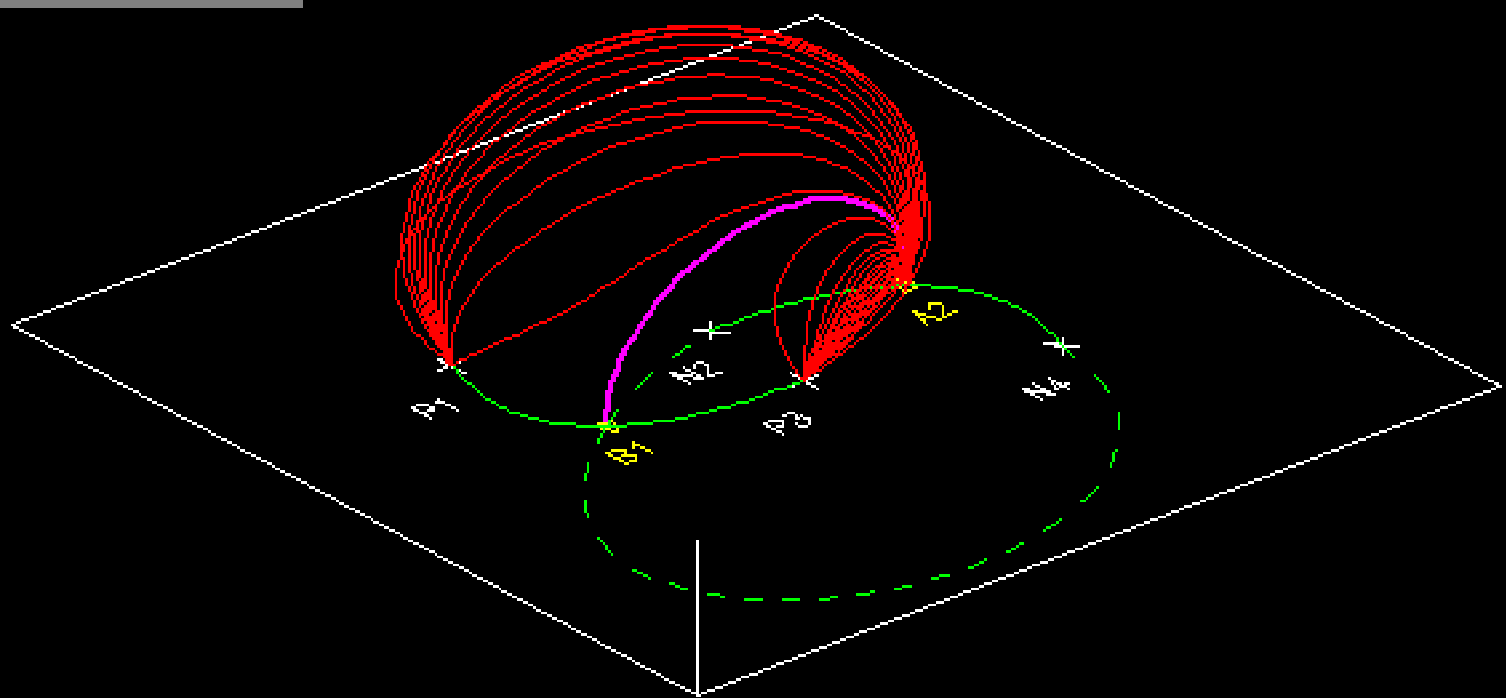

P1
N2
P3
N4

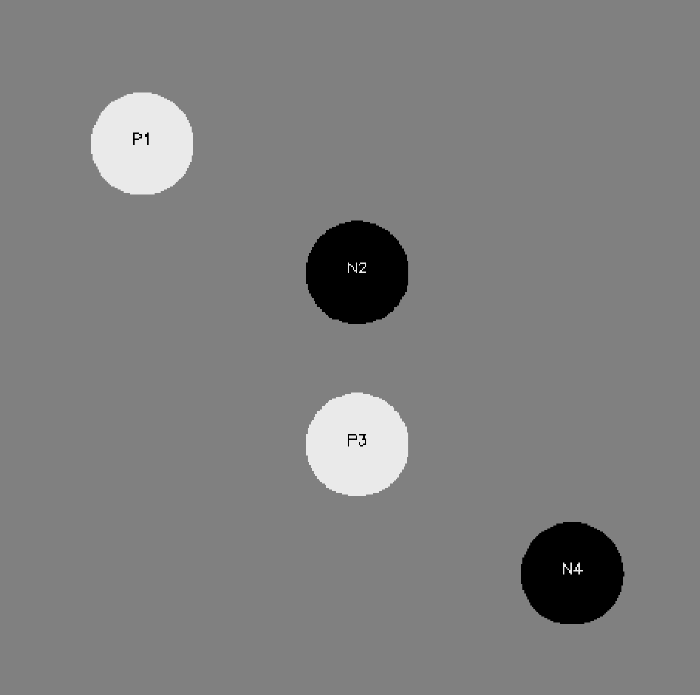

