





	WIND Ulysses Cassini Stereo			NRH Tremsdorf Culgoora Hiraiso GB/SRBS (FASR)		NoRH OVSA SSRT VLA BIMA NoRP (FASR)		JCMT CSO (ALMA)		Sac Peak BBSO La Palma (ATST) STEREO Hinode SDO		Ya Si Ti Si H	Yohkoh SOHO TRACE STEREO Hinode SDO		Yohkoh CGRO RHESSI				
10 <sup>-10</sup>	10 <sup>-9</sup>	10 <sup>-8</sup>	10 <sup>-7</sup>	10 <sup>-6</sup>	10 <sup>-5</sup>	10 <sup>-4</sup>	10	3 En 3 10	ęrgy ( 10	eγ) ) 10	<sup>0</sup> 10	) <sup>1</sup> 10	<sup>2</sup> 10	<sup>3</sup> 1	04	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>	
	DH/Kilometer			de cimeter/meter		centimeter/millimeter		submmillimeter	far-infrared	near-infrared	opucar ultraviolet	extreme ultraviolet	soft X-rays		hard X-rays		gamma rays		
l <sup>4</sup> 1	0 <sup>5</sup> 1	0 <sup>6</sup>	10 <sup>7</sup>	10 <sup>8</sup> 1	10 <sup>9</sup>	1010 10	D <sup>11</sup>	10 <sup>12</sup> Freq	10 <sup>13</sup> uency	10 <sup>14</sup> (Hz)	10 <sup>15</sup>	10 <sup>16</sup>	10 <sup>17</sup>	10 <sup>18</sup>	10	<sup>19</sup> 1	0 <sup>20</sup> 1	0 <sup>21</sup>	10 <sup>2</sup>





















In the absence of emission, absorption, or scattering the specific intensity along a ray does not change. However, if emission and absorption occur, the radiative transfer equation is

$$\frac{d\mathcal{I}_{\nu}}{ds} = -\alpha_{\nu}\mathcal{I}_{\nu} + j_{\nu}$$

where  $\alpha_v$  is the absorption coefficient (units cm<sup>-1</sup>) and  $j_v$  is the emission coefficient (units ergs cm<sup>-3</sup> s<sup>-1</sup> sr<sup>-1</sup> Hz<sup>-1</sup>). Defining the optical depth  $\tau_v = \alpha_v ds$  and the source function  $S_v = j_v / \alpha_v$  the transfer equation can be rewritten

$$\frac{d\mathcal{I}_{\nu}}{d\tau_{\nu}} = -\mathcal{I}_{\nu} + S_{\nu}$$

For an isolated and homogeneous source the solution is simply

$$\mathcal{I}_{\nu}(\tau_{\nu}) = \mathcal{S}_{\nu}(1 - e^{-\tau_{\nu}})$$

When  $\tau_v >> 1$ , the source is said to be optically thick and  $\mathcal{I}_{\nu} = S_{\nu}$ ; when  $\tau_v << 1$ , the source is said to be optically thin and  $\mathcal{I}_{\nu} \approx \tau_{\nu} S_{\nu}$ 

For a system of matter and radiation in thermodynamic equilibrium it is characterized everywhere by a single temperature *T* and the differential density distribution is given by the Maxwell-Boltzmann distribution

$$n(E)dE = \left(\frac{2}{\pi}\right)^{1/2} \frac{N}{k_B T} \left(\frac{E}{k_B T}\right)^{1/2} \exp\left[-\frac{E}{k_B T}\right] dE$$

The specific intensity, referred to under these circumstances as blackbody radiation, is described by the Planck function

$$B_{\nu}(T) = \frac{2h\nu^{3}/c^{2}}{e^{h\nu/k_{B}T} - 1}$$

Note that at radio wavelengths we have  $hv \ll k_B T$  and so

$$B_{\nu}(T) = I_{\nu}(T) \approx \frac{2\nu^2}{c^2} kT$$

a relationship known as the Rayleigh-Jeans Law.

