Planetary magnetic fields and dynamos

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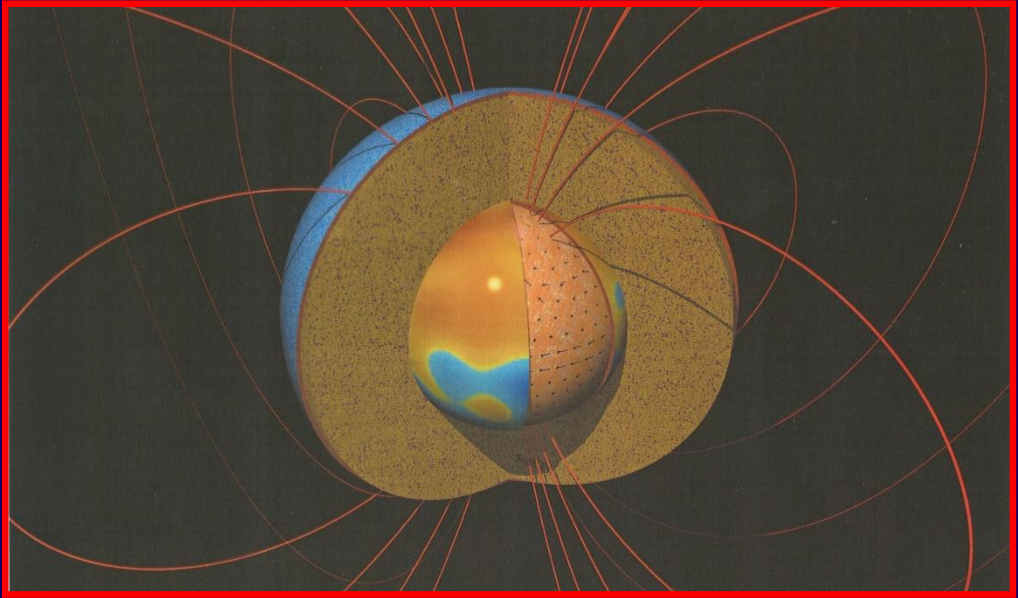

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- Secular variation
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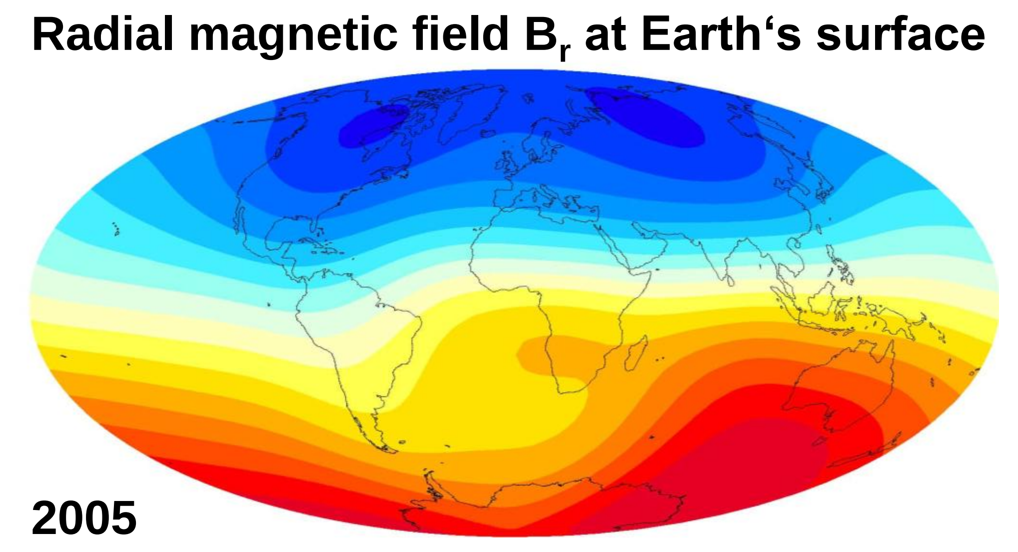
- Geodynamo models vs. observed field
- Scaling of magnetic field strength
- Comparison with low-mass stars
- Dynamo models for planets other than Earth

Geomagnetic field

Mapping with high spatial resolution from orbit:

- Magsat (1980)
- Ørsted (1999 -)
- Champ (2000 -)


Radial magnetic field B_r at Earth‘s surface

2005

$$\mathbf{B} = -grad\ V \qquad V = R_p \sum_{n=1}^{\infty} \left(\frac{R_p}{r} \right)^{n+1} \sum_{m=0}^{n} P_n^m(\cos\theta) \left(g_n^m \cos m\lambda + h_n^m \sin m\lambda \right)$$

Gauss (1838): Representation of field **B** by scalar potential V expanded in spherical harmonic functions. Gauss coefficients g_{nm}, h_{nm}

Power spectrum

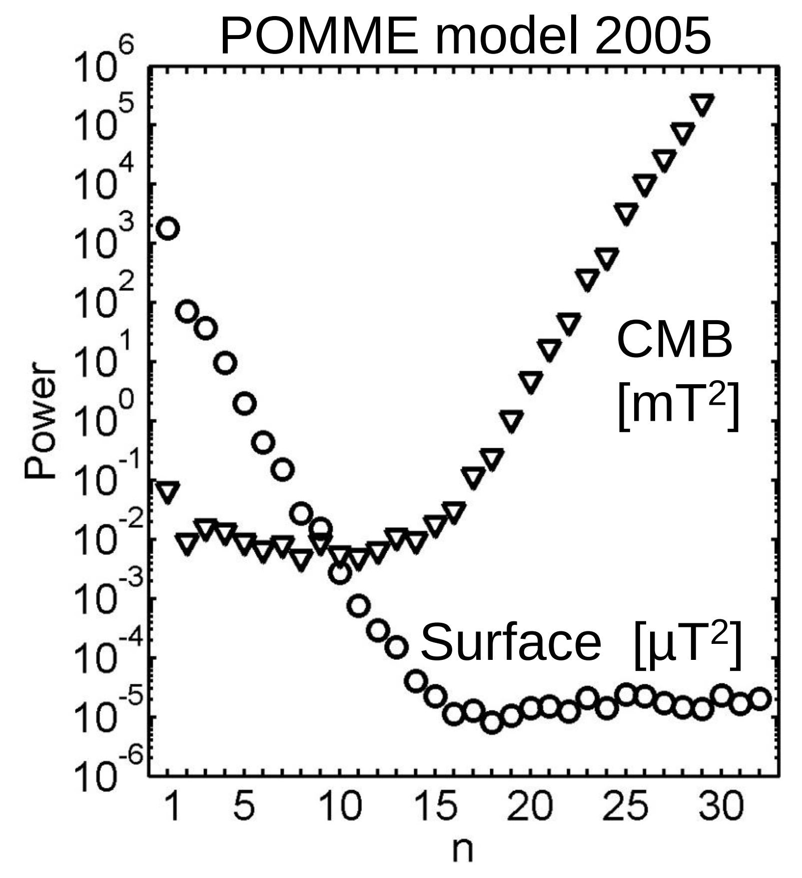
Degree power

$$P_n = (n+1)\left(\frac{R_p}{r}\right)^{2n+4} \sum_{m=0}^{n} \left[\left(g_n^m\right)^2 + \left(h_n^m\right)^2 \right]$$

If no field sources in crust and mantle downward continue field to core

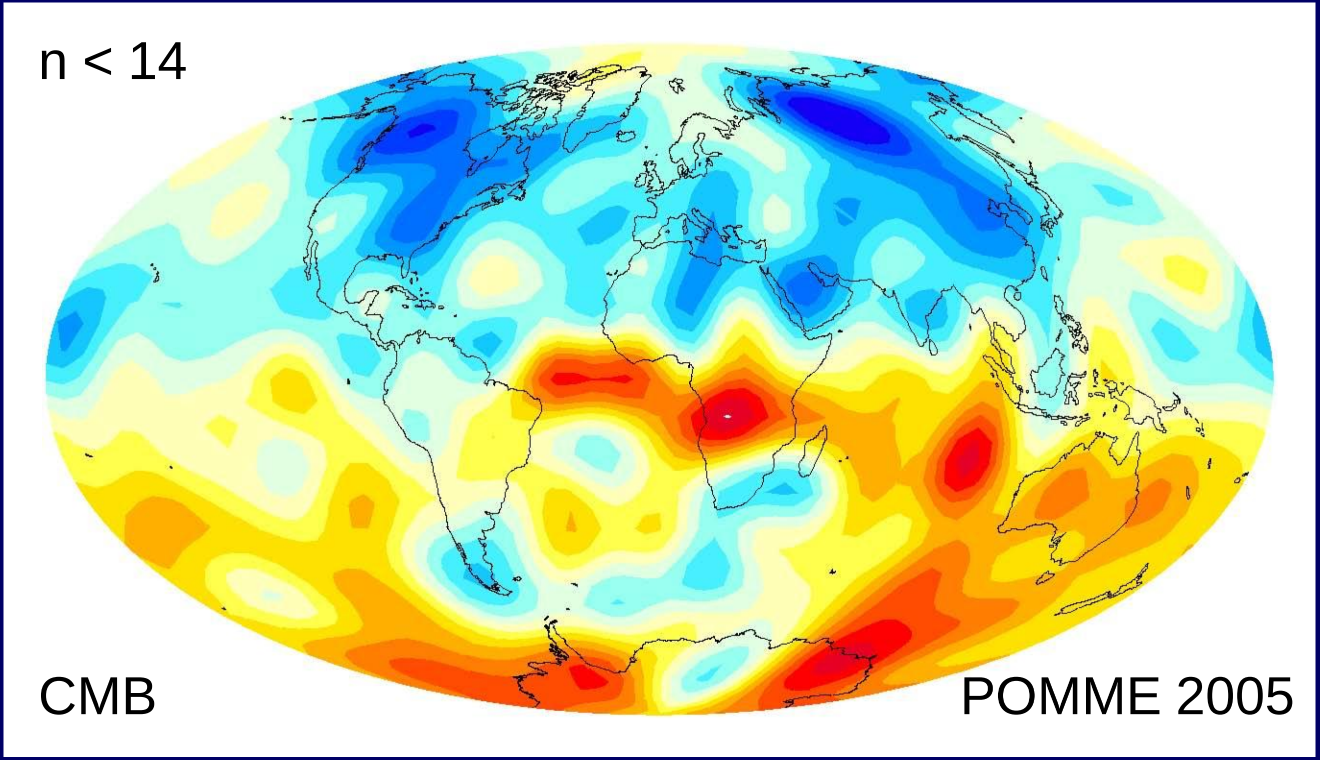
At Earth‘s surface: strong drop up to harmonic degree ~ 14, white spectrum beyond

At core-mantle boundary (CMB, 2900 km depth): white spectrum (dipole sticking out by factor 5) up to n ≈ 14, blue spectrum beyond.



Interpretation: Observed field up to n=14 dominated by core field, for n>14 dominated by field of inhomogeneous magnetization of Earth‘s crust

Field at top of the core



Dipole still dominant, but more structured field. Scales < 1500 km unknown.

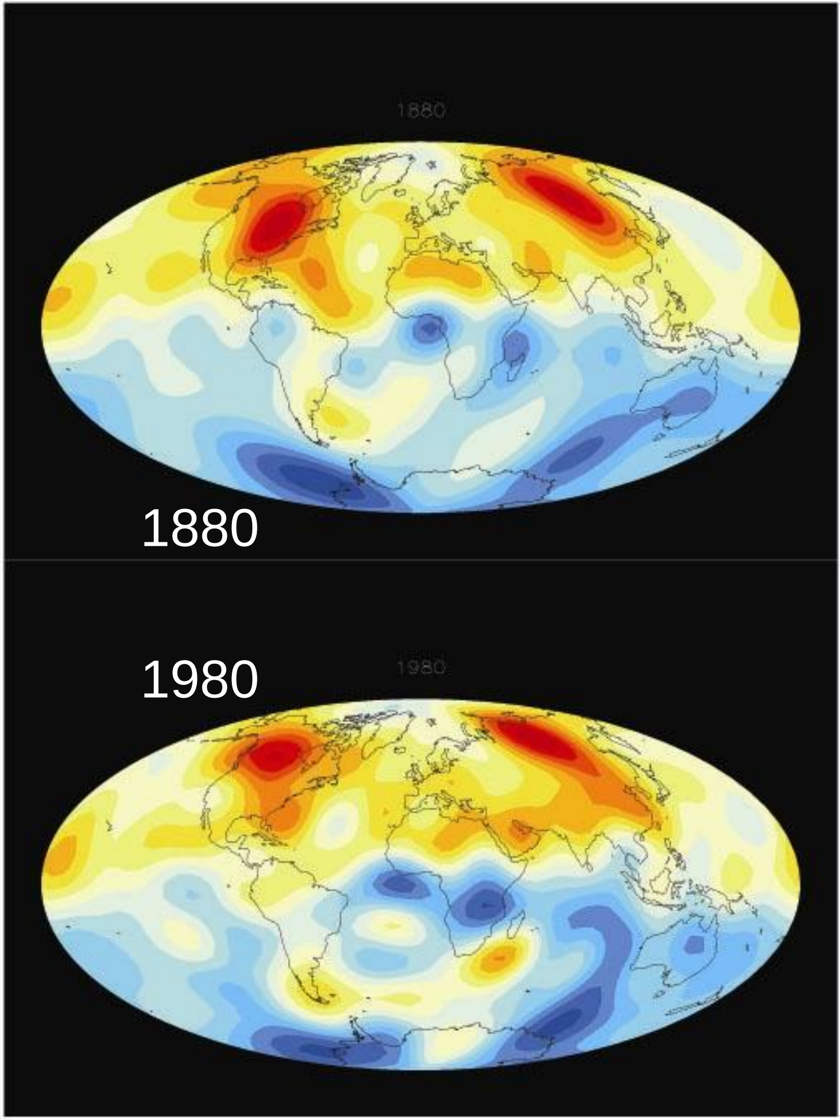
Four high latitude flux lobes (65º) at same longitudes in both hemispheres

Weak or reversed flux at rotation poles. Low latitude patches of both polarities.

rms – field strength at top of core in degrees 1-13 is 0.39 mT (3.9 Gauss).

Internal field strength in core ~ 1 - 4 mT ?   (Toroidal field ~ Poloidal field).