P1
N2
P3
N4

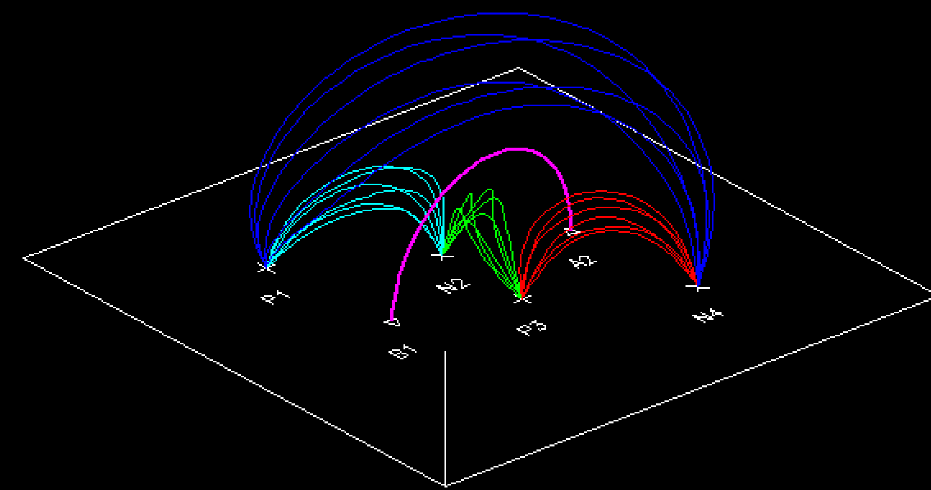

P1
N2
P3
N4