Given the simplicity of the Planckian in the Rayleigh-Jeans regime, it is useful to can define the brightness temperature:

$$\mathcal{I}_{\nu} = \mathcal{B}_{\nu}(T_B) = \frac{2\nu^2}{c^2} k T_B$$

and the effective temperature:

$$\mathcal{S}_{\nu} = \frac{2\nu^2}{c^2} k T_{eff}$$

Note that the transfer equation can now be written:

$$\frac{dT_B}{d\tau_{\nu}} = -T_B + T_{eff}$$

with solutions  $T_B = T_{e\!f\!f} = T$  when the source is optically thick and  $T_B = \tau_v T_{e\!f\!f}$  when the source is optically thin.

Radiation from a Maxwellian particle distribution is referred to as thermal radiation whereas radiation from a non-Maxwellian particle distribution is referred to as nonthermal radiation.

Example of a nonthermal distribution:

$$n(E)dE = CNE^{-\delta}dE$$
$$C = (\delta - 1)E_{C}^{\delta - 1}$$





Larmor formulae: radiation from an accelerated charge

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} \mathbf{a}^2 \sin^2 \theta \qquad P = \frac{2q^2}{3c^3} \mathbf{a}^2$$

Relativistic Larmor formulae

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} \frac{(a_\perp^2 + \gamma^2 a_\parallel^2)}{(1 - \beta \cos \theta)^4} \sin^2 \theta$$

$$P = \frac{2q^2}{3c^3}\gamma^4(a_{\perp}^2 + \gamma^2 a_{\parallel}^2)$$

Cases relevant to radio and HXR/gamma-ray emission:

Acceleration experienced in the Coulomb field: bremsstrahlung Acceleration experience in a magnetic field: gyromagnetic radiation





























Plasma radiation at  $v = v_{pe}$  (fundamental) is produced when Langmuir waves interact with ion sound waves:

 $L+s \to T$ 

Plasma radiation at  $v=2v_{pe}$  (harmonic) is produced when two Langmuir waves coalesce:

$$L + L' \longrightarrow T$$

Such Langmuir waves must essentially collide head-on in order to conserve momentum. Note that an important ancillary reaction is the decay of Langmuir waves into a secondary Langmuir wave and an ion sound wave.  $L \longrightarrow L' + s$ 

Two important examples of plasma radiation phenomena relevant to heliophysics:

- Type II radio burst: plasma radiation associated with a coronal or IP shock
- Type III radio bursts: plasma radiation associated with an electron beam







### Nonthermal bremsstrahlung radiation

A HXR photon is emitted when an energetic electron scatters off the Coulomb field of a proton (or other ion).

Consider an energetic electron in a uniform, fully ionized hydrogen plasma with a proton density  $n_p$ . The photon production rate is then just  $n_p \sigma_e(E) v(E)$ . Consider a distribution of energetic electrons N(E)*dE*. Then the total photon rate produced by the (optically thin) source volume *V* is

$$n_p V \int_{\epsilon}^{\infty} \sigma_{\epsilon}(E) v(E) N(E) dE$$

and the photon count rate at Earth (R = 1 AU) is then

$$S(\epsilon) = \frac{n_p V}{4\pi R^2} \int_{\epsilon}^{\infty} \sigma_{\epsilon}(E) v(E) N(E) dE$$

in units of photons cm<sup>-2</sup> s<sup>-1</sup> per unit energy.

This is referred to as "thin target" emission because the time scale for Coulomb losses by the energetic electrons is much greater than any other time scale relevant to the flare.

