Solar and stellar dynamo models

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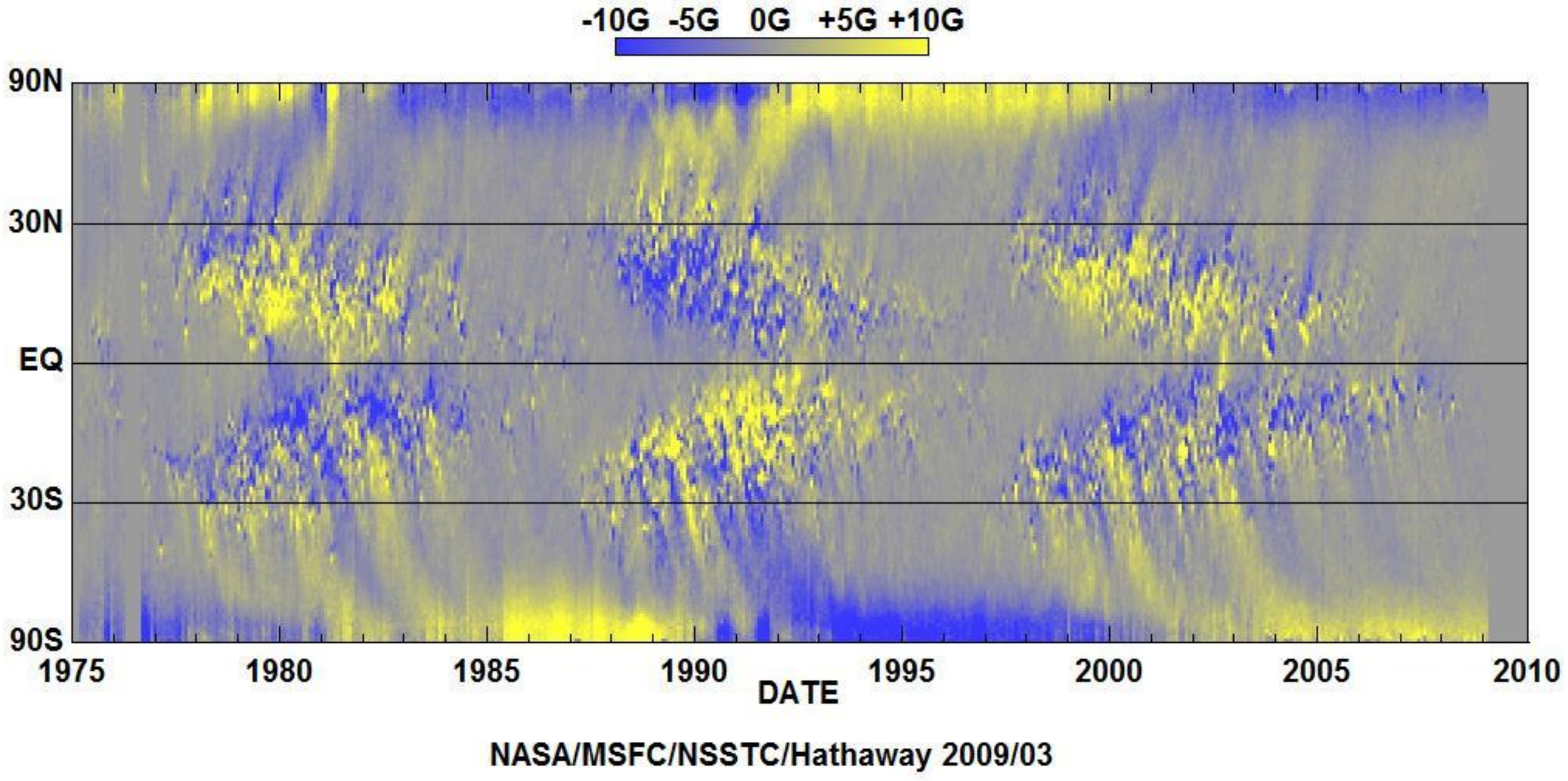
From MHD to simple dynamo models
Mean-field models
Babcock-Leighton models
Stochastic forcing
Cycle forecasting
Stellar dynamos

The solar magnetic cycle



The MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \ ,$$

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \frac{1}{\mu_0 \rho}(\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{1}{\rho}\nabla \cdot \boldsymbol{\tau} \ ,$$

$$\frac{\mathrm{D}e}{\mathrm{D}t} + (\gamma - 1)e\nabla \cdot \mathbf{u} = \frac{1}{\rho}\left[\nabla \cdot \Big((\chi + \chi_r)\nabla T\Big) + \phi_\nu + \phi_B\right] \ ,$$

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \ .}$$

Model setup

Solve MHD induction equation in spherical polar coordinates for large-scale (~R), axisymmetric magnetic field in a sphere of electrically conducting fluid:

$$\mathbf{B}(r,\theta,t) = \nabla \times (A(r,\theta,t)\hat{\mathbf{e}}_\phi) + B(r,\theta,t)\hat{\mathbf{e}}_\phi$$

Evolving under the influence of a steady, axisymmetric large-scale flow:

$$\mathbf{u}(r,\theta) = \frac{1}{\rho}\nabla \times (\Psi(r,\theta)\hat{\mathbf{e}}_\phi) + \varpi\Omega(r,\theta)\hat{\mathbf{e}}_\phi$$

Match solutions to potential field in $r > R$.

Kinematic axisymmetric dynamo

$$\frac{\partial A}{\partial t} = \underbrace{\eta \left(\nabla^2 - \frac{1}{\varpi^2} \right) A}_{\text{resistive decay}} - \underbrace{\frac{\mathbf{u}_p}{\varpi} \cdot \nabla (\varpi A)}_{\text{advection}} \qquad [+\ \text{Source}]\ ,$$

$$\frac{\partial B}{\partial t} = \underbrace{\eta \left(\nabla^2 - \frac{1}{\varpi^2} \right) B}_{\text{resistive decay}} + \underbrace{\frac{1}{\varpi} \frac{\partial (\varpi B)}{\partial r} \frac{\partial \eta}{\partial r}}_{\text{diamagnetic transport}} - \underbrace{\varpi \mathbf{u}_p \cdot \nabla \left(\frac{B}{\varpi} \right)}_{\text{advection}}$$

$$- \underbrace{B \nabla \cdot \mathbf{u}_p}_{\text{compression}} + \underbrace{\varpi (\nabla \times (A \hat{\mathbf{e}}_\phi)) \cdot \nabla \Omega}_{\text{shearing}}\ .$$

Kinematic axisymmetric dynamo

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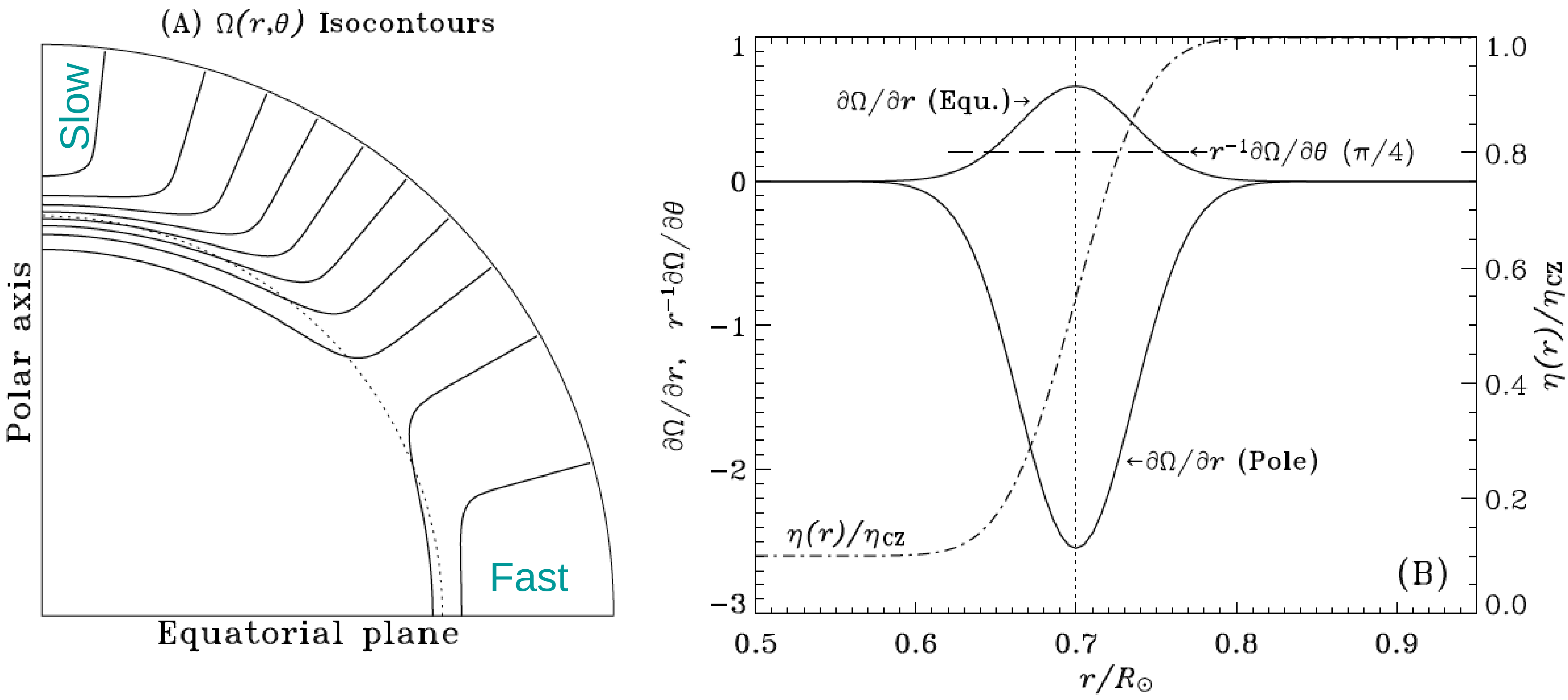
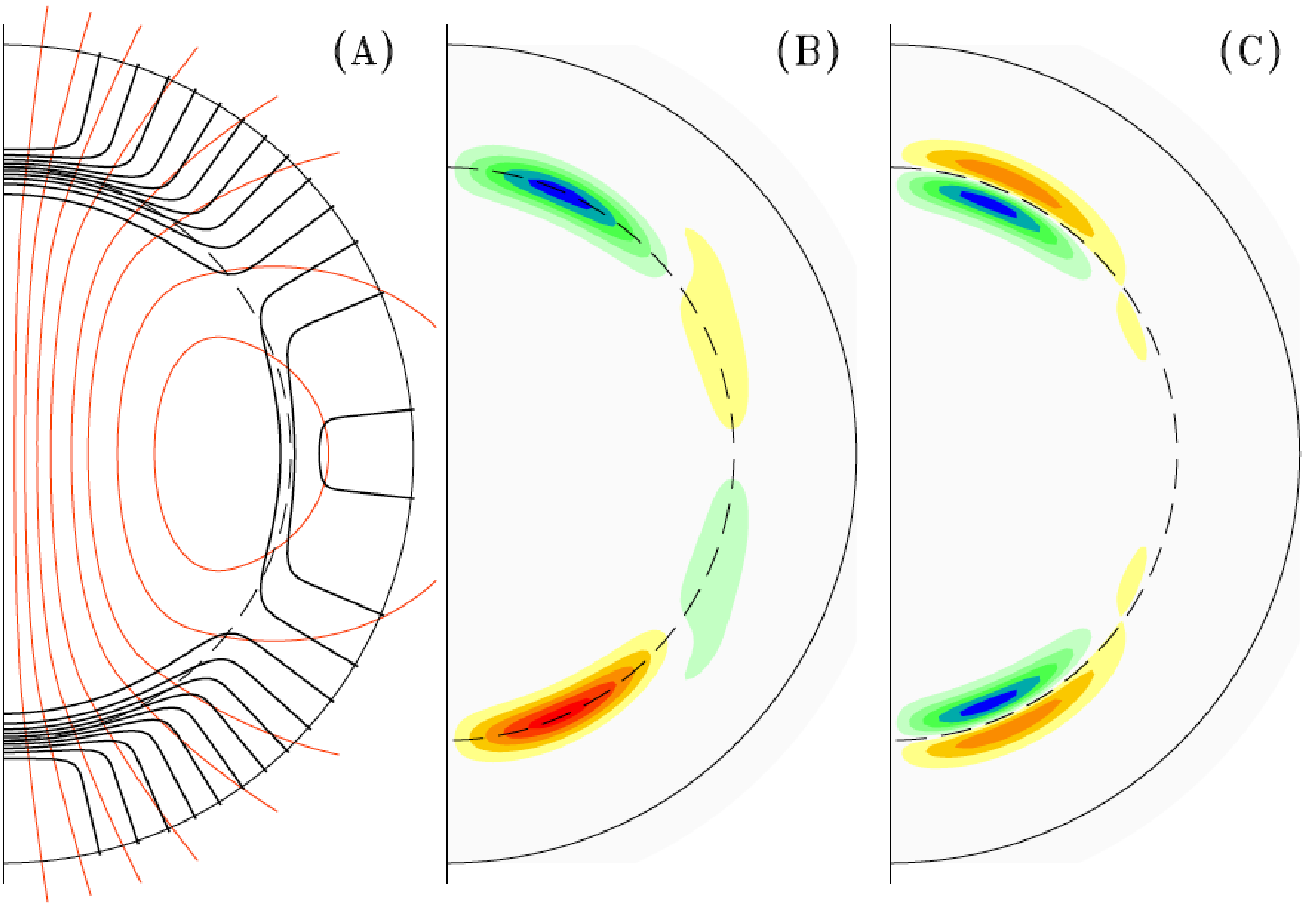
$$- \underbrace{B\nabla\cdot\mathbf{u}_p}_{\text{compression}} + \underbrace{\varpi(\nabla\times(A\hat{\mathbf{e}}_\phi))\cdot\nabla\Omega}_{\text{shearing}}\ .$$

Differential rotation



Shearing by axisymmetric differential rotation



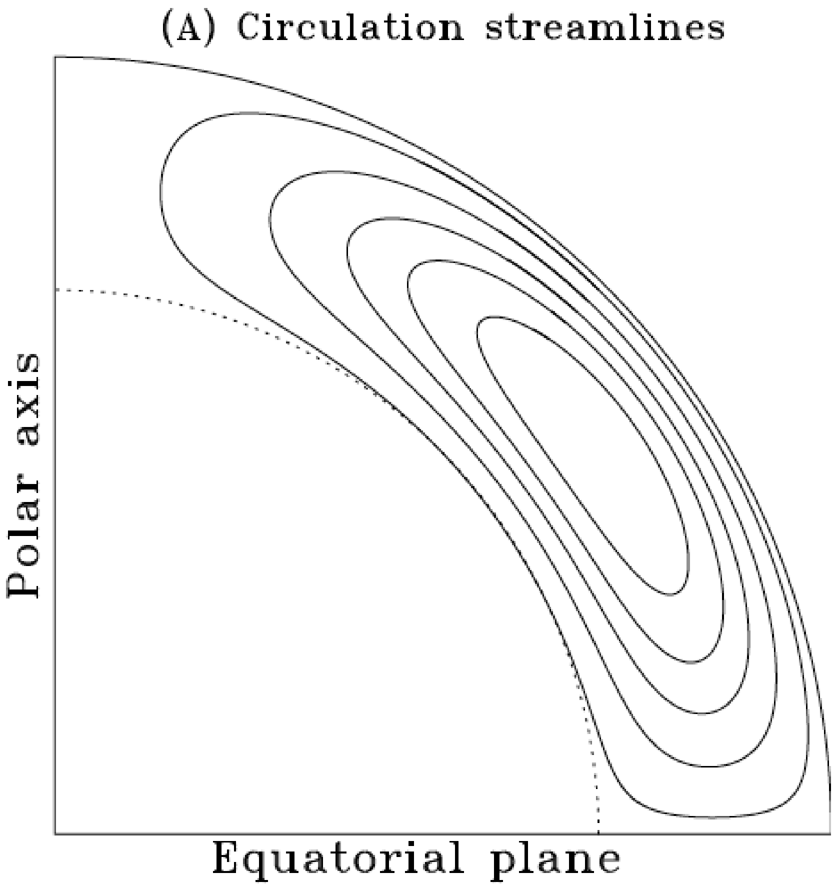
Kinematic axisymmetric dynamo

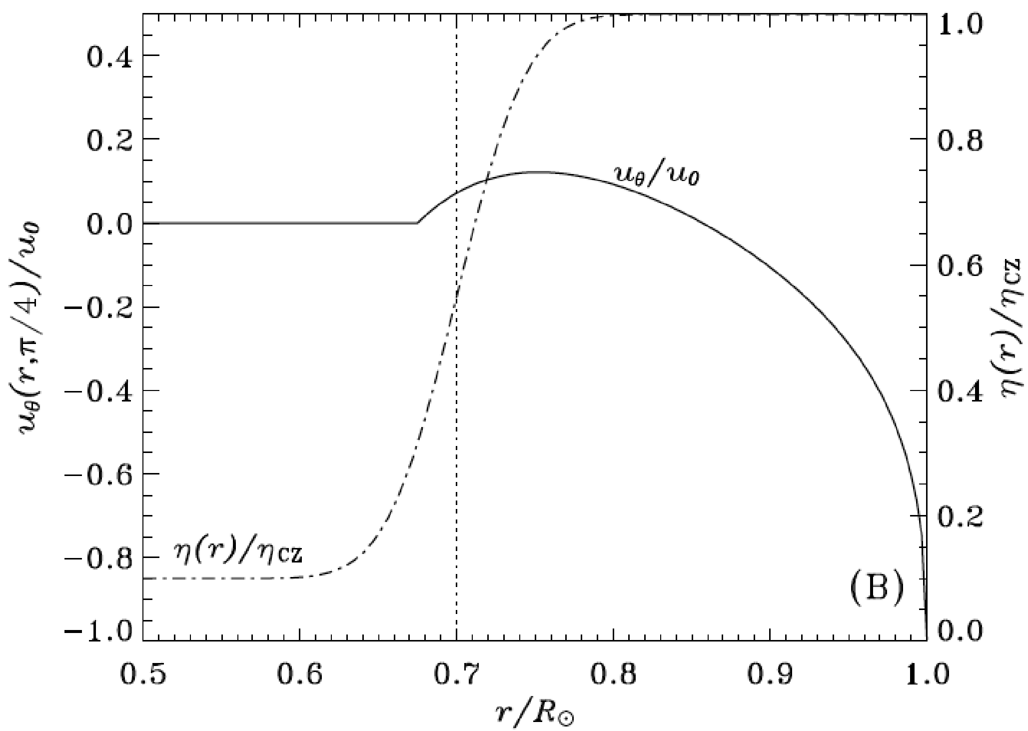
$$\frac{\partial A}{\partial t} = \underbrace{\eta \left(\nabla^2 - \frac{1}{\varpi^2} \right) A}_{\text{resistive decay}} \underbrace{- \frac{\mathbf{u}_p}{\varpi} \cdot \nabla (\varpi A)}_{\text{advection}} \qquad [+ \text{Source}] \ ,$$

$$\frac{\partial B}{\partial t} = \underbrace{\eta \left(\nabla^2 - \frac{1}{\varpi^2} \right) B}_{\text{resistive decay}} + \underbrace{\frac{1}{\varpi} \frac{\partial (\varpi B)}{\partial r} \frac{\partial \eta}{\partial r}}_{\text{diamagnetic transport}} \underbrace{- \varpi \mathbf{u}_p \cdot \nabla \left(\frac{B}{\varpi} \right)}_{\text{advection}}$$

$$- \underbrace{B \nabla \cdot \mathbf{u}_p}_{\text{compression}} + \underbrace{\varpi (\nabla \times (A \hat{\mathbf{e}}_\phi)) \cdot \nabla \Omega}_{\text{shearing}} \ .$$

Meridional circulation





Kinematic axisymmetric dynamos

$$\frac{\partial A}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)A}_{\text{resistive decay}} - \underbrace{\frac{\mathbf{u}_p}{\varpi}\cdot\nabla(\varpi A)}_{\text{advection}} \quad [+\ \text{Source}]\ ,$$

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$$- \underbrace{B\nabla\cdot\mathbf{u}_p}_{\text{compression}} + \underbrace{\varpi(\nabla\times(A\hat{\mathbf{e}}_\phi))\cdot\nabla\Omega}_{\text{shearing}}\ .$$

Poloidal source terms

1. Turbulent alpha-effect
2. Active region decay (Babcock-Leighton mechanism)
3. Helical hydrodynamical instabilities
4. Magnetohydrodynamical instabilities
   (flux tubes, Spruit-Tayler)

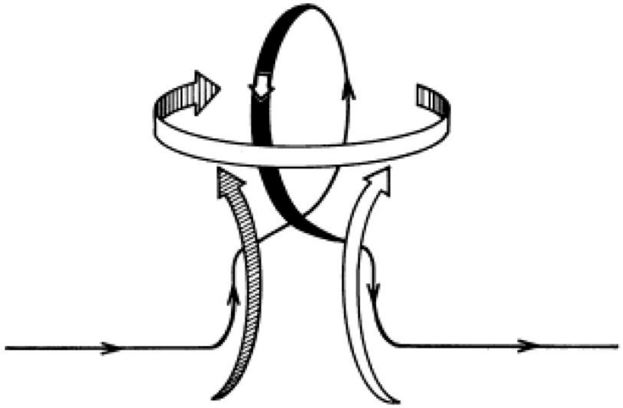
Mean-field electrodynamics and dynamo models

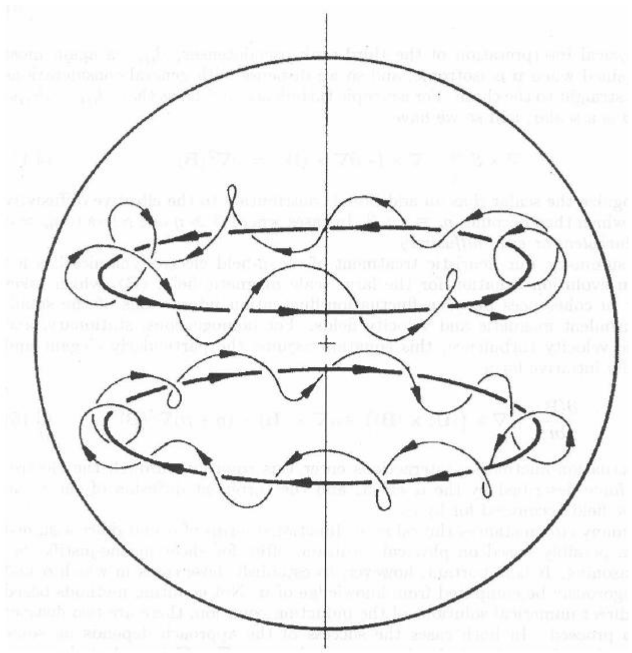
[see also Rempel chapter, vol. 1]

The basic idea

[Parker, E.N., ApJ, **122**, 293 (1955)]

Cyclonic convective updraft/downdrafts acting on a pre-existing toroidal magnetic field will twist the fieldlines into poloidal planes (in the high R*m* regime)





The collective effect of many such events is the production of an electrical current flowing parallel to the background toroidal magnetic field; such a current system contributes to the production of a poloidal magnetic component

The turbulent EMF (1)

Separate flow and magnetic field into large-scale, « laminar » component, and a small-scale, « turbulent » component:

$$\mathbf{U} = \langle \mathbf{U} \rangle + \mathbf{u} \qquad\qquad \mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}$$

Assume now that a good separation of scales exists between these two components, so that

$$\langle \mathbf{u} \rangle = \langle \mathbf{b} \rangle = 0.$$

Substitute into MHD induction equation and apply averaging operator:

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times \big(\langle \mathbf{U} \rangle \times \langle \mathbf{B} \rangle \big) + \nabla \times \boldsymbol{\mathcal{E}} + \eta \nabla^2 \langle \mathbf{B} \rangle$$

with : $\boldsymbol{\mathcal{E}} = \langle \mathbf{u} \times \mathbf{b} \rangle$ TURBULENT ELECTROMOTIVE FORCE !