Secular variation



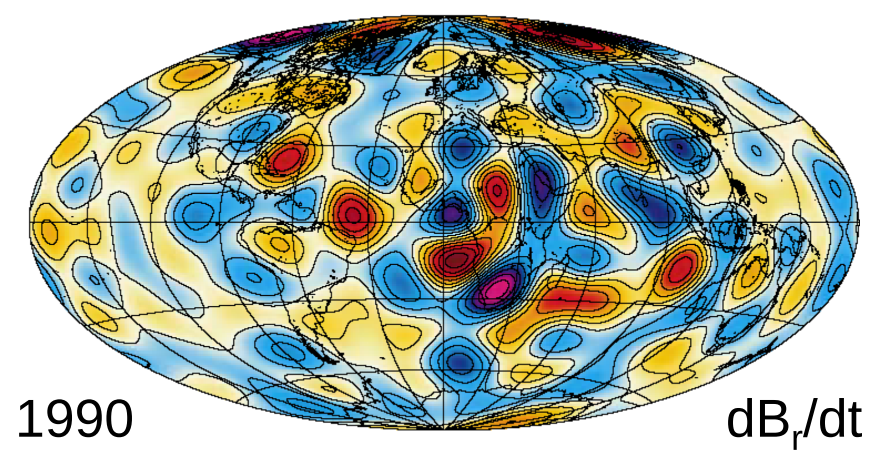
Dipole dropped by 9% since 1840

Reconstructions of core field morphology 1590 - 2009

Fluctuations of non-dipole parts on time scales 50 – 400 yrs

Stability of high-latitude flux lobes

Westward drift in Atlantic / Africa

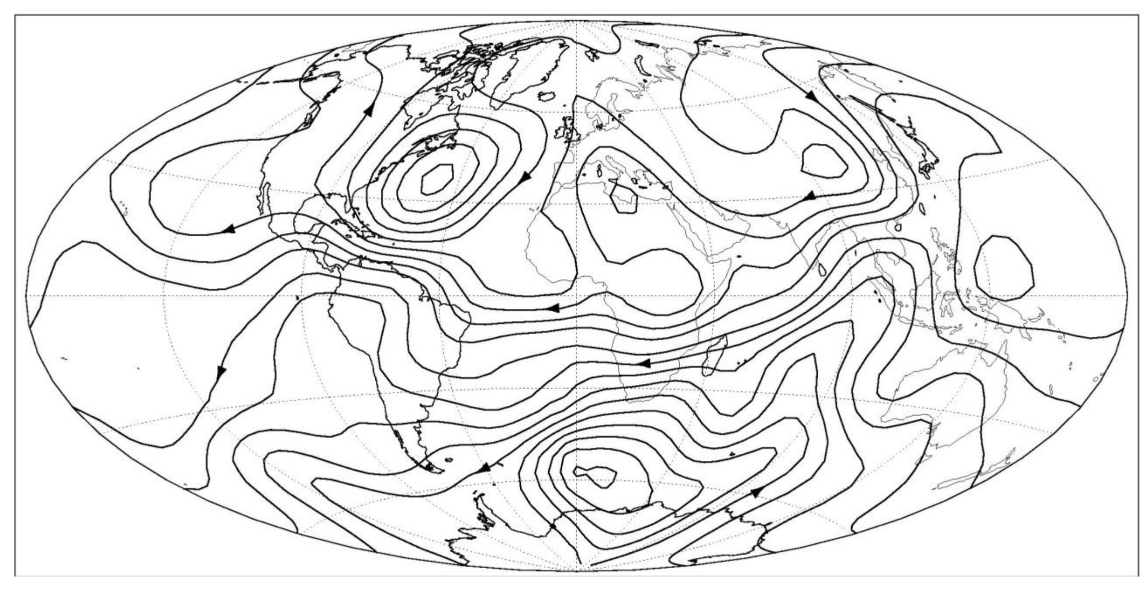


Inversion for core flow

Frozen flux assumption (negligible diffusion in magnetic induction equation) leads to simple equation connecting B_r, $\partial B_r/\partial t$ and the horizontal flow $\mathbf{u}_h$ at the CMB.

$$\frac{\partial B_r}{\partial t} = -\nabla_h \bullet (\mathbf{u}_h B_r)$$

One equation with two unknowns: additional assumption needed, for example purely toroidal flow $\nabla_h \cdot \mathbf{u}_h = 0$ (plus damping of the inversion).



Westward flow under Africa and South Atlantic.

Typical core flow velocities are of order 0.5 mm/s at large scale

$E_{mag} = B^2/2\mu_o \sim 2\ \mathrm{Jm}^{-3}$

$E_{kin} = \rho u^2/2 \sim 10^{-2}\ \mathrm{Jm}^{-3}$

Paleomagnetism

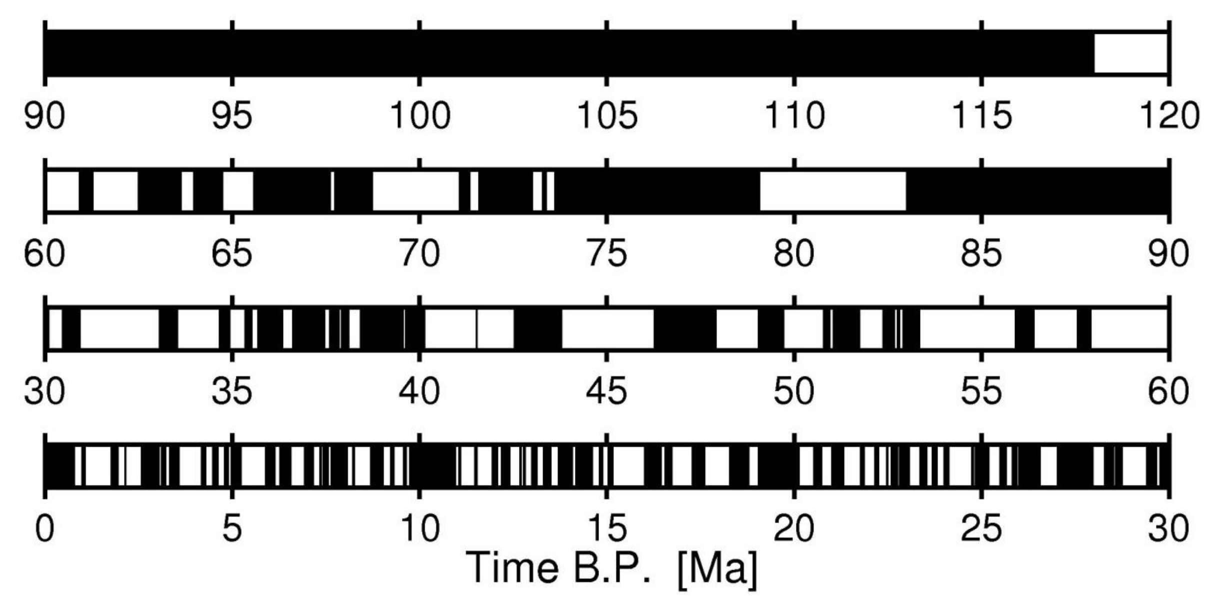
- Rocks contain ferromagnetic minerals, e.g. magnetite Fe_3O_4
- During their formation rocks acquire remanent magnetisation.
- Thermoremanence when cooled below Curie (blocking) temperature.
- Demagnetisation of rock samples in laboratory to distinguish primary from secondary (later acquired) magnetisation that may overprint.
- Information on direction and intensity of magnetising field at time of formation (determined radiometrically)

Results:

- Earth‘s field existed since at least 3.2 billion years.
- Intensity fluctuated within a factor of 2-5, but without long-term trend.
- During the past ~ 5 Myr, the field was dominated by the axial dipole with moderate contributions from multipoles, similar as present field
- For earlier times this is more difficult to prove because of continental drift, but available evidence is in favor of it.
- Dipole field reverses direction

Geomagnetic reversals

- On average a few reversals per million years
- Stochastic (not nearly periodic as in case of the Sun)
- Duration of reversal short (several 1000 yrs) compared to duration of stable polarity periods (several 100,000 yrs)
- Earth surface field during reversal weaker (factor 0.1-0.3), multipolar
- Reversal frequency varies on 100 Myr time scale (mantle influence ?)



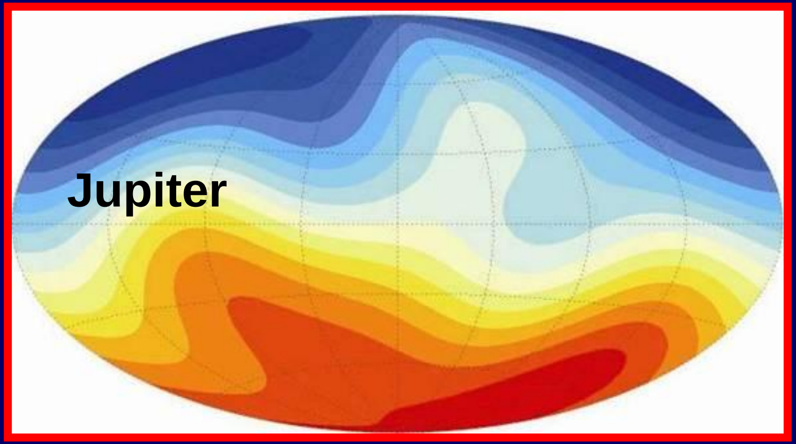
Magnetic fields of solar system planets

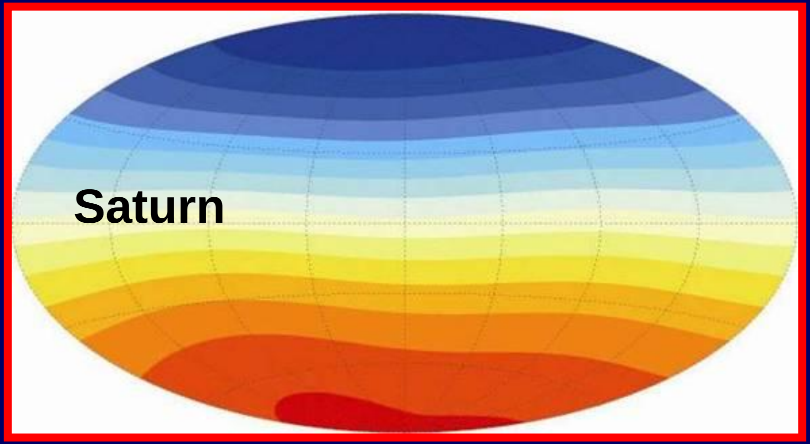
Spacecraft detected magnetic fields at most (but not all) major planets

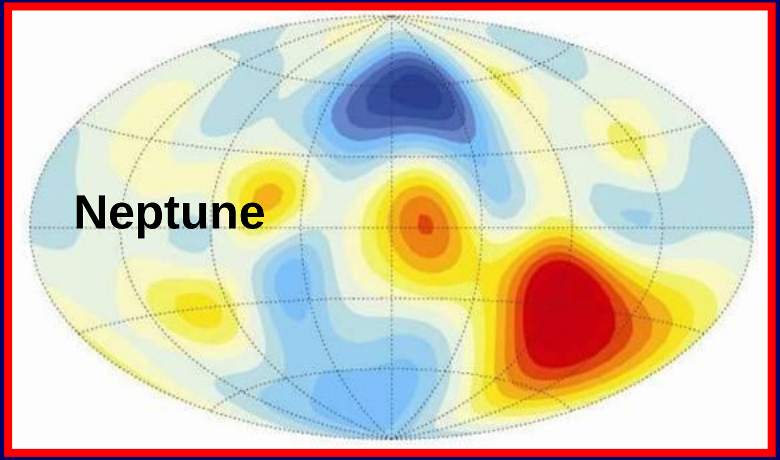
Planet	Dynamo	R_c/R_p	B_s [μT]	Dip. tilt	Quadr / Dipole
Mercury	Yes (?)	0.75	0.35	$<5^o$?	0.1-0.5
Venus	No	0.55			
Earth	Yes	0.55	44	10.4^o	0.04
Moon	No	0.2 ?			
Mars	No, but in past	0.5			
Jupiter	Yes	0.84	640	9.4^o	0.10
Saturn	Yes	0.6	31	0^o	0.02
Uranus	Yes	0.75	48	59^o	1.3
Neptune	Yes	0.75	47	45^o	2.7
Ganymede	Yes	0.3 ?	1.0	$< 5^o$?	?

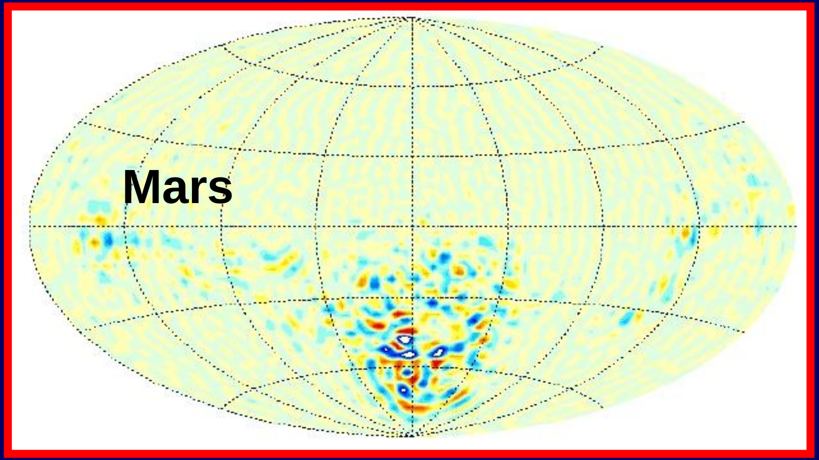
$R_{c,}$/ R_p: core / planetary radius, B_s: Mean field at planet‘s surface, Quadr. / dipole power at R_c

Diversity of planetary magnetic fields









Interior of planets:

Fundamental requirements for dynamo

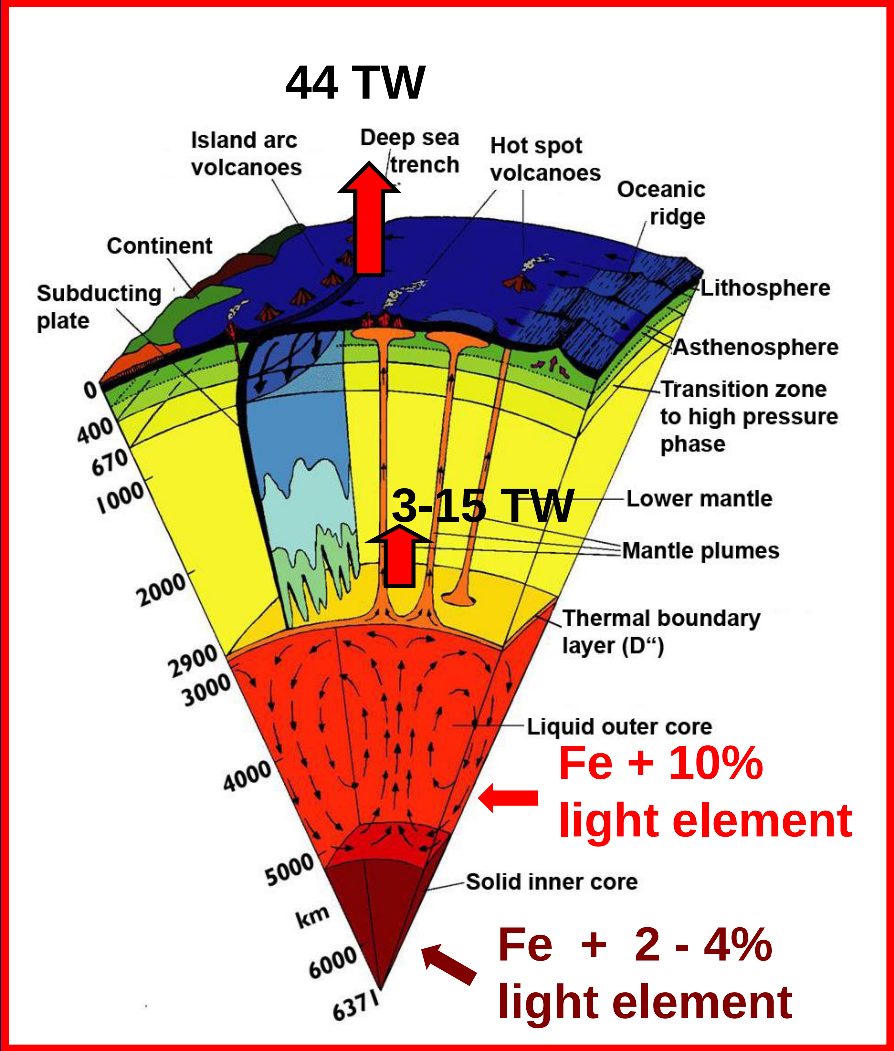
- Electrically conducting fluid layer

- Motion in this layer with a sufficient velocity.
  Magnetic Reynolds number $Rm=UR_c/\lambda > 50$
  Convection likely source of motion.

