Waves in Planetary Atmospheres

R. L. Walterscheid

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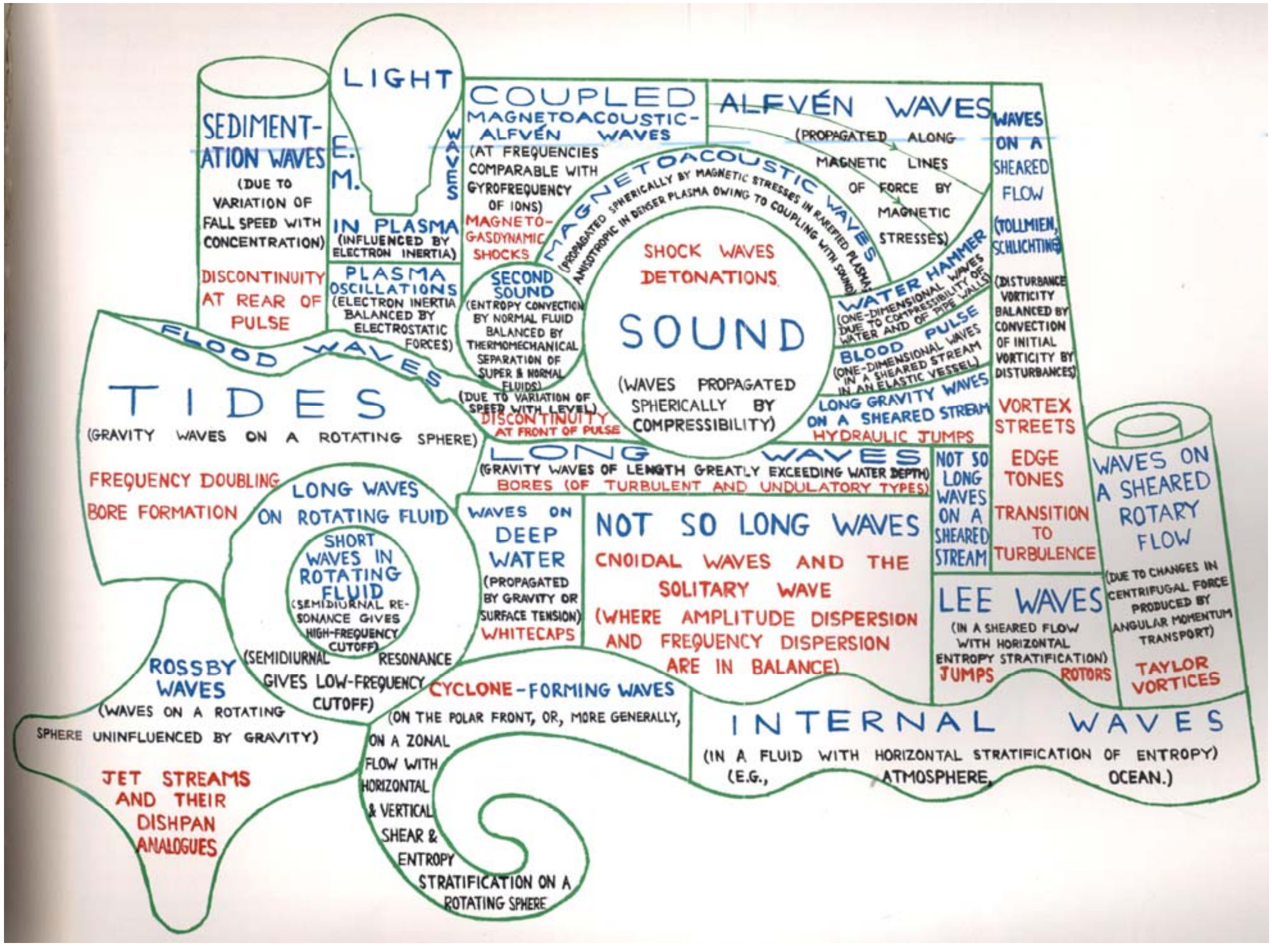
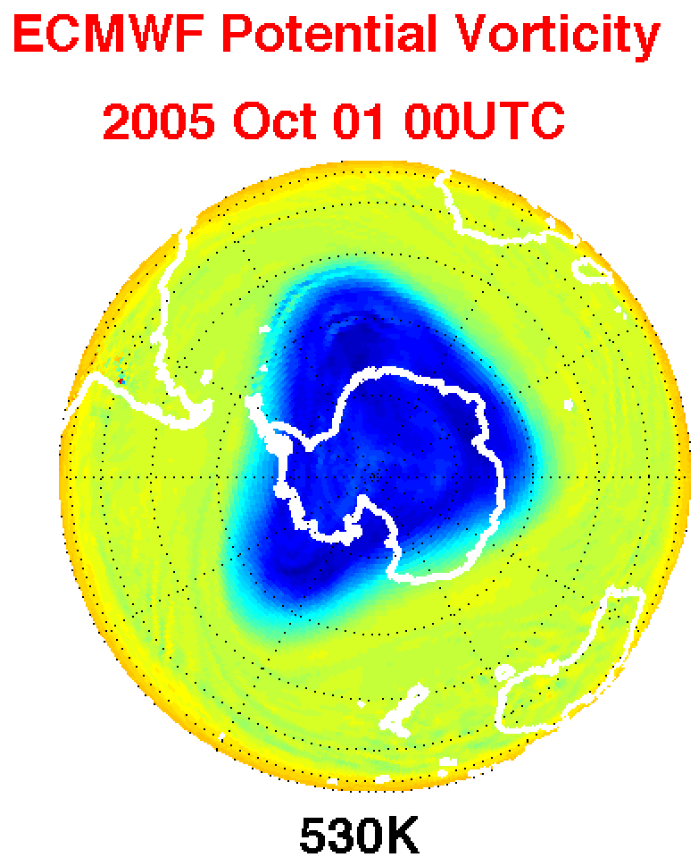
The Wave Zoo



Lighthill, Comm. Pure Appl. Math., 20, 1967

Wave-Deformed Antarctic Vortex



Courtesy of VORCORE Project, Vial et al., 2007

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Atmospheric Waves

- Some waves that are common to rotating bodies with sensible atmospheres
  - Rossby-Haurwitz
  - Gravity
  - Tides
  - Kelvin
  - Acoustic
  - Inertial

Large-Scale Waves

- Here I focus on two broad classes of large-scale waves
- Planetary (Rossby-Haurwitz) waves
  - "Planetary" refers to both the scale of the wave and to dynamics peculiar to rotating bodies
  - Planetary waves are waves that depend in some way on the effects of rotation – in particular on the variation of the vertical component of rotation with latitude
- Gravity waves
  - Wave in a stratified medium and for which gravity is the restoring force
- Atmospheric tides include both classes