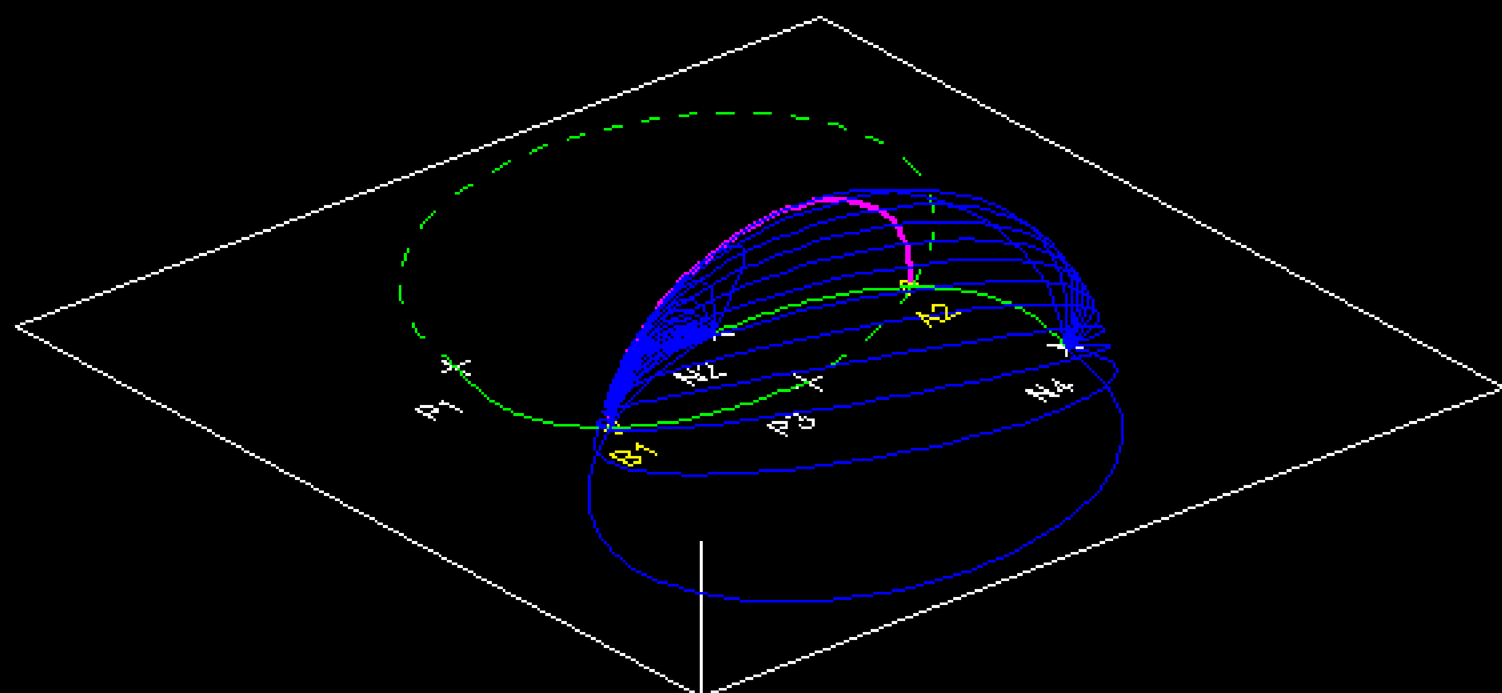

P1
N2
P3
N4


P1
N2
P3
N4


P1
N2
P3
N4
B1
A2


P1
N2
P3