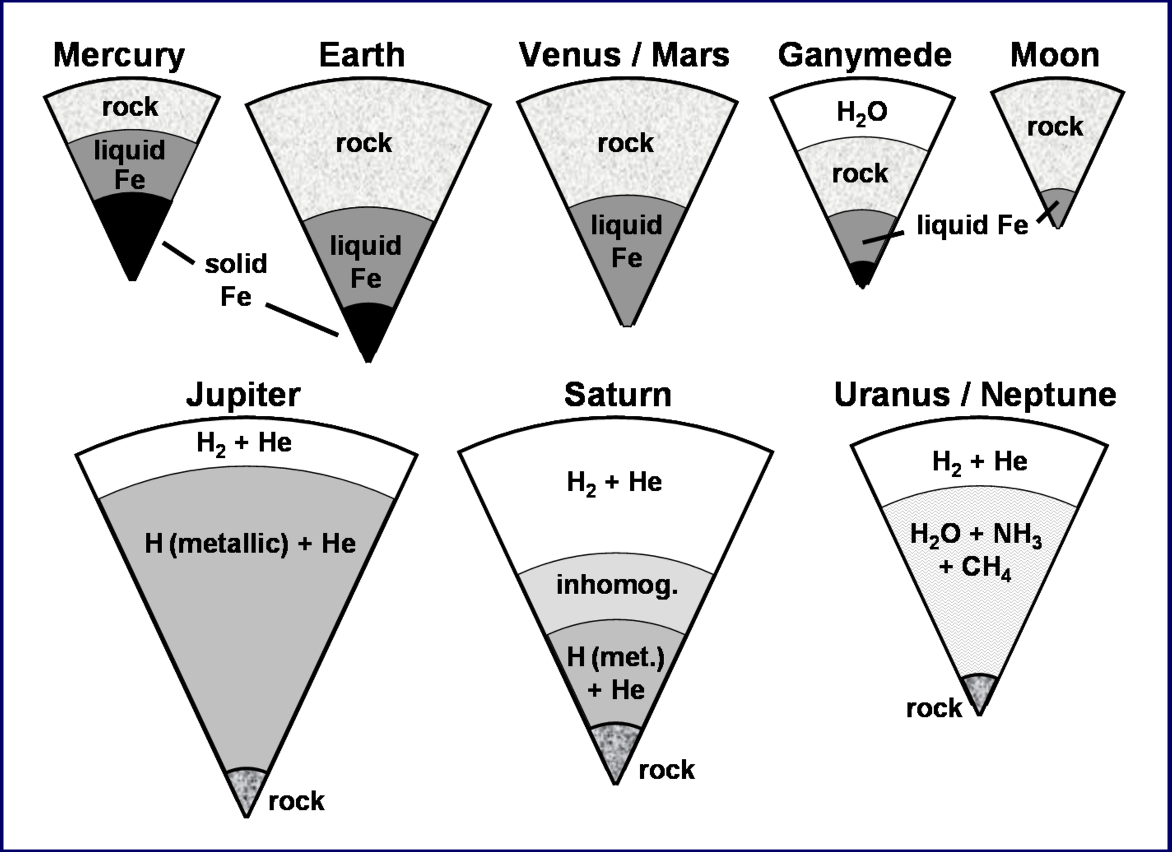
- Motion must have suitable geometry (e.g. helical). Rotation (Coriolis force) important.

Earth: Internal structure & energetics

- Seismology: Dense core with $R_c/R_p=0.55$
- Fe only cosmochemically abundant element matching density
- No shear waves in outer core, hence it is liquid
- Solid inner core with $0.35R_c$
- ~10% light element (Si, S, O, ...) in outer core, less in inner core
- Earth heat flow 44 TW. Core fraction estimated 3-15 TW
- Core heat flow mostly due to secular cooling (radioactive ^{40}K in core ?)



Planetary interiors: a comparison



Dynamo region:

Liquid iron in Earth-like planets and Ganymede. Solid inner core uncertain.

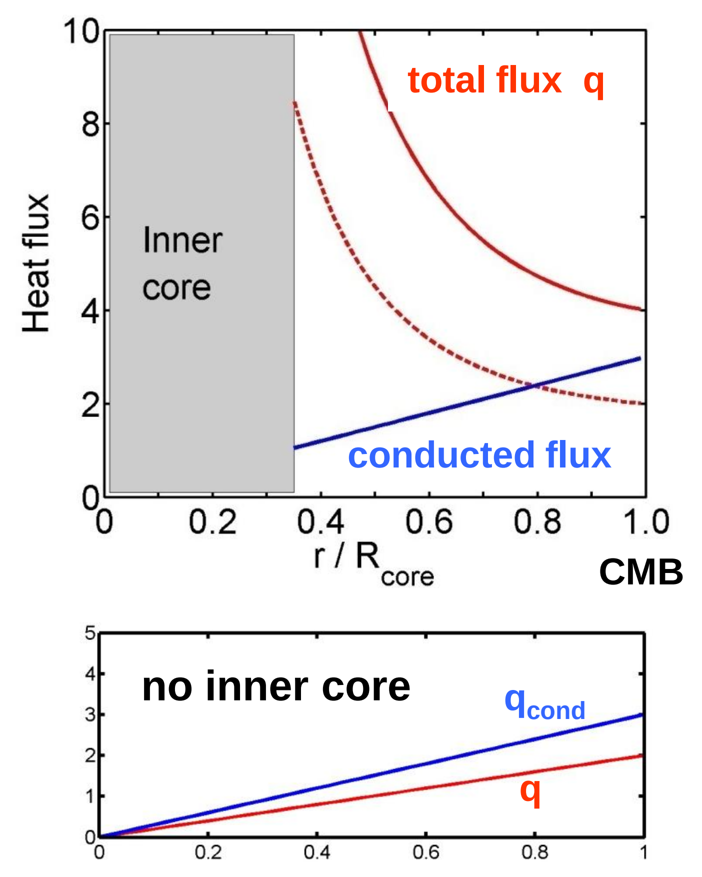
Metallic hydrogen in Jupiter & Saturn

“Ices” with ionic conductivity in Uranus & Neptune

Heat flux: uncertain for rocky planets other than Earth. For gas planets deduced from excess infrared radiation.

Thermal & compositional convection in the cores of terrestrial planets

- For convection, the temperature gradient must exceed the adiabatic gradient $(dT/dr)_{ad} = \alpha g(r)T/c_p = T/H_T$
- The core heat flux of a terrestrial planet is controlled by the mantle
- Significant heat can be conducted along adiabatic temperature gradient
  Earth's CMB: $q_{cond} = 20 - 30\ \mathrm{mWm^{-2}}$
  Total CMB flux: $q = 30 – 150\ \mathrm{mWm^{-2}}$
- Core heat mostly due to secular cooling
- If a growing solid inner core exists, latent heat of freezing contributes to driving thermal convecting and release of light element drives compositional convection



Planetary versus solar dynamo I

Magnetic Reynolds number Rm = $\mu_o \sigma UD$ = UD/η

- **in the sun: O(10^9)**

- **in terrestrial planets: O(10^3)**

- **in hydrogen planets: O(10^4) – O(10^5)**

In planets the magnetic Reynolds number low enough to allow direct numerical simulation of the induction process $\Rightarrow$ cause for success of geodynamo models ?

But: Hydrodynamic Reynolds number too large to resolve turbulent flow in any of these objects.

σ: conductivity η: magnetic diffusivity U: characteristic velocity D: shell thickness

Planetary vs solar dynamo II

Sun: Compressibility (stratification) important

- **considered essential to generate flow helicity**
- **convection zone covers many density scale heights**
- **Coriolis force and nonlinear inertial term similar order Rossby number $Ro = U/(\Omega L) \approx O(1)$**

Planetary cores:

- **Density scale height > R_{core} Boussinesq models.**
- **Magnetic pressure and magnetic buoyancy play small role**
- **Coriolis force >> Inertial force (on large scales) $Ro \ll 1$**

Ω rotation rate L: characteristic length scale

Nondimensional Boussinesq equations

$$\left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u}\right) + 2\vec{e}_z \times \vec{u} + \vec{\nabla} P = E\, \nabla^2 \vec{u} + Ra^* \frac{\vec{r}}{r_o} T + (\vec{\nabla} \times \vec{B}) \times \vec{B}$$

Inertia **Coriolis** **Viscosity** **Buoyancy** **Lorentz**

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \frac{E}{Pr} \nabla^2 T$$

Advection **Diffusion**

$$\frac{\partial \vec{B}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{B} = \vec{B} \cdot \vec{\nabla} \vec{u} + \frac{E}{Pm} \nabla^2 \vec{B}$$

Advection **Induction** **Diffusion**

$$\vec{\nabla} \cdot \vec{u} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

Control parameters

Definition	**Name**	**Force balance**	**Earth value**	**Model values**
$Ra^* = \alpha g \Delta T / \Omega^2 D$	Rayleigh number	Buoyancy Rotational forces	5000 x critical ?	< 100 x critical
$E = \nu / \Omega D^2$	Ekman number	Viscosity Coriolis force	10^{-14}	$\geq 10^{-6}$
$Pr = \nu / \kappa$	Prandtl number	Viscosity Thermal diffusion	0.1 - 1	0.1 – 10
$Pm = \nu / \eta$	Magnetic Prandtl #	Viscosity Magnetic diffus.	10^{-6}	0.06 - 20

α: therm. expansivity, g: gravity, D: shell thickness, Ω: rotation rate, ΔT: driving temperature contrast, ν: viscosity, κ: thermal diffusvity, η: magnetic diffusivity

Diagnostic numbers

	Name	Ratio of	Earth	Model
$Re = UD/\nu$	Reynolds number	Nonlinear inertia Viscosity	10^8	10 - 2000
$Rm = UD/\eta$	Magnetic Reynold#	Advection Magnet. diffus.	10^3	40 - 3000
$Ro = U/\Omega D$	Rossby number	Nonlinear inertia Coriolis	5×10^{-6}	10^{-4} - 1
$Nu = q/q_{cond}$	Nusselt number	Total heat flow Conductive heat	? (>>1)	2 - 30
$\Lambda=\sigma B^2/2\rho\Omega$	Elsasser number	Lorentz force Coriolis force	0.3 - 5	.03 - 100

Nondimensional Boussinesq equations

$$\left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u}\right) + 2\vec{e}_z \times \vec{u} + \vec{\nabla} P = E\, \nabla^2 \vec{u} + Ra^* \frac{\vec{r}}{r_o} T + (\vec{\nabla} \times \vec{B}) \times \vec{B}$$

Inertia **Coriolis** **Viscosity** **Buoyancy** **Lorentz**

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \frac{E}{Pr} \nabla^2 T$$

Advection **Diffusion**

$$\frac{\partial \vec{B}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{B} = \vec{B} \cdot \vec{\nabla} \vec{u} + \frac{E}{Pm} \nabla^2 \vec{B}$$

Advection **Induction** **Diffusion**

$$\vec{\nabla} \cdot \vec{u} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

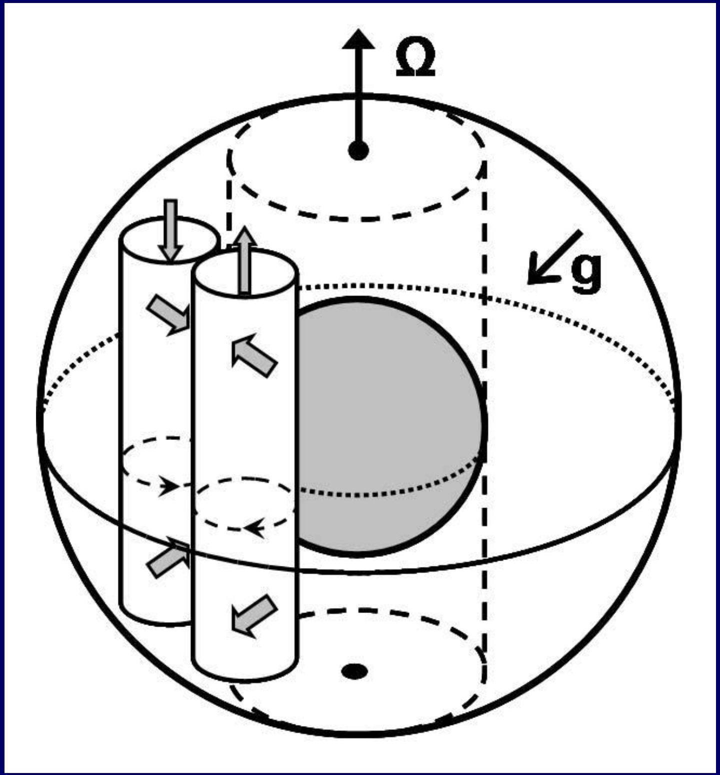
Geostrophic flow in spherical shell

Balance Coriolis force ~ pressure gradient force

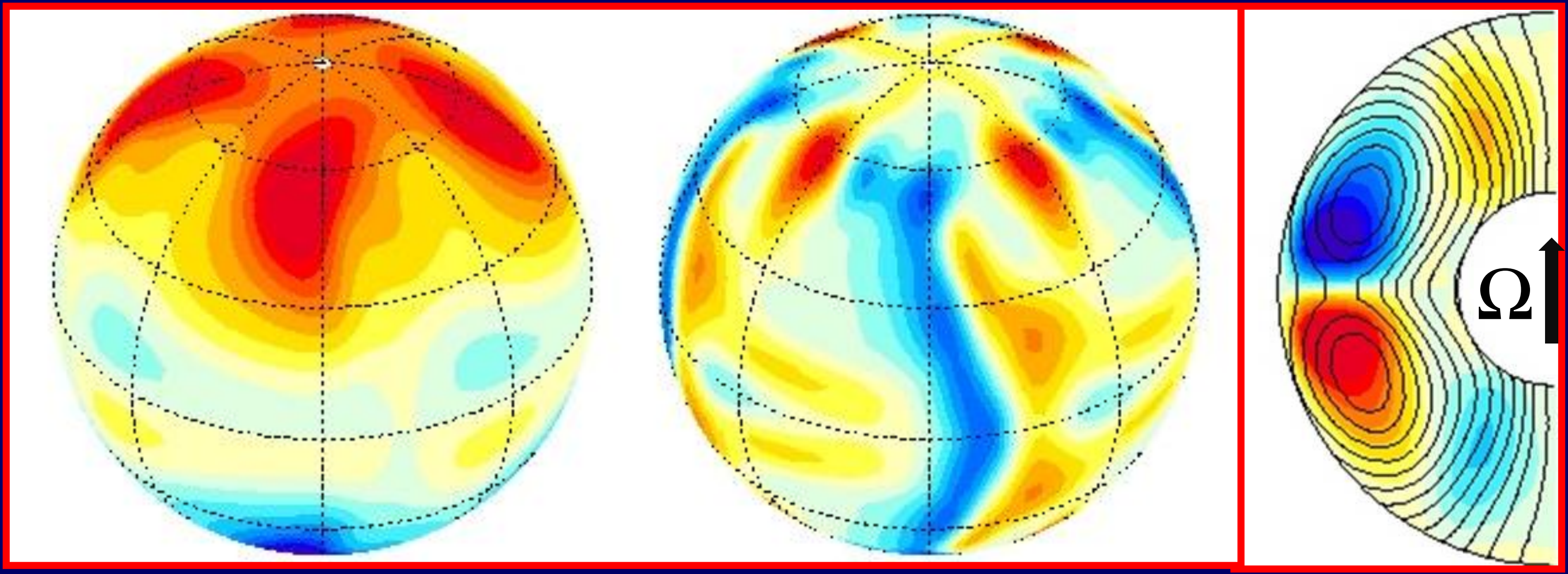
$2\,\rho\,\Omega\times u = \nabla p$ **Take curl** $\Rightarrow$ $(\Omega \bullet \nabla)\,u = 0$

Proudman Taylor theorem

- **When E << 1 Ro << 1 and Λ << 1 convection in spherical shells in columns aligned with rotation axis outside the inner core tangent cylinder**
- **Must violate P-T-theorem, but does so as little as necessary**
- **Columnar flow is helical: secondary circulation along center of columns**