Components of Motion

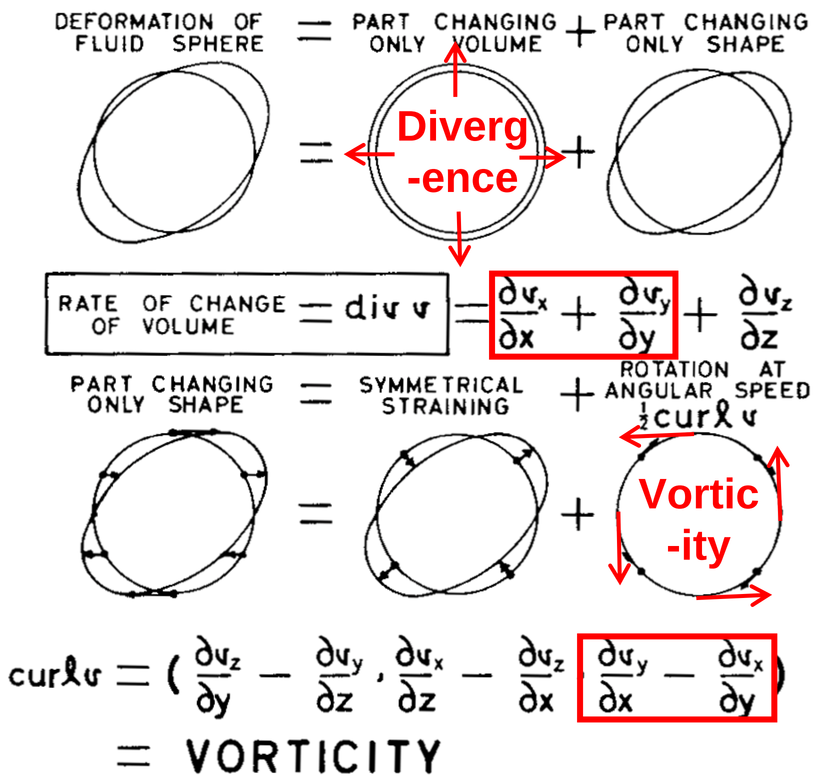


Figure 1. Analysis of deformation of a small spherical element of fluid in non-uniform motion.

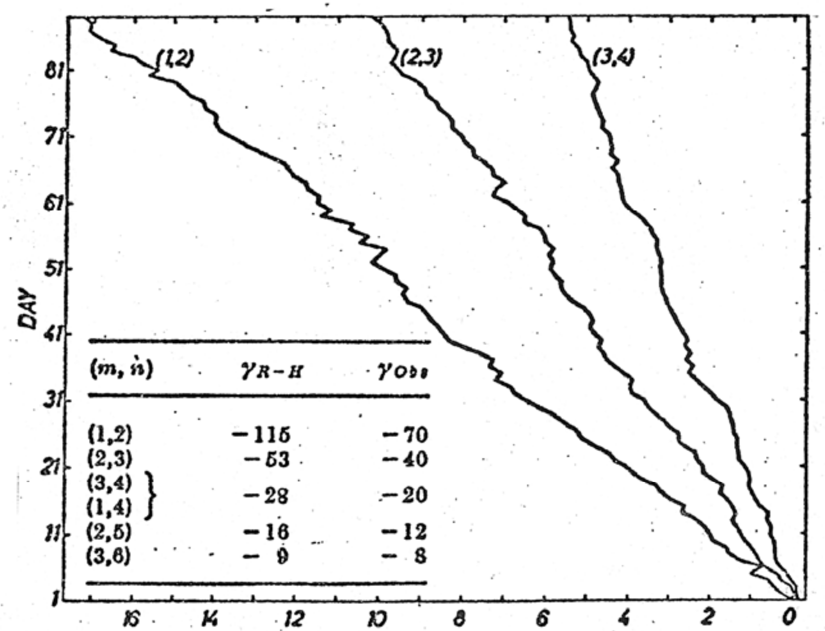
Gravity Waves

Rossby Waves

Lighthill, Comm. Pure Appl. Math., 20, 1967

Observational Introduction
Traveling Free Waves

E. ELIASEN AND B. MACHENHAUER



(m, n)	γ_{R-H}	γ_{Obs}
(1,2)	−115	−70
(2,3)	−53	−40
(3,4) (1,4)	−28	−20
(2,5)	−16	−12
(3,6)	−9	−8

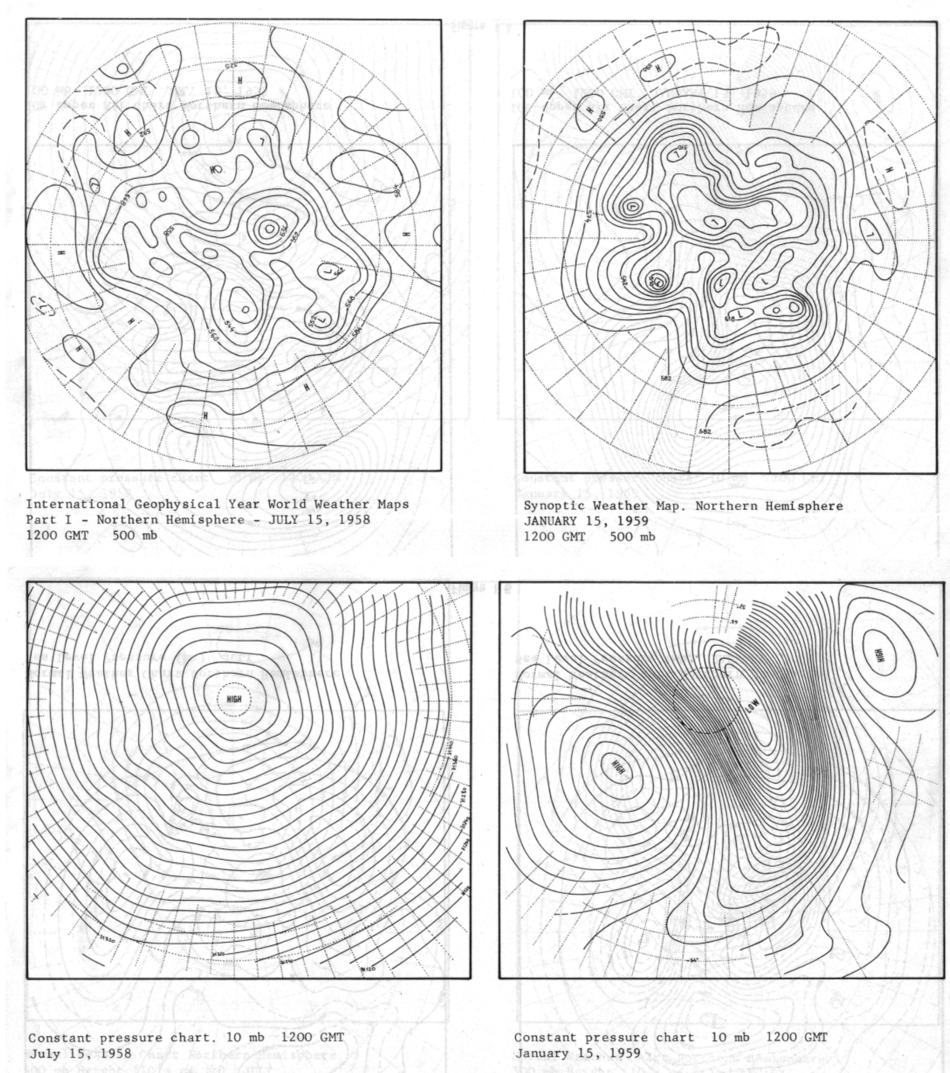
FIG. 3. Successive daily values of the phase angle for the 24 hours tendency field, $\psi_{m,n}(t+1)-\psi_{m,n}(t)$, at the 500 mb level for the components $(m,n)=(1,2)$, (2,3), and (3,4), during the 90 days period, beginning 1 December 1956. Abscissae are the number of westward circulations round the earth after the first passage of the Greenwich meridian.