The turbulent EMF (2)

Now, the whole point of the mean-field approach is NOT to have to deal explicitly with the small scales; since the PDE for **b** is linear, with the term $\nabla \times \left(\mathbf{u} \times \langle \mathbf{B} \rangle\right)$acting as a source; therefore there must exit a linear relationship between **b** and **B**, and also between **B** and $\langle \mathbf{u} \times \mathbf{b} \rangle$
We develop the mean emf as

$$\mathcal{E}_i = \alpha_{ij} \langle B \rangle_j + \beta_{ijk} \partial_k \langle B \rangle_j + \gamma_{ijkl} \partial_j \partial_k \langle B \rangle_l + \cdots,$$

Where the various tensorial coefficients can be a function of $\langle \mathbf{U} \rangle$
of the statistical properties of **u**, on the magnetic diffusivity, but
NOT of $\langle \mathbf{B} \rangle$

Specifying these closure relationships is the crux of the mean-field approach

The alpha-effect (1)

Consider the first term in our EMF development:

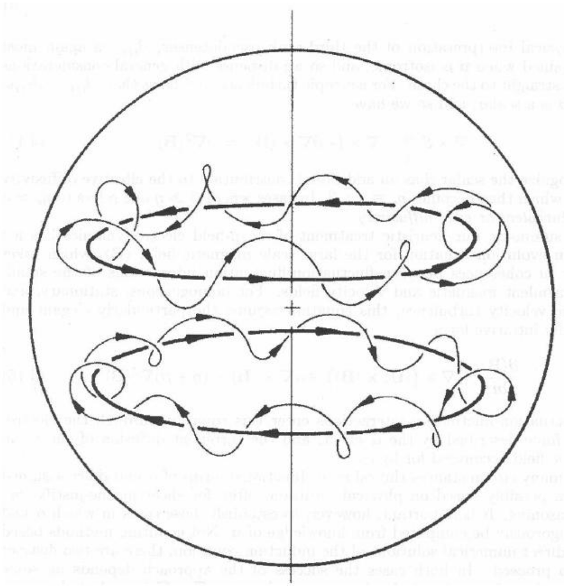
$$\mathcal{E}_i^{(1)} = \alpha_{ij} \langle B \rangle_j$$

If **u** is an isotropic random field, there can be no preferred direction in space, and the alpha-tensor must also be isotropic:

$$\alpha_{ij} = \alpha \delta_{ij}$$

This leads to:

$$\boldsymbol{\mathcal{E}}^{(1)} = \alpha \langle \mathbf{B} \rangle$$

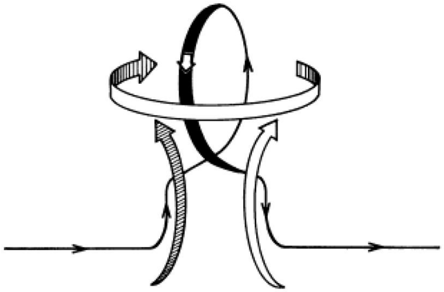


The mean turbulent EMF is parallel to the mean magnetic field!
This is called the « **alpha-effect** »

The alpha-effect (2)

Computing the alpha-tensor requires a knowledge of the statistical properties of the turbulent flow, more precisely the cross-correlation between velocity components; under the assumption that **b** << **B**, if the turbulence is only mildly anisotropic and inhomogeneous, the so-called Second-Order Correlation Approximation leads to

$$\alpha = -\frac{1}{3}\tau_c \langle \mathbf{u} \cdot (\nabla \times \mathbf{u}) \rangle \,, \qquad [\mathrm{m\,s^{-1}}]$$



where τ_c is the correlation time for the turbulence.

The alpha-effect is proportional to the fluid helicity!

If the mild-anisotropy is provided by rotation, and the inhomogeneity by stratification, then we have

$$\alpha = -\frac{1}{3}\tau_c^2 u^2 \mathbf{\Omega} \cdot \nabla \ln(\rho u)$$

Turbulent diffusivity

Turn now to the second term in our EMF development:

$$\mathcal{E}_i^{(2)} = \beta_{ijk}\partial_k\langle B\rangle_j$$

In cases where **u** is isotropic, we have $\beta_{ijk} = \beta\epsilon_{ijk}$, and thus:

$$\nabla \times \mathcal{E}^{(2)} = \nabla \times \left(-\beta\nabla \times \langle\mathbf{B}\rangle\right) = \beta\nabla^2\langle\mathbf{B}\rangle.$$

The mathematical form of this expression suggests that β can be interpreted as a **turbulent diffusivity** of the large-scale field. for homogeneous, isotropic turbulence with correlation time τ_c it can be shown that

$$\beta = \frac{1}{3}\tau_c\langle\mathbf{u}^2\rangle\ , \qquad [\mathrm{m}^2\mathrm{s}^{-1}]$$

This result is expected to hold also in mildly anisotropic, mildly inhomogeneous turbulence. In general, $\beta \gg \eta$

Scalings and dynamo numbers

Length scale: solar/stellar radius: R

Time scale: turbulent diffusion time: $\tau = R^2/\eta_0$

$$\frac{\partial A}{\partial t} = \eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)A - \frac{\mathrm{R}_m}{\varpi}\mathbf{u}_p\cdot\nabla(\varpi A) + \boxed{C_\alpha \alpha B},$$

$$\frac{\partial B}{\partial t} = \eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)B + \frac{1}{\varpi}\frac{\mathrm{d}\eta}{\mathrm{d}r}\frac{\partial(\varpi B)}{\partial r}$$

$$-\mathrm{R}_m\varpi\nabla\cdot\left(\frac{B}{\varpi}\mathbf{u}_p\right) + \boxed{C_\Omega\varpi(\nabla\times A)\cdot(\nabla\Omega)} + \boxed{C_\alpha\hat{\mathbf{e}}_\phi\cdot\nabla\times[\alpha\nabla\times(A\hat{\mathbf{e}}_\phi)]}$$

Three dimensionless groupings have materialized:

$$C_\alpha = \frac{\alpha_0 R}{\eta_0},$$

$$C_\Omega = \frac{\Omega_0 R^2}{\eta_0},$$

$$\mathrm{R}_m = \frac{u_0 R}{\eta_0},$$

The mean-field zoo

α^2 dynamo

The alpha-effect is the source of both poloidal and toroidal magnetic components; works without a large-scale flow! planetary dynamos are believed to be of this kind.

$\alpha\Omega$ dynamo

Rotational shear is the sole source of the toroidal component; the alpha-effect is the source of only the poloidal component. the solar dynamo is believed to be of this kind.

$\alpha^2\Omega$ dynamo

Both the alpha-effect and differential rotation shear contribute to toroidal field production; stellar dynamos could be of this kind if differential rotation is weak, and/or if dynamo action takes place in a very thin layer.

Linear alpha-Omega solutions (1)

Solve the axisymmetric kinematic mean-field alpha-Omega dynamo equation in a differentially rotating sphere of electrically conducting fluid, embedded in vacuum; in spherical polar coordinates:

$$\frac{\partial A}{\partial t} = \left(\nabla^2 - \frac{1}{\varpi^2} \right) A + C_\alpha B \ ,$$

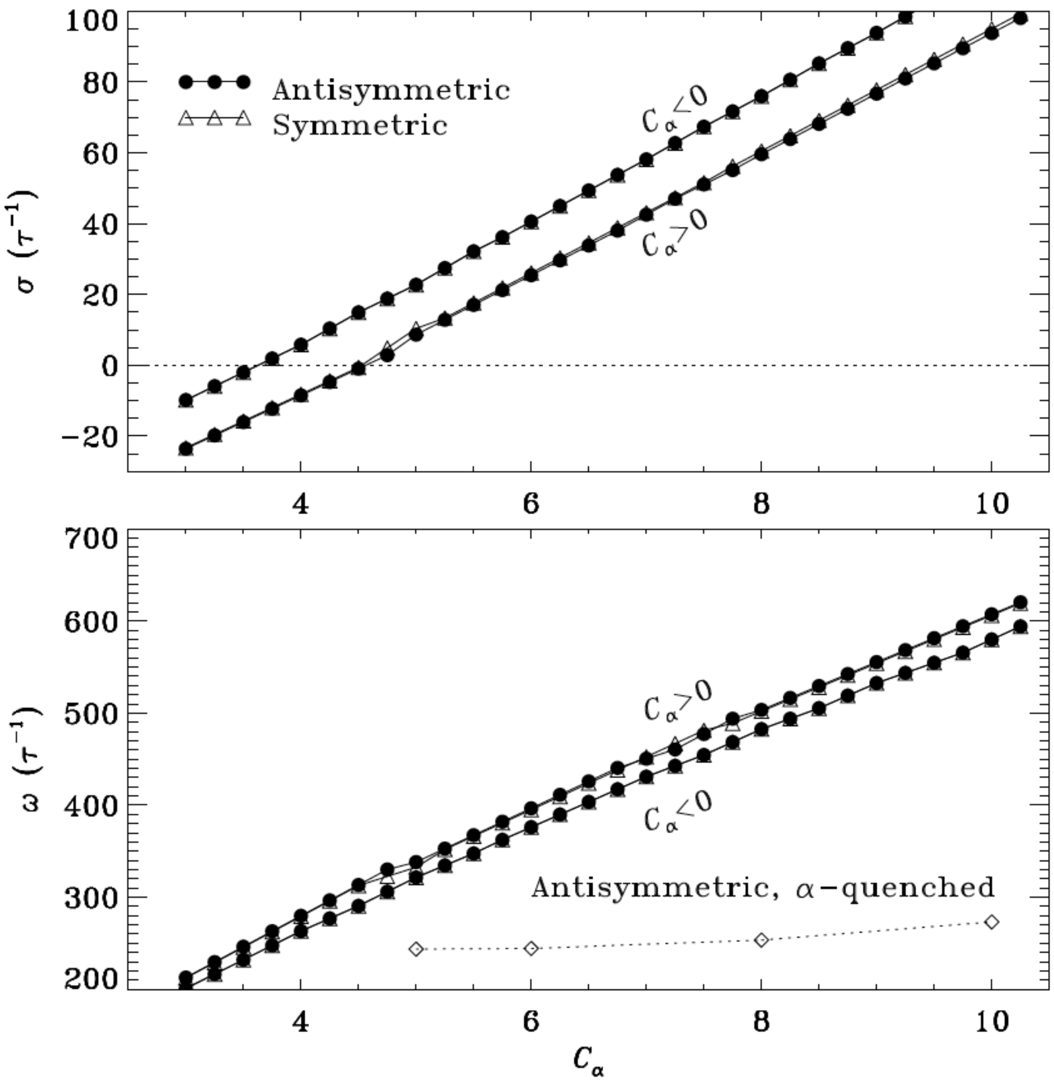
$$\frac{\partial B}{\partial t} = \left(\nabla^2 - \frac{1}{\varpi^2} \right) B + C_\Omega \varpi (\nabla \times A \hat{\mathrm{e}}_\phi) \cdot (\nabla \Omega) + \frac{1}{\varpi} \frac{\mathrm{d}\eta}{\mathrm{d}r} \frac{\partial (\varpi B)}{\partial r}$$

Choice of alpha: $\alpha(r, \theta) = f(r) g(\theta) \ ,$

$$g(\theta) = \cos\theta$$

$$f(r) = \frac{1}{4} \left[1 + \mathrm{erf} \left(\frac{r - r_c}{w} \right) \right] \left[1 - \mathrm{erf} \left(\frac{r - 0.8}{w} \right) \right]$$

Linear alpha-Omega solutions (2)



$$\begin{bmatrix} A(r,\theta,t) \\ B(r,\theta,t) \end{bmatrix} = \begin{bmatrix} a(r,\theta) \\ b(r,\theta) \end{bmatrix} e^{\lambda t}$$

$$\lambda = \sigma + i\omega$$

The growth rate, frequency, and eigenmode morphology are completely determined by the product of the two dynamo numbers

$$\boxed{D \equiv C_\alpha \times C_\Omega = \frac{\alpha_0 \Omega_0 R^3}{\eta_0^2}}$$

Linear alpha-Omega solutions (3)

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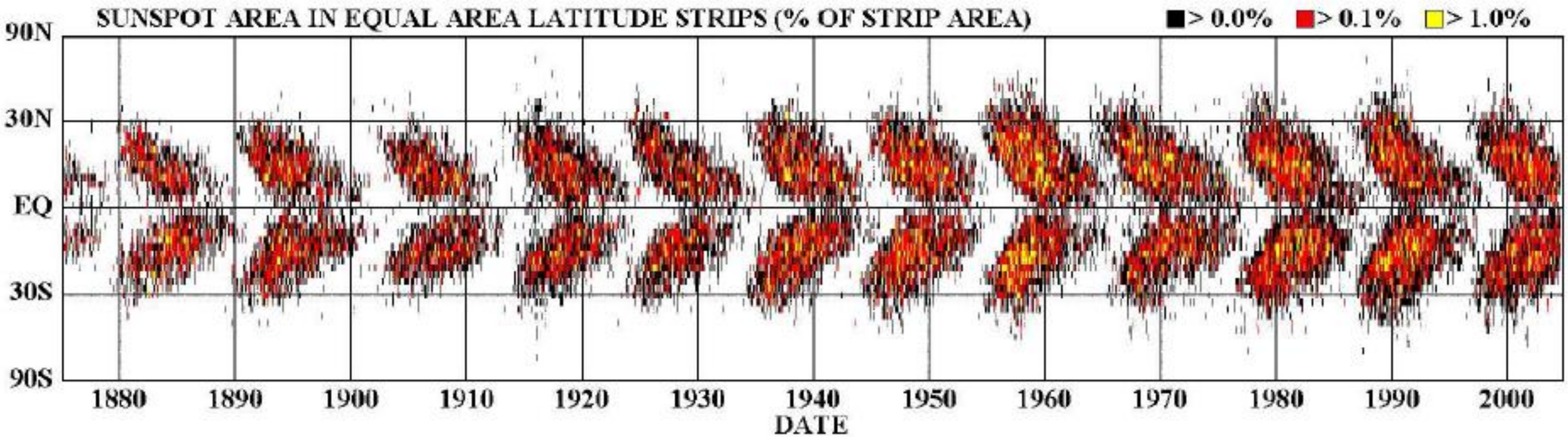
Positive alpha-effect

Negative alpha-effect

Linear alpha-Omega solutions (4)

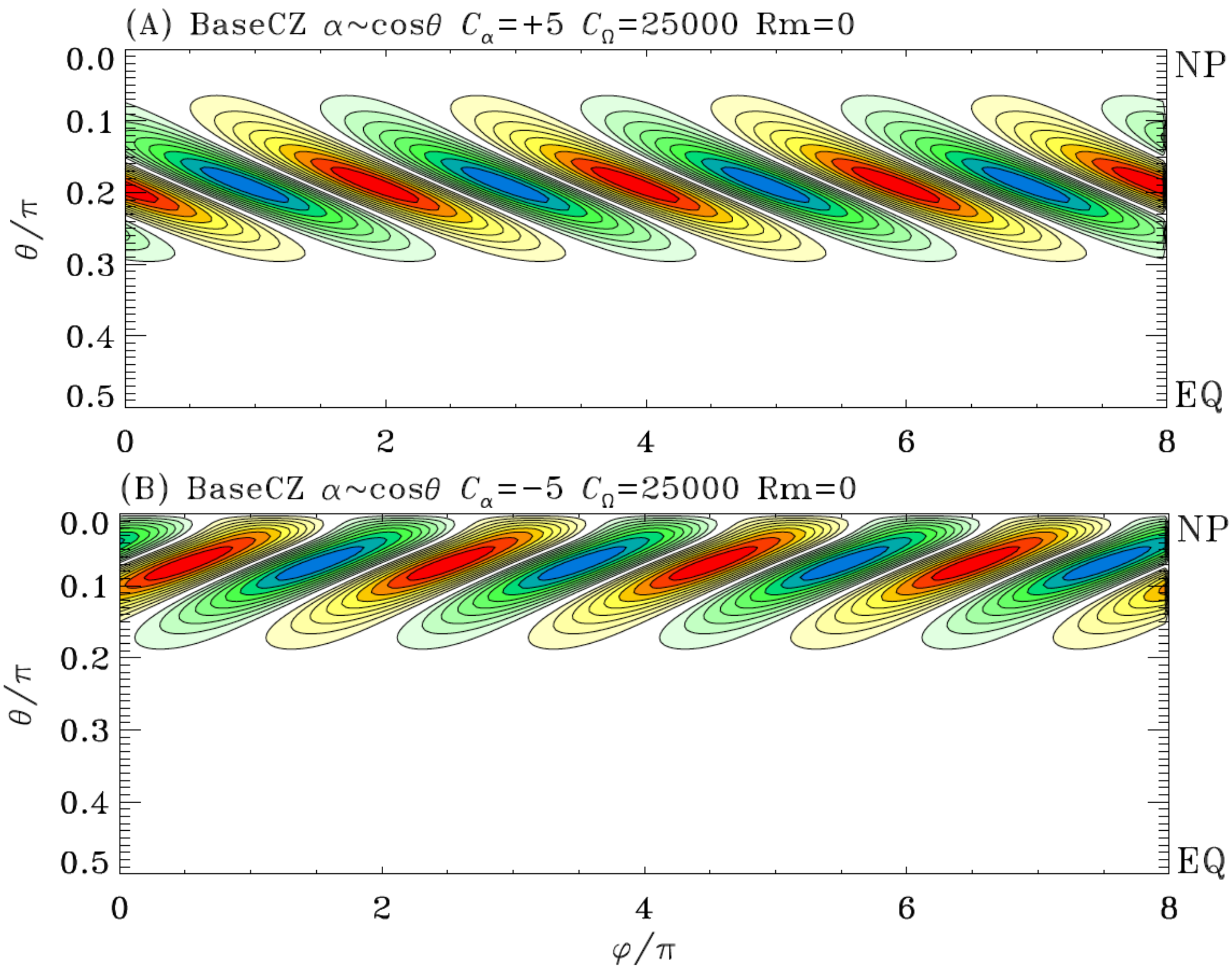
Time-latitude « butterfly » diagram

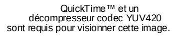


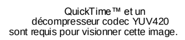
[http://solarscience.msfc.nasa.gov/images/bfly.gif]

Equivalent in axisymmetric numerical model: constant-*r* cut at *r/R*=0.7, versus latitude (vertical) and time (horizontal)

Linear alpha-Omega solutions (5)







Stix-Yoshimura sign rule

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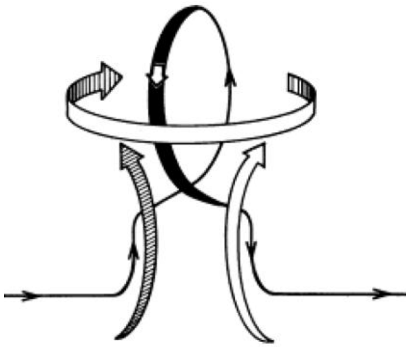
Propagation direction of « dynamo waves » given by:

$$\boxed{\mathbf{s} = \alpha \nabla \Omega \times \hat{\mathbf{e}}_{\phi}}$$

Nonlinear models: alpha-quenching (1)

We expect that the Lorentz force should oppose the cyclonic motions giving rise to the alpha-effect;



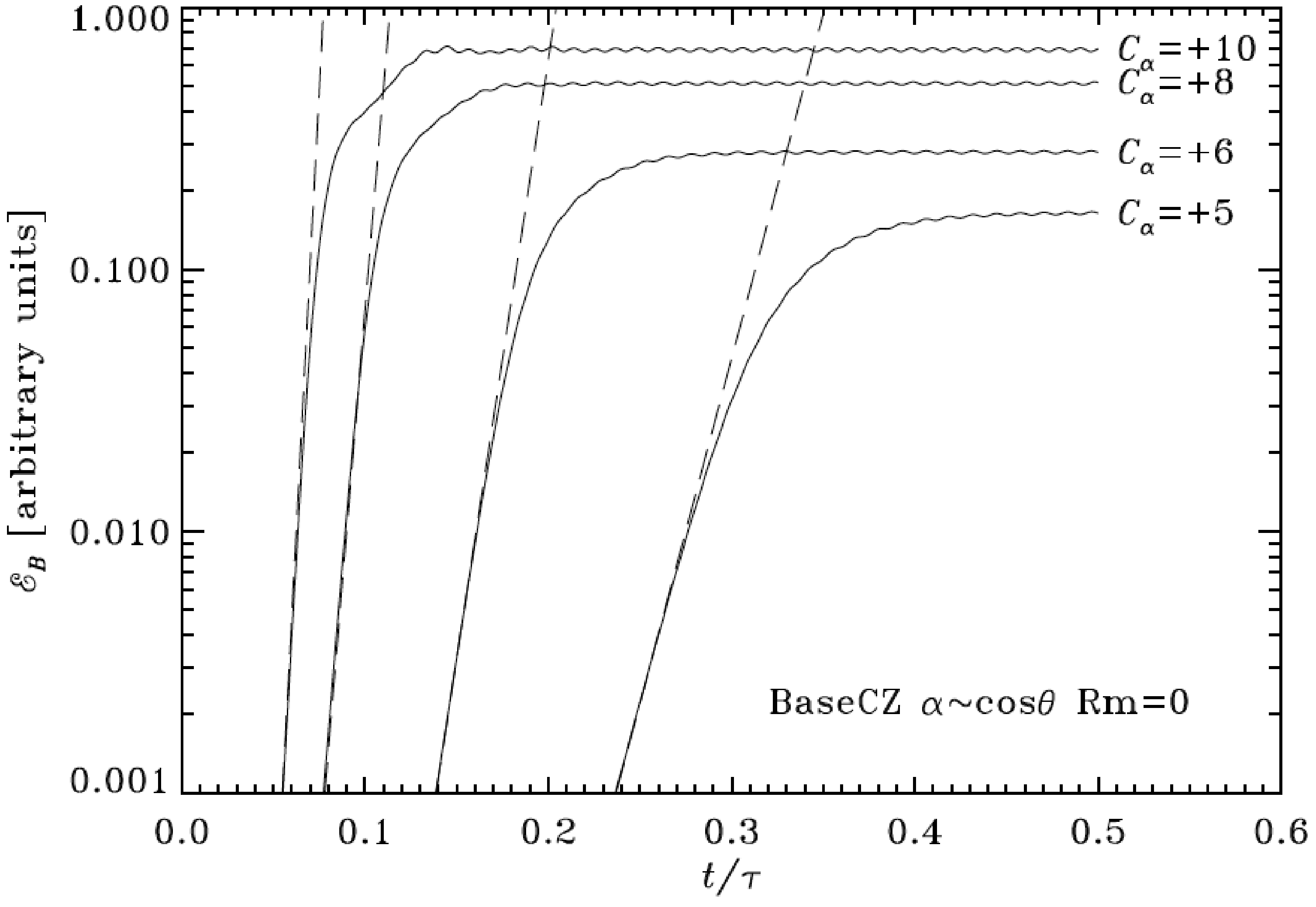
We also expect this to become important when the magnetic energy becomes comparable to the kinetic energy of the turbulent fluid motions, i.e.:

$$\frac{B_{\rm eq}^2}{2\mu_0} = \frac{\rho u_t^2}{2} \quad \rightarrow \quad B_{\rm eq} = u_t\sqrt{\mu_0\rho}$$