A simple geodynamo model



Radial magnetic field

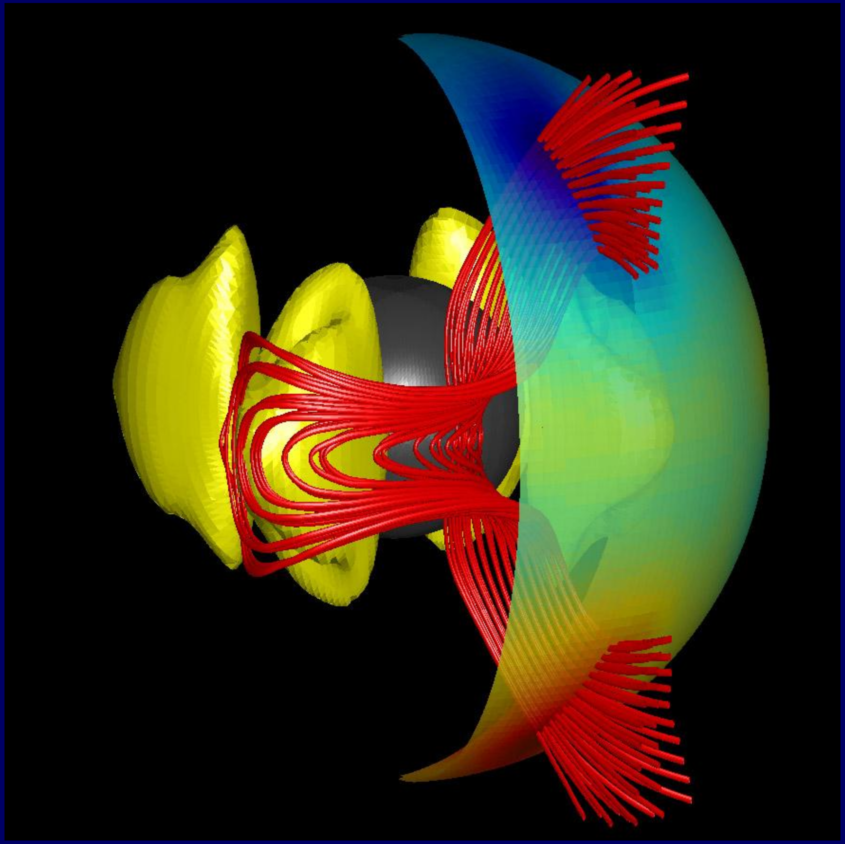
Radial velocity

Axisymmetric field

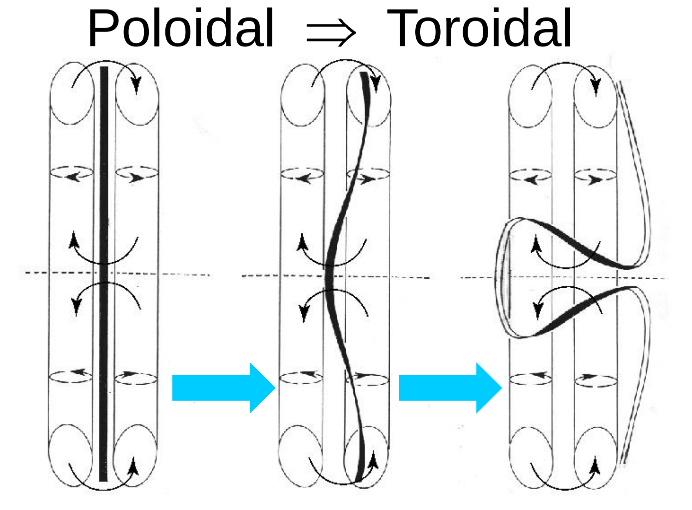
$E=10^{-3}$ $Ra/Ra_c=1.8$ $Pm=5$ $Rm=39$

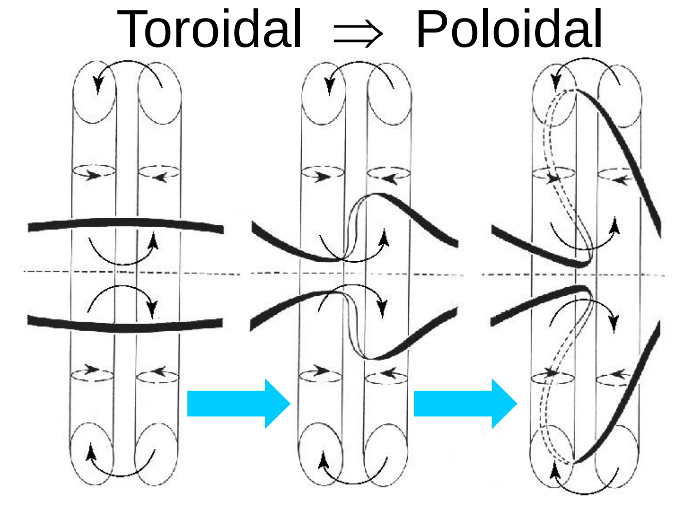
Quasi-stationary flow and magnetic field

Field generation mechanism

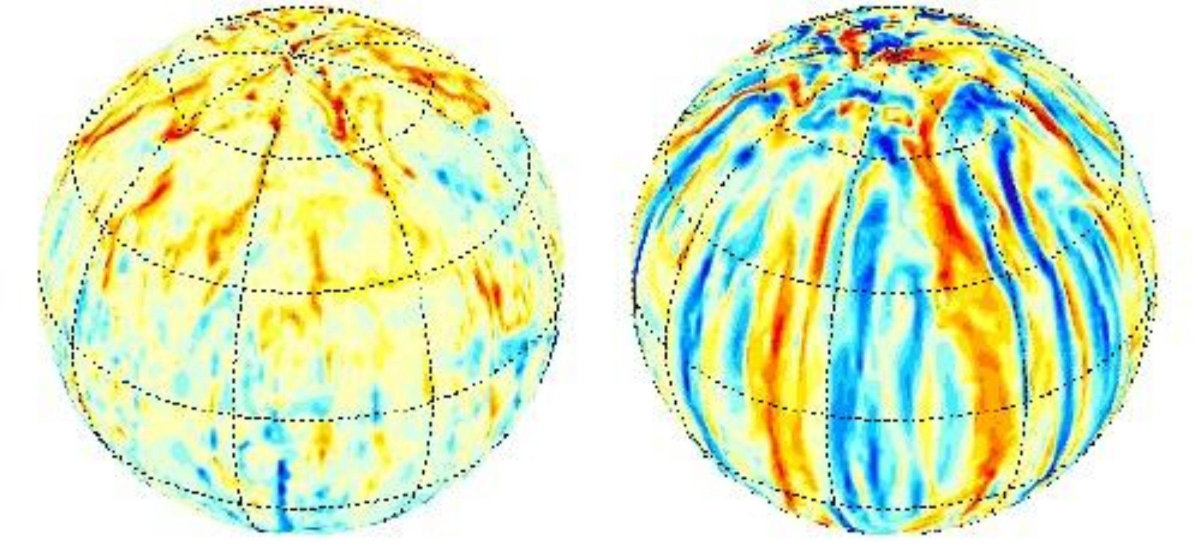


„Macroscopic“
α^2-dynamo





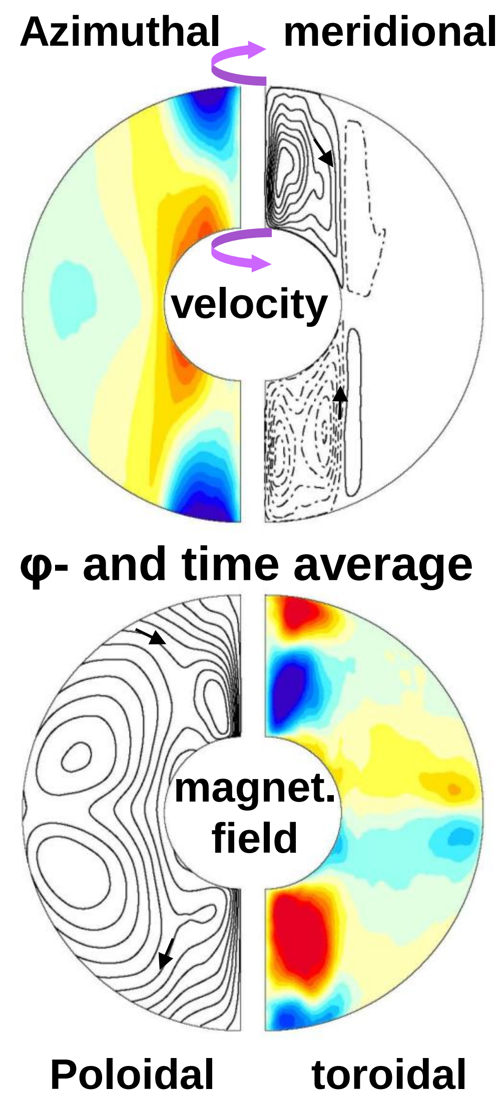
An advanced model



Radial magnetic field

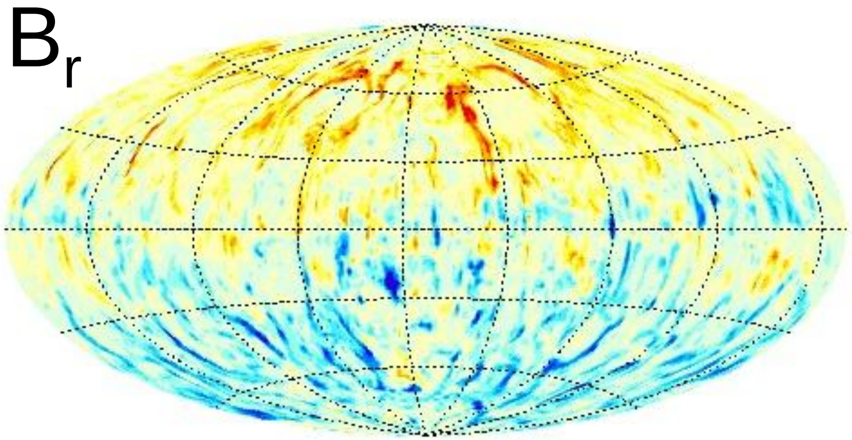
Radial velocity

$E=10^{-5}$ $Ra/Ra_c=114$ Pm=0.8 Rm=914

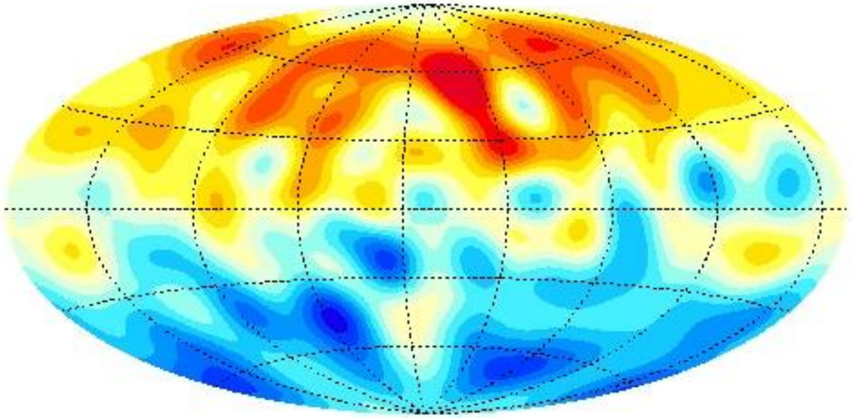


- **Flow columnar outside tangent cylinder**
- **Vigorous flow inside tangent cylinder; polar plumes**
- **Strong toroidal field inside tangent cylinder**
- **$\alpha^2\Omega$ – dynamo ? (Ω-effect inside tangent cylinder)**

Comparison with Earth: Field morphology

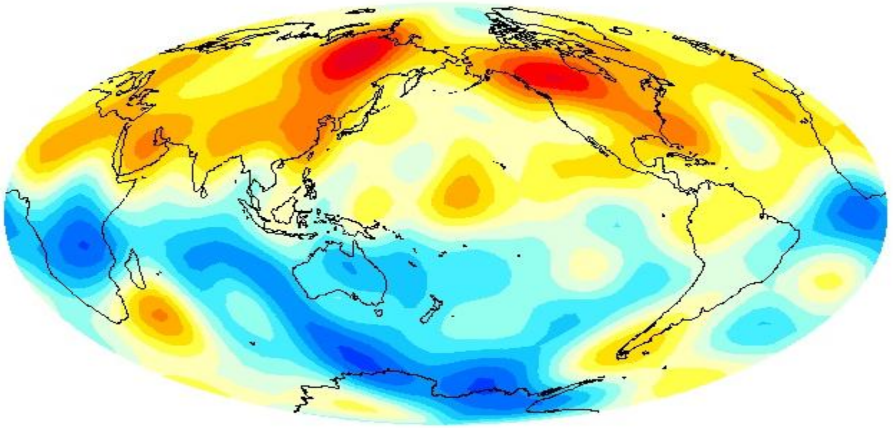


Dynamo model, full resolution



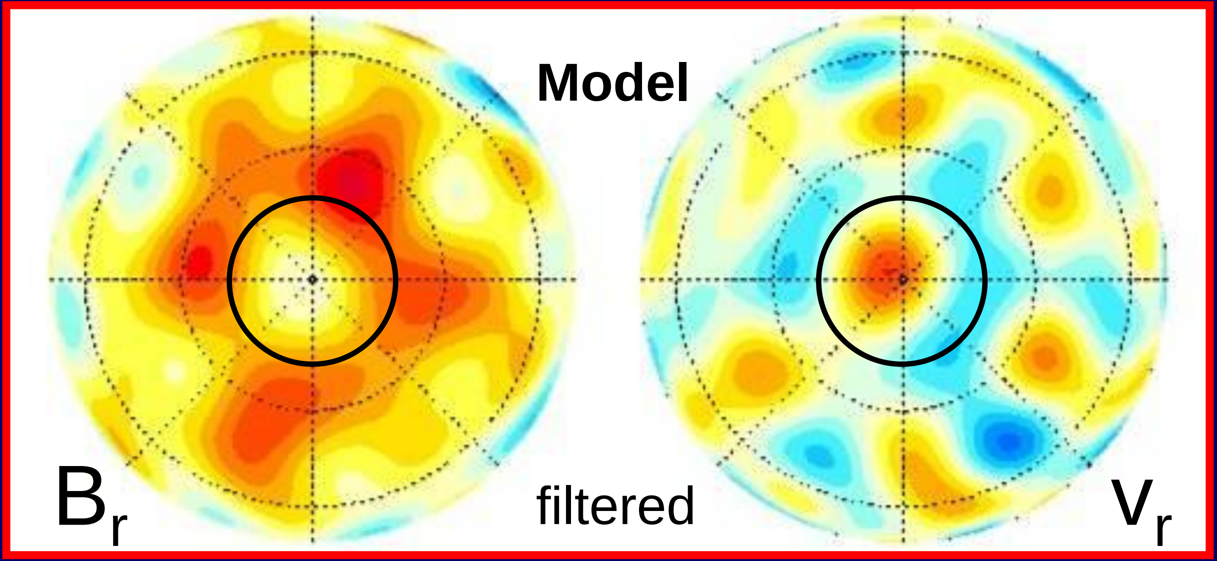
Dynamo model, filtered to n < 13

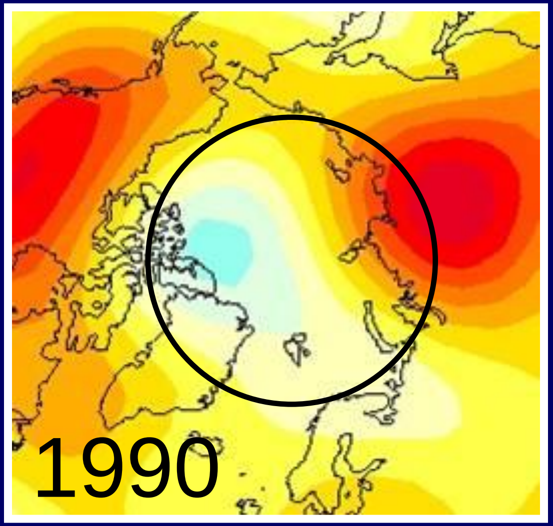
- Flux lobes at 60-70^o latitude
- Weak flux at poles
- Flux spots of both polarities at low latitude. Expulsion of toroidal field bundles ?
- Westward vortex flow in polar cap