Eliasen and Machenhauer, 1965 (Tellus, 17, p 220-238)

Observational Introduction

Stationary Waves

Charney, Dynamic Meteorology, D. Reidel, 1973



500 hPa

10 hPa

Summer **Winter**

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Observational Introduction

Gravity Waves

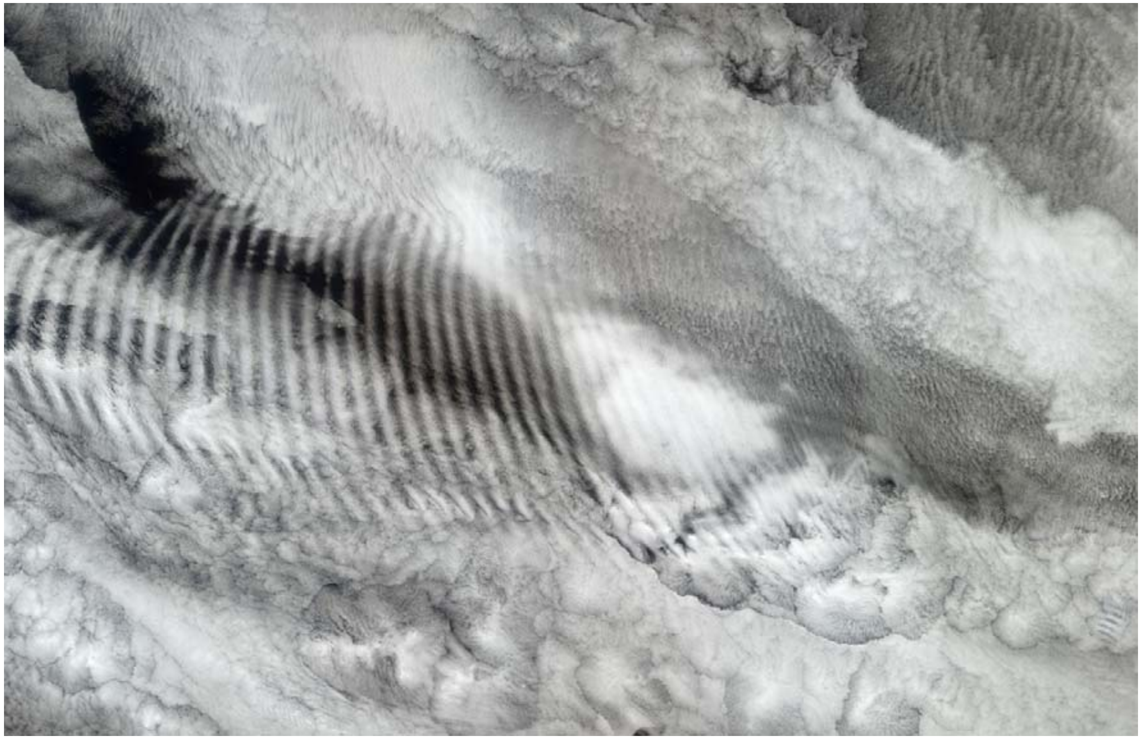


Image courtesy
NASA/GSFC/LaRC/JPL,MISR Team

Observational Introduction
Atmospheric Tides

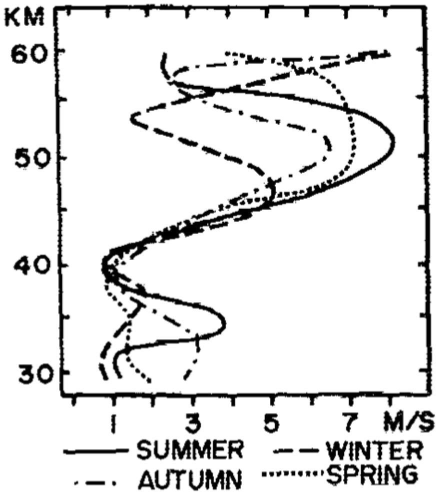


FIG. 9. Amplitude of the diurnal tide in the meridional wind at Ascension Island for each Southern Hemisphere season.

Nastrom and Belmont, J. Atmos. Sci., 1976

Dynamics of Planetary Waves
Vorticity

- **Vorticity**
  - **Curl of velocity**
  - **Mainly interested in vertical component**
  - **For a body in solid rotation**

$$\varsigma = 2\Omega$$

- **Absolute vorticity**
  - **Sum of relative vorticity and planetary vorticity**

Planetary Waves
Mechanism for Wave Motion (1)

$$f + \varsigma = \text{absolute vorticity} \sim \text{constant}$$

$$f = 2\Omega \sin \varphi \qquad \text{(planetary vorticity)}$$

$$\varsigma = \partial v/\partial x - \partial u/\partial y \quad \text{(relative vorticity)}$$

Rotational part of wind field

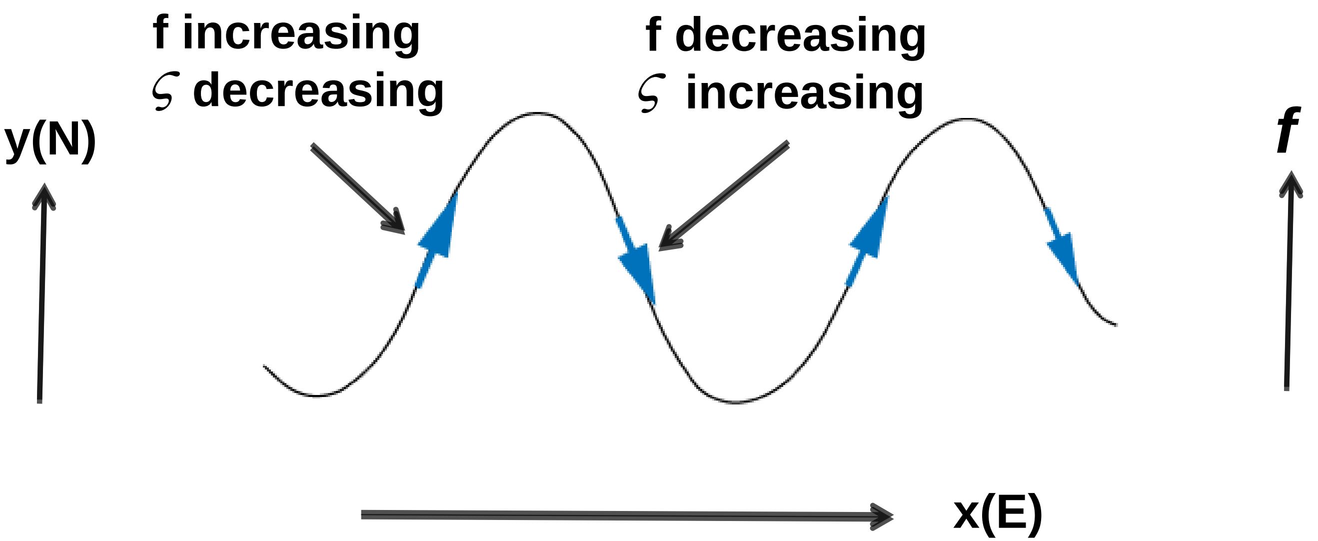
$$\mathbf{u}_\psi = -\hat{\mathbf{k}} \times \nabla \psi$$