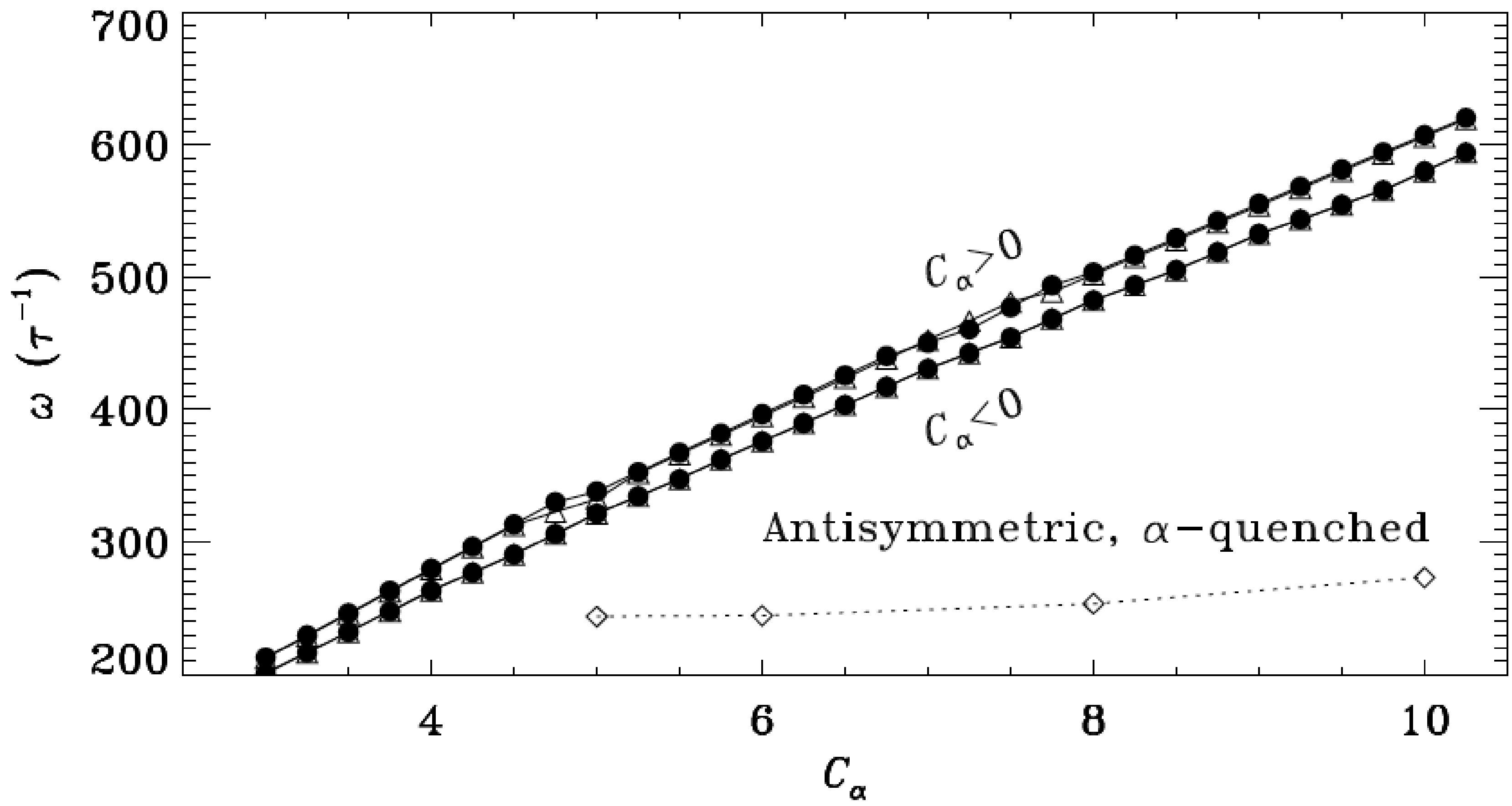
This motivates the following *ad hoc* expression for « alpha-quenching »:

$$\boxed{\alpha \rightarrow \alpha(B) = \frac{\alpha_0}{1+(B/B_{\rm eq})^2}}$$

Nonlinear models: alpha-quenching (2)



Nonlinear models: alpha-quenching (3)



The magnetic diffusivity is the primary determinant of the cycle period

Nonlinear models: alpha-quenching (4)

Magnetic fields concentrated at too high latitude;
Try instead a latitudinal dependency for alpha: $g(\theta) = \sin^2 \cos\theta$

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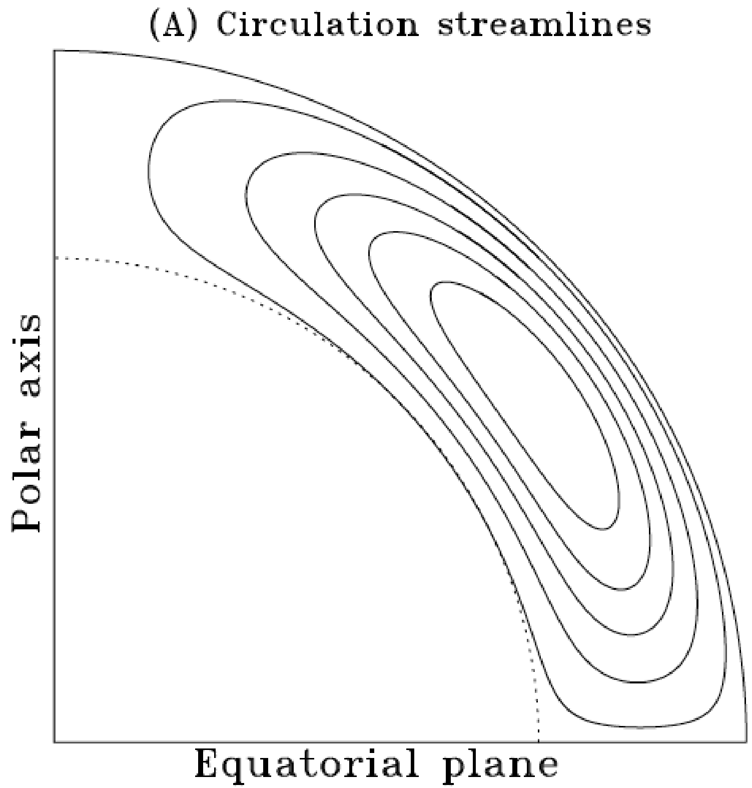
Alpha-Omega dynamos with meridional circulation (1)



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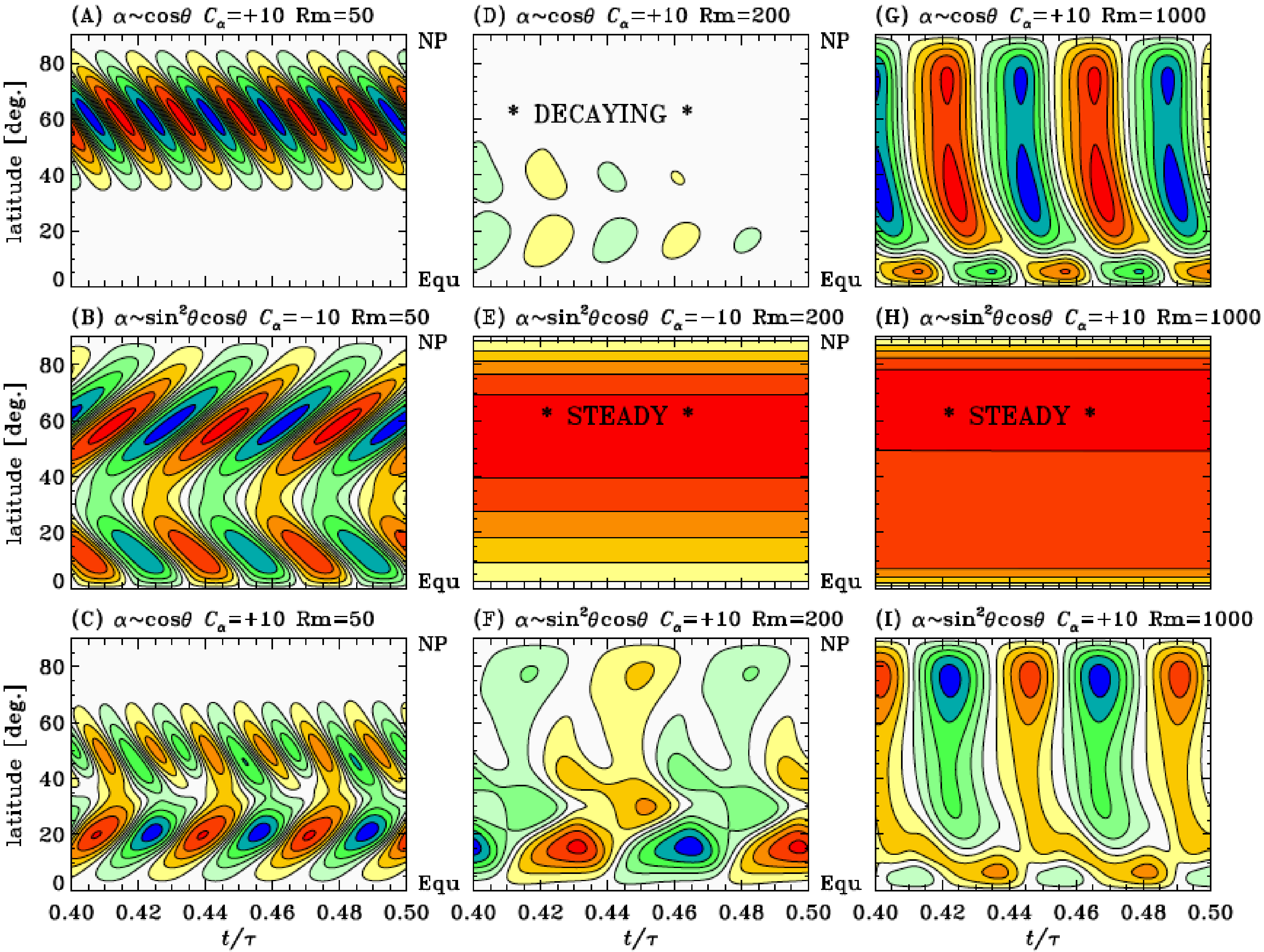
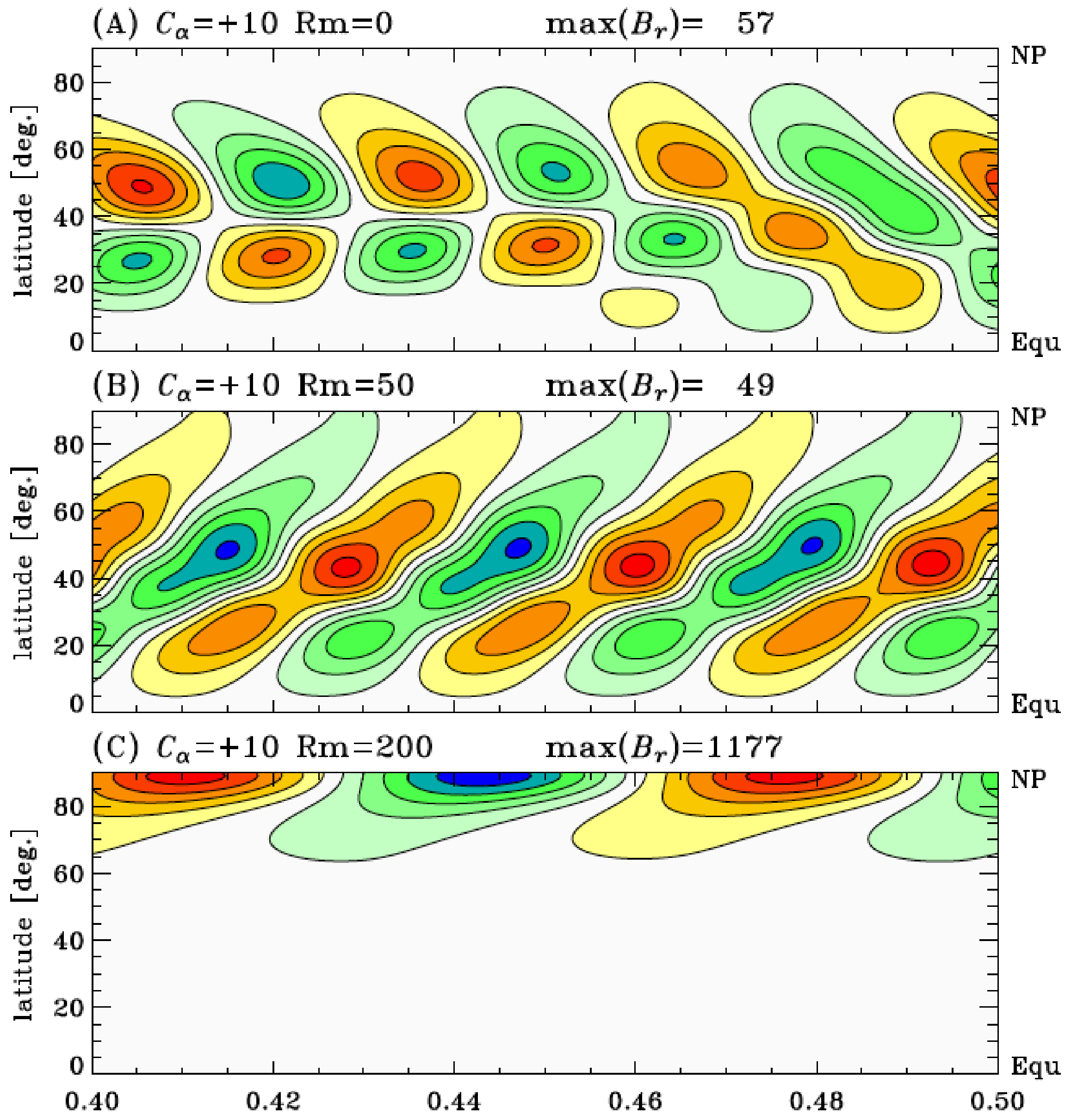
Equatorward propagation of the deep toroidal field is now due to advection by the meridional flow, not « dynamo waves » effect.

Models with meridional circulation (2)



Models with meridional circulation (3)

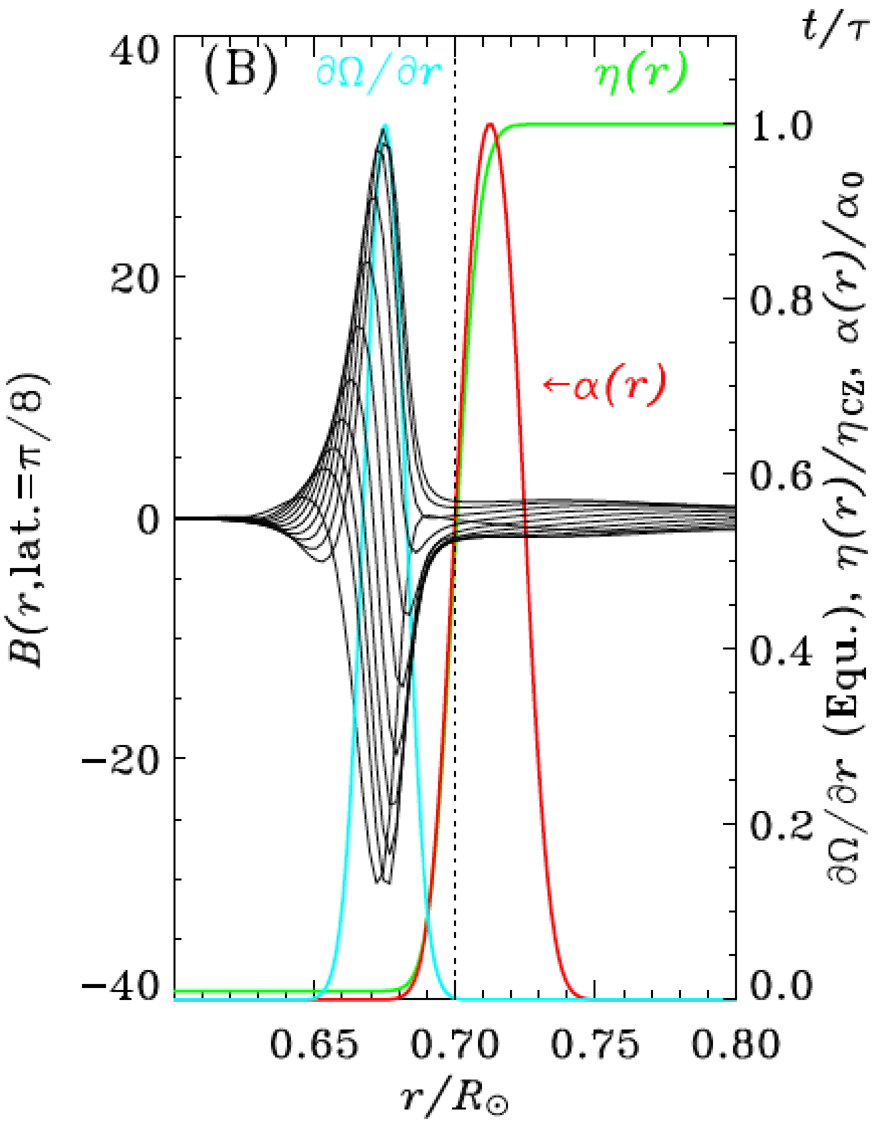


Interface dynamos (1)

Many measurements of the alpha effect in MHD numerical simulations suggest that its quenching assumes a form labeled « catastrophic »:

$$\alpha \rightarrow \alpha(B) = \frac{\alpha_0}{1 + \mathrm{R}_m (B/B_{\mathrm{eq}})^2}$$

The dynamo saturates at toroidal field strengths that are absolutely minuscule!



Interface dynamos (2)

Parker (1993): ratio of toroidal field strength in the two regions scales as:

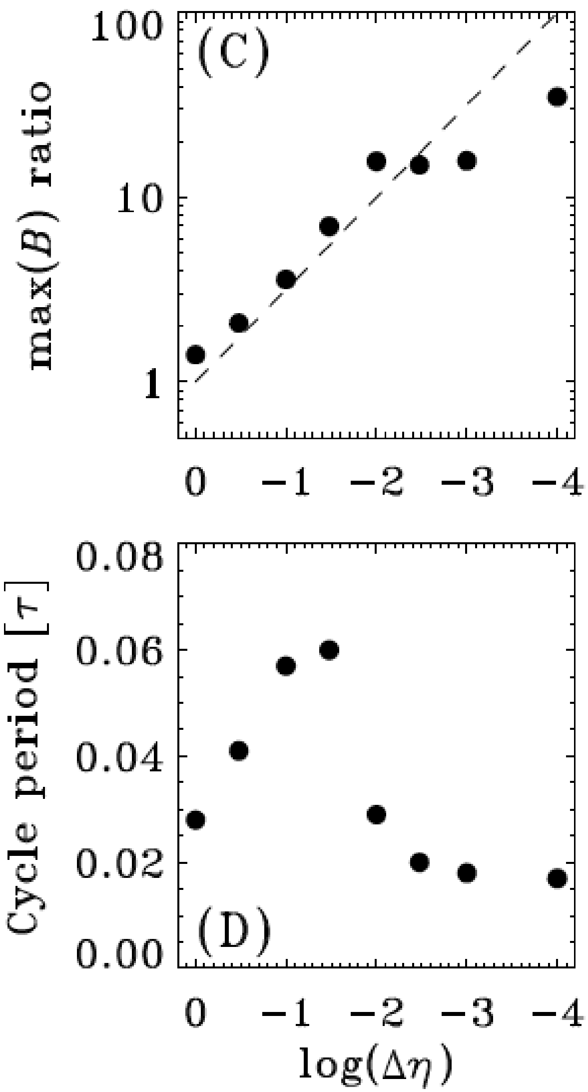
$$\frac{\max(B_2)}{\max(B_1)} \sim \left(\frac{\eta_2}{\eta_1}\right)^{-1/2}$$

For a turbulent diffusivity

$\eta_2 \sim \ell v$

Then:

$$\frac{\max(B_2)}{\max(B_1)} \sim \ (v\ell/\eta_1)^{1/2} \equiv \mathrm{R}_m^{1/2}$$



Nonlinear magnetic backreaction through the Lorentz force

Nonlinear magnetic backreaction through the Lorentz force

Problem: differential rotation and meridional circulation are powered by thermally-driven convective turbulence, for which we are lacking a model simple enough for inclusion in mean-field-like dynamo models.

Trick: large-scale flows are separated into two contributions, with only the second reacting to the Lorentz force:

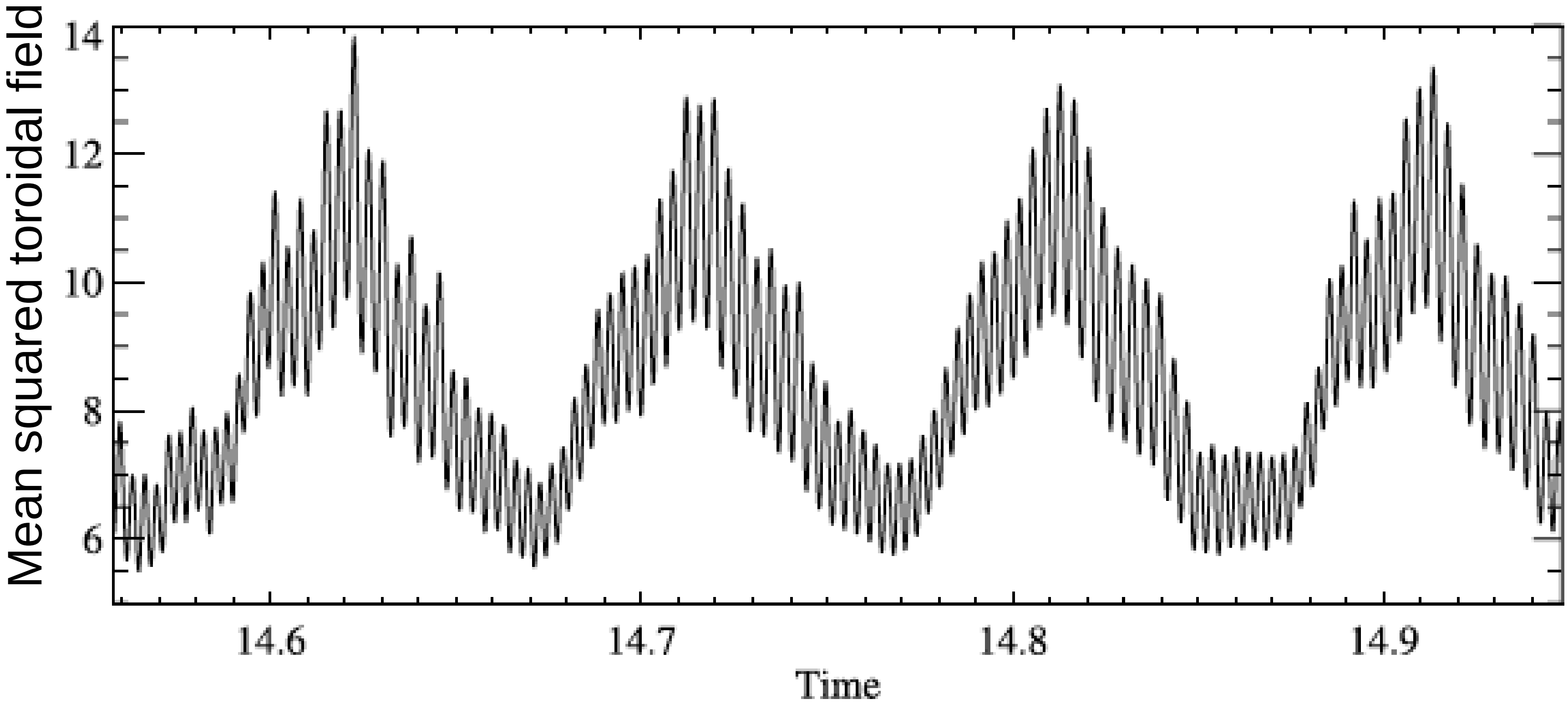
$$\mathbf{u} = \mathbf{U}(\mathbf{x}) + \mathbf{U}'(\mathbf{x}, t, \mathbf{B})$$

It is now a matter of solving an equation of motion only for this second, time-varying component, together with the usual dynamo equations:

$$\frac{\partial \mathbf{U}'}{\partial t} = \frac{\Lambda}{4\pi\rho}(\nabla \times \mathbf{B}) \times \mathbf{B} + \mathrm{P}_m \nabla^2 \mathbf{U}$$