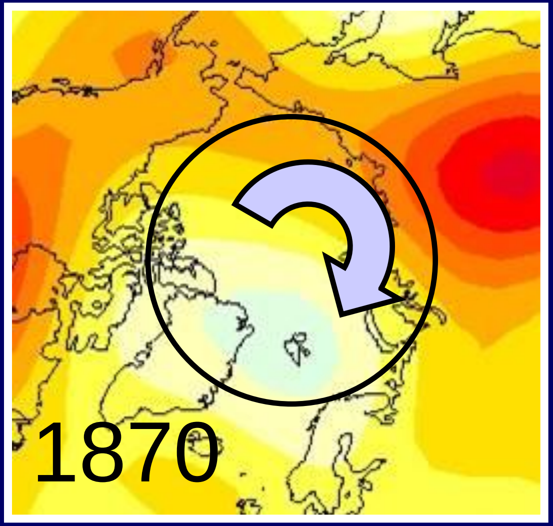


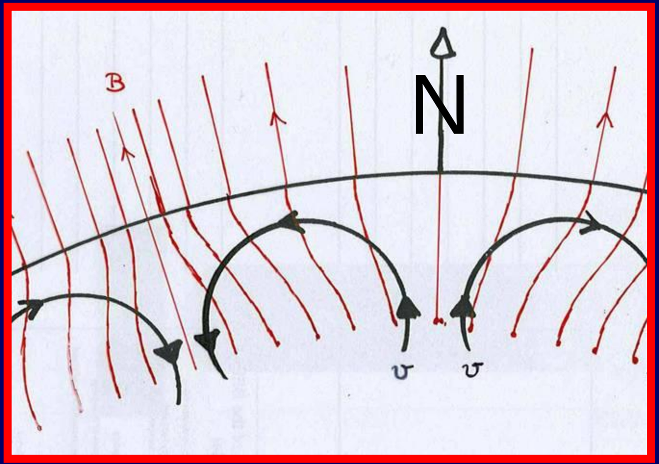
Earth's field at core mantle boundary

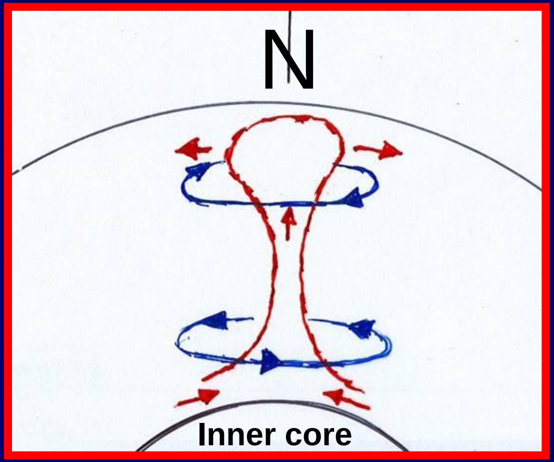
Field structure & core dynamics



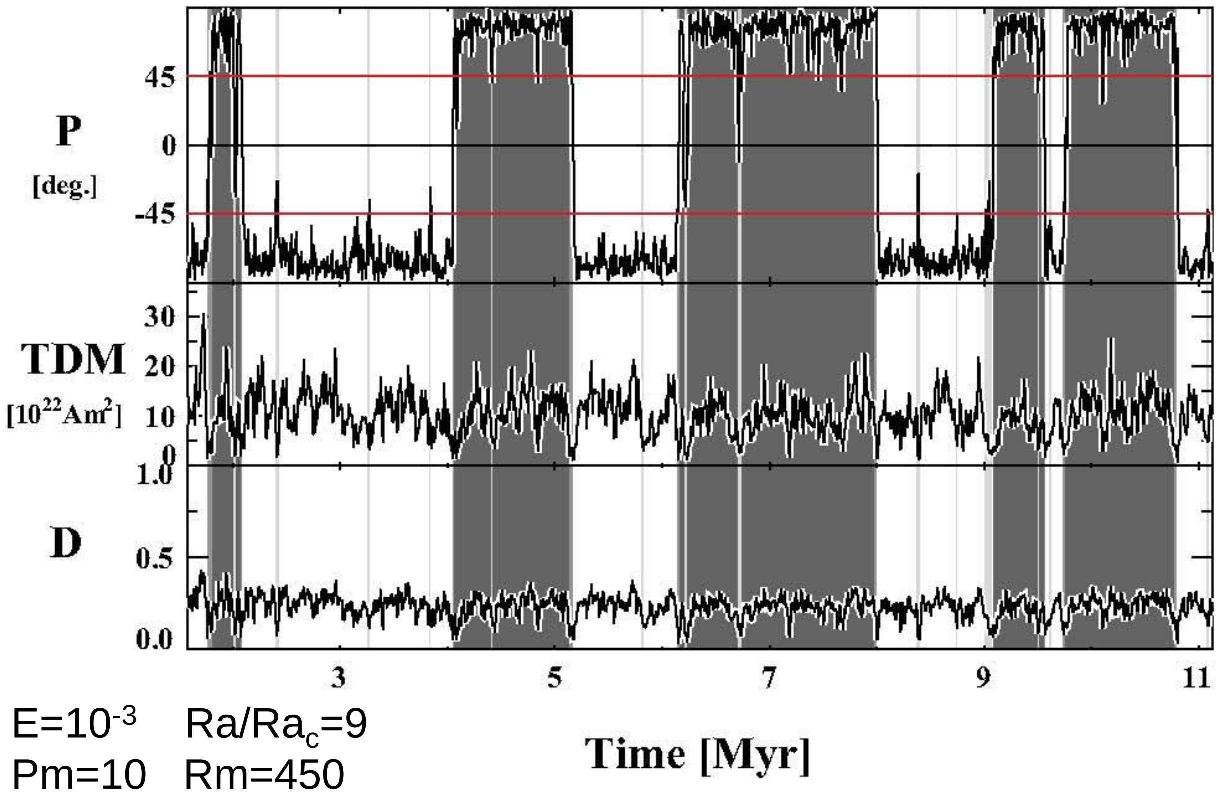








Comparison with Earth: Reversals

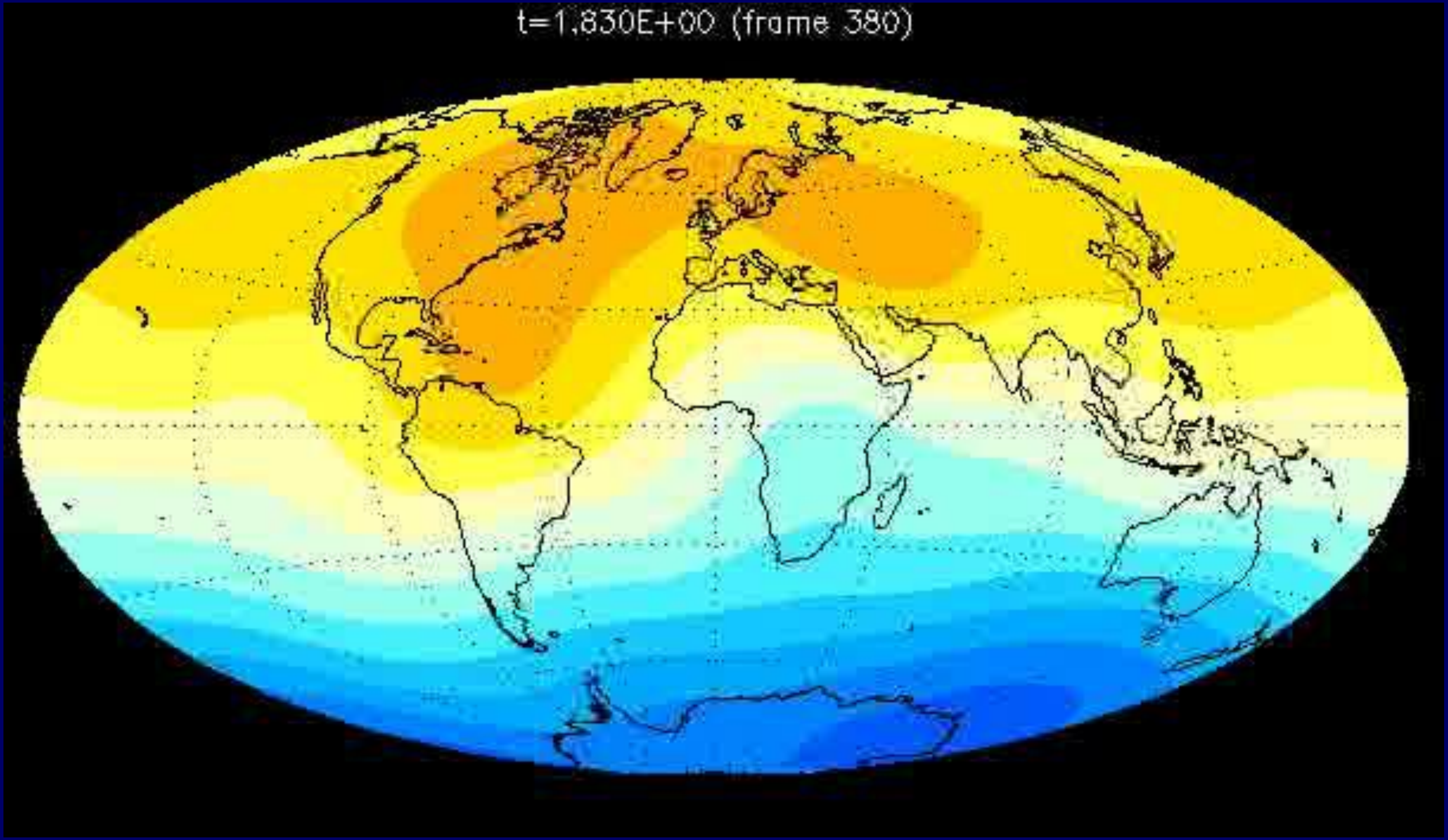


$E=10^{-3}$ $Ra/Ra_c=9$
$Pm=10$ $Rm=450$

Stochastic reversals found in some dynamo models. Systematic model studies require very long runs, possible only at moderate parameter values.

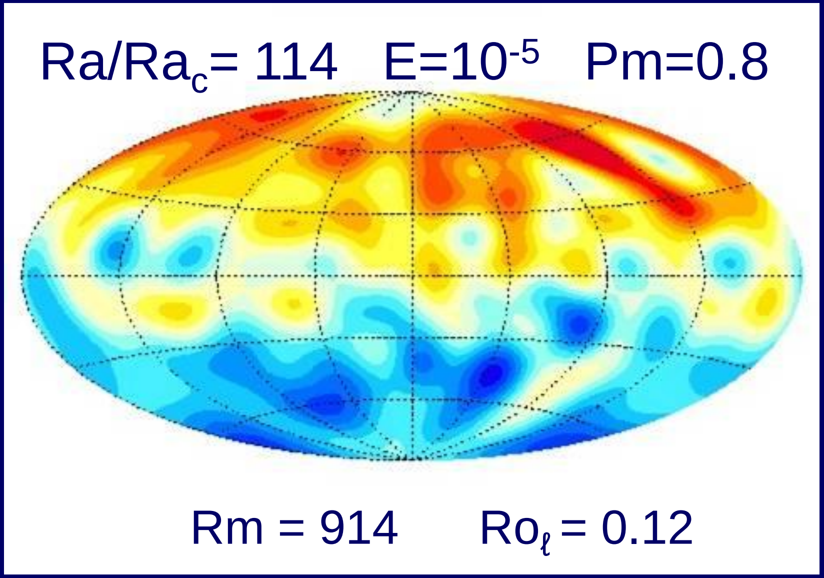
During reversal the true dipole moment TDM drops, whereas the non-dipole field is little affected. The ‘dipolarity’ D (= dipole field / total CMB field) decreases strongly as a consequence.

A simulated reversal



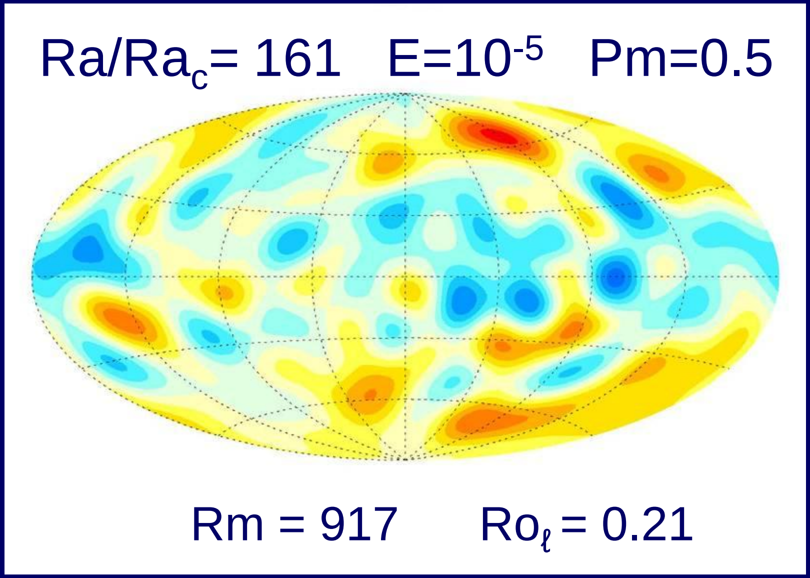
Magnetic field of dynamo model at Earth’s surface

Field morphology: two regimes



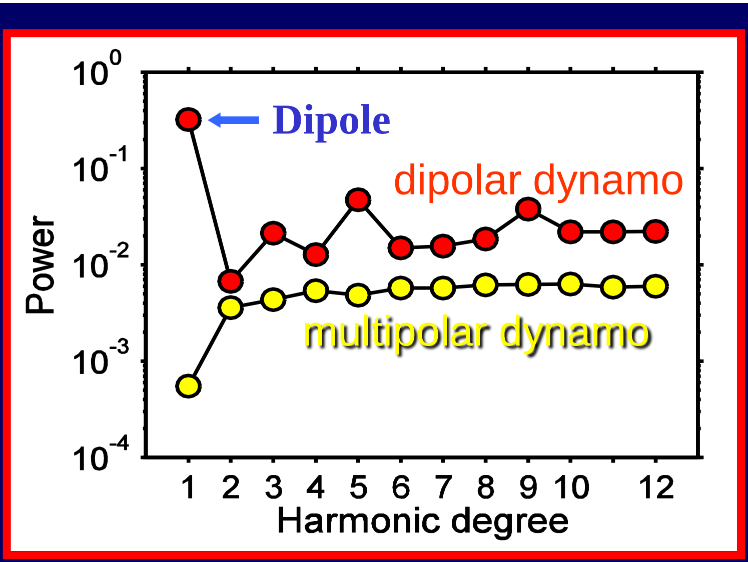
$Ra/Ra_c = 114$ $E=10^{-5}$ $Pm=0.8$

$Rm = 914$ $Ro_\ell = 0.12$



$Ra/Ra_c = 161$ $E=10^{-5}$ $Pm=0.5$

$Rm = 917$ $Ro_\ell = 0.21$

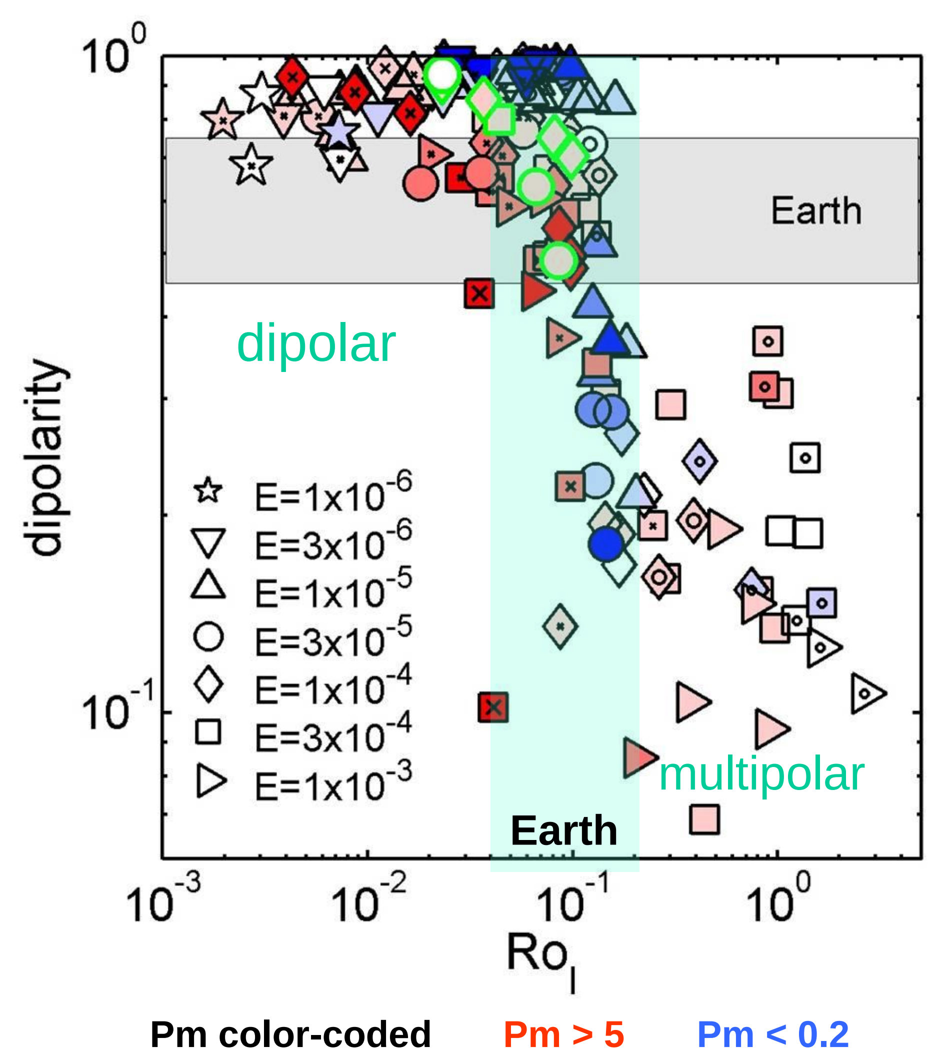


Power spectrum at dynamo surface nearly white from degrees n=3 to n>12.