$$\psi = \text{Stream funtion}$$

$$\boxed{\varsigma = \nabla^2 \psi}$$

$\Rightarrow$**Vorticity a function of curvature along stream line**

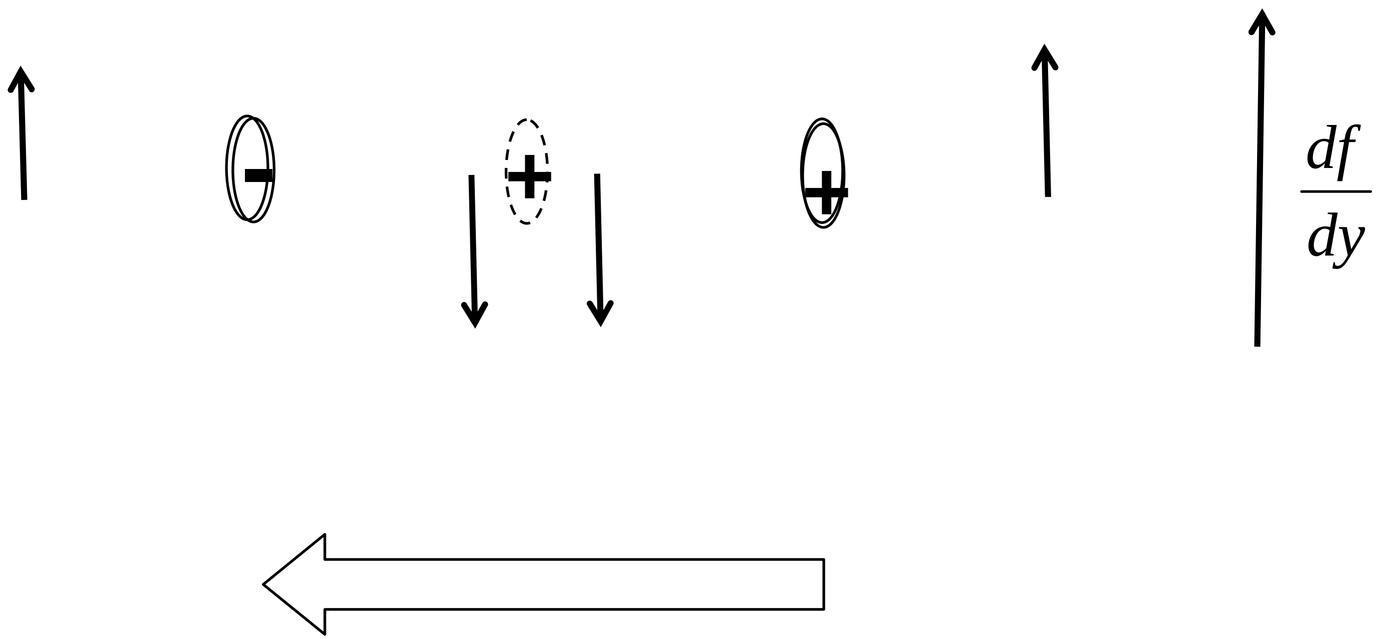
Planetary Waves
Mechanism for Wave Motion (2)



Restoring mechanism is latitudinal gradient of planetary vorticity

Propagation of Rossby Waves

Mechanism of Wave Propagation



Vorticity tendency due to advection of planetary vorticity induces westward motion

Prototype Rossby Waves

- **Non-divergent barotropic motion on a β-plane**

$$f = f_0 + \beta(y - y_0)$$
$$\beta = (\partial f / \partial y)_0 = \text{constant}$$

- **Dispersion relation**

$$\omega = -\beta/k$$

- **Phase velocity**

$$c = -\beta/k^2 \quad \text{(westward)}$$

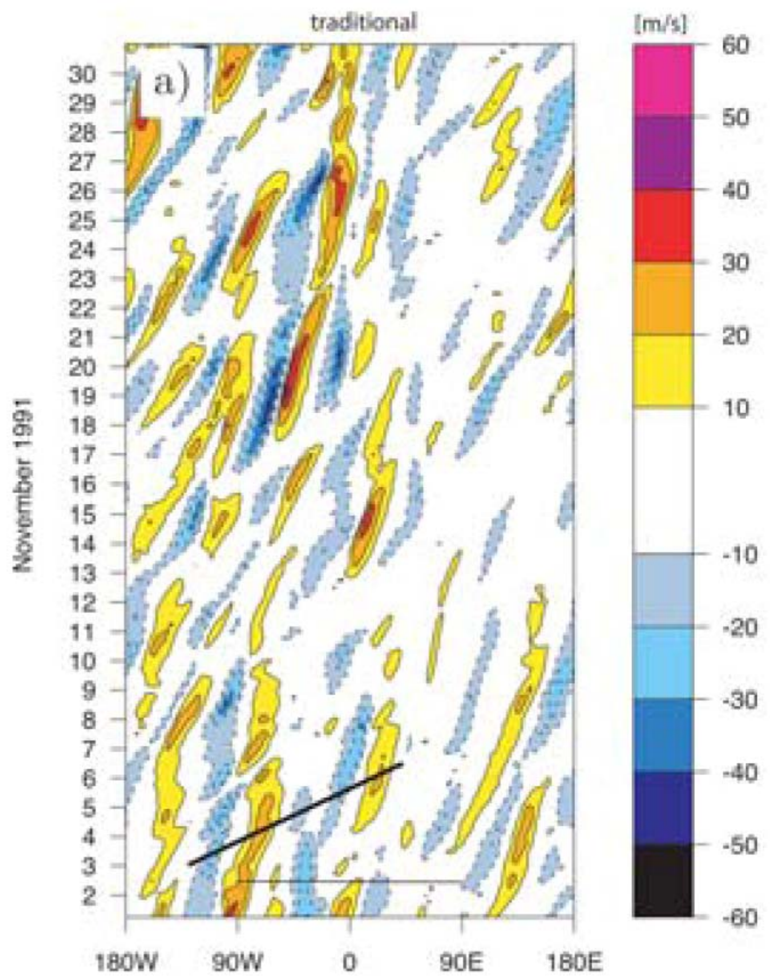
- **Group velocity**

$$u_g = +\beta/k^2 \quad \text{(eastward)}$$

$$\boxed{u_g = -c}$$

Rossby Wave Group Propagation

Hovmöller Diagram



Martius et al., Tellus, 2006

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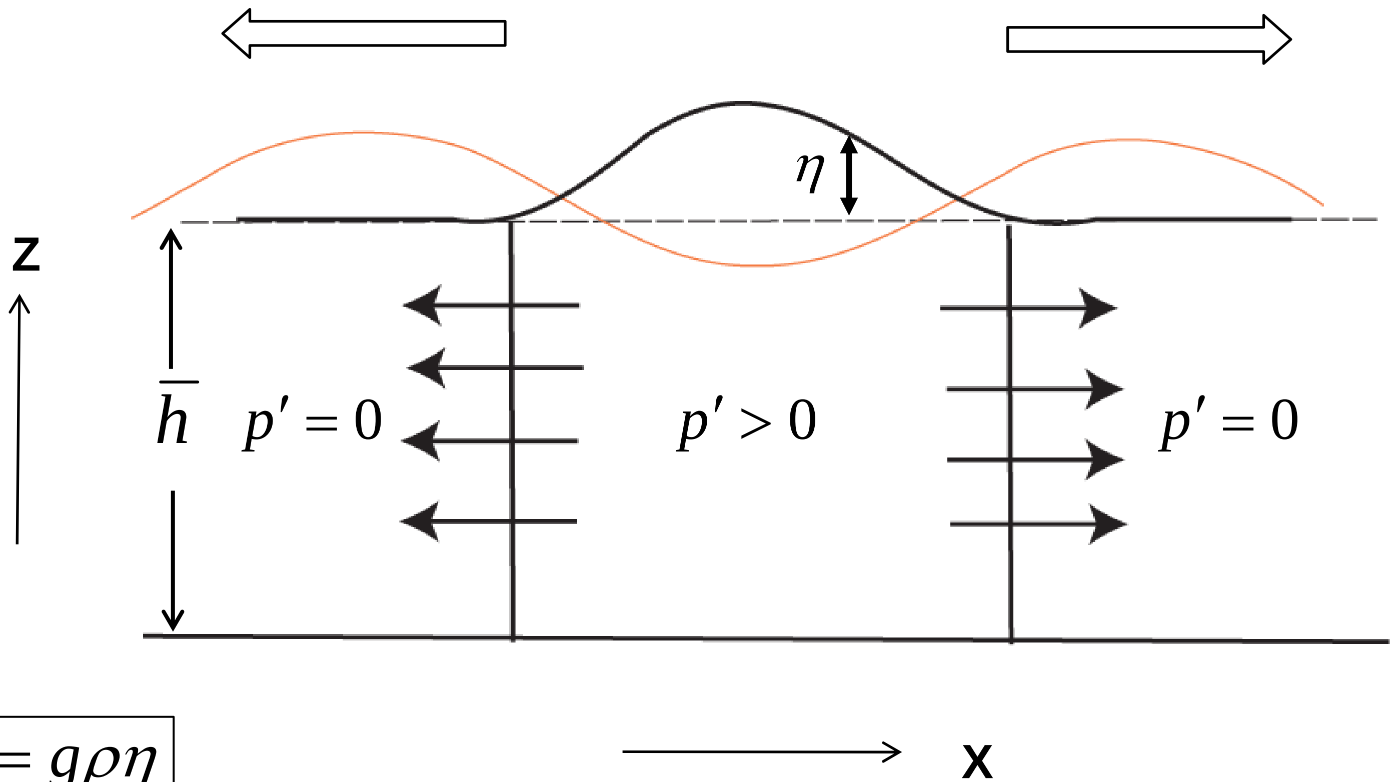
Dynamics of Gravity Waves