The photon cross section used for HXR emission is the *Bethe-Heitler* cross section, valid for non-relativistic to weakly relativistic electrons:

$$\begin{aligned} \sigma_{\epsilon}(E) &= \frac{8}{3} \alpha r_{\circ}^2 \frac{m_e c^2}{\epsilon E} \log \frac{1 + \sqrt{1 - \epsilon/E}}{1 - \sqrt{1 - \epsilon/E}} \\ &= \frac{16}{3} \alpha r_{\circ}^2 \frac{m_e c^2}{\epsilon E} \log \left( \sqrt{E/\epsilon} + \sqrt{1 - E/\epsilon} \right) \end{aligned}$$

Rewriting the photon count rate,

$$S(\epsilon) = \frac{2\beta}{\epsilon} \int_{\epsilon}^{\infty} \frac{N(E)}{\sqrt{E}} \log\left(\sqrt{E/\epsilon} + \sqrt{1 - E/\epsilon}\right) dE$$
$$\beta = \frac{2}{3} \alpha n_{\circ} \left(\frac{r_{\circ}}{R}\right)^2 m_e c^2 \sqrt{\frac{2}{m_e}}$$

where

Suppose the energetic electron distribution function is a power law:

 $N(E) = KE^{-\delta}$ 

With a change of variables from *E* to  $E/\varepsilon$  we then have

$$KE^{-\delta} = K\epsilon^{-\delta}u^{-\delta}, \qquad u = E/\epsilon$$

and the photon count at 1 AU is recast as

$$S(\epsilon) = 2\beta K \epsilon^{-(\delta+1/2)} \int_{1}^{\infty} u^{-(\delta+1/2)} \log\left(\sqrt{u} + \sqrt{1-u}\right) du$$
$$S(\epsilon) \propto \epsilon^{-(\delta+1/2)}$$

This result was obtained by Brown (1971), although instead it was formulated in terms of the observed photon spectrum, taken to be a power law,

 $S(\epsilon) = K_2 \epsilon^{-\gamma}$ 

from which  $N\!(\!E\!)$  was inferred through inversion:  $N(E) \propto E^{-(\gamma-1/2)}$ 

Now consider the case where we have a continuous injection of fast electrons into the source volume where they suffer energy losses via collisions on free (and bound) electrons and are brought to a stop.

For a fully ionized plasma we have

$$\frac{dE}{dt} = -n_p v(E) E \sigma_{ee}(E)$$

where

An electron injected with an energy  $E_o$  can radiate photons with energy  $\varepsilon$  via bremsstrahlung until the electron's energy has fallen below  $\varepsilon$ . It's photon production rate is then given by:

 $\sigma_{ee}(E) = \frac{2\pi e^4}{E^2} \Lambda_{ee}(E); \qquad \Lambda_{ee}(E) = \log\left(\frac{E}{e^2} b_{max}\right)$ 

$$\nu(\epsilon, E_{\circ}) = \int_{t(E=E_{\circ})}^{t(E=\epsilon)} \sigma_{\epsilon}(E) n_p v(E) dt = \frac{1}{C} \int_{\epsilon}^{E_{\circ}} E \sigma_{\epsilon}(E) dE$$

where  $C = 2\pi e^4 \Lambda_e e(E)$ 

Now if a distribution of  $F(E_o)$  electrons per unit energy are injected into the source volume each second, the total photon emission rate is

$$\nu_{tot}(\epsilon) = \int_{\epsilon}^{\infty} F(E_{\circ})\nu(\epsilon, E_{\circ})dE_{\circ}$$

and the photon flux per unit energy at Earth is then

 $\phi(E) = \int_{E}^{\infty} F(E_{\circ}) dE_{\circ}$ 

$$S(\epsilon) = \frac{2\beta}{\epsilon} \frac{1}{Cn_{\circ}} \sqrt{\frac{m_e}{2}} \int_{\epsilon}^{\infty} F(E_{\circ}) \left[ \int_{\epsilon}^{E_{\circ}} \log\left(\sqrt{E/\epsilon} + \sqrt{1 - E/\epsilon}\right) dE \right] dE_{\circ}$$