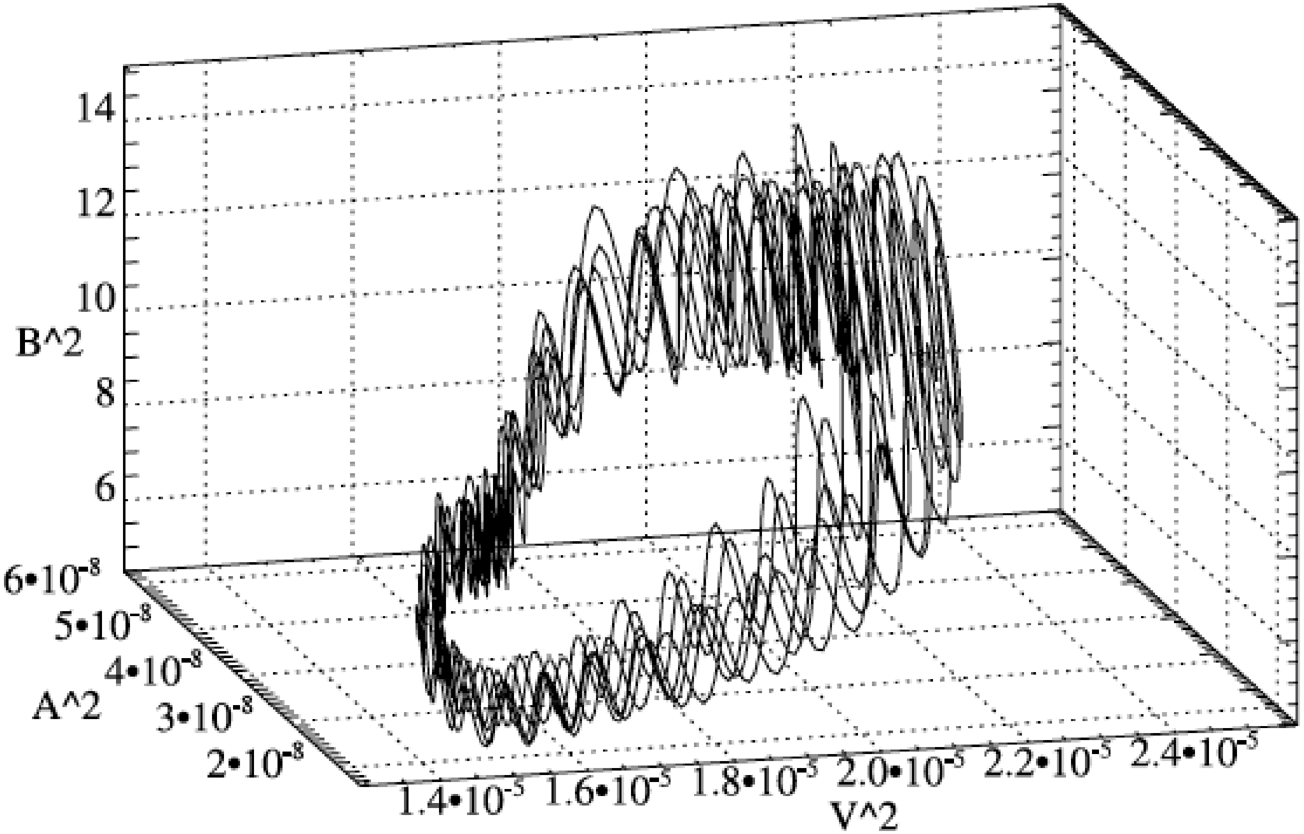
Amplitude modulation (1)

The primary cycle picks up a longer modulation, with period controlled by the magnetic Prandtl number (ratio of viscosity to magnetic diffusivity; ~0.01 for microscopic values)



Bushby & Tobias 2007, ApJ, **661**, 1289.

Amplitude modulation (2)



Bushby & Tobias 2007, ApJ, 661, 1289.

Pm must be large, otherwise excessively large velocity modulation ensues; but if Pm is too large, the modulation period becomes too long.

Amplitude and parity modulations

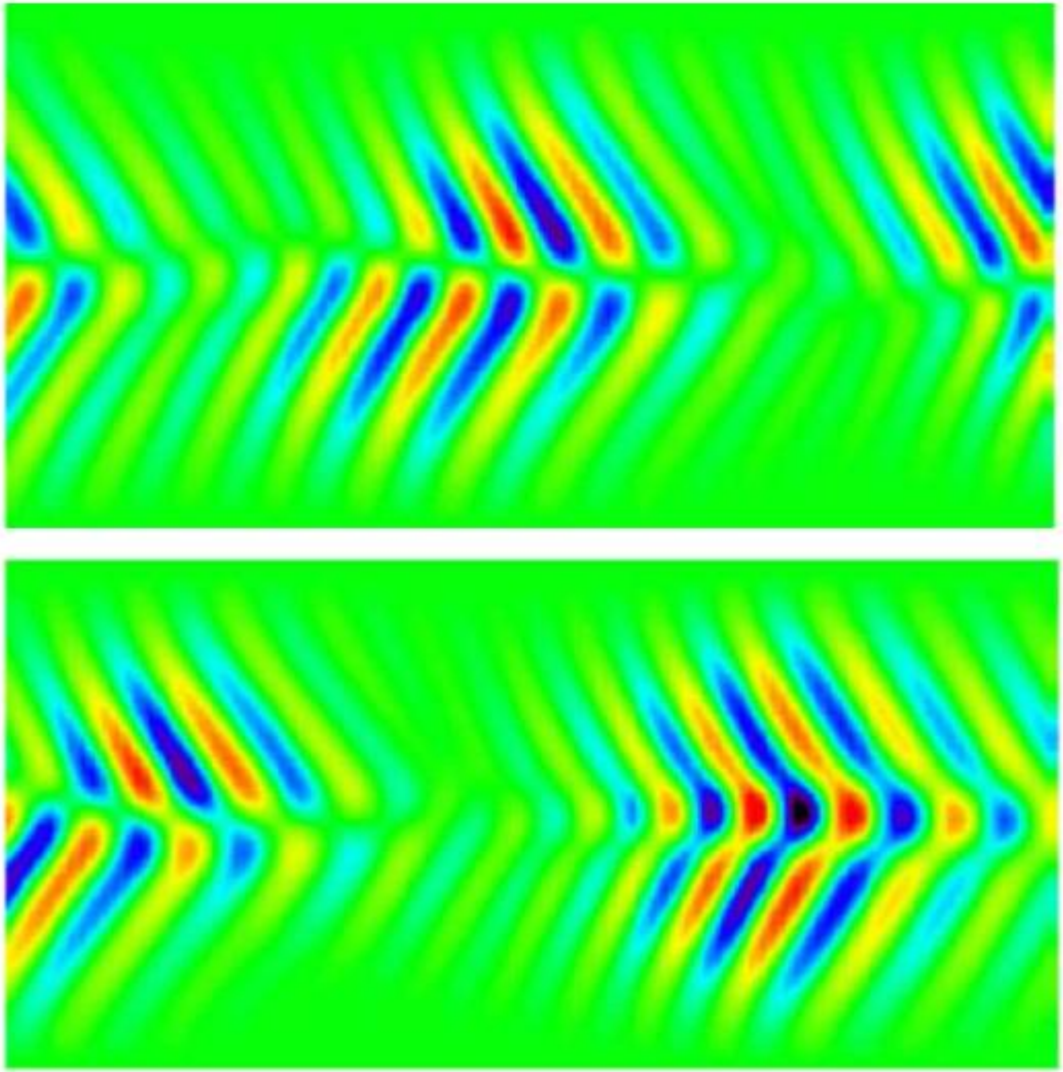
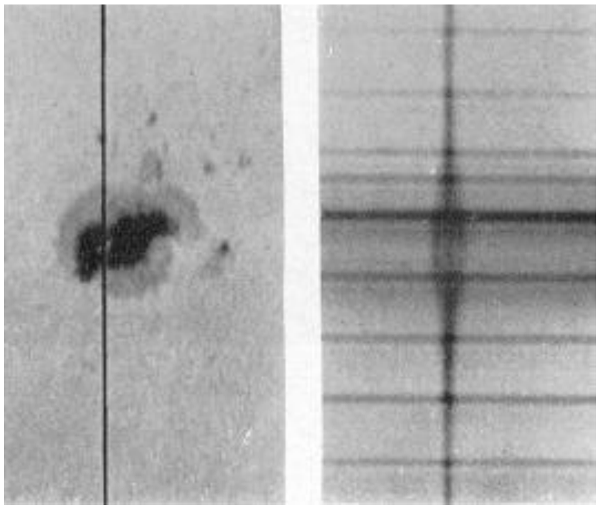


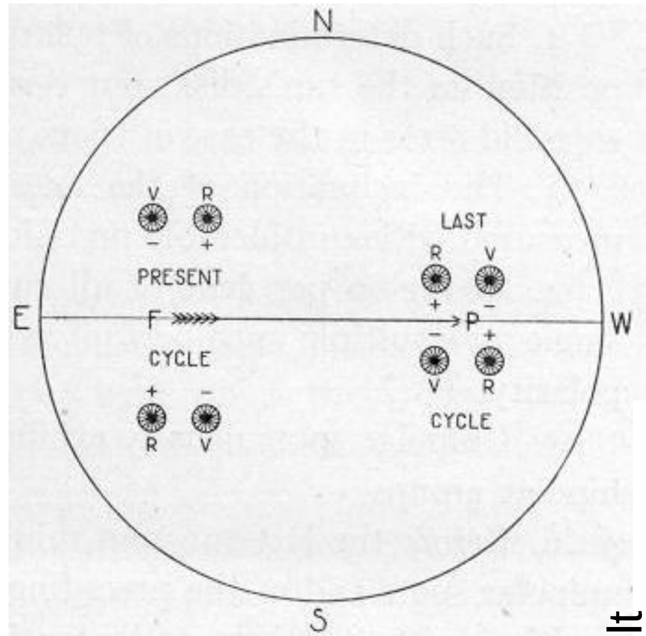
Figure courtesy S. Tobias

Models based on the Babcock-Leighton mechanism

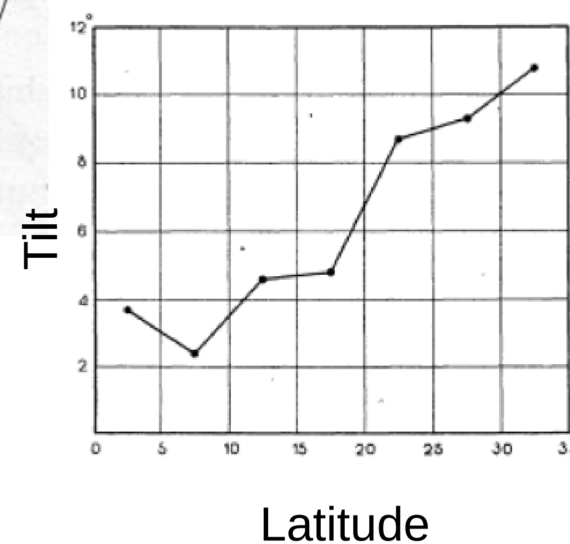
Sunspot as as emerging toroidal flux ropes

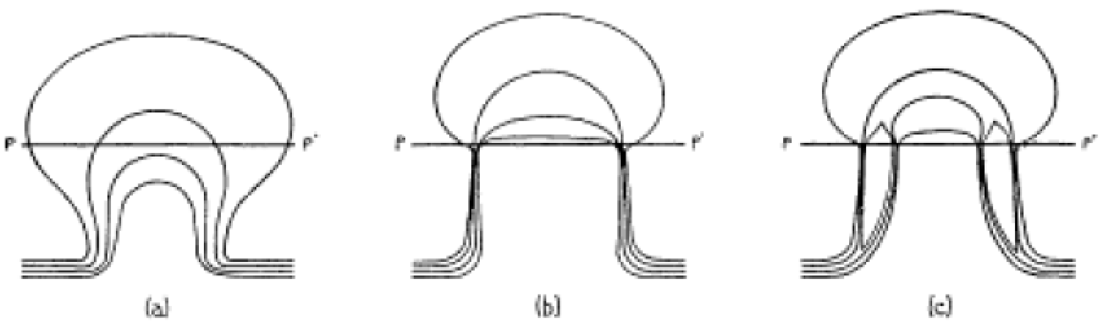
Hale et al. 1919, ApJ, **49**, 153





$$\sin\tilde{\theta} = 0.48\cos(\theta) + 0.03$$





Parker 1955, ApJ, **121**, 491

Buoyant rise of toroidal flux ropes (1)

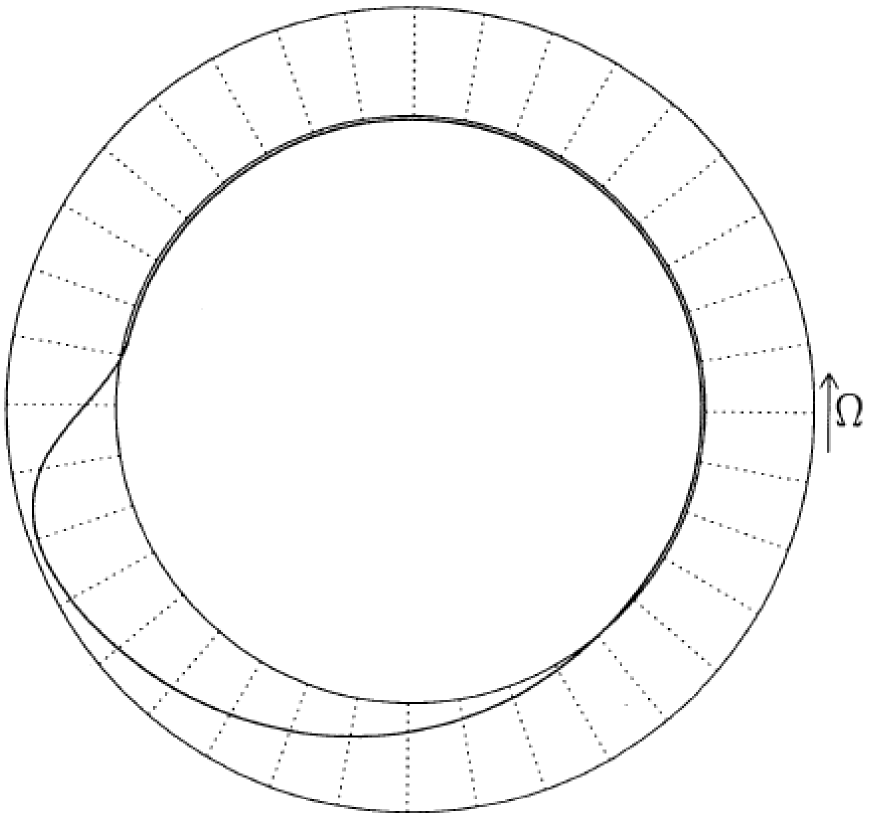
[Caligari et al. 1995, ApJ, **441**, 886]

Destabilization and buoyant rise of thin toroidal magnetic flux tubes stored immediately below the core-envelope interface (overshoot)

Non-axisymmetric modes of low order (m=1 or m=2) are most easily destabilized;

Conservation of angular momentum generates a flow along the axis of the rising loop;

The Coriolis force acting on the flow in the legs of the loop imparts a twist that shows up as an E-W tilt upon emergence through the surface.



Buoyant rise of toroidal flux ropes (2)

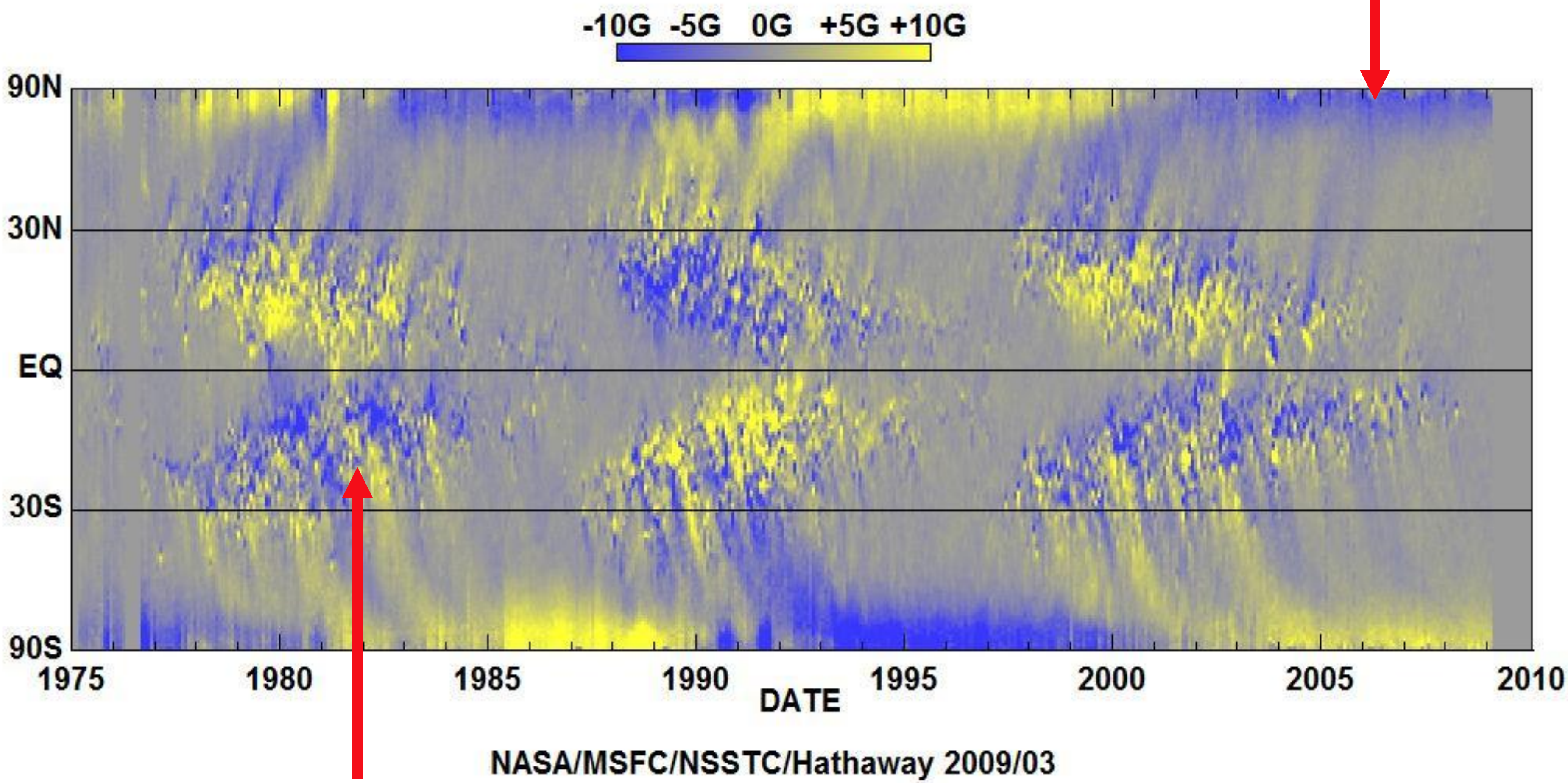
What we have learned from such simulations:

1. Strongly magnetized toroidal flux ropes must be stored immediately below the core-envelope interface
2. The flux ropes must have field strength above a few T, otherwise they emerge at too high latitudes and/or « explode » before reaching the surface.
3. The flux ropes must have field strength below 20 T, otherwise they would emerge without the observed E-W tilt.

In the overshoot layer: $6 < B < 20$ T

Active region decay (1)

Peak polar cap flux: ~10^{14} Wb



Toroidal flux emerging in active regions in one cycle: ~10^{17} Wb

Active region decay (2)

$$\frac{\partial B_r}{\partial t} = \frac{2u_0}{R}(1-\mu^2)\left[B_r + \mu\frac{\partial B_r}{\partial \mu}\right] - \Omega_S(1-\mu^2)^{1/2}\frac{\partial B_r}{\partial \phi}$$

$$+\frac{\partial}{\partial \mu}\left[\frac{\eta}{R^2}\frac{\partial B_r}{\partial \mu}\right] + \frac{\partial}{\partial \phi}\left[\frac{\eta}{R^2(1-\mu^2)}\frac{\partial B_r}{\partial \phi}\right] \; ,$$

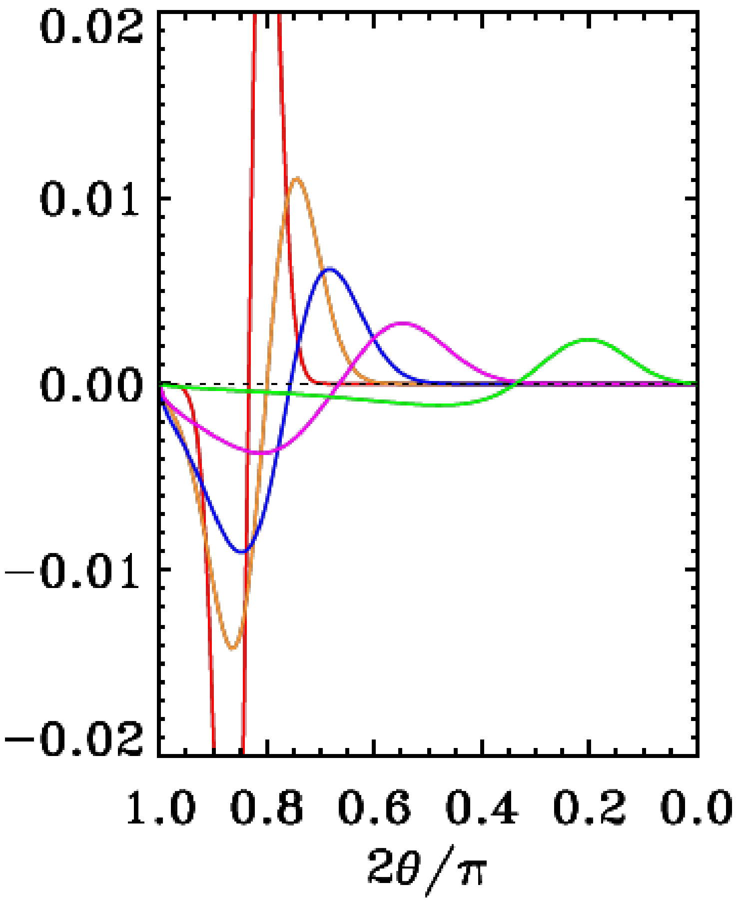
Solve *r*-component of MHD induction equation on a spherical surface; flow includes surface differential rotation and meridional circulation.

QuickTime™ et un
décompresseur codec YUV420
sont requis pour visionner cette image.