- Exist in stratified fluids in gravitational fields
- Propagate horizontal divergence

Gravity Waves



$$\boxed{p' = g\rho\eta}$$

Wave propagates through interplay between horizontal divergence and pressure gradients in a gravitational field

Prototype Gravity Waves

Wave motion of an irrotational nondivergent fluid with a free surface

- **Dispersion relation for waves in shallow water ($l_x >> \overline{h}$)**

$$\omega^2 = g\overline{h}k^2$$

- **Phase speed**

$$c_x = \pm\sqrt{g\overline{h}}$$

- **Group speed**

$$u_g = c_x$$

Quantitative Theory of Oscillations on a Rotating Sphere

Oscillations of an Ideal Ocean

- **Oscillations of the free surface of an ocean of uniform undisturbed depth covering a rotating planet**
- **Shallow water equations**
  - **Shallow compared to horizontal scale of the motion**
  - **Quasi-static**

Quantitative Theory of Oscillations on a Rotating Sphere

- **Laplace's Tidal Equations**
  - **Linearized equations for motions on a background state of rest**
  - **Describes a variety of wave types**
    - **Rossby-Haurwitz waves**
    - **Gravity waves**
    - **Kelvin**
    - **Tides**

Laplace’s Tidal Equations for an Ideal Ocean

$$\frac{\partial u'}{\partial t} - (2\Omega \sin\varphi) v' = -\frac{\partial \phi'}{a \cos\varphi \, \partial\lambda} \qquad (1)$$

$$\frac{\partial v'}{\partial t} + (2\Omega \sin\varphi) u' = -\frac{\partial \phi'}{a \partial\varphi} \qquad (2)$$

$$\frac{1}{a\cos\varphi}\left[\frac{\partial u'}{\partial\lambda} + \frac{\partial(\cos\varphi \, v')}{\partial\varphi}\right] + \frac{1}{g\bar{h}}\frac{\partial\phi'}{\partial t} = 0 \qquad (3)$$

$\varphi = \text{Latitude}$

$\lambda = \text{Longitude}$

$\phi' = \mathrm{g}\eta$

$\eta = h - \bar{h}$

$h = \text{Height of the free surface}$

$\psi' = \psi - \overline{\psi}$

$u = a\cos\varphi\,\dot{\lambda}$

$v = a\,\dot{\varphi}$

$a = \text{Planetary radius}$

$\Omega = \text{Planetary rotation rate}$

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Waveform Solutions

Assume solutions of the form

$$\phi'(\varphi, \lambda, t) = \Phi(\varphi)\exp[i(\omega t - s\lambda)]$$

where

$$s = \text{Zonal Wavenumber}$$

$$\omega = \text{Wave frequency}$$

Laplace's Tidal Equation

$$F\left[\Theta^{\omega,s}(\varphi)\right] = \varepsilon\,\Theta^{\omega,s}(\varphi)$$

$$F = \frac{d}{d\mu}\left(\frac{1-\mu^2}{f^2-\mu^2}\frac{d}{d\mu}\right) - \frac{1}{f^2-\mu^2}\left[\frac{s}{f}\left(\frac{f^2+\mu^2}{f^2-\mu^2}\right) + \frac{s}{1-\mu^2}\right]$$

$$\varepsilon = \frac{(2\Omega a)^2}{g\bar{h}}$$

where

$$\mu = \sin\varphi$$

$$a = \text{Planetary radius}$$

$$\varepsilon = \text{Lambs parameter}$$

$$f = \text{Nondemensional frequency}\,(= \omega/2\Omega)$$

$$\Omega = \text{Planetary angular rotation rate}$$

Eigensolutions

- **Eigenvalues**
  - ***f*** **(eigenfrequency)**
- **Eigenfunction**
  - **Hough Functions**
- **Solutions depend parametrically on *s* and ε**
  - **For every s and ε there is a complete set of modes each with a different frequency**

Oscillations on a Sphere

Wave Types (Limiting Cases)

- **Small ε*f***
  - **Large $f \Rightarrow$ Class 1 (Irrotational gravity waves on a sphere)**

$$f_n^{\,2} = \varepsilon n(n+1) \qquad \Theta_n^s(\mu) = P_n^s(\mu)$$

    - **Eastward and westward propagation**
  - **Small $f \Rightarrow$ Class 2 (Nondivergent Rossby-Haurwitz waves)**