N4


P1
N2
P3
N4
B1
A2

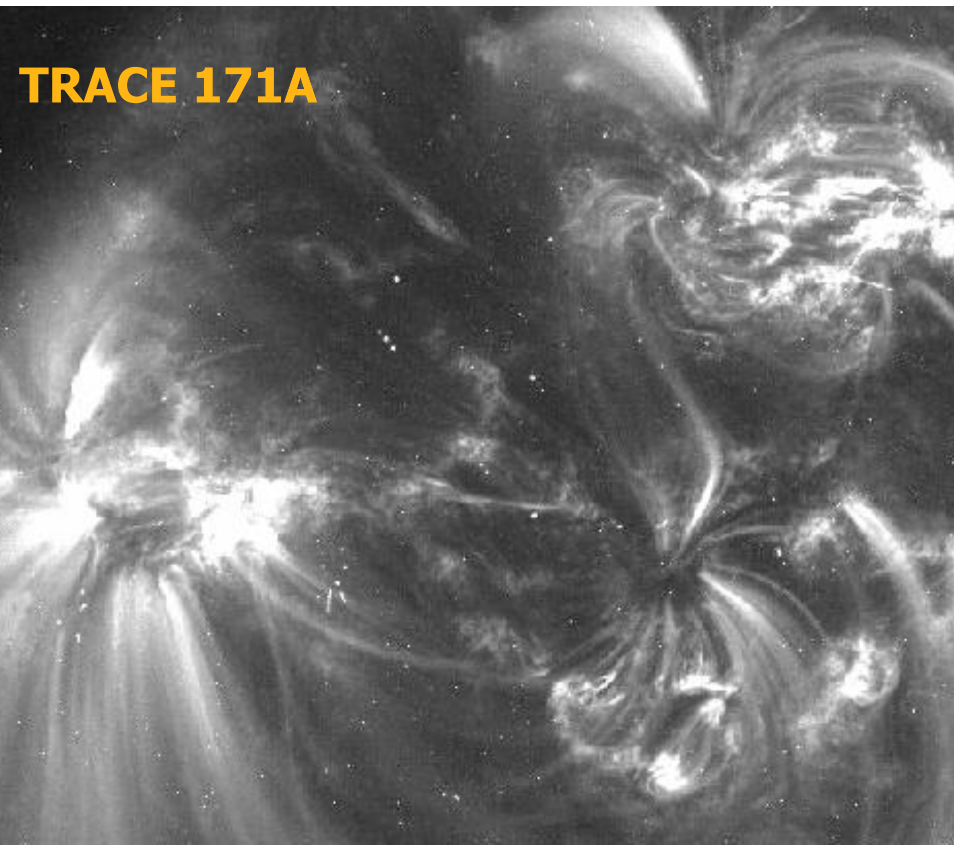

N2
N4