Dipolar regime: dipole is clearly stronger than multipoles.

Multipolar regime: dipole is weaker than multipoles.

Morphology controlled by rotation



Inertial vs. Coriolis force:

Local Rossby number Ro_ℓ calculated with mean length scale ℓ in the kinetic energy spectrum

$$Ro_\ell = U/\Omega\ell$$

Regime boundary at $Ro_\ell \approx 0.12$

Scaling laws

- For convection-driven dynamos in rotating spheres, how do characteristic properties, in particular the characteristic magnetic field strength, vary with control parameters ?

- Does the dynamical regime change between parameter values accessible in numerical models and planetary values ?

- Do planetary dynamos and (some) stellar dynamos follow the same scaling rules ?

Elsasser number rule

Balance Coriolis – Lorentz (Magnetostrophic)

$$2\rho\,\Omega \quad u \quad \sim \quad j \quad B$$

Generalized Ohm‘s law: $j = \sigma\,(E + u \quad B)$

Ignore electric field

$$J \sim \sigma UB \qquad \Rightarrow \qquad 2\rho\Omega U \sim \sigma UB^2$$

Elsasser number $\Lambda = \sigma B^2/(2\rho\Omega) \sim 1$

$$B^2/2\mu_o \approx \rho\eta\Omega$$

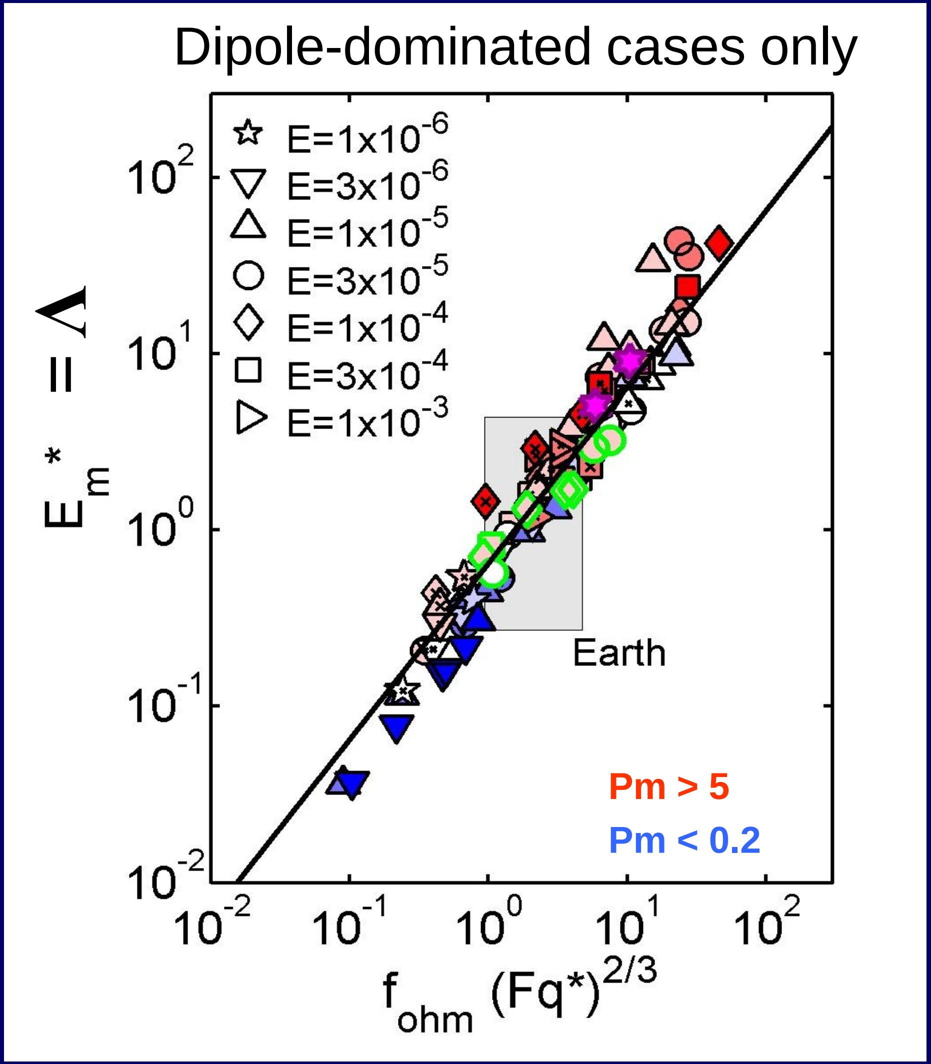
Power-controlled field strength

Hypothesis: The magnetic energy density depends on thermodynamically available energy flux, that is the part of the energy flux that can be converted to magnetic energy and can balance ohmic dissipation The field strength is independent of rotation rate, conductivity, viscosity,...

$$B^2/2\mu_o \sim f_{ohm}\,\rho^{1/3}\,(L/H_T\;q_c)^{2/3}$$

q_c: convected heat flux, $H_T = c_p/(\alpha g)$: temp. scale height, L: charact. radial length scale,
ρ: density, f_{ohm}: ratio ohmic dissipation / total dissipation

Scaling law vs. model results



q* : non-dimensional heat flux

F: thermodynamic efficiency

f_o: ohmic / total dissipation

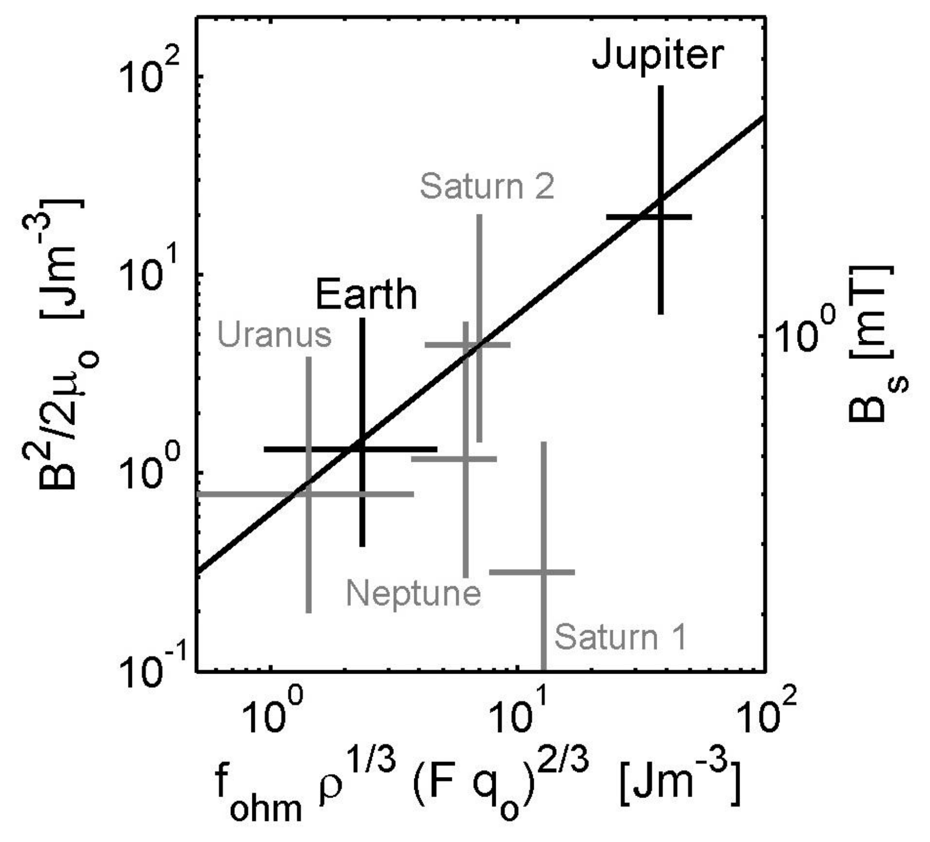
E_m*: non-dim. magnetic energy density = Elsasser #

$$E_m^* = 0.63\, f_o\, (Fq^*)^{2/3}$$

in dimensional form:

$$B^2 = 1.2\mu_o\, f_o\, \rho^{1/3}\, (Fq)^{2/3}$$

Comparison with planetary fields



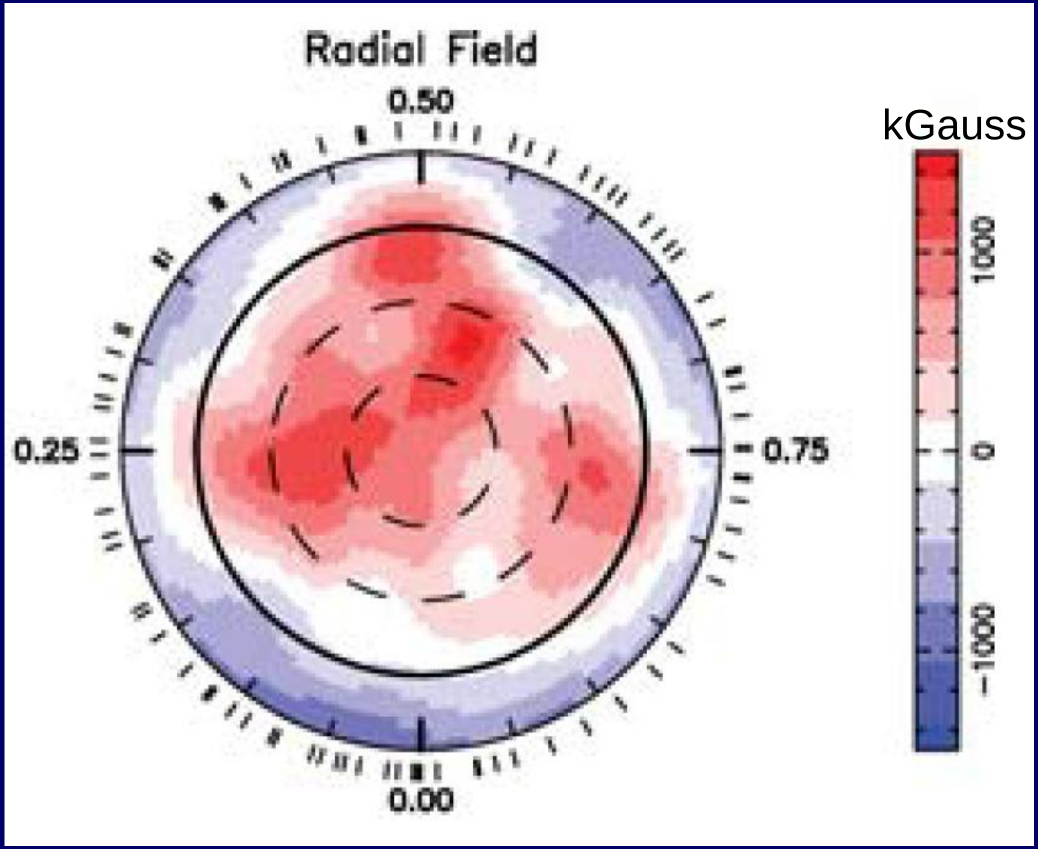
Field strength vs. heat flux

Assume ratio between total internal field and dipole field at CMB in range 4 - 15 (from dynamo models)

Saturn 1: $R_c/R_p = 0.6$

Saturn 2: $R_c/R_p = 0.4$

Magnetic fields of stars



V374 Peg Donati et al., MNRaS, 2006

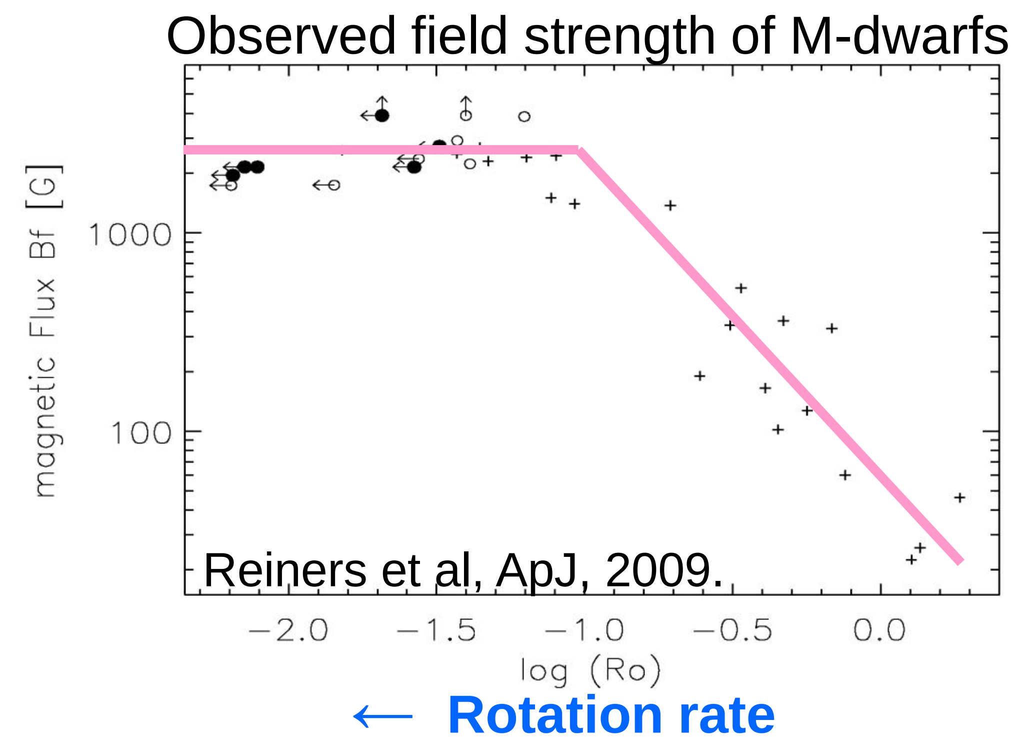
M = 0.28 M_{sun} Rotation period 0.45 d

Field mapped by Zeeman Doppler tomography

Slowly rotating solar-type stars: small-scale field

Rapidly rotating low-mass stars: significant large scale field component

M-dwarfs: surface field vs. rotation


Observed field strength of M-dwarfs

Reiners et al, ApJ, 2009.