Kinematic simulation:
the radial magnetic component behaves like a passive scalar, advected by the flows and mixed by diffusion

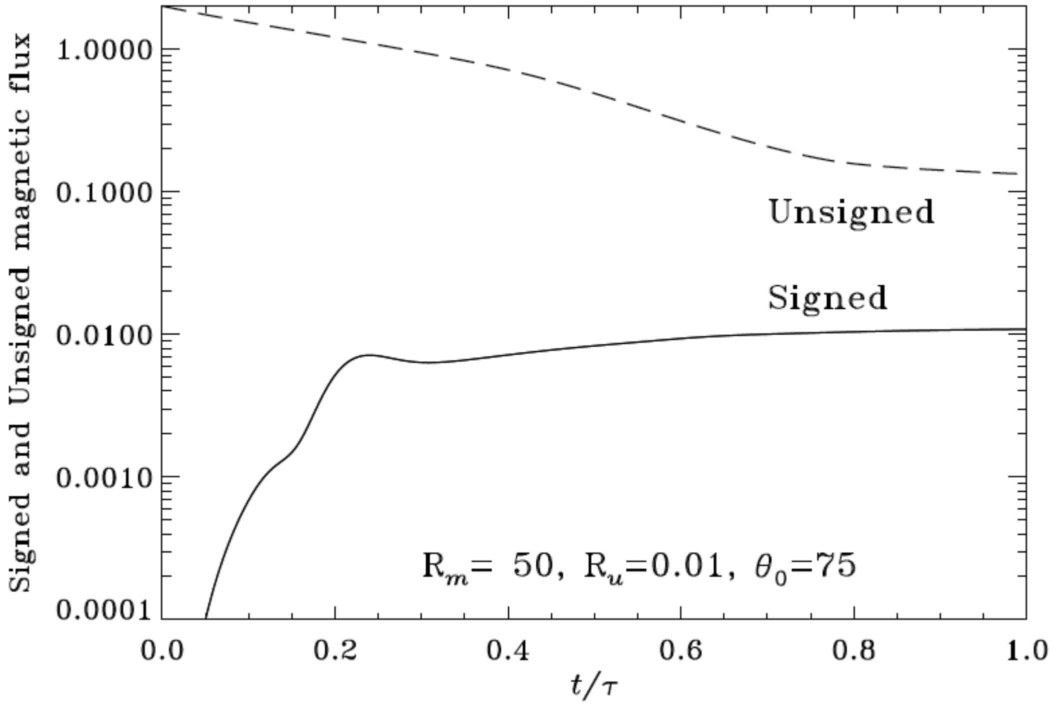
Active region decay (3)

Zonal means

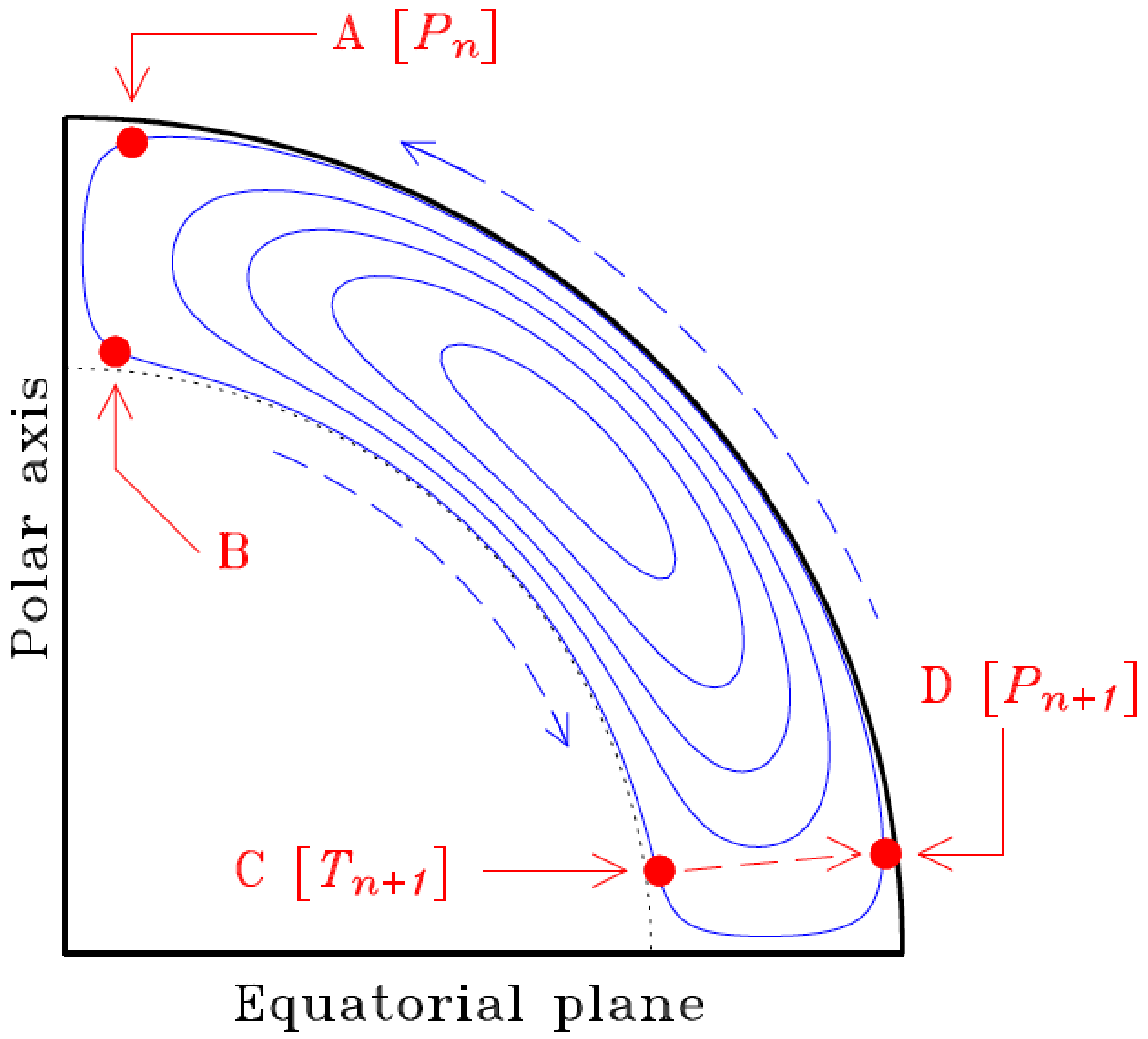


$$\langle B_r \rangle = -\int_0^{2\pi}\int_{-1}^{+1} B_r \mathrm{d}\mu \mathrm{d}\phi$$

$$\Phi = |\langle B_r \rangle| \ , \qquad F = \langle | B_r | \rangle$$



Babcock-Leighton dynamo model (1)



Kinematic axisymmetric dynamo

$$\frac{\partial A}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)A}_{\text{resistive decay}} - \underbrace{\frac{\mathbf{u}_p}{\varpi}\cdot\nabla(\varpi A)}_{\text{advection}} \quad [+\ \text{Source}]$$

$$\frac{\partial B}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)B}_{\text{resistive decay}} + \underbrace{\frac{1}{\varpi}\frac{\partial(\varpi B)}{\partial r}\frac{\partial \eta}{\partial r}}_{\text{diamagnetic transport}} - \underbrace{\varpi\mathbf{u}_p\cdot\nabla\left(\frac{B}{\varpi}\right)}_{\text{advection}}$$

$$- \underbrace{B\nabla\cdot\mathbf{u}_p}_{\text{compression}} + \underbrace{\varpi(\nabla\times(A\hat{\mathbf{e}}_\phi))\cdot\nabla\Omega}_{\text{shearing}} \ .$$

Babcock-Leighton dynamo model (2)

A Babcock-Leighton source term for the axisymmetric dynamo equations:

$$S(r,\theta,B(t)) = s_0 f(r) g(\theta)\,\mathrm{erf}\left(\frac{B(r_c,\theta,t)-B_1}{w_1}\right)\left[1-\mathrm{erf}\left(\frac{B(r_c,\theta,t)-B_2}{w_2}\right)\right] B(r_c,\theta,t)$$

$$g(\theta) = \sin\theta\cos\theta\ ,$$

Peaking at mid-latitudes

Non-local in *B*

$$f(r) = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{r-r_2}{d_2}\right)\right]\left[1-\mathrm{erf}\left(\frac{r-r_3}{d_3}\right)\right]$$

Concentrated in surface layers

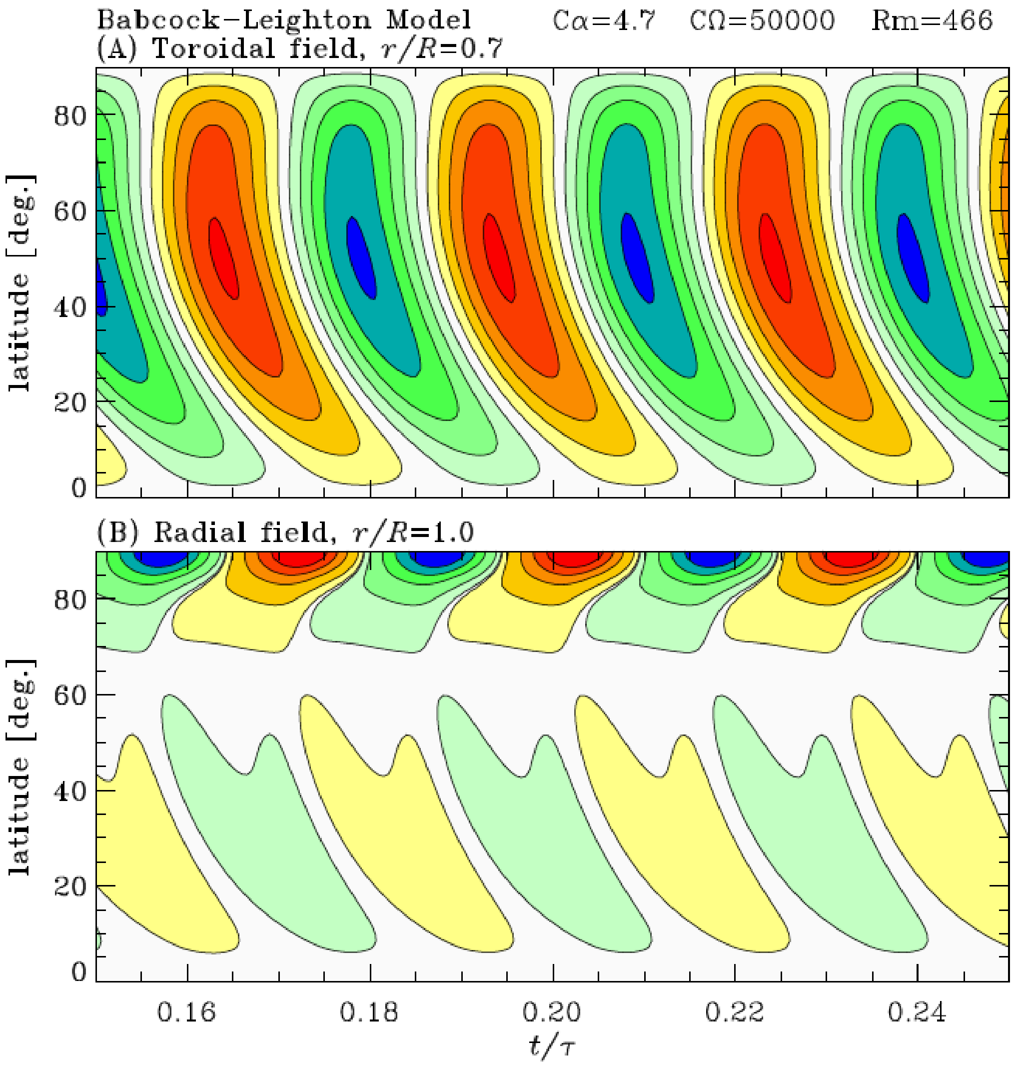
The source term operates only in a finite range of toroidal field strengths.

Babcock-Leighton dynamo model (3)

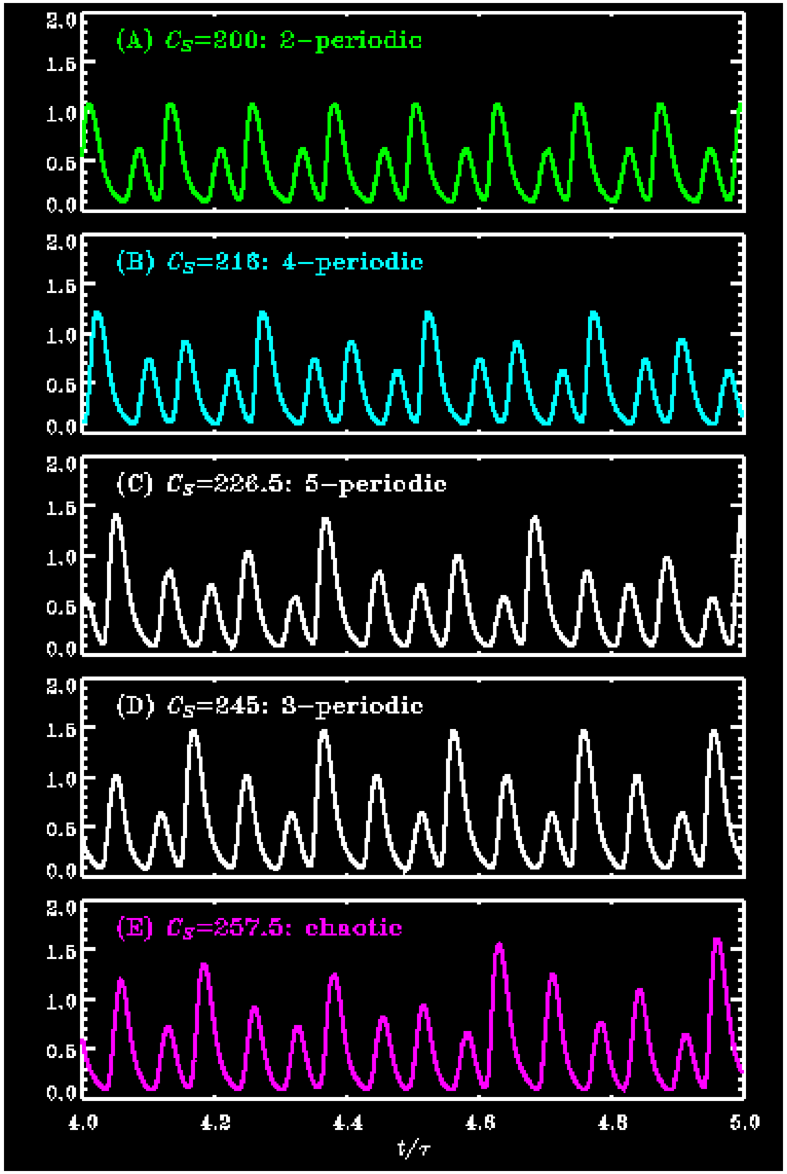
QuickTime™ et un
décompresseur codec YUV420
sont requis pour visionner cette image.

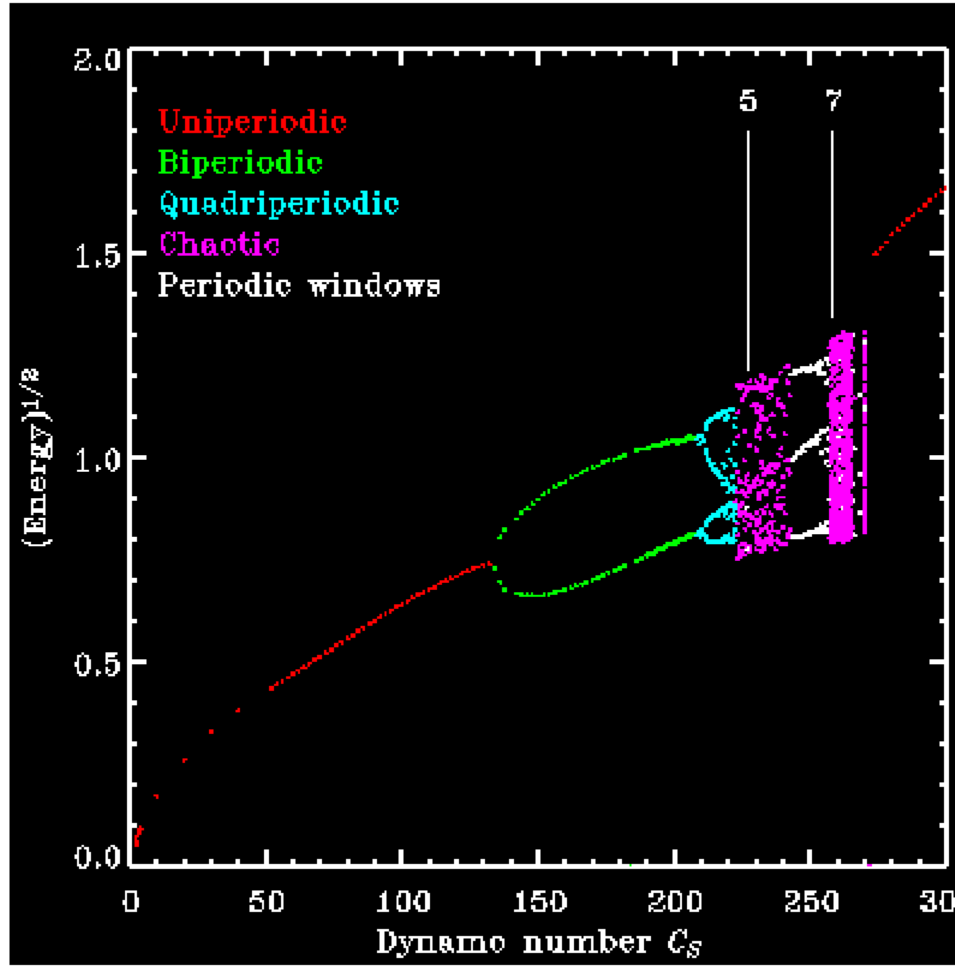
The turnover time of the meridional flow
is the primary determinant of the cycle period

Babcock-Leighton dynamo model (4)



Bifurcations in numerical solutions





Charbonneau et al., ApJ, **619**, 613 (2005)

Babcock-Leighton versus alpha-effect

There are serious potential problems with the operation of the alpha-effect at high field strength; not so with the B-L mechanism

The B-L mechanism operates only in a finite range of field strength; potentially problematic in the presence of large cycle amplitude fluctuations.

Both models can produce tolerably solar-like toroidal field butterfly diagrams, and yield the proper phase relationship between surface poloidal and deep toroidal components (with circulation included in the mean-field model)

The alpha-effect (or something analogous) appears unavoidable in stratified, rotating turbulence.

A decadal period arises « naturally » in B-L models; in mean-field models, it requires tuning the value of the turbulent magnetic diffusivity

Stochastic forcing

Stochastic forcing

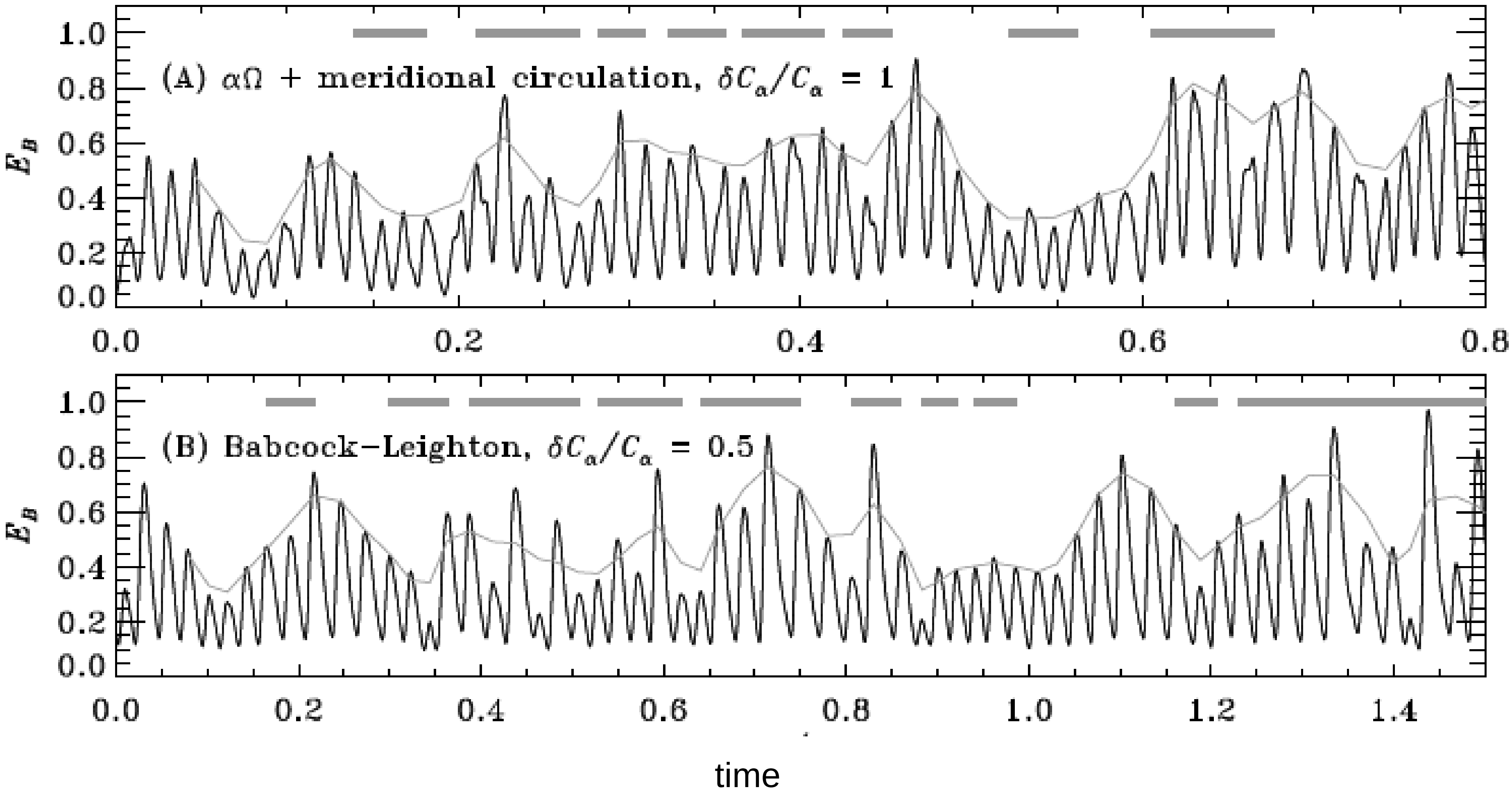
The solar dynamo operates in part or in totality in a strongly turbulent environment; all large-scale flows contributing to field amplification will be characterized by strong fluctuations about the mean.

Also, mean-field coefficients or other source terms result from a process of averaging over many elementary « events », and therefore will also fluctuate in time about their mean.