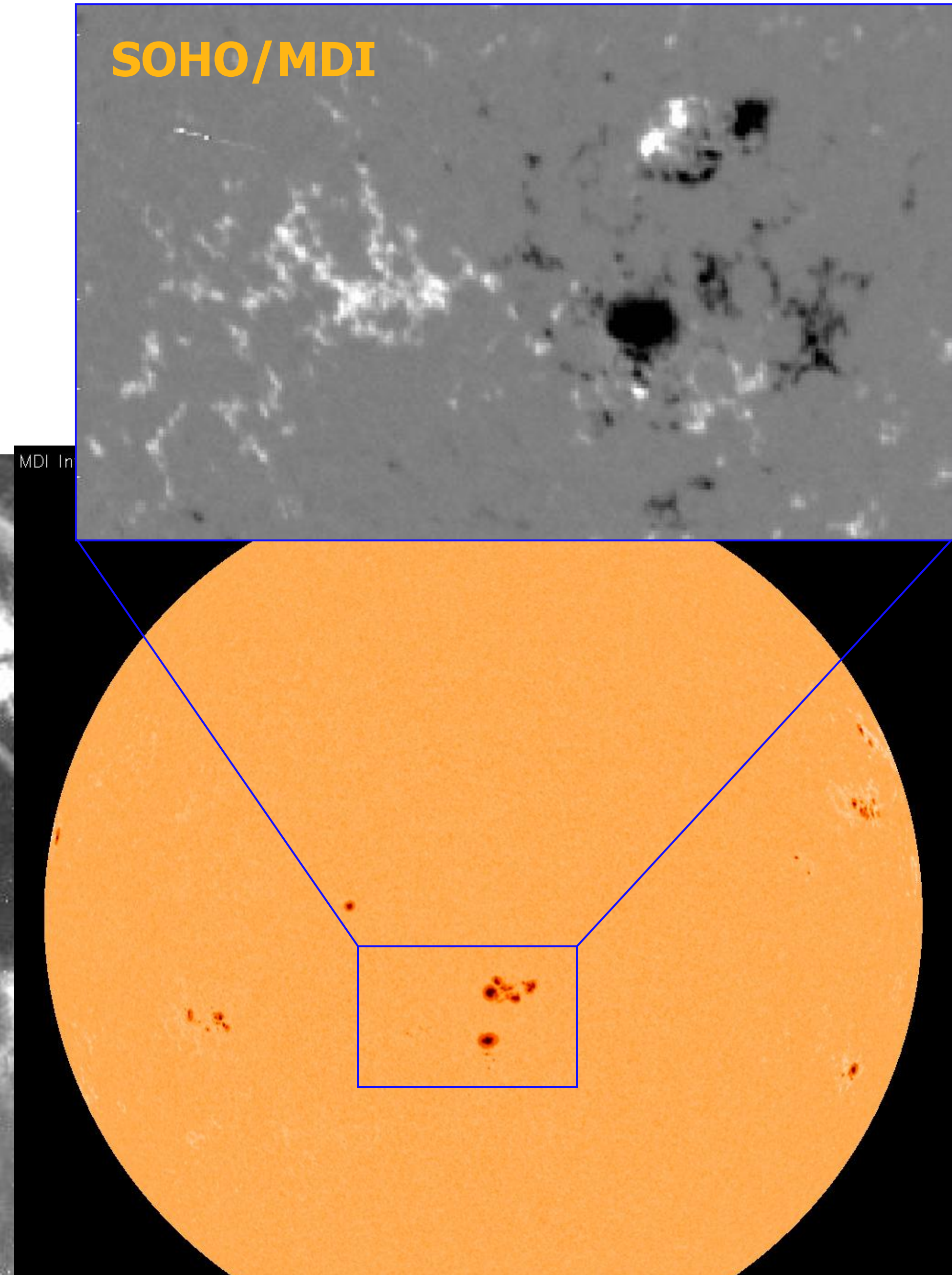
A Real Example

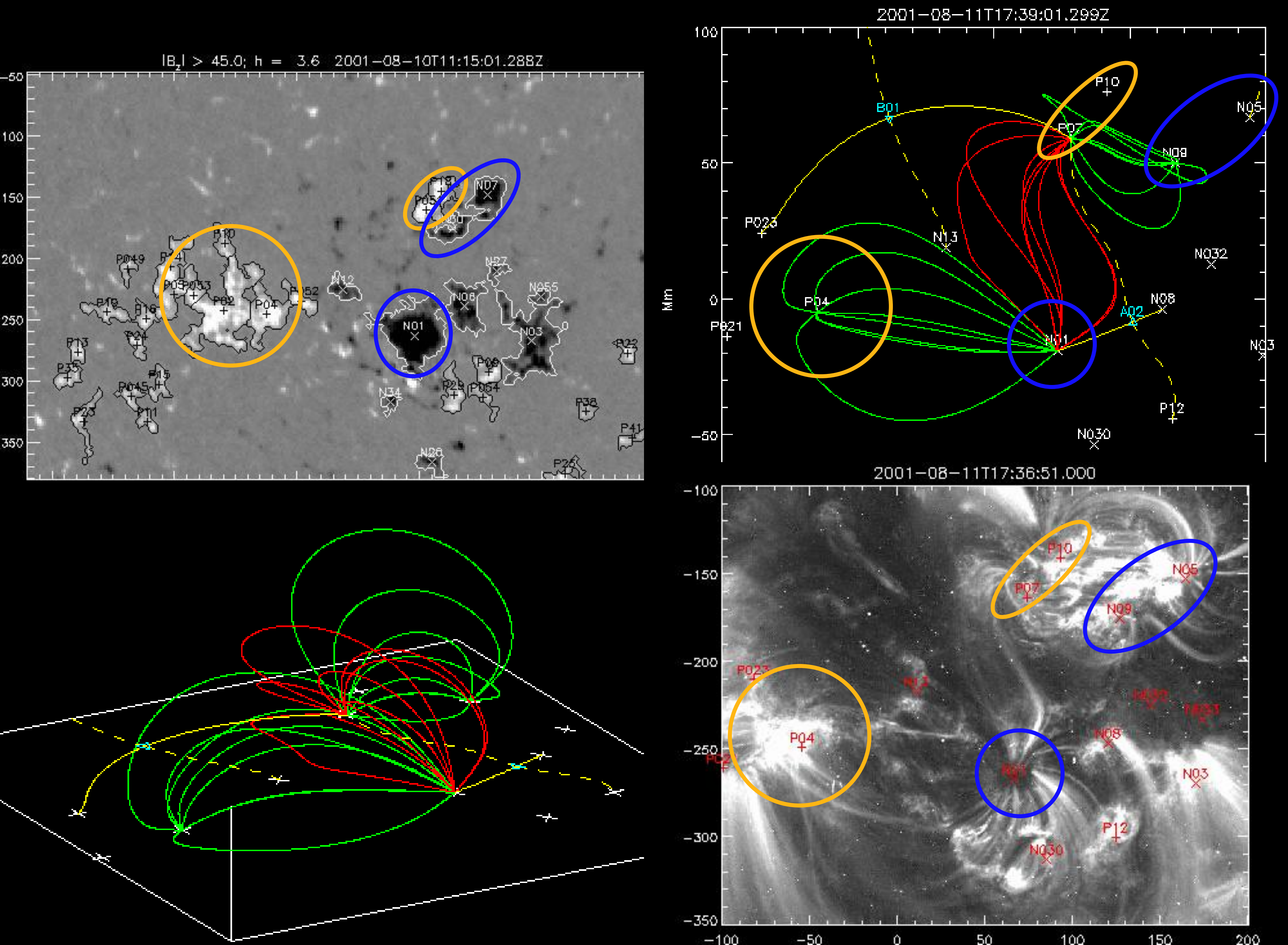
[movie]