$$f_n = -\frac{s}{n(n+1)} \qquad \Theta_n^s(\mu) = P_n^s(\mu)$$

    - **Propagate only westward**

Eigensolutions

Eigenfrequencies

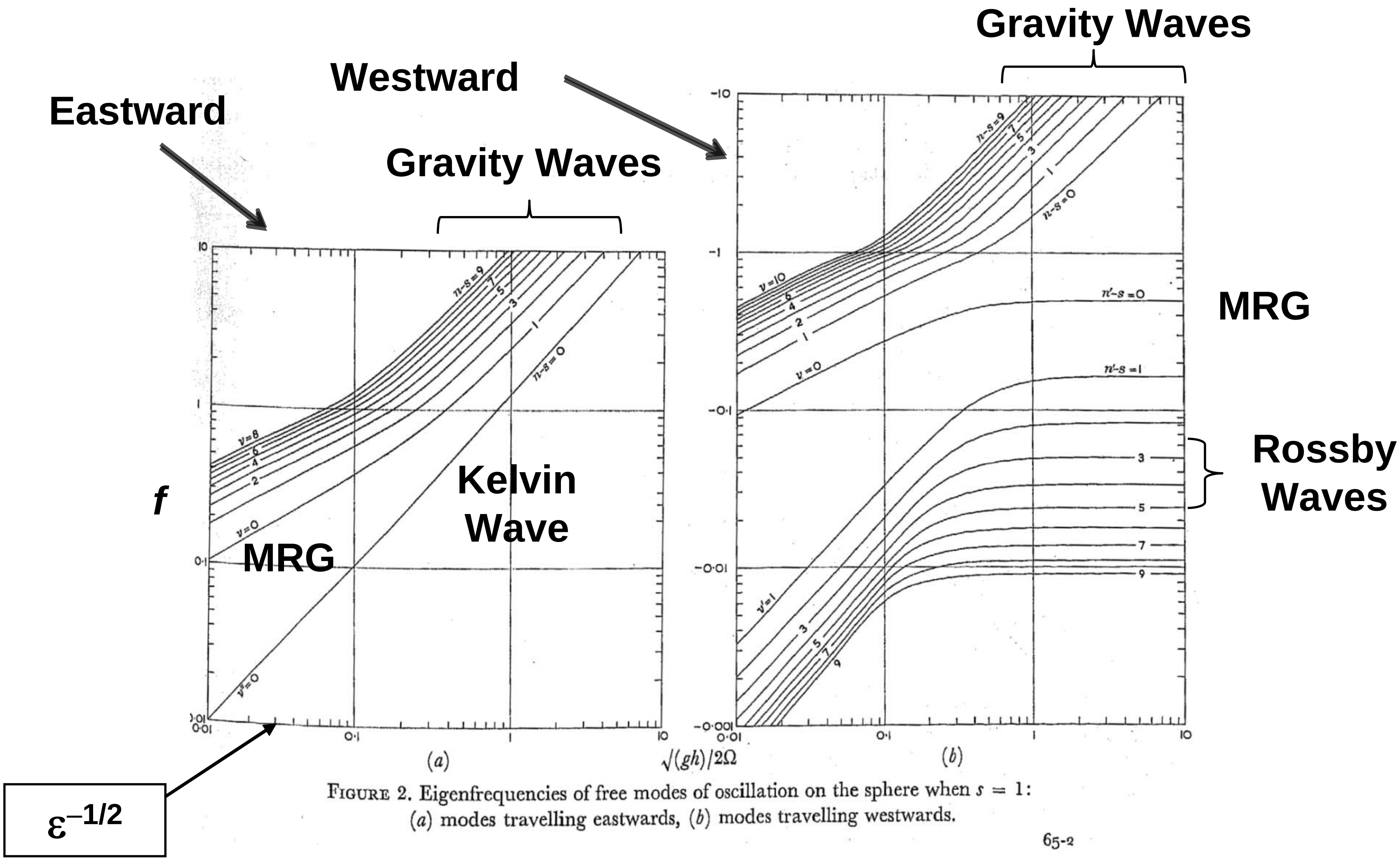

FIGURE 2. Eigenfrequencies of free modes of oscillation on the sphere when $s = 1$: (a) modes travelling eastwards, (b) modes travelling westwards.

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Longuet-Higgins, Trans. Roy. Soc. London, Ser. A, 1968

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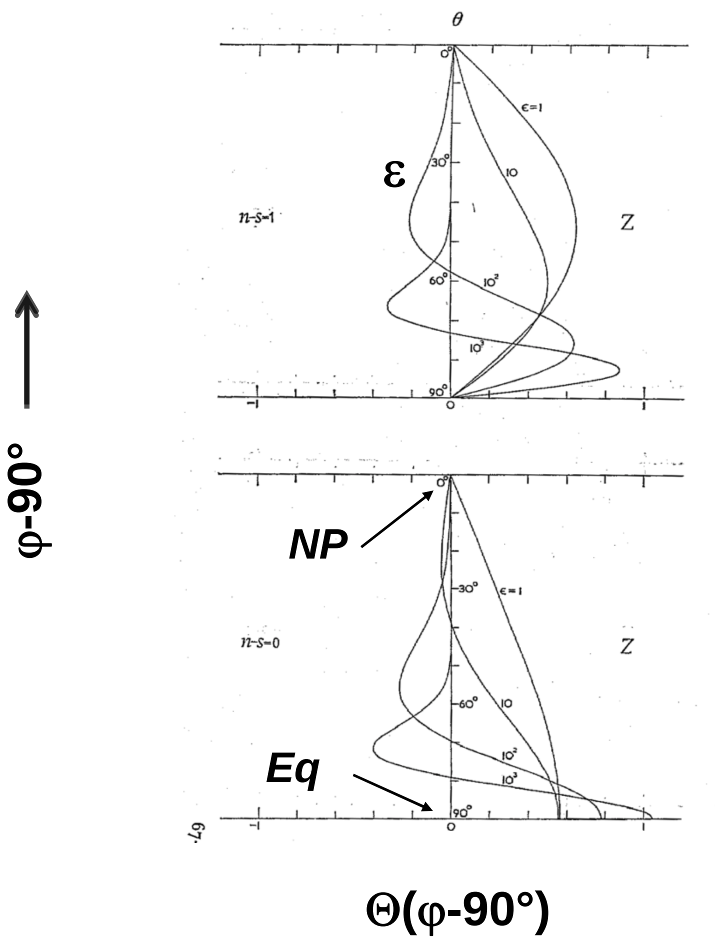
Eigensolutions

Eigenfunctions



As $\mathcal{E}$ increases the eigenfunctions become increasingly equatorially trapped

Longuet-Higgins, Trans. Roy. Soc. London, Ser. A, 1968

Tidal Equations for an Atmosphere

$$\frac{\partial u'}{\partial t} - (2\Omega \sin\varphi) v' = -\frac{\partial \phi'}{a\cos\varphi\,\partial\lambda} \qquad (1)$$

$$\frac{\partial v'}{\partial t} + (2\Omega \sin\varphi) u' = -\frac{\partial \phi'}{a\partial\varphi} \qquad (2)$$

$$\frac{\partial \phi'_z}{\partial t} + Sw' = 0 \qquad (3)$$

$$\frac{1}{a\cos\varphi}\left[\frac{\partial u'}{\partial\lambda} + \frac{\partial(\cos\varphi\, v')}{\partial\varphi}\right] + \frac{\partial\left(w' e^{-z}\right)}{\partial z} = 0 \qquad (4)$$

$$z = \log(p_0/p)$$

$$w = \dot{z}$$

$$\phi' = \mathrm{g}\,\eta$$

$$\eta = h - \overline{h}$$

$$h = \text{Height of constant } z \text{ surface}$$

$$S(z) = g\overline{H}\,\frac{d\log\overline{\theta}}{dz}$$

$$\overline{H} = R\overline{T}/g$$

$$\overline{\theta} = \overline{T}e^{\kappa z}$$

Traveling Oscillations in an Atmosphere

Assume solutions of the form

$$w'(\varphi, \lambda, z, t) = W(\varphi, z)\exp[i(\omega t - s\lambda)]$$

Traveling Oscillations in an Atmosphere

PDE for Oscillations on a Motionless Background State

$$\left[\frac{\partial}{\partial z}-1\right]\frac{\partial}{\partial z}W(\varphi,z)+\frac{S(z)}{4a^2\Omega^2}F\left[W(\varphi,z)\right]=0$$

Separable in height and latitude

$$W(\varphi,z)=\hat{w}(z)\theta(\varphi)$$

Traveling Oscillations in an Atmosphere

Separation of Variables

Vertical structure equation

$$\left[\frac{d}{dz}-1\right]\frac{d}{dz}\hat{w}(z)+\frac{S(z)}{gh}\hat{w}(z)=0$$