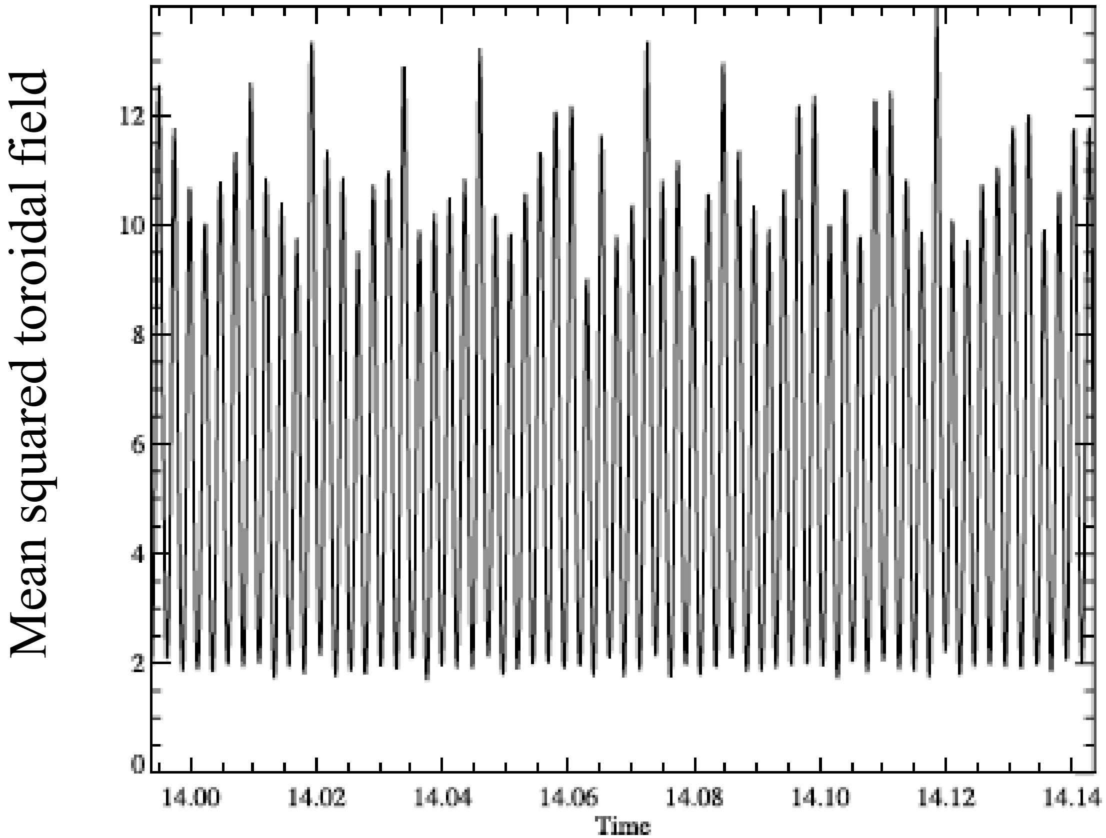
Introduce this latter effect in mean-field-like models as:

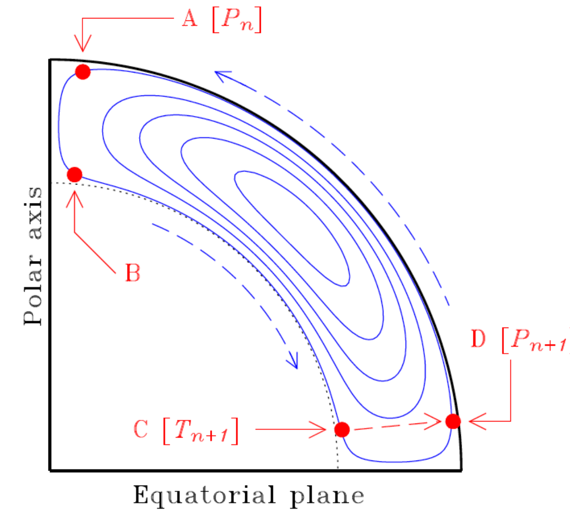
$$C_\alpha \rightarrow \bar{C}_\alpha + \rho \times \delta C \ , \qquad \rho \in [-1, 1] \ , \qquad \text{if}(t \bmod \tau_c) = 0 \ .$$

Amplitude modulation by stochastic forcing (1)



Amplitude modulation by stochastic forcing (2)



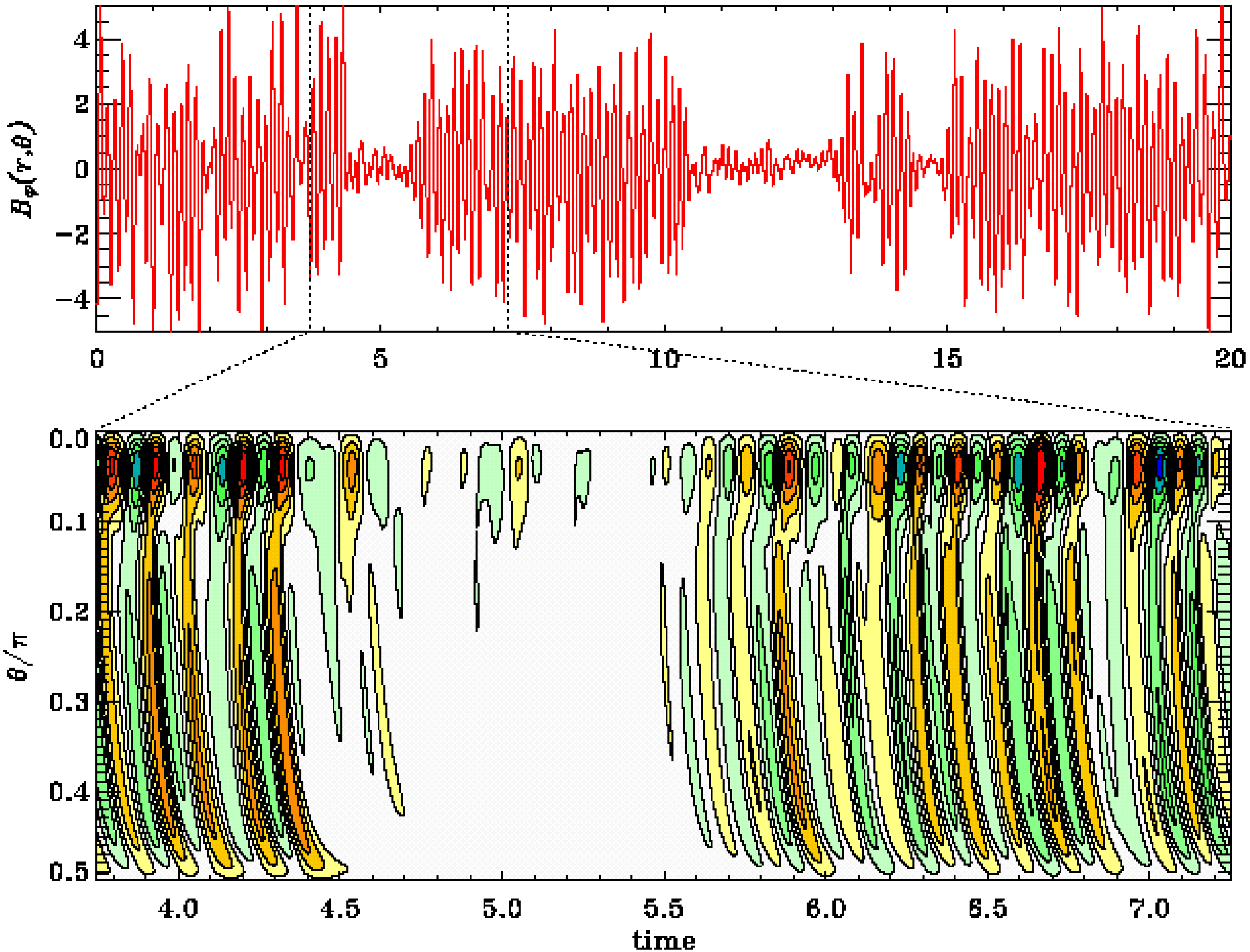


The lower extent of the meridional flow cell fluctuates stochastically by 1%, with coherence time of one month.

Bushby & Tobias 2007, ApJ, **661**, 1289.

Intermittency (1)

[Charbonneau et al., ApJ **616**, L183 (2004)]



Intermittency (2)

[Ossendrijver, A&A **359**, 364 (2000)]

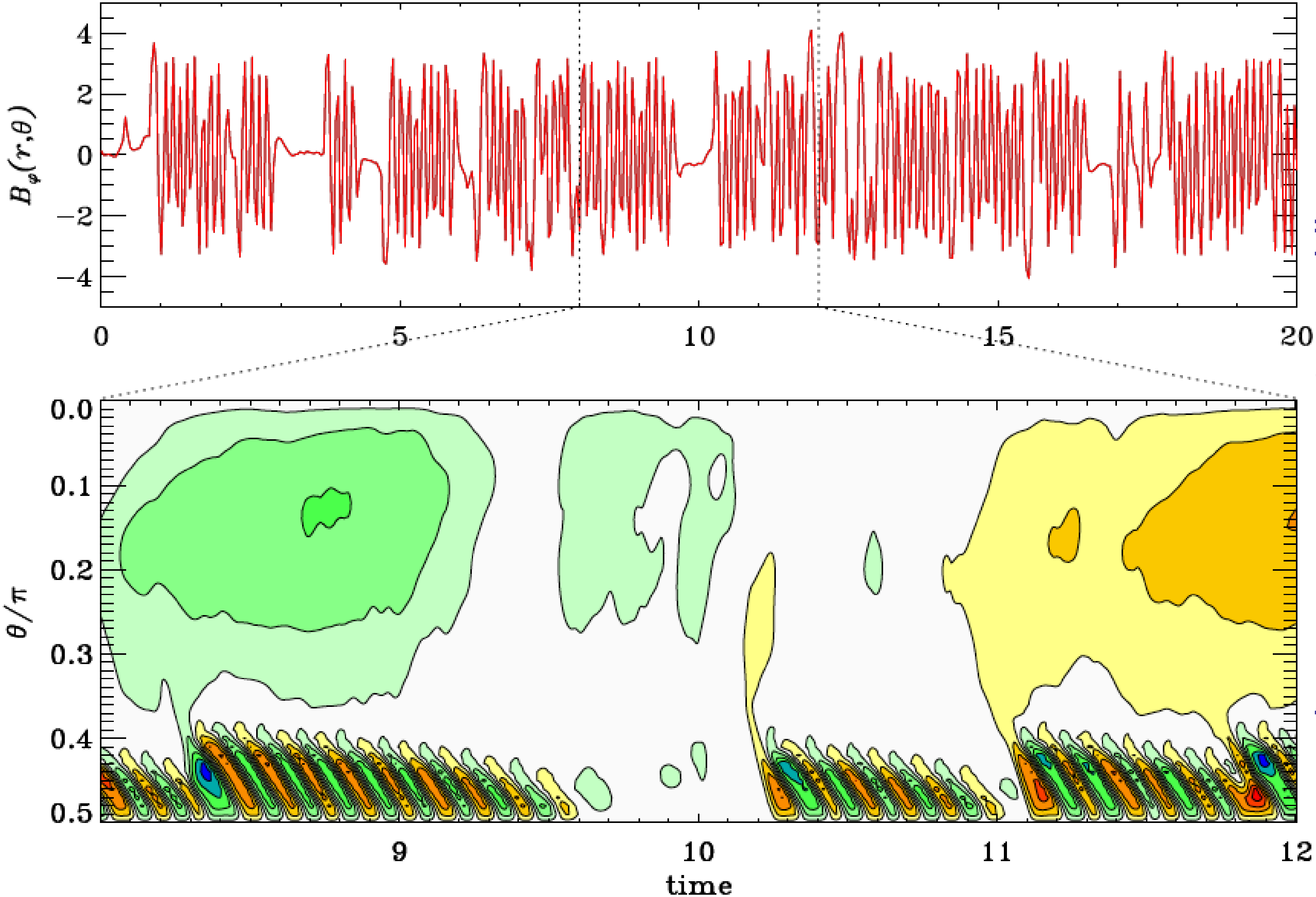


Figure courtesy M. Ossendrijver

Solar cycle forecasting

The polar field as a precursor (1)

VOL. 5, NO. 5 GEOPHYSICAL RESEARCH LETTERS MAY 1978

USING DYNAMO THEORY TO PREDICT

THE SUNSPOT NUMBER DURING SOLAR CYCLE 21

Kenneth H. Schatten, Philip H. Scherrer, Leif Svalgaard and John M. Wilcox

Institute for Plasma Research, Stanford University, Stanford, California

Abstract. On physical grounds it is suggested that the sun's polar field strength near a solar minimum is closely related to the following cycle's solar activity. Four methods of estimating the sun's polar magnetic field strength near solar minimum are employed to provide an estimate of cycle 21's yearly mean sunspot number at solar maximum of 140 ± 20. We think of this estimate as a first order attempt to predict the cycle's activity using one parameter of physical importance based upon dynamo theory.

Introduction

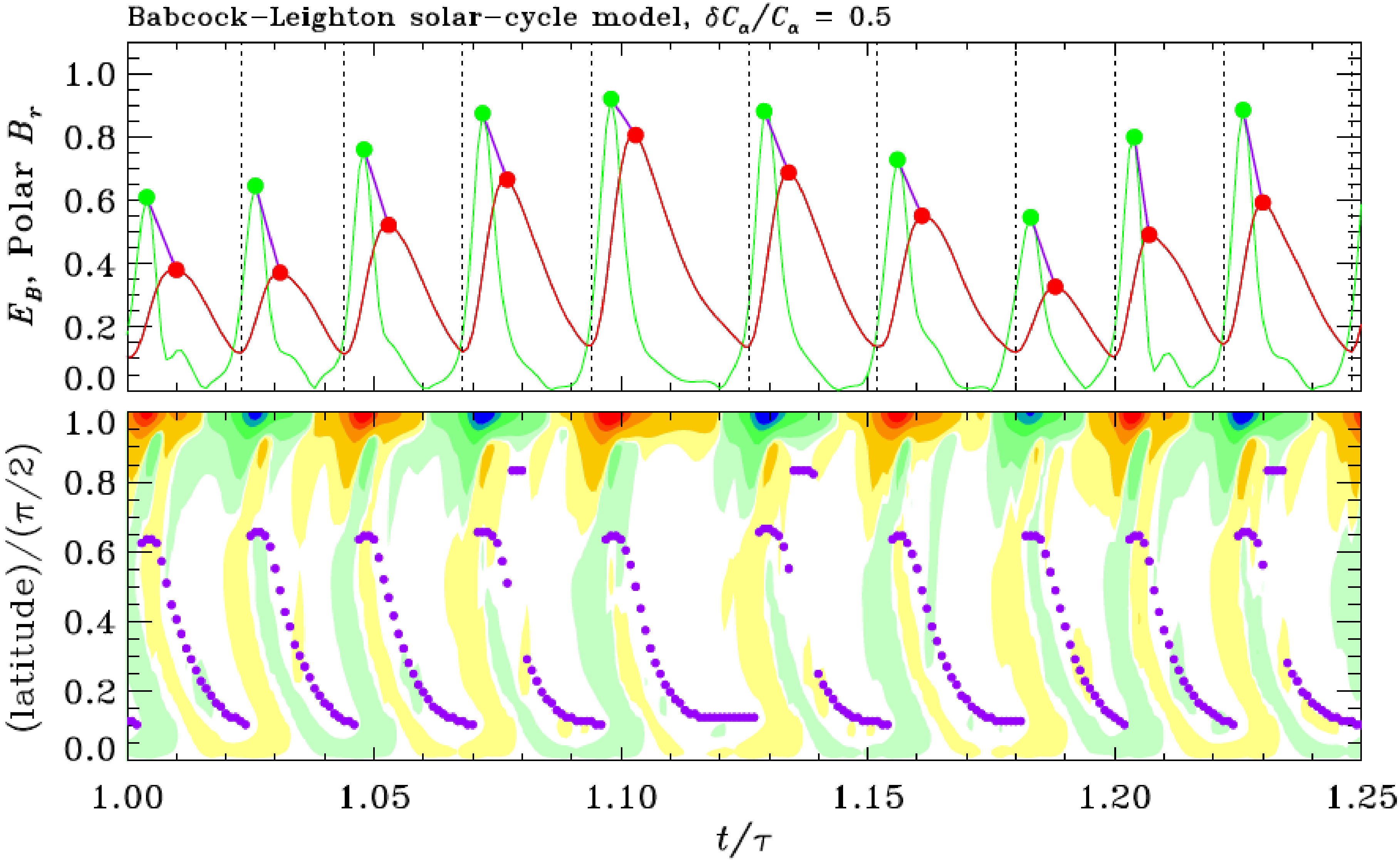
A variety of methods have been used by many scientists to predict solar activity (Sargent, 1978 and references contained therein). Often they rely on time series analyses which assume implicitly that the solar dynamo has basic periodicities. These methods are questionable

Polar Field Strength

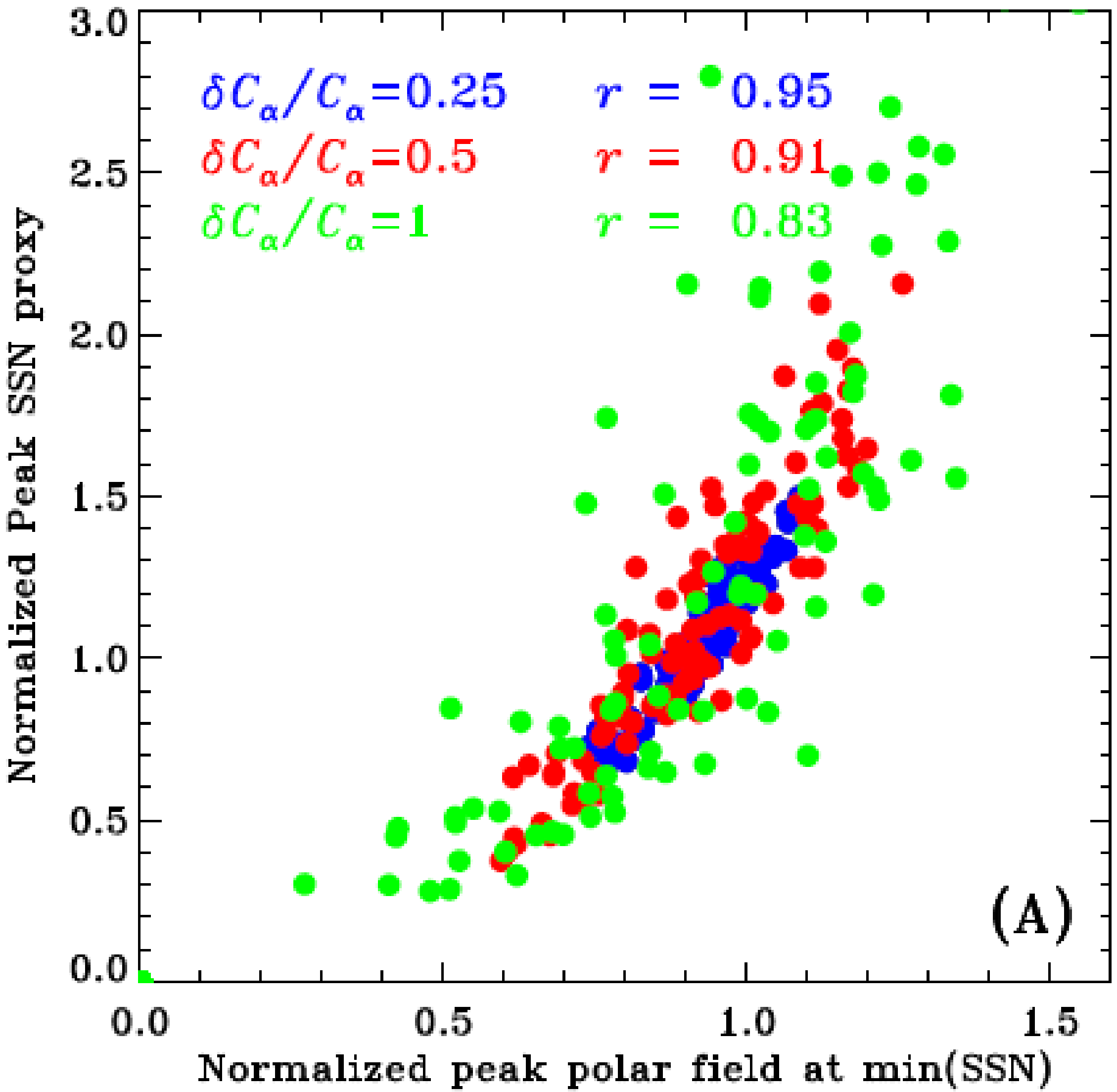
Estimates of the polar magnetic field strength near sunspot minimum may be obtained from the shape of the corona at the time of solar eclipses, or by the amount of flattening of the "warped current sheet" at 1AU as obtained from interplanetary magnetic field measurements analysed in accordance with the methods of Rosenberg and Coleman (1969). A further and more direct estimate of polar field strength is obtained by observing the number of polar faculae.

The shape of the corona at eclipses may be used to obtain a measure of polar field strength in the following two ways. The Ludendorf index (Billings, 1966) may be used as a measure of coronal flattening and hence of polar field strength. Figure 1a shows the mean sunspot number versus the Ludendorf index. Each point represents the Ludendorf index for an eclipse

The polar field as a precursor (2)

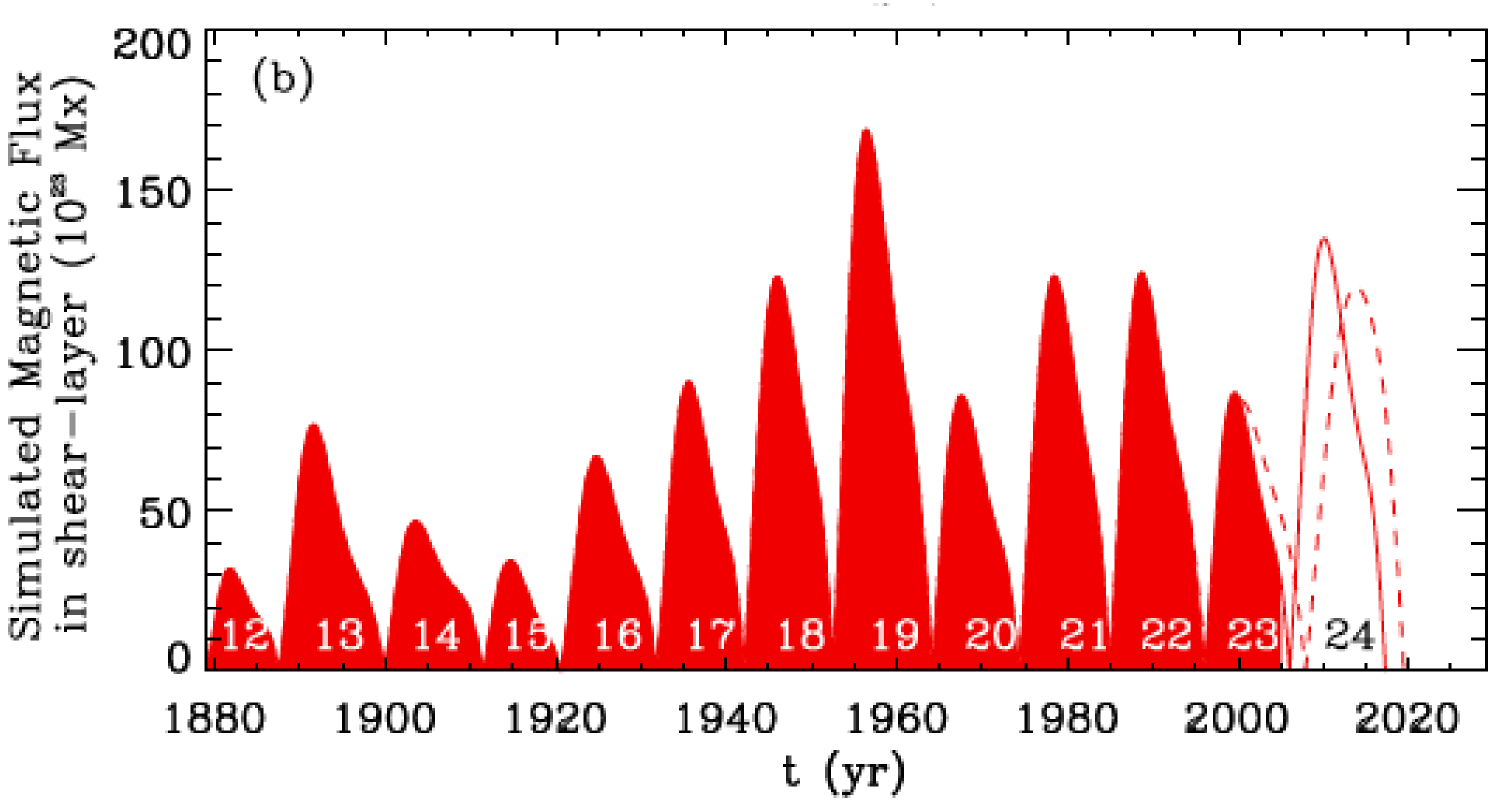


The polar field as a precursor (3)



The Dikpati et al. forecasting scheme

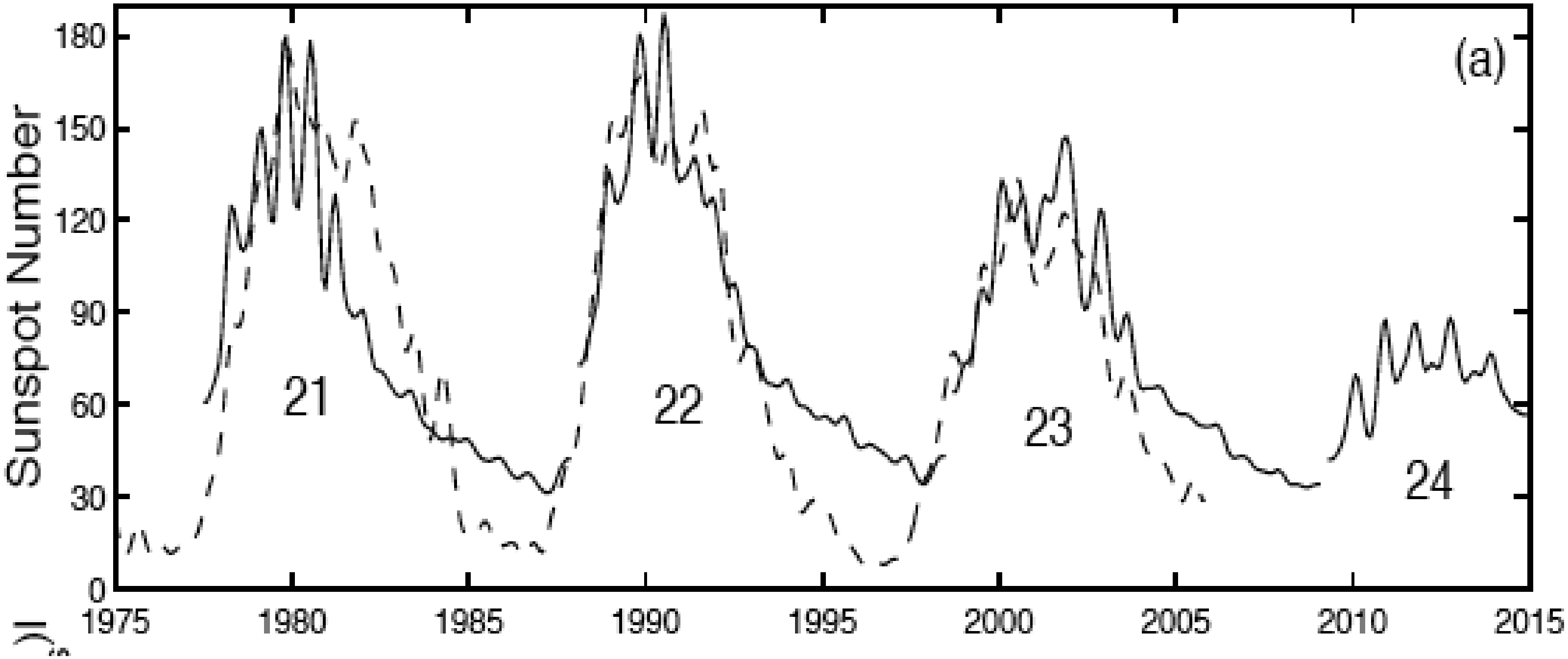
[Dikpati, DeToma & Gilman 2006, GeoRL 33(5), L05102]



- Forecast is for a cycle 24 amplitude 30-50% higher than cycle 23

The Choudhuri et al. forecasting scheme

[Choudhuri, Chatterjee & Jiang 2007, PhRvL 98(13),131103]



- Forecast is for sunspot cycle 24 amplitude 35% lower than cycle 23

Stellar dynamos

Back to basics…

What have we learned?