PHOTOSPHERE

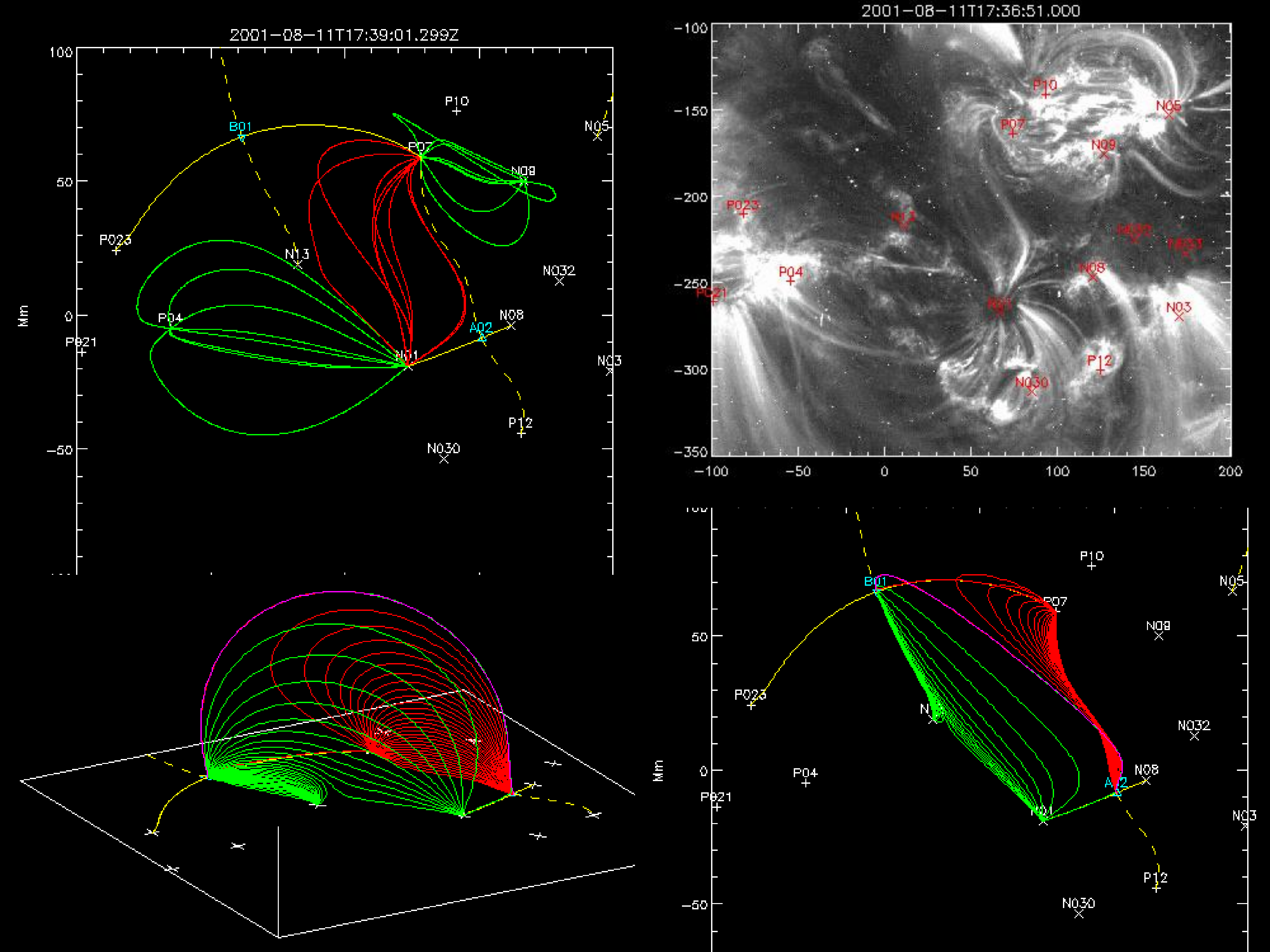
CORONA





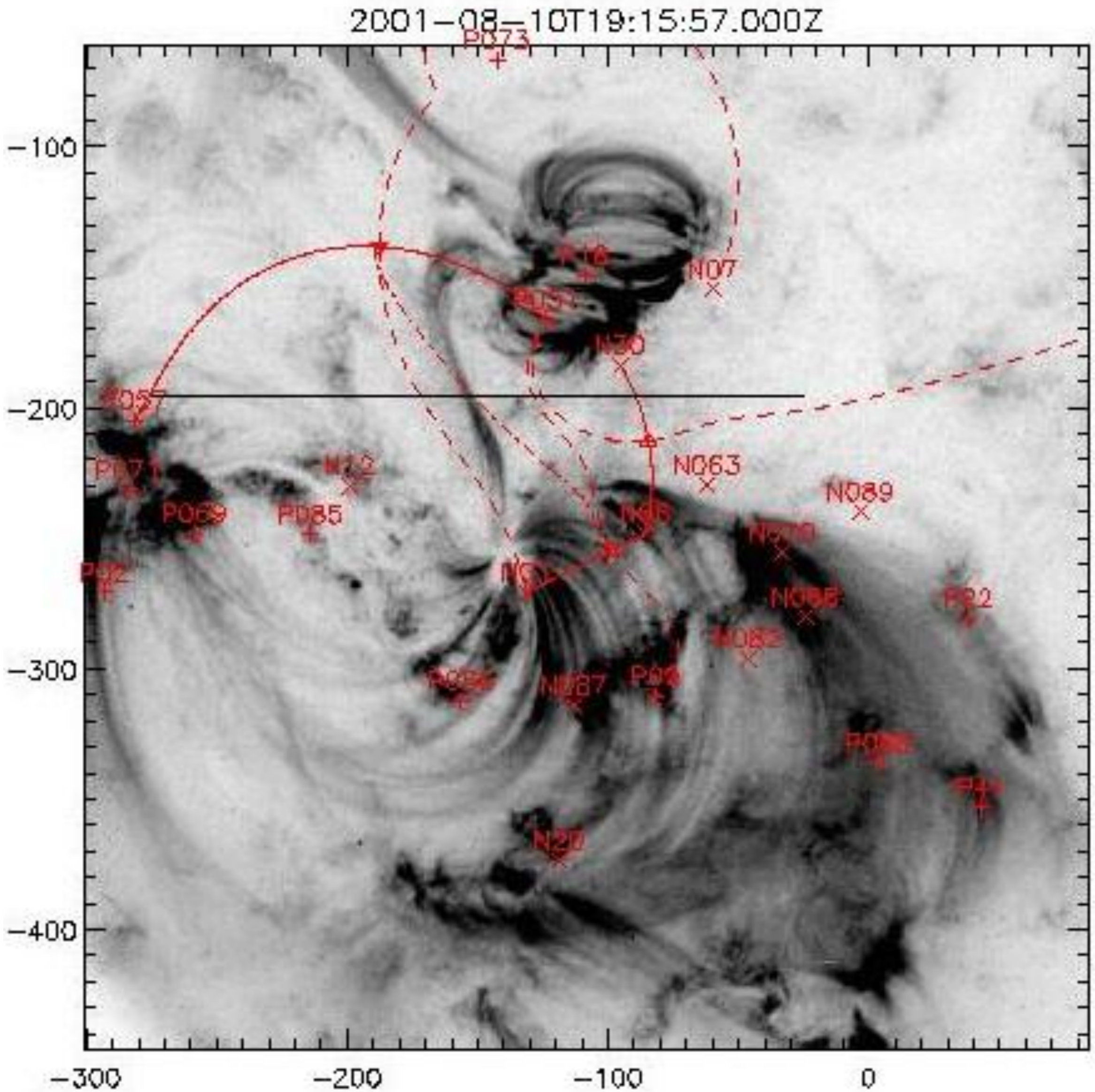


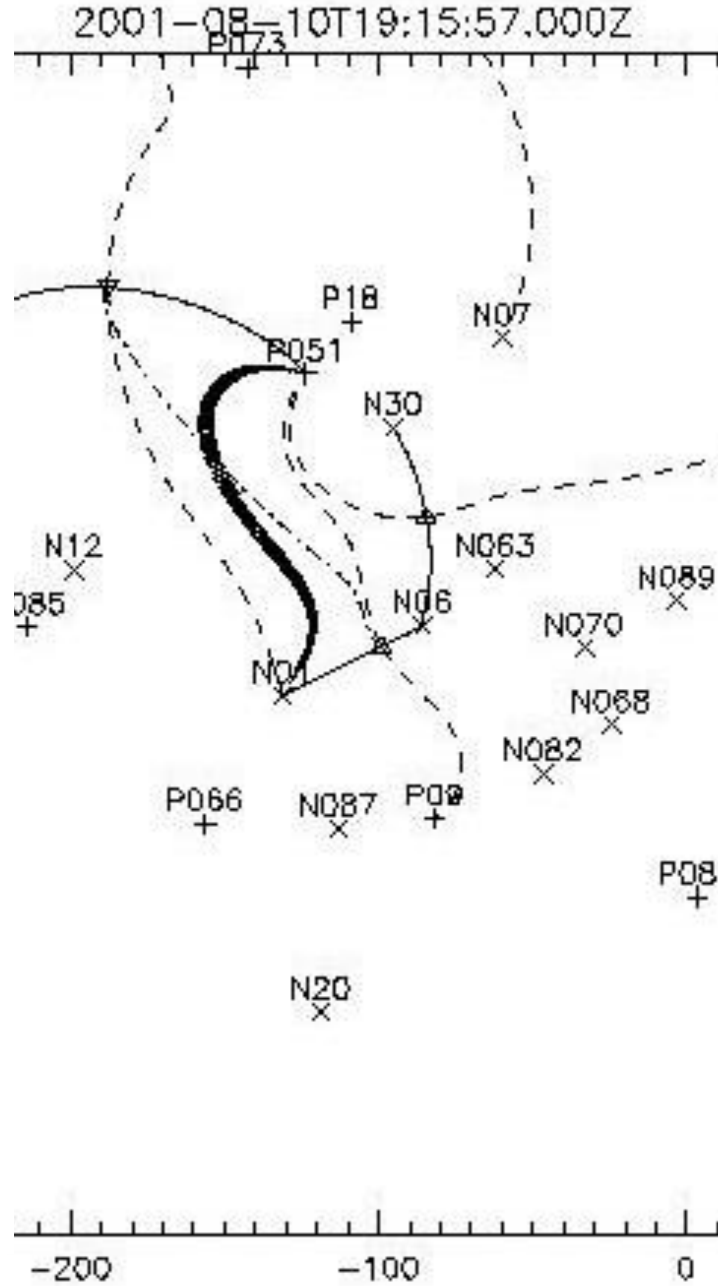
|B_z| > 45.0; h = 3.6 2001-08-10T11:15:01.288Z
2001-08-11T17:39:01.299Z
2001-08-11T17:36:51.000
Mm
P10
P07
N05
N09
P04
N01
N08
N03
N13
N032
N030
P12
P023
P021
B01
A02
N07
N12
N27
N055
N34
N28
P41
P38
P25
P13
P23
P45
P11
P15
P24
P16
P18
P049
P52