Horizontal structure equation

$$F\left[\Theta(\varphi)\right]=\varepsilon\Theta(\varphi)$$

Equivalent depth (separation constant)

$$h=(2\Omega a)^2\varepsilon / g$$

Refractive Index for Vertical Propagation

The vertical structure equation can be placed in canonical form by transforming the dependent variables

$$\frac{d^2}{dz^2}\tilde{w}(z) + m^2(z)\tilde{w}(z) = 0$$

$$\boxed{m^2 = \frac{S(z)}{gh} - \frac{1}{4}}$$

$$\tilde{w}(z) = \hat{w}(z)\exp(-z/2)$$

Dimensionally

$$m^2 = \frac{N^2}{gh} - \frac{1}{4H^2}$$

For *m² constant*

$$m^2 > 0 \Rightarrow \tilde{w} = A\exp(imz) + B\exp(-imz)$$

$$m^2 < 0 \Rightarrow \tilde{w} = A\exp(+\sqrt{m^2}z) + B\exp\left(-\sqrt{m^2}z\right)$$

Vertical Structure

- Waves in a continuously stratified fluid (atmospheres) may be internal or external
  - Internal waves exhibit wave-like behavior in the vertical direction
    - Transfer energy vertically
    - Conservative waves maintain nearly constant energy density with altitude
    - Implies that wave amplitude grows exponentially with altitude
    - Small amplitude waves originating in the lower atmosphere may achieve large amplitudes in the upper atmosphere
  - External waves have constant phase with height
    - Wave amplitude decays in height (evanescent)

Traveling Oscillations in an Atmosphere
Method of Solution

- **Find eigensolution of VSE** $\rightarrow$
  - **Vertical structure** $w'(z)$
  - **Equivalent depth** ***h***
- **Find eigensolutions of HSE** $\rightarrow$
  - **Eigenfunctions** $\theta(\varphi)$
  - **Eigenfrequencies** ω

Eigensolutions

Eigenfrequencies

Longuet-Higgins, Trans. Roy. Soc. London, Ser. A, 1968

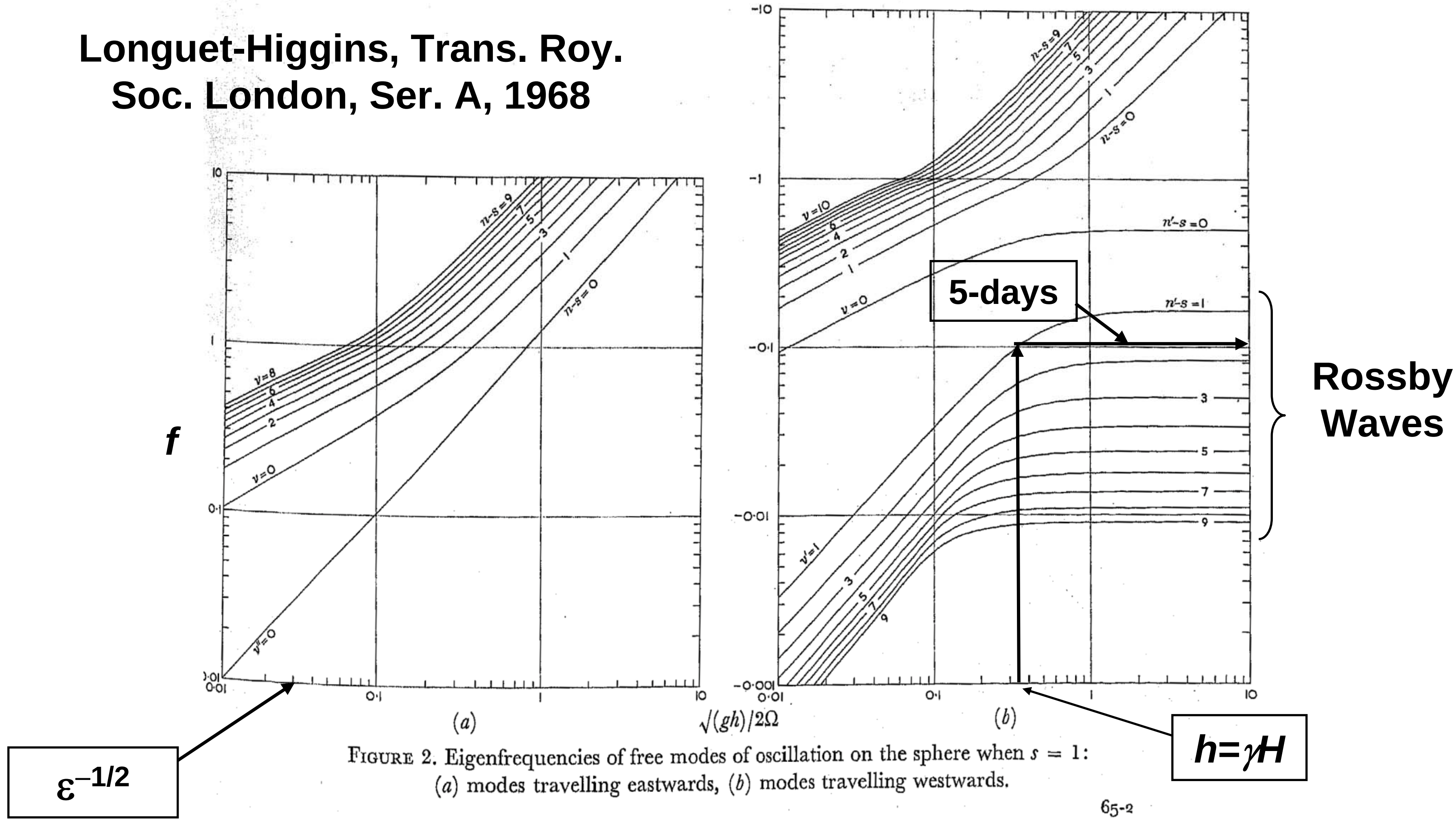


FIGURE 2. Eigenfrequencies of free modes of oscillation on the sphere when $s = 1$: (a) modes travelling eastwards, (b) modes travelling westwards.

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Traveling Oscillations in an Atmosphere
Lamb Waves

- For the terrestrial atmosphere there is only one eigensolution for the VSE
  - Lamb waves (waves for which the vertical velocity is identically zero)
    - Propagate horizontally as pure compression waves
  - Vertical structure is evanescent
    - Wave amplitude $\propto \exp\left[\kappa z\right]$
- Combined solutions (vertical + horizontal) are Lamb-Rossby waves

Traveling Oscillations in an Atmosphere
Sources

- Free waves are easy to excite and maintain
- May be excited by
  - Selective response to random fluctuations
  - Wave-wave interactions
  - Parametric instabilities
- Main free waves in the terrestrial atmosphere
  - 2, 5 and 16 day waves

Traveling Oscillations in an Atmosphere

The 5-Day Wave (Latitude-Height Structure)

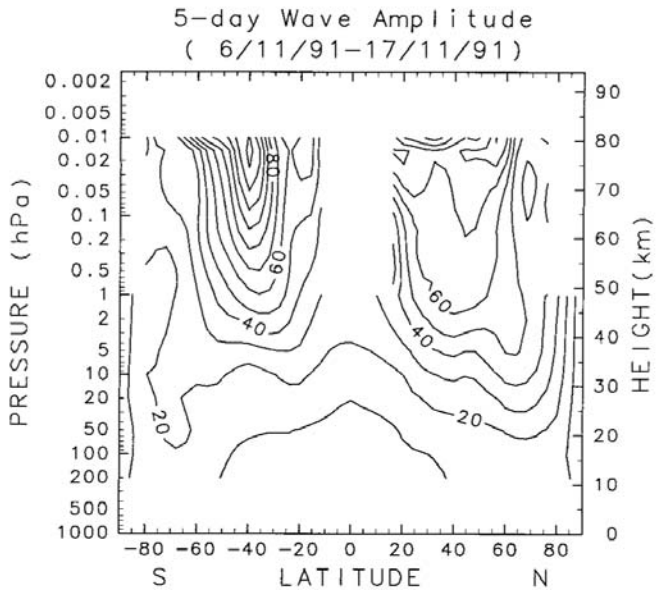

FIG. 4. Latitude–height section of mean amplitude of the 5-day wave during 6–17 November 1991. Units are in meters; the contour interval is 10 m.