Exchanging the order of integration, this can be rewritten

$$= \frac{2\beta}{\epsilon} \frac{1}{Cn_{\circ}} \sqrt{\frac{m_e}{2}} \int_{\epsilon}^{\infty} \phi(E) \log \left(\sqrt{E/\epsilon} + \sqrt{1 - E/\epsilon}\right) dE$$

where

As before, we assume a power-law distribution of electrons is injected into the source. Then we have

$$\phi(E) = \frac{K_1}{\delta - 1} E^{-(\delta - 1)} = K_1 \epsilon^{-(\delta - 1)} u^{-(\delta - 1)}$$

With the same change of variables as before we can then write

$$S(\epsilon) = \frac{2\beta K_1}{Cn_{\circ}} \sqrt{\frac{m_e}{2}} \epsilon^{-(\delta-1)} \int_1^\infty u^{-(\delta-1)} \log\left(\sqrt{u} + \sqrt{1-u}\right) du$$

and it's seen for "thick target" emission,

$$S(\epsilon) \propto \epsilon^{-(\delta-1)}$$

or, inverting a power-law photon spectrum,

 $F(E) \propto E^{-(\gamma+1)}$ 

In order to directly compare with the thin target case we note that

$$F(E) \sim v(E)N(E) \propto E^{1/2}N(E) \propto E^{-(\gamma-1)}$$

The inferred index of the injection spectrum to be flatter by 2 for the thin target case ( $\gamma$ -1) than inferred for the thick target case ( $\gamma$ +1).

Alternatively, for a given electron injection spectrum with an index  $\delta$ , the resulting photon spectral index is steeper by 2 for a thin target source ( $\delta$ +1) than it is for a thick target source ( $\delta$ -1).





## Other bremsstrahlung contributions

At higher (relativistic) energies, corrections to the electron-proton photon cross section must be included. Moreover, additional contributions to the bremsstrahlung photon spectrum may need to be included. These are:

Electron-electron bremsstrahlung (e.g., Haug 1975; Kontar 2007)
Positron-electron bremsstrahlung (e.g., Haug 1985)

Another point: in principle, protons accelerated in a flare also stream down to the chromosphere and photosphere where they should produce bremsstrahlung by scattering on ambient electrons. That is,

 $_{\odot}$  Proton-electron bremsstrahlung (Emslie & Brown 1985; Haug 2003)

This "inverse bremsstrahlung" is not widely believed to be significant, however.











### Gamma-ray Emission

Gamma-rays are produced by the interaction of energetic protons, alpha particles, and heavy nuclei with the ambient chromospheric and photospheric plasma.

Consider a particle species j that has been accelerated to a high energy and is incident on an ambient particle species i. The interaction rate can be written as

$$\nu_{ij} = n_i \int_0^\infty N_j(E) \sigma_{ij}(E) v(E) dE$$

We again formulate the problem in terms of thin- or thick-target emission. Thin target processes are relevant to particles that escape into the IPM. Here, we discuss thick-target processes. The yield of particles (e.g., neutrons, positrons, pions) from a particular interaction in the thick target case is given by (Ramaty et al 1986):

$$Q = \frac{1}{m_p} \sum_{ij} \frac{n_i}{n_H} \int_0^\infty \bar{N}_j(E) dE \int_0^\infty \frac{\sigma_{ij}(E')}{(dE'/dx)_j} dE'$$

Or, reversing the order of integration,

$$Q = \frac{1}{m_p} \sum_{ij} \frac{n_i}{n_H} \int_0^\infty \frac{\sigma_{ij}(E)}{(dE/dx)_j} dE \int_E^\infty \bar{N}_j(E') dE'$$

In our treatment of HXR bremsstrahlung we specified the dominant energy loss term (collisions with electrons) as a function of time. Here, the energy loss term(s) are left unspecified and are expressed as a function of range – depth into the source. Since energy loss by protons and ions is dominated by losses on H and He,

$$\left(\frac{dE}{dx}\right)_{j} \approx \left(\frac{dE}{dx}\right)_{j,H} \left[1 + \frac{n_{He}}{n_{H}} \frac{m_{He}}{m_{p}} \frac{(dE/dx)_{j,He}}{(dE/dx)_{j,H}}\right]$$

The terms in square brackets is approximately 1.13, and is nearly independent of energy.

$$\left(\frac{dE}{dx}\right)_{j,H} \approx 630 \left(\frac{Z_{eff}^2}{A}\right)_j E^{-0.8} \qquad Z_{eff} = Z[1 - \exp(-\beta/\alpha Z^{2/3})]$$

Again, the detailed physics is embodied in  $\sigma_{ii}(E)$ .