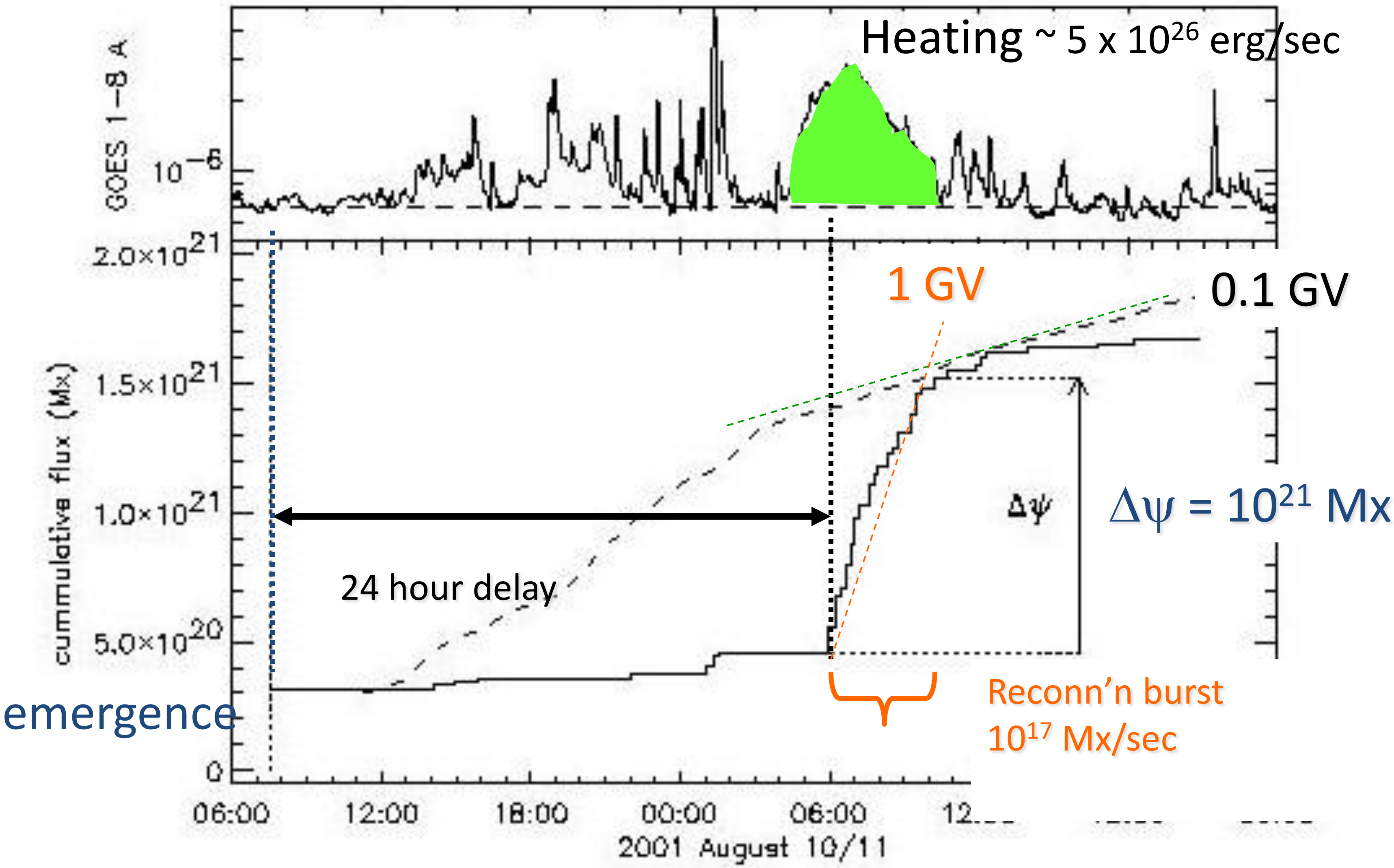
2001-08-11T17:39:01.299Z
2001-08-11T17:36:51.000
Mm
100
50
0
-50
-100
-150
-200
-250
-300
-350
200
P10
N05
B01
P07
N09
P023
N13
N032
P04
N08
A02
P021
N01
N03
P12
N030

The separator





Reconnection observed



Summary

- Plasma behaves as "perfect conductor" on large* scales
- Perfect conduction leads to spontaneous development of small scales – sows seeds of its own violation
- Non-ideal response: **reconnection** attempts to eliminate small scales

* In space, all scales are large