← **Rotation rate**

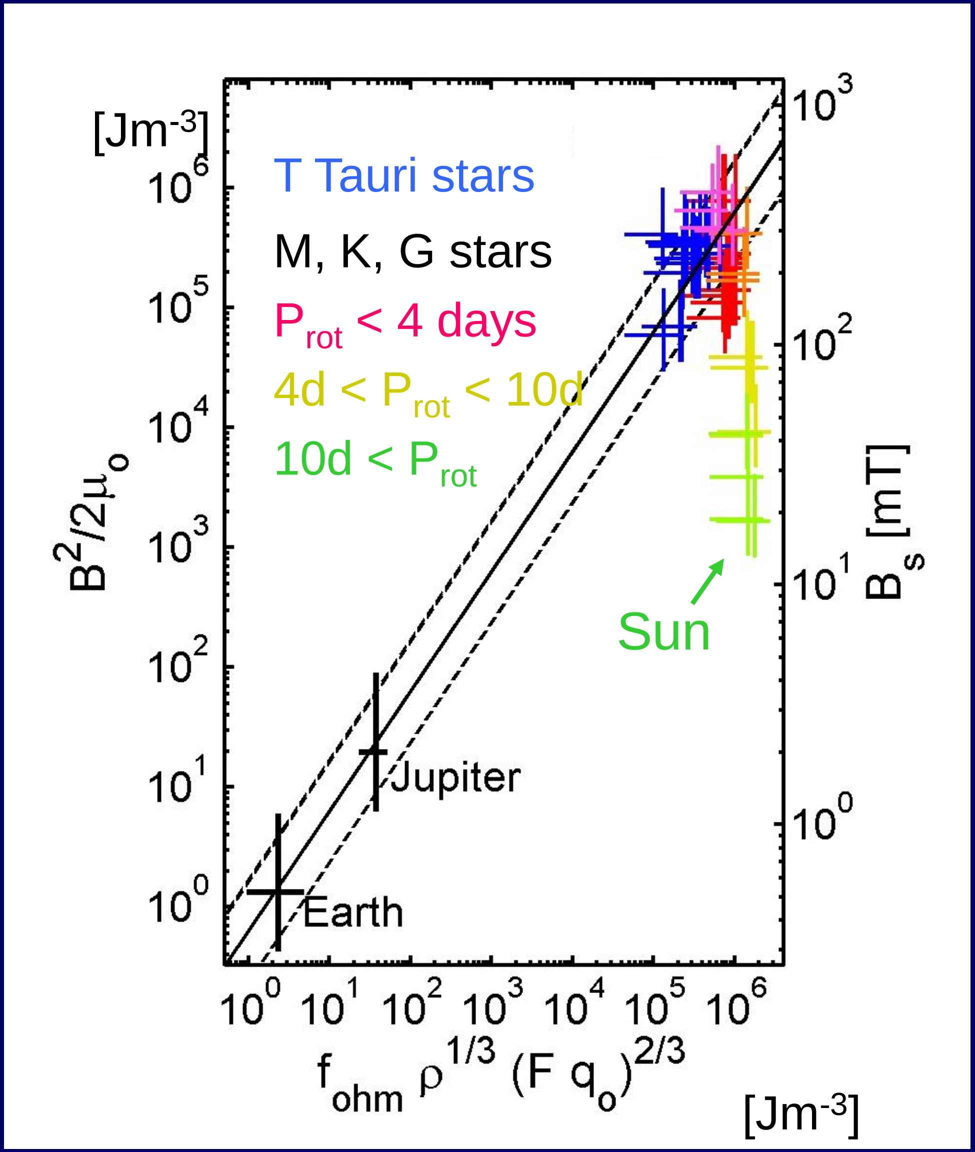
Magnetic field strength at surface of M-stars increases with rotation rate at high Rossby # but saturates at low Ro

(Reiners et al., ApJ, 2009)

The field is dominated by axial dipole at low Rossby number, but less so at high Ro

(Morin et al., MNRaS, 2008)

Comparison with planets and stars



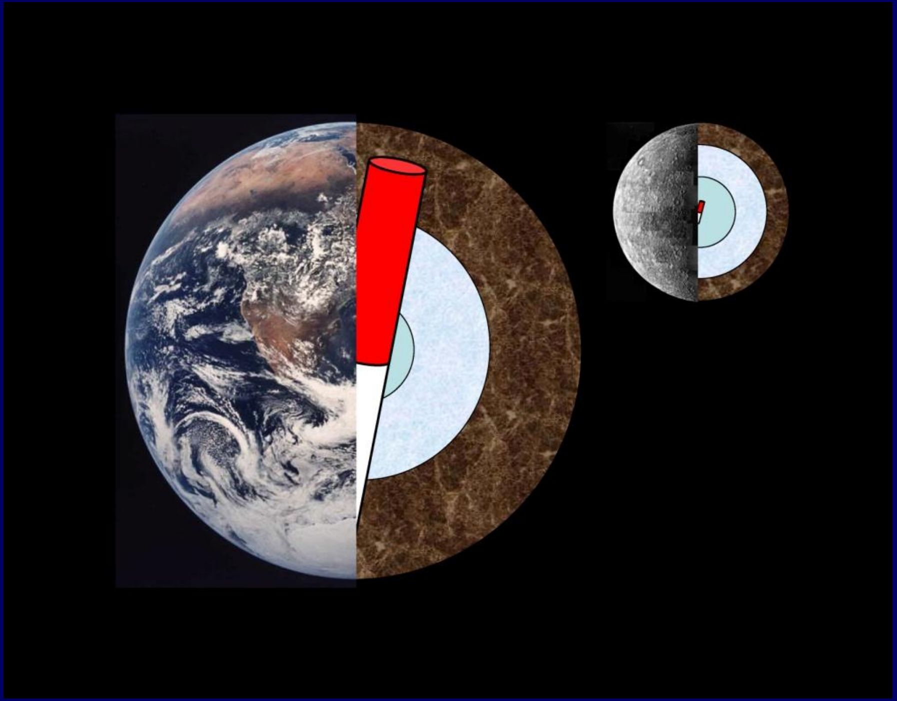
The observed fields of rapidly rotating low-mass stars agree with the prediction as well as that of Jupiter and Earth

⇒ confirmation for scaling law

⇒ dynamos in planets and (some) stars may be similar

Christensen et al, Nature, 2009

Mercury and its magnetic field



Slow rotation (T = 59d)

Large iron core

Core (partially) liquid from forced libration

Solid inner core likely, but size very uncertain

Magnetic field ~ 320 nT (1 % of Earth‘s strength) at surface

Dominantly dipolar with tilt < 5°

Quadrupole / Dipole ratio uncertain (0.1 – 0.5)

Mercury‘s dynamo

Why is Mercury‘s surface field so weak? Is it generated by an Earth-like dynamo?

Thermoelectric dynamo (Stevenson, 1987)

Remanent magnetisation with systematic depth variation of Curie surface (Aharonson et al., 2004)

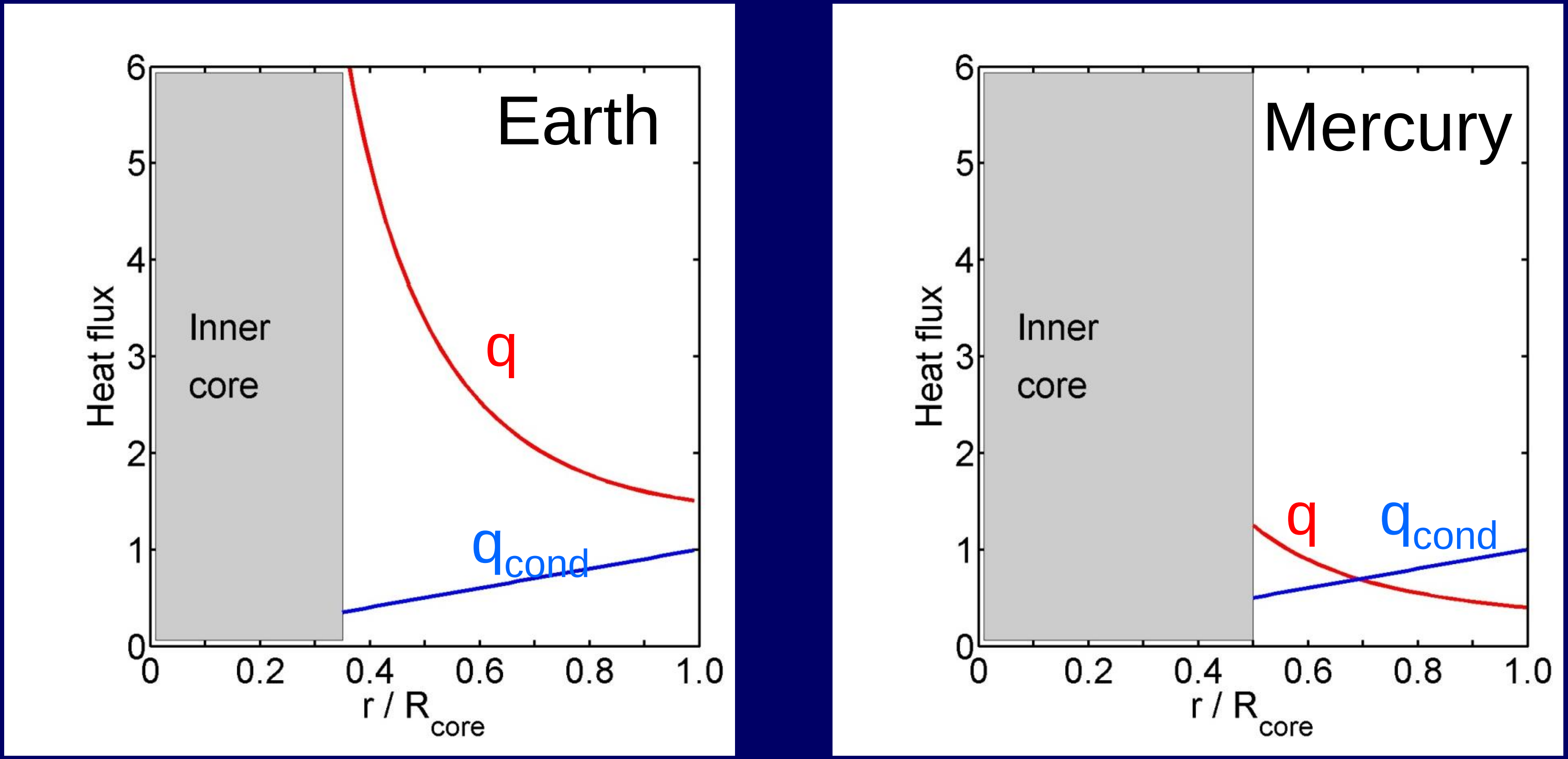
Dynamo affected by negative feedback from magnetospheric magnetic field (Glassmeier et al., 2007)

Dynamo in thin liquid shell with high toroidal/poloidal ratio (Stanley et al., 2005) or with low dipole/multipole ratio (Takahashi & Matsushima, 2006)

Dynamo with very small inner core (Heimpel et al. 2005)

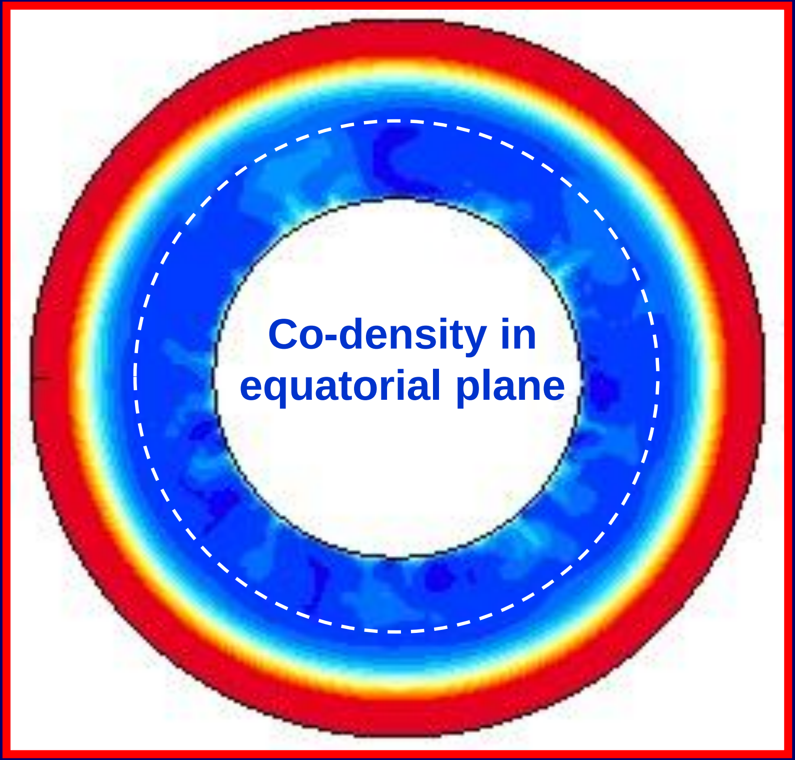
Dynamo below stably stratified layer at top of the core (Christensen, 2006; Christensen & Wicht, 2008)