1. When rotation is present, a turbulent flow in a stratified environment can produce a large-scale magnetic field

2. Differential rotation is an excellent mechanism to produce magnetic fields organised on large spatial scales

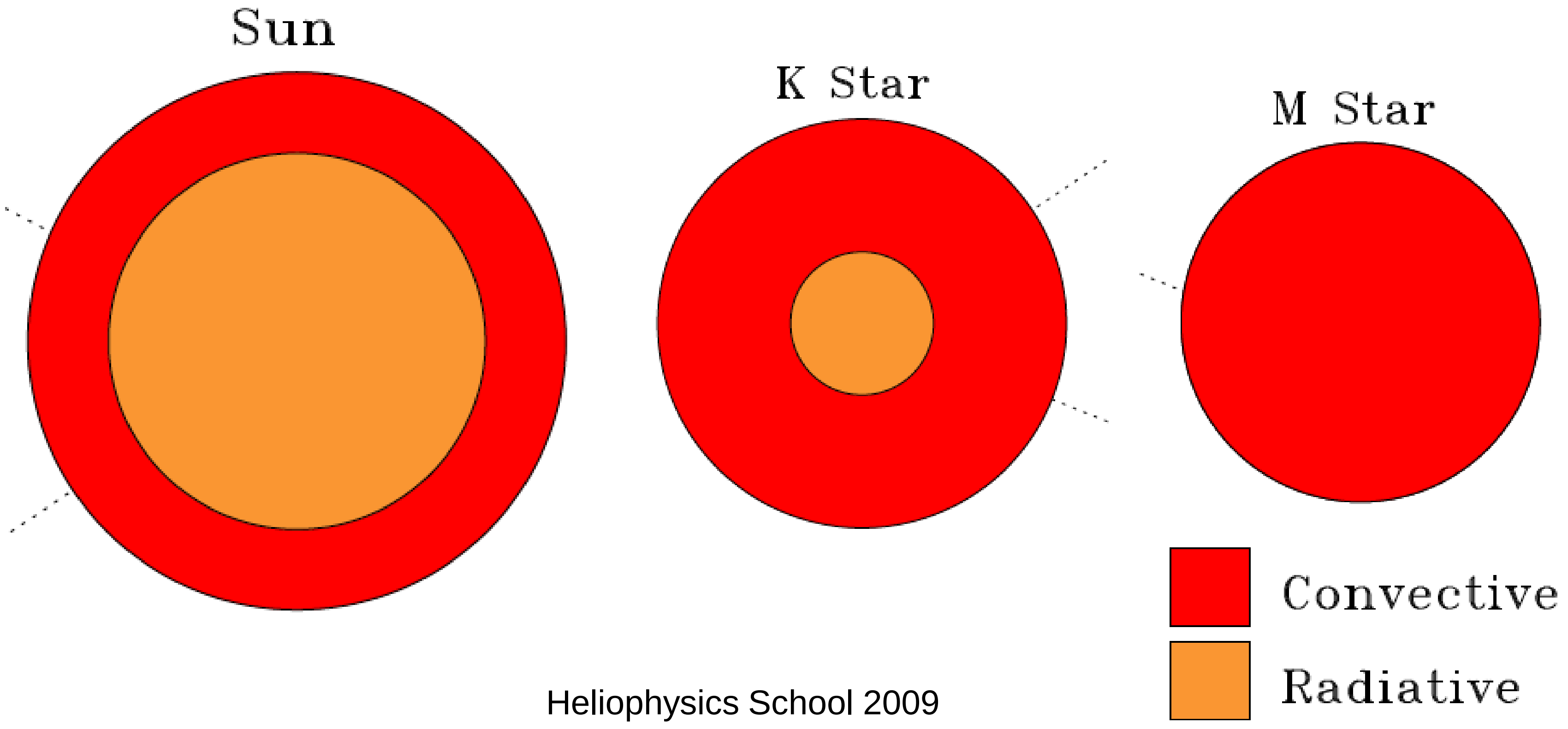
This is good because:

1. Most stars convect somewhere in their interior

2. Most stars rotate significantly

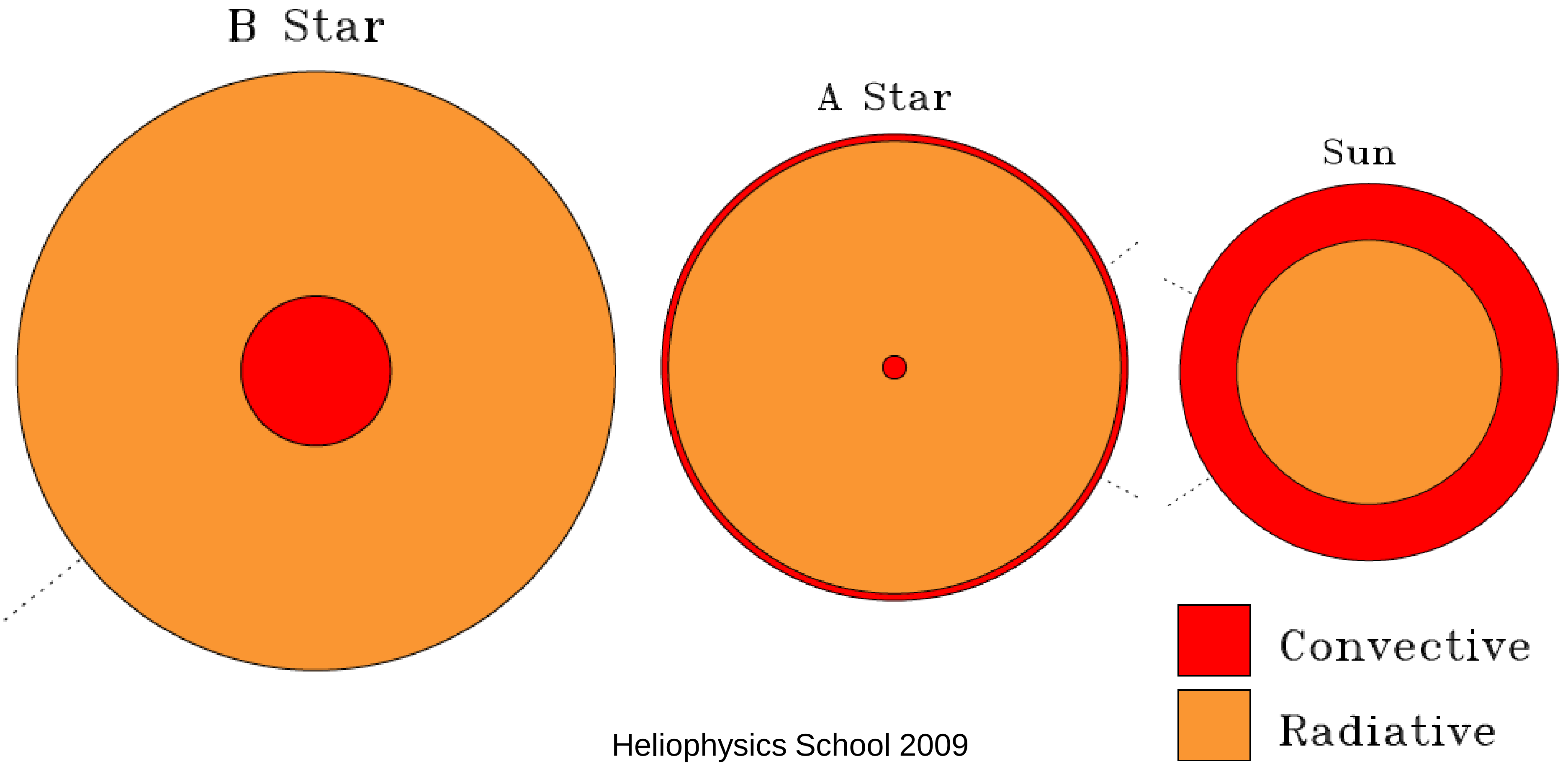
Convection in main-sequence star (1)

In « cool » stars, the convective envelope deepens as the surface temperature/mass decreases; stars are fully convective around M5



Convection in main-sequence stars (2)

In « hot » stars, the convective envelope disappears, but a convective core builds up as mass/effective temperature increases



Solar-type stars

Dynamo modelling in solar-type stars (1)

To « extrapolate » solar dynamo models to solar-type stars,
we must specify:

1. What is the mechanism responsible for poloidal field regeneration, and in what regime is it operating?
2. What is the star's internal structure (convection zone depth, etc)
3. How do the form and magnitude of differential rotation vary with stellar parameters (rotation, luminosity, etc) ?
4. How does meridional circulation vary with stellar parameters?
5. How do the alpha-effect, turbulent diffusivity, Babcock-Leighton source term, etc, vary with stellar parameters
6. Which nonlinear effect quenches the growth of the dynamo magnetic field?

Dynamo modelling in solar-type stars (2)

$$P_{\rm cyc} \propto \alpha_0^{-0.54} (\Delta\Omega)^{-0.57} \ , \qquad [\text{Nonlinear Mean} - \text{Field}]$$

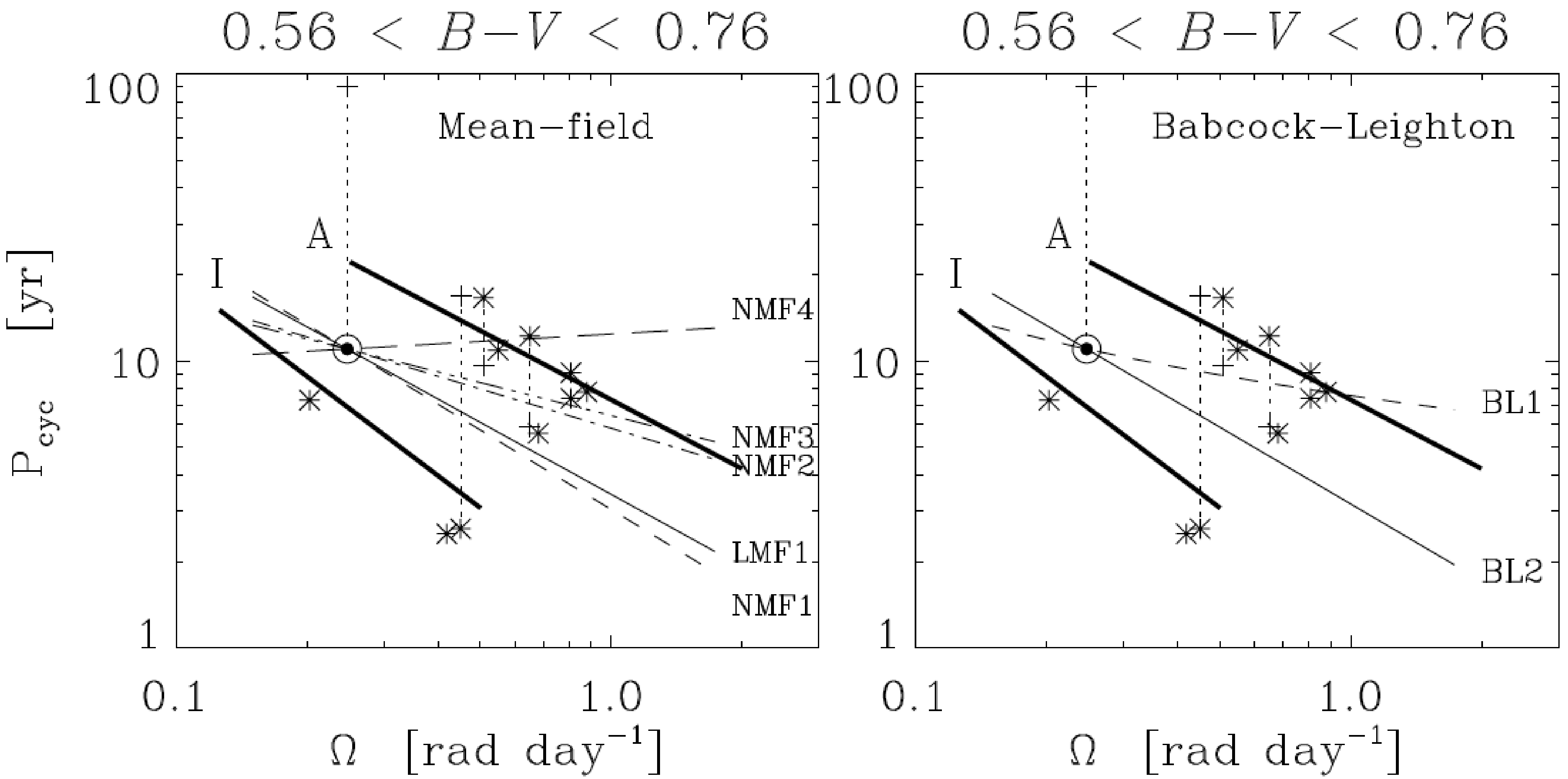
$$P_{\rm cyc} \propto s_0^{-0.13} \eta_T^{+0.22} u_m^{-0.89} \ , \qquad [\text{Babcock} - \text{Leighton}]$$

Table 1. Ingredients for $P_{\rm cyc}$–Ω theoretical relationships of Fig. 3

Curve	$P_{\rm cyc}$ relation	α-scaling	$\Delta\Omega$-scaling	u_m-scaling
LMF1	Eq. (2)	$\alpha_0 \propto \Omega$	$\Delta\Omega \propto \Omega^{+0.67}$	none
NMF1	Eq. (3)	$\alpha_0 \propto \Omega$	$\Delta\Omega \propto \Omega^{+0.67}$	none
NMF2	Eq. (3)	$\alpha_0 \propto \Omega$	$\Delta\Omega \propto \Omega^{-0.15}$	none
NMF3	Eq. (3)	none	$\Delta\Omega \propto \Omega^{+0.67}$	none
NMF4	Eq. (3)	none	$\Delta\Omega \propto \Omega^{-0.15}$	none
BL1	Eq. (4)	N/A	none	$u_m \propto \log(\Omega)$
BL2	Eq. (4)	N/A	none	$u_m \propto \Omega$

[Joint work with S. Saar, Harvard/CfA]

Dynamo modelling in solar-type stars (3)



[Joint work with S. Saar, Harvard/CfA]

Dynamo modelling in solar-type stars (4)

Other questions:

1. Moving down the main-sequence, what happens when we hit fully convective stars ?
2. Without a tachocline, is the Babcock-Leighton mechanism possible? Are there still starspots?
3. As the convective envelope gains in depth, are there « transitions » in dynamo operating modes (alpha-Omega to alpha^2-Omega to alpha^2) ?
4. How do nonlinearities play into all this?

Intermediate mass stars

Fossil fields versus dynamo action

The absence of observed temporal variability is compatible with the idea that a fossil field, **OR** a field produced during a convective phase during pre-main-sequence evolution

There exists dynamo mechanisms driven by MHD instabilities of large-scale internal fossil fields, which could operate in stellar radiative envelopes if significant differential rotation is present.

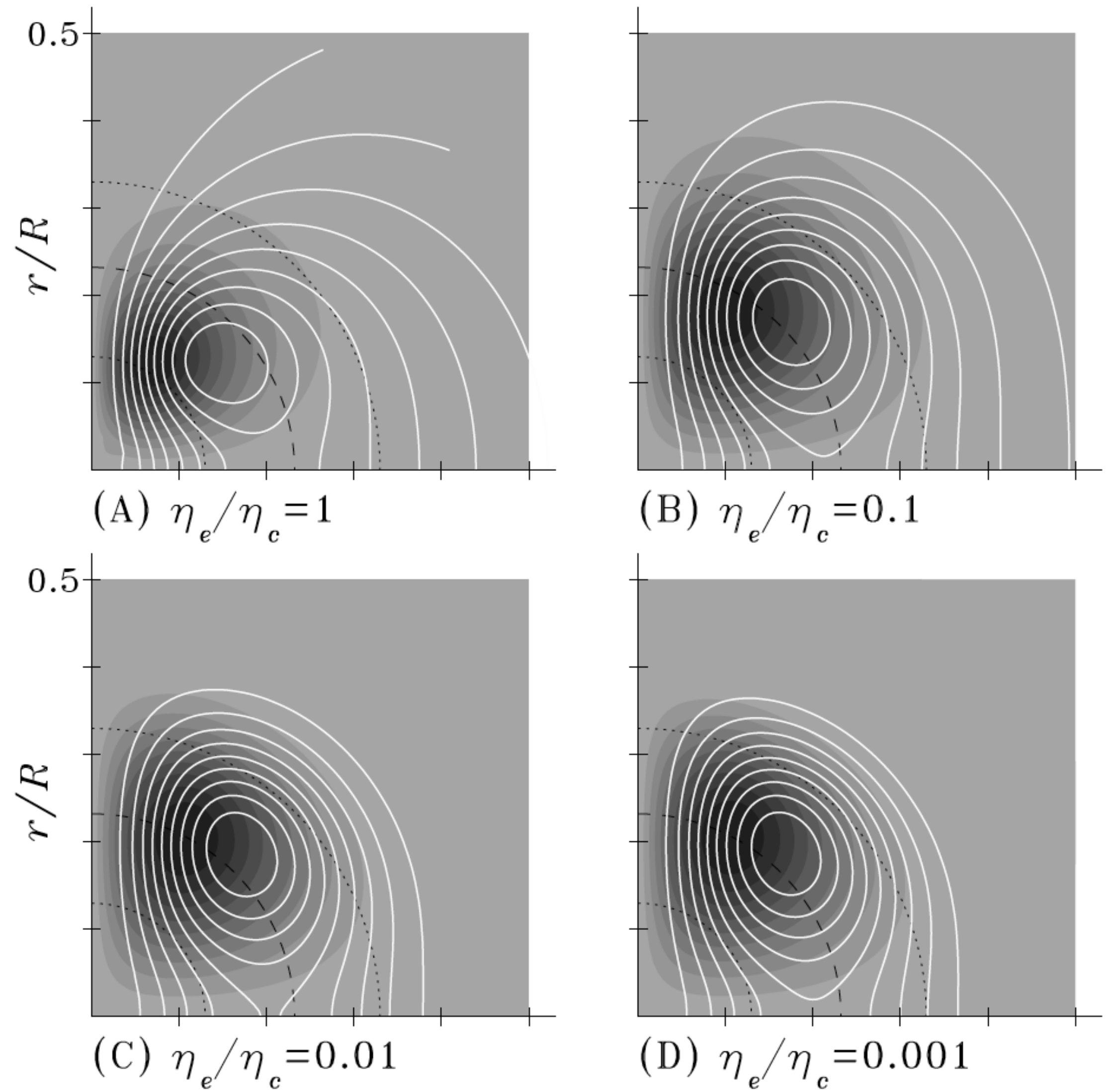
Early-type stars

alpha^2 dynamo solutions

The magnetic field produced by a kinematic alpha^2 dynamo is usually steady in time.

The magnetic field remains « trapped » in the deep interior if a strong magnetic diffusivity contrast exists between the core and envelope.

A very strong field may exist in the deep interior, without being visible at the surface !!