# Pion decay

There are three types of  $\pi$ -mesons, or pions:  $\pi^{0}$ ,  $\pi^{+}$ , and  $\pi^{-}$ . Neutral pions have a rest mass of about 135 MeV whereas the charged pions have a rest mass of about 140 MeV. The threshold energy for pion production is therefore ~300 MeV nuc<sup>-1</sup>.

An example of an interaction that produces pions is proton-proton collisions:

 $p + p \longrightarrow \pi^+ + {}^2H$ 

In the case of neutral pions, most (99%) decay directly into two photons. In the case of charged pions, they first decay to muons, then electrons and positrons.

 $\pi^{o} \longrightarrow 2\gamma$  (each photon ~67 MeV, strongly Doppler broadened)

 $\begin{array}{cccc} \pi^+ & \longrightarrow & \mu^+ + \nu_\mu \longrightarrow e^+ + \nu_e + & \overline{\nu}_\mu \\ \pi^- & \longrightarrow & \mu^- + \overline{\nu}_\mu \longrightarrow & e^- + & \overline{\nu}_e + \nu_\mu \end{array}$ 

Secondary electrons  $\rightarrow$  bremsstrahlung continuum Secondary positrons  $\rightarrow$  bremsstrahlung plus annihilation radiation



# Positron annihilation

Positrons are produced by two types of processes:

 $\pi^{+}$  decay (thresholds of 100s of MeV)  $\beta$ -decay of radioactive isotopes of C, N, and O (thresholds of 1-10s MeV)

Positrons are "born" relativistic. They initially lose energy to collisions with electrons and through ionization of neutrals until they have slowed sufficiently to:

1. annihilate on electrons, producing two photons, each 511 keV, OR

2. become bound to an ambient electron, forming positronium (Ps)

The subsequent decay of Ps depend on whether it is formed as spin-0 para-positronium (singlet state <sup>1</sup>Ps) or spin-1 orthopositronium (triplet state <sup>3</sup>Ps), the singlet state forming ¼ of the time, the latter forming ¾ of the time. The singlet state decays into two photons, each 511 keV. The triplet state decays into three photons, each with  $\varepsilon$  < 511 keV.

Direct annihilation of positrons and <sup>1</sup>Ps decay produce the 511 keV annihilation line; <sup>3</sup>Ps produces a continuum contribution below 511 keV.







#### Neutron capture line

A large number of processes can yield fast neutrons: proton-proton, protonalpha, alpha-proton, alpha-alpha, as well as protons and alphas on heavy nuclei – e.g.,  ${}^{13}C(\alpha,n){}^{16}O$  – and the inverse reactions. The threshold energy for these processes ranges from ~1 MeV nuc<sup>-1</sup> (heavies) to 100s of MeV nuc<sup>-1</sup> (proton-proton).

There are four possible fates for neutrons so produced:

- 1. They escape from the Sun
- 2. They decay:  $n \longrightarrow p + e^{-} + v_e$  (photons only emitted via secondary electrons)
- They charge exchange with <sup>3</sup>He, producing tritium: <sup>3</sup>He(n,p)<sup>3</sup>H (no photon emitted)
- They are captured onto H, producing deuterium: <sup>1</sup>H(n,p)<sup>2</sup>H (photon emitted at 2.223 MeV)

Since the cross section for elastic scattering is much larger than that for either charge exchange or capture, neutrons thermalize before either (3) or (4) occurs, leading to a time delay of 10s to 100s of seconds before the 2.223 MeV line appears.





