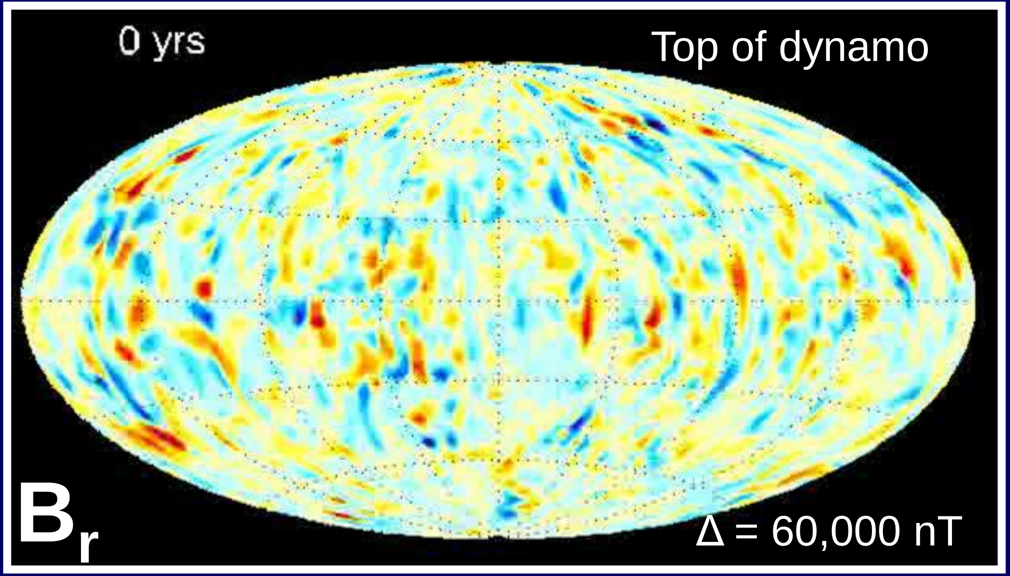
Heat flux vs. radius

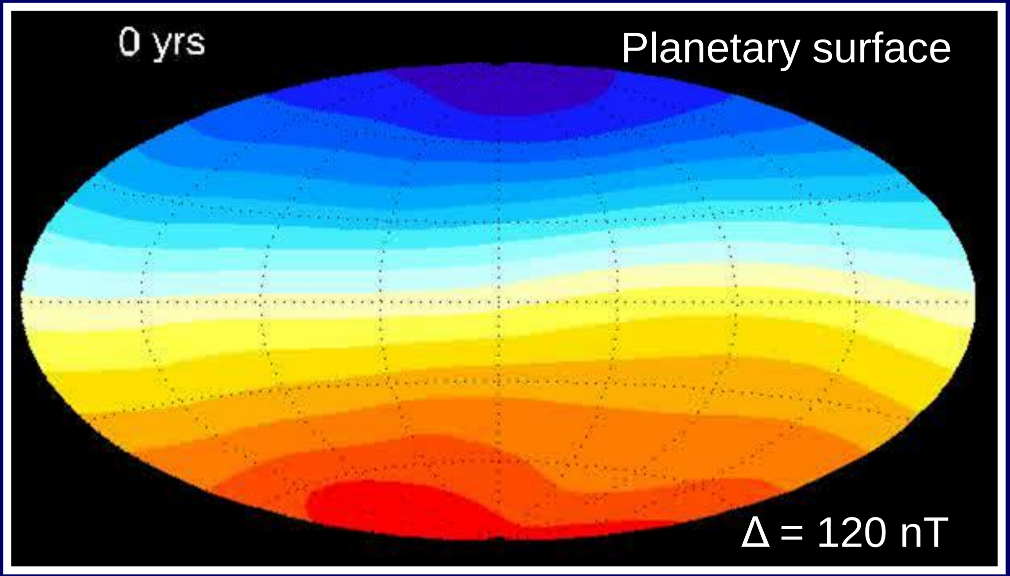


At Mercury's core-mantle boundary heat flux $q < q_{cond}$ likely

Dynamo below stable fluid layer





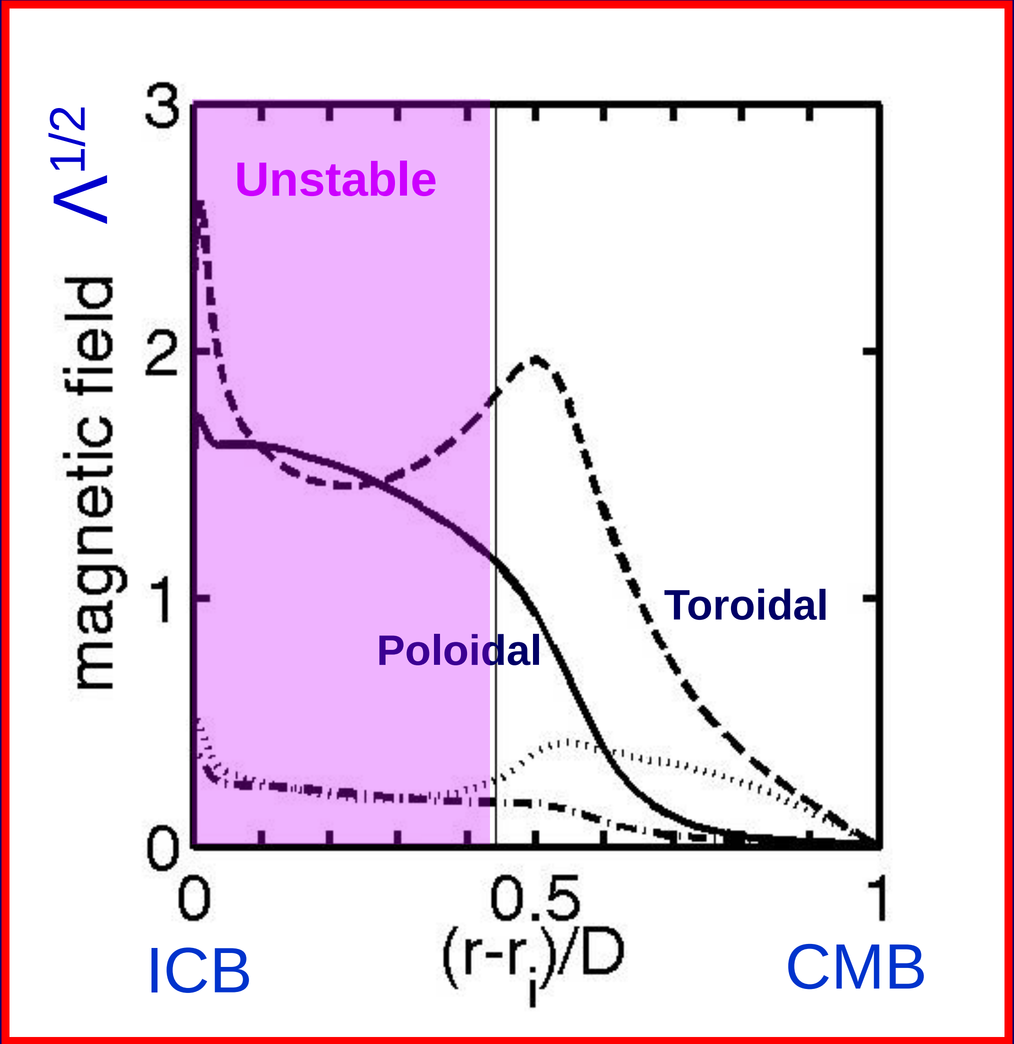


- Internal field strong & small-scale
- Surface field weak & large-scale

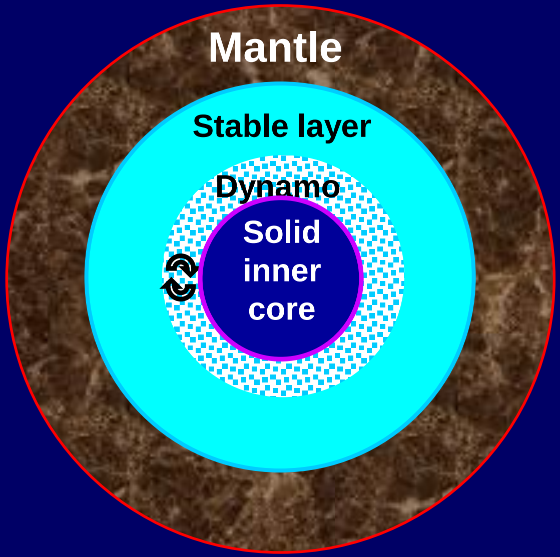
Christensen, Nature, 2006;
Christensen & Wicht, Icarus, 2008.

Magnetic field vs. radius inside core

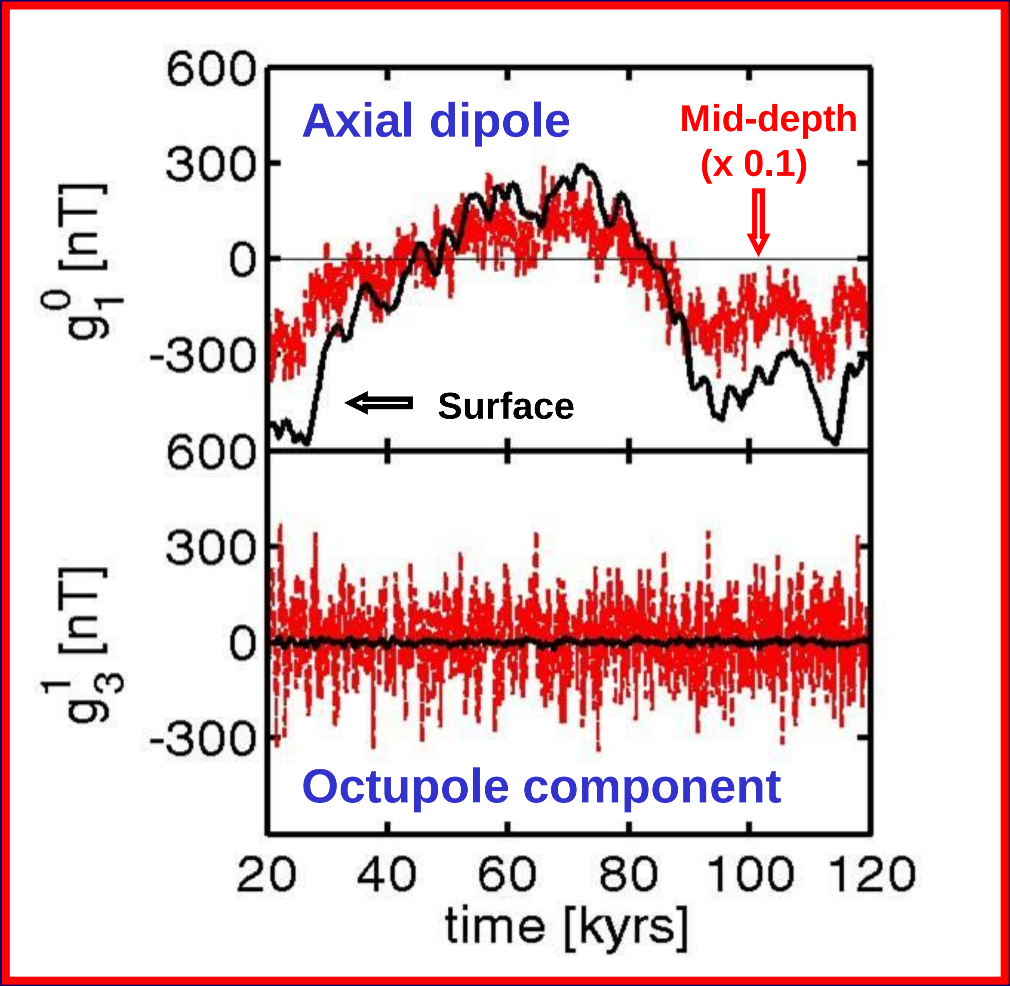
- Field in dynamo region is strong (250,000 nT)
- Poloidal field strength drops drastically from top of unstable layer to core-mantle boundary (1,000 nT)



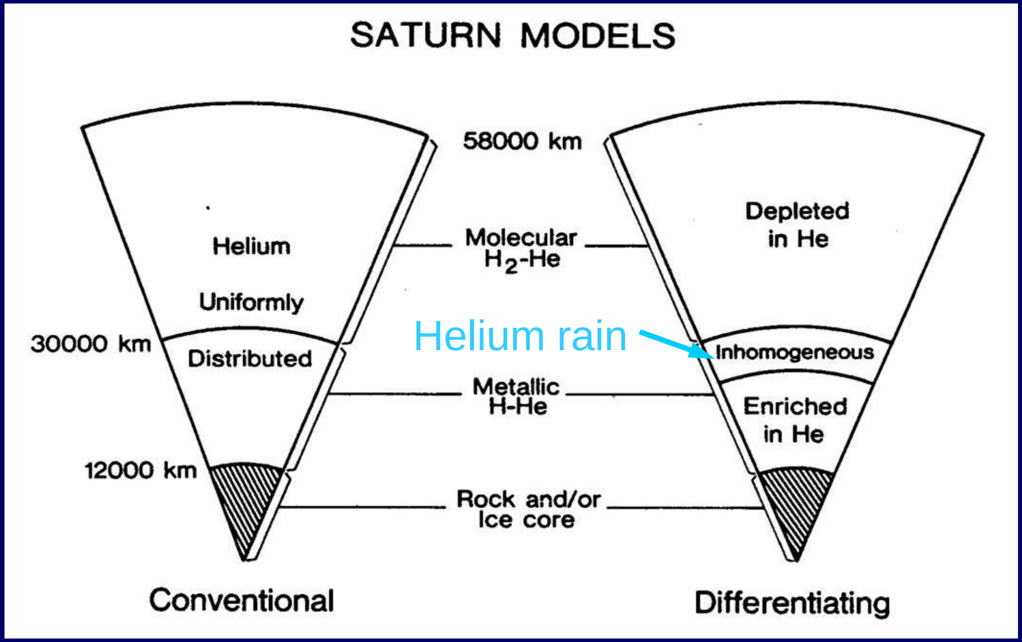
Skin effect



- Dynamo field must penetrate through stagnant conductor
- High frequencies damped.
- Higher multipoles fluctuate rapidly in dynamo region $\Rightarrow$ low amplitude at surface.
- Dipole varies slowly and penetrates stagnant layer.



Saturn‘s axisymmetric field

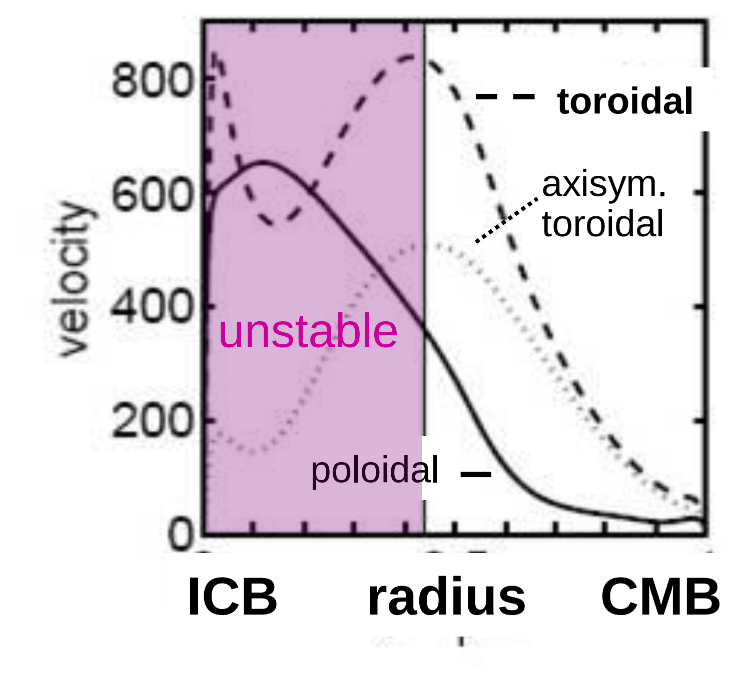


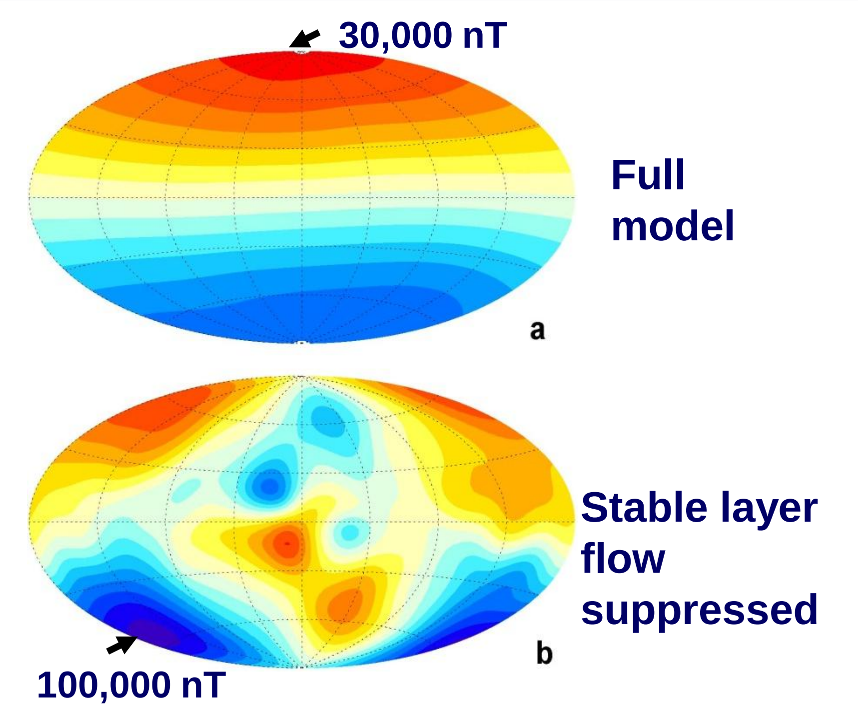
All field observations to date can be fitted within uncertainties by an axisymmetric model (g_1^0, g_2^0, g_3^0)

Layer of helium immiscibility at top of metallic region
$\Rightarrow$ stably stratified conducting region

Differential rotation in stable layer supresses non-axi-symmetric part of dynamo field (Stevenson, 1980, 1982)

Effect of „wind“ in stable layer





- **Strong toroidal flow in stable layer (mostly differential rotation)**
- **In dynamo models with a dipole dominated field inside the dynamo, the external field is strong and very axisymmetric**
- **When flow in stable layer is suppressed, the external field has significant non-axisymmetric components (Christensen & Wicht, 2008)**

Summary

- Magnetic Reynolds number much lower than in Sun: DNS
- Compressibility plays less role, rotation more than in Sun
- Scaling laws from numerical dynamo models:
  - Energy flux controls field strength
  - Rotation rate controls field morphology
- Rapidly rotating stars may follow same rules
- Stably stratified conducting layers may play important role for dynamos in Mercury, Saturn, Uranus & Neptune.

Discussion and homework

(1) Calculate the stand-off distance of the magnetopause for different planets, field configurations and solar wind conditions (handout)

(2) Could the present-day magnetic field of Mars be due to an unusual dynamo? Could Mercury‘s field be caused by remanent magnetisation of the crust?

(3) Discuss possible causes why Mars and Venus do not have an active dynamo at present.

(4) Could you think of a way to find out if Jupiter‘s dipole field has reversed in the past as Earth‘s field did?

(5) YOUR favorite subject for discussion.