3D MHD core dynamo action (1)

THE ASTROPHYSICAL JOURNAL, 629:461–481, 2005 August 10

SIMULATIONS OF CORE CONVECTION IN ROTATING A-TYPE STARS: MAGNETIC DYNAMO ACTION

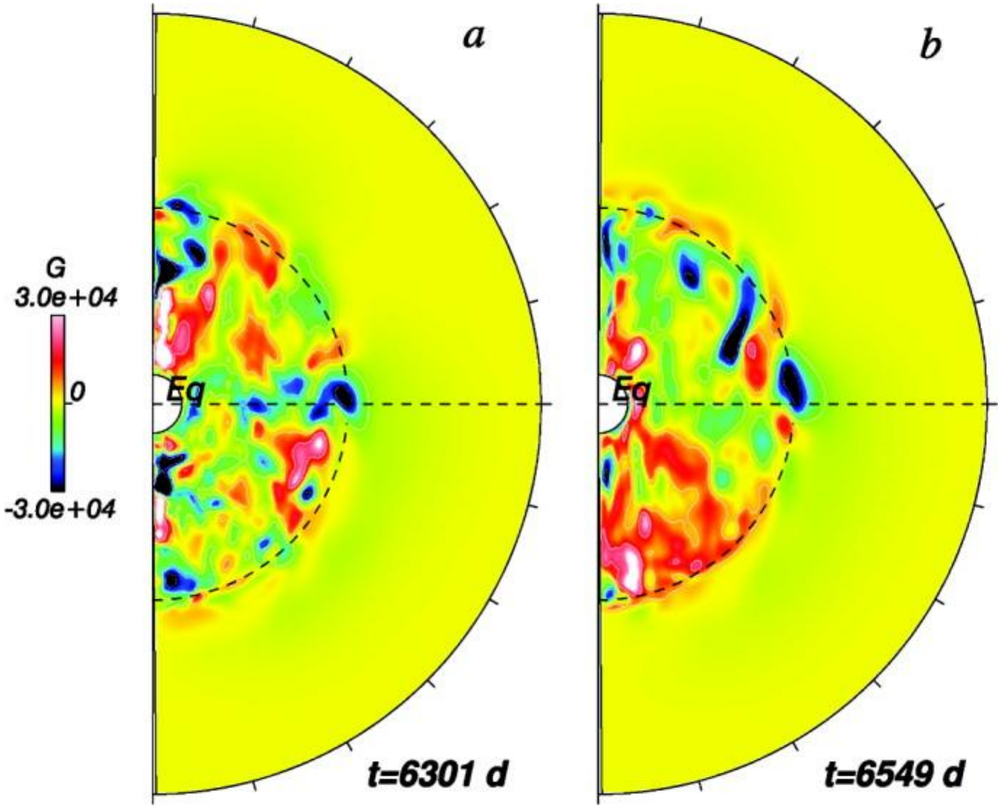
ALLAN SACHA BRUN,[1,2] MATTHEW K. BROWNING,[2] AND JURI TOOMRE[2]
Received 2004 September 24; accepted 2005 March 21

ABSTRACT

Core convection and dynamo activity deep within rotating A-type stars of 2 $M_\odot$ are studied with three-dimensional nonlinear simulations. Our modeling considers the inner 30% by radius of such stars, thus capturing within a spherical domain the convective core and a modest portion of the surrounding radiative envelope. The magnetohydrodynamic (MHD) equations are solved using the anelastic spherical harmonic (ASH) code to examine turbulent flows and magnetic fields, both of which exhibit intricate time dependence. By introducing small seed magnetic fields into our progenitor hydrodynamic models rotating at 1 and 4 times the solar rate, we assess here how the vigorous convection can amplify those fields and sustain them against ohmic decay. Dynamo action is indeed realized, ultimately yielding magnetic fields that possess energy densities comparable to that of the flows. Such magnetism reduces the differential rotation obtained in the progenitors, partly by Maxwell stresses that transport angular momentum poleward and oppose the Reynolds stresses in the latitudinal balance. In contrast, in the radial direction we find that the Maxwell and Reynolds stresses may act together to transport angular momentum. The central columns of slow rotation established in the progenitors are weakened, with the differential rotation waxing and waning in strength as the simulations evolve. We assess the morphology of the flows and magnetic fields, their complex temporal variations, and the manner in which dynamo action is sustained. Differential rotation and helical convection are both found to play roles in giving rise to the magnetic fields. The magnetism is dominated by strong fluctuating fields throughout the core, with the axisymmetric (mean) fields there relatively weak. The fluctuating magnetic fields decrease rapidly with radius in the region of overshooting, and the mean toroidal fields less so due to stretching by rotational shear.

Subject headings: convection — MHD — stars: evolution — stars: interiors — stars: magnetic fields

3D MHD core dynamo action (2)



[Azimuthally-averaged toroidal magnetic component, in meridional plane]

From the core to the surface (1)

THE ASTROPHYSICAL JOURNAL, 586:480–494, 2003 March 20

MAGNETIC FIELDS IN MASSIVE STARS. II. THE BUOYANT RISE OF MAGNETIC FLUX TUBES THROUGH THE RADIATIVE INTERIOR

K. B. MACGREGOR[1] AND J. P. CASSINELLI[1,2]
Received 2001 November 8; accepted 2001 November 21

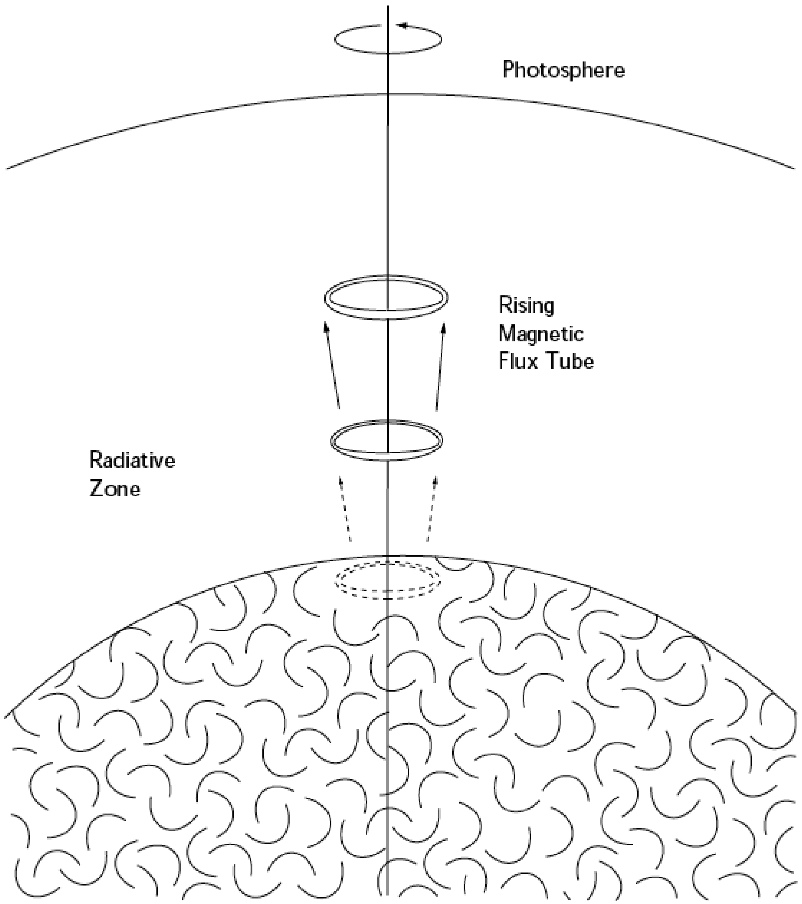
ABSTRACT

We present results from an investigation of the dynamical behavior of buoyant magnetic flux rings in the radiative interior of a uniformly rotating, early-type star. Our physical model describes a thin, axisymmetric, toroidal flux tube that is released from the outer boundary of the convective core and is acted on by buoyant, centrifugal, Coriolis, magnetic tension, and aerodynamic drag forces. We find that rings emitted in the equatorial plane can attain a stationary equilibrium state that is stable with respect to small displacements in radius, but is unstable when perturbed in the meridional direction. Rings emitted at other latitudes travel toward the surface along trajectories that largely parallel the rotation axis of the star. Over much of the ascent, the instantaneous rise speed is determined by the rate of heating by the absorption of radiation that diffuses into the tube from the external medium. Since the timescale for this heating varies like the square of the tube cross-sectional radius, for the same field strength, thin rings rise more rapidly than do thick rings. For a reasonable range of assumed ring sizes and field strengths, our results suggest that buoyancy is a viable mechanism for bringing magnetic flux from the core to the surface, being capable of accomplishing this transport in a time that is generally much less than the stellar main-sequence lifetime.

Subject headings: convection — stars: atmospheres — stars: early-type — stars: interiors — stars: magnetic fields

From the core to the surface (2)

[MacGregor & Cassinelli, ApJ, 586, 480 (2003)]

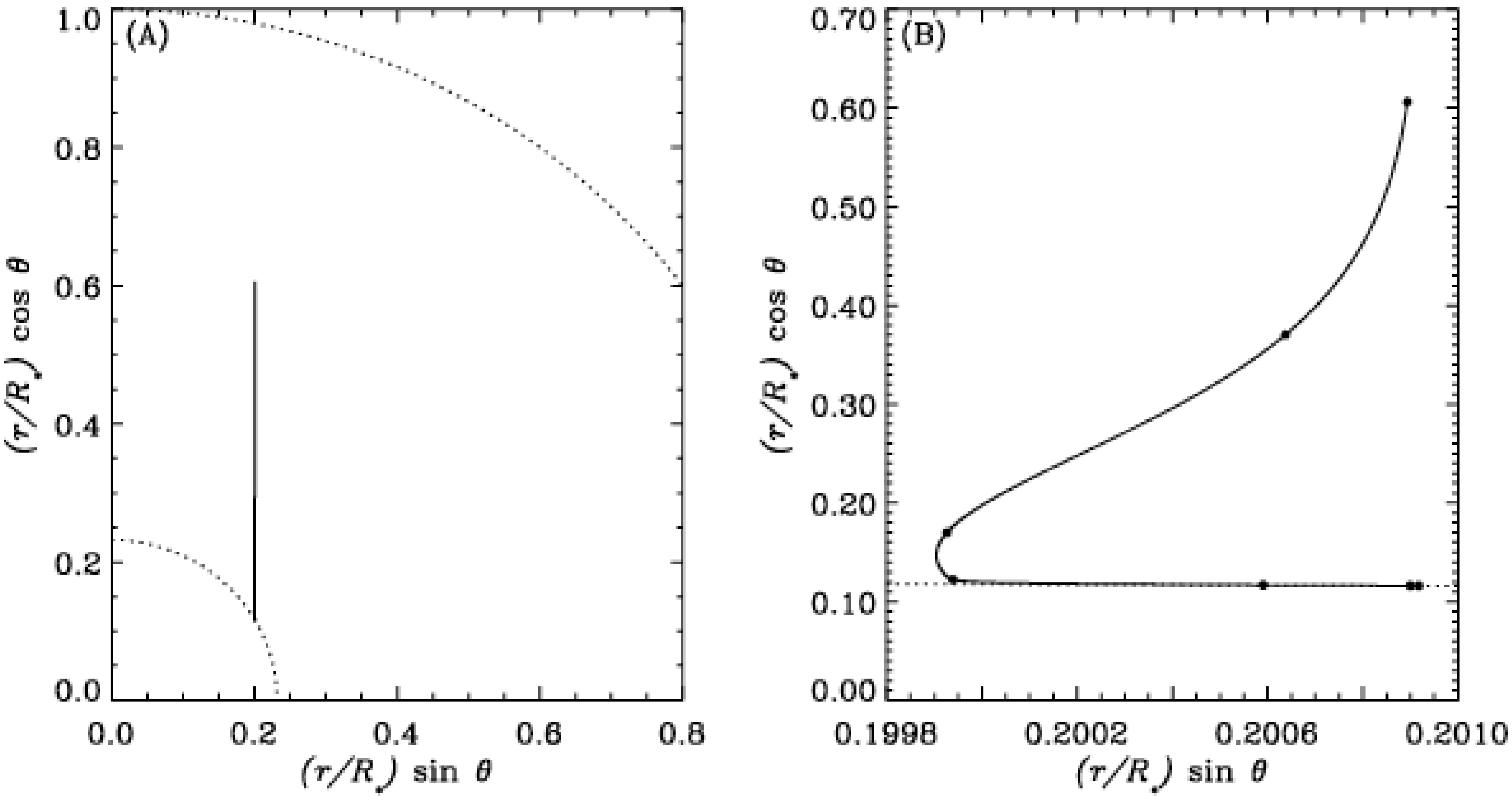


In analogy to what we think happens at the solar core-envelope interface, could toroidal flux ropes here also form at the core boundary, and if so could they rise buoyantly to the surface

From the core to the surface (3)

[MacGregor & Cassinelli, ApJ, 586, 480 (2003)]

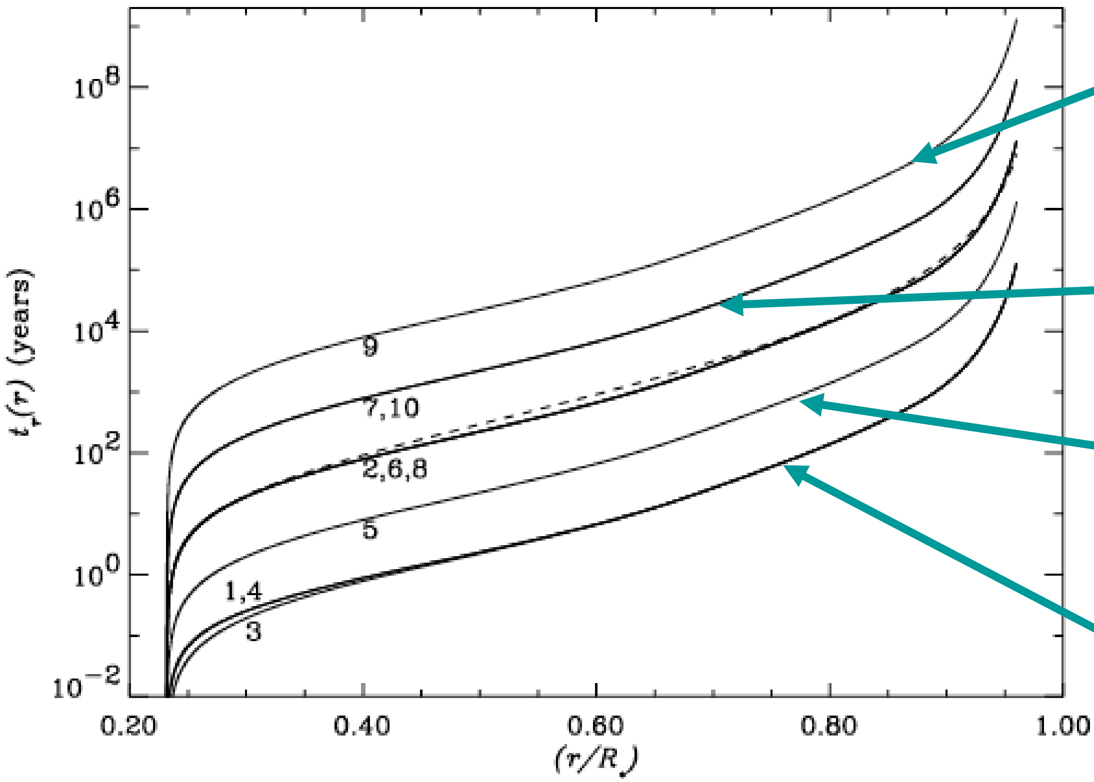
Thin tube (0.0001h), 58T, with drag



From the core to the surface (4)

[MacGregor & Cassinelli, ApJ, 586, 480 (2003)]

If toroidal magnetic flux ropes do form at the boundary of the convective core, magnetic buoyancy can lift them up to a tenth of a stellar radius under the surface, **under the most optimistic working hypotheses.**



Curve « 9 »: thick tube, 58T, without drag

Curve « 7 »: thick tube, 18T, with drag

Curve « 5 »: thin tube, 18T, with drag

Curve « 1 »: thin tube, 58T, with drag

Alternatives to core dynamo action

1. Dynamo action powered by MHD instabilities in the radiative envelope (e.g., Spruit-Tayler); could contribute to internal angular momentum redistribution and to chemical mixing

2. Dynamo action in outer convective layers produced by iron opacities

Fully convective stars

Fully convective stars (1)

THE ASTROPHYSICAL JOURNAL, 638:336–347, 2006 February 10

MAGNETIC FIELD GENERATION IN FULLY CONVECTIVE ROTATING SPHERES

WOLFGANG DOBLER,[1,2] MICHAEL STIX,[1] AND AXEL BRANDENBURG[3]
Received 2004 October 26; accepted 2005 October 7

ABSTRACT

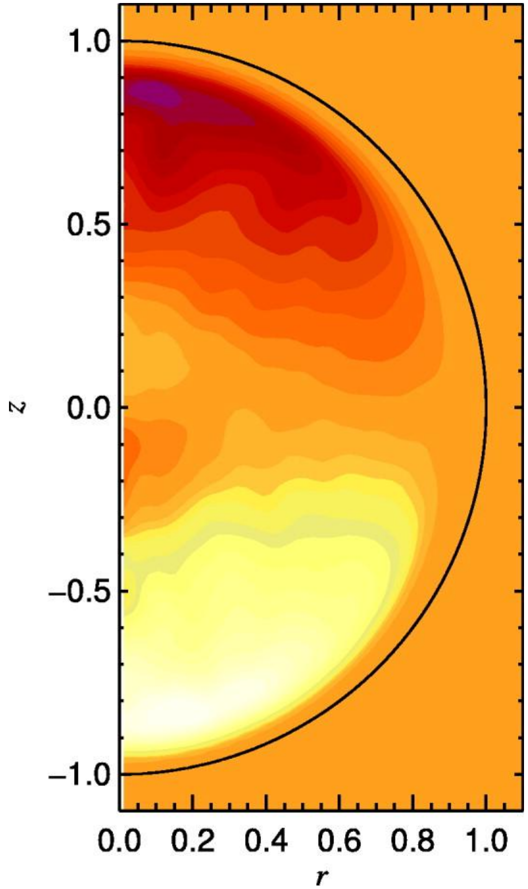
Magnetohydrodynamic simulations of fully convective, rotating spheres with volume heating near the center and cooling at the surface are presented. The dynamo-generated magnetic field saturates at equipartition field strength near the surface. In the interior, the field is dominated by small-scale structures, but outside the sphere, by the global scale. Azimuthal averages of the field reveal a large-scale field of smaller amplitude also inside the star. The internal angular velocity shows some tendency to be constant along cylinders and is "antisolar" (fastest at the poles and slowest at the equator).

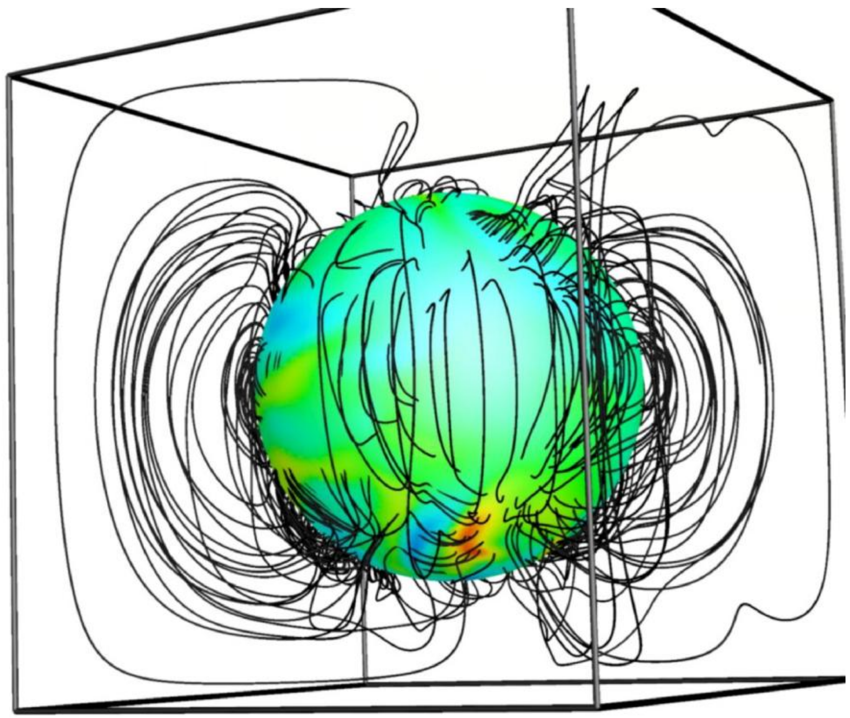
Subject headings: convection — MHD — stars: low-mass, brown dwarfs — stars: magnetic fields — stars: pre–main-sequence — turbulence

Online material: color figures

Fully convective stars (2)

Kinetic helicity





Dynamo problems

The kinematic dynamo problem:

HARD

TURBULENCE

« To find a flow **u** that can lead to field amplification when substituted in the MHD equation »

The self-excited dynamo problem:

MUCH HARDER

TURBULENCE

« To find a flow **u** that can lead to field amplification when substituted in the MHD equation, while being dynamically consistent with the fluid equations including the Lorentz force »

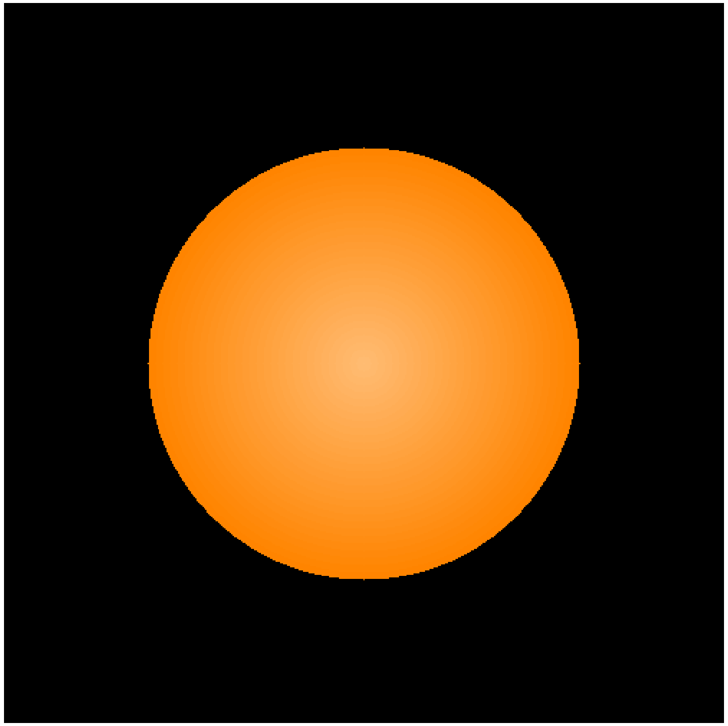
The solar/stellar dynamo problem(s):

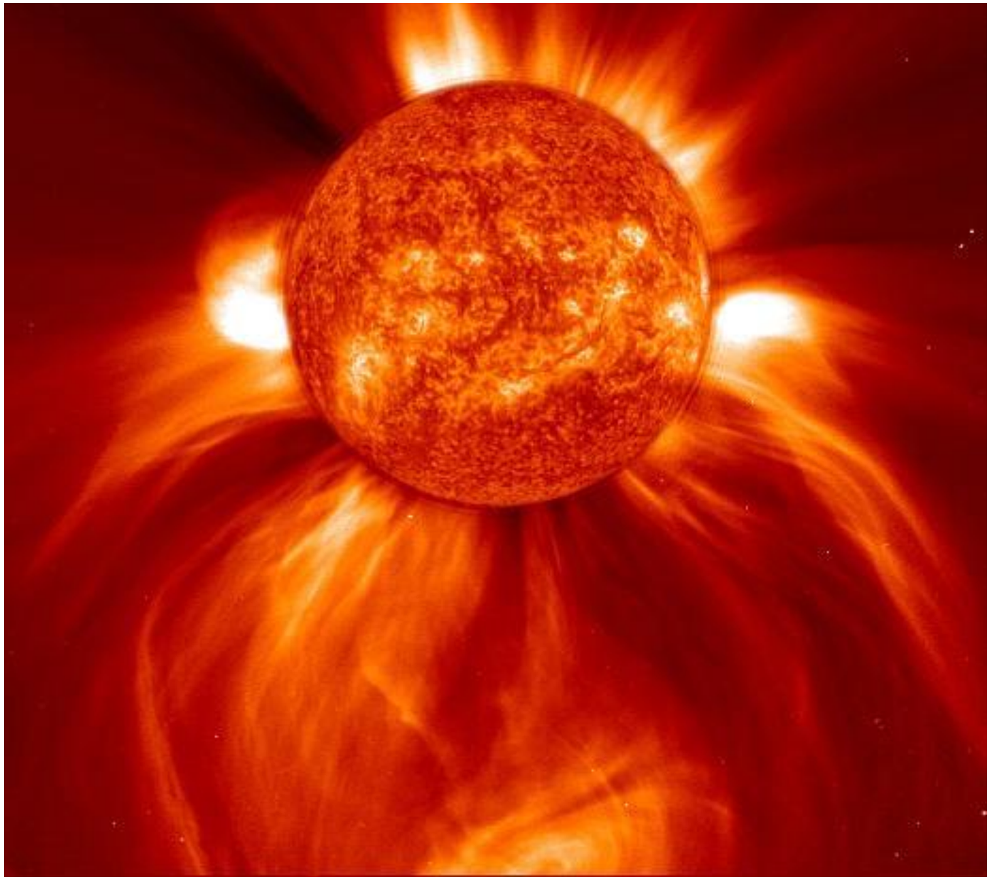
HARDEST

????????

« To find a flow **u** that leads to a magnetic field amplification and evolution in agreement with observational inferences for the Sun and stars »

Solar/stellar magnetism





« If the sun did not have a magnetic field, it would be as boring a star as most astronomers believe it to be »

(Attributed to R.B. Leighton)