Hirooka, J. Atmos. Sci., 2000

Traveling Oscillations in an Atmosphere

The 5-Day Wave (Vertical Structure)

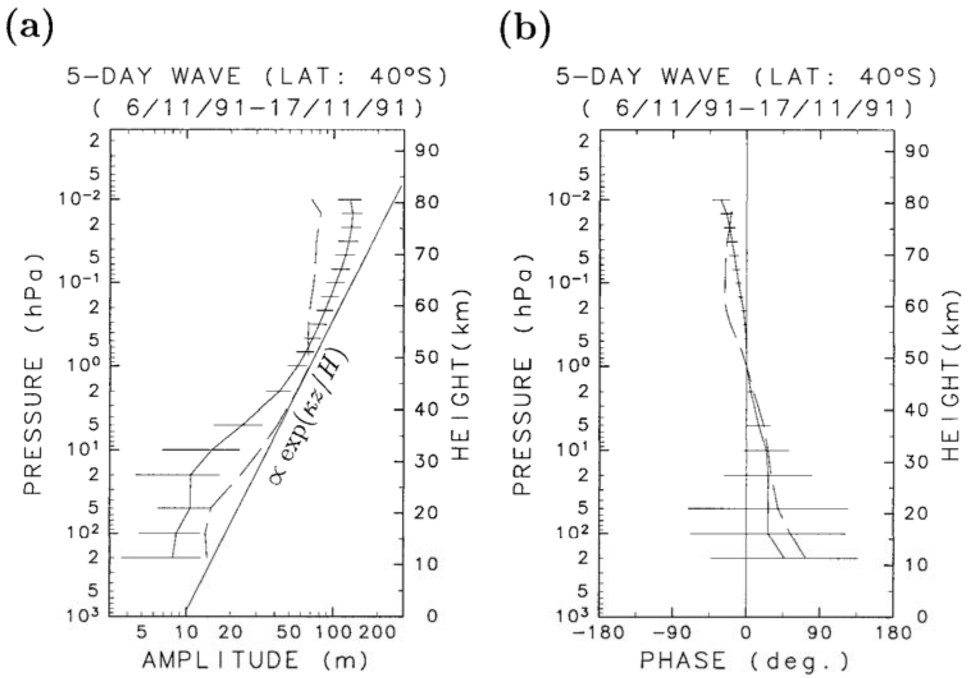

FIG. 6. Vertical structure of the 5-day wave at 40°S (solid lines) and 40°N (broken lines) during the same large-amplitude period as in Fig. 4: (a) average amplitude, along with the theoretical expectation of the Lamb mode; (b) average relative phase differences between the 1-hPa and other levels. The standard deviation at 40°S is denoted by horizontal bars and that at 40°N is comparable.

Hirooka, J. Atmos. Sci., 2000

Forced Waves

- **May have internal wave structure**
- **The forcing determines the frequency**
  - **Rather than one equivalent depth associated with many frequencies there are many equivalent depths associated with a given frequency**
  - **Equivalent depths found from HSE**
  - **Vertical structure depends on *h* as a parameter**

Forced Solutions

Tides

For forced waves negative equivalent depths are also possible

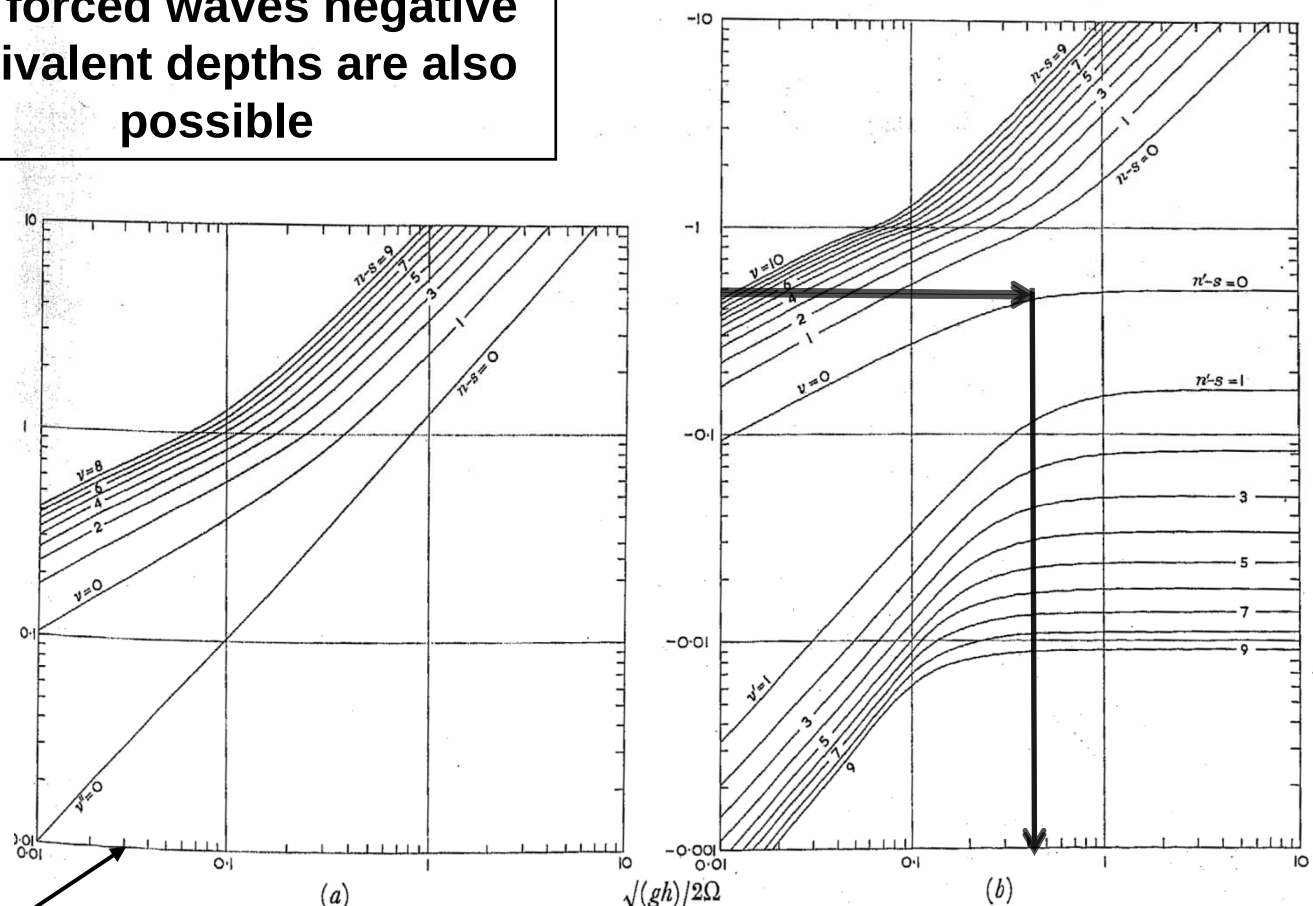

Figure 2. Eigenfrequencies of free modes of oscillation on the sphere when $s = 1$: (a) modes travelling eastwards, (b) modes travelling westwards.

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Diurnal s=1 Tide

$\varepsilon^{-1/2}$

$$h = (2\Omega a)^2 \varepsilon / g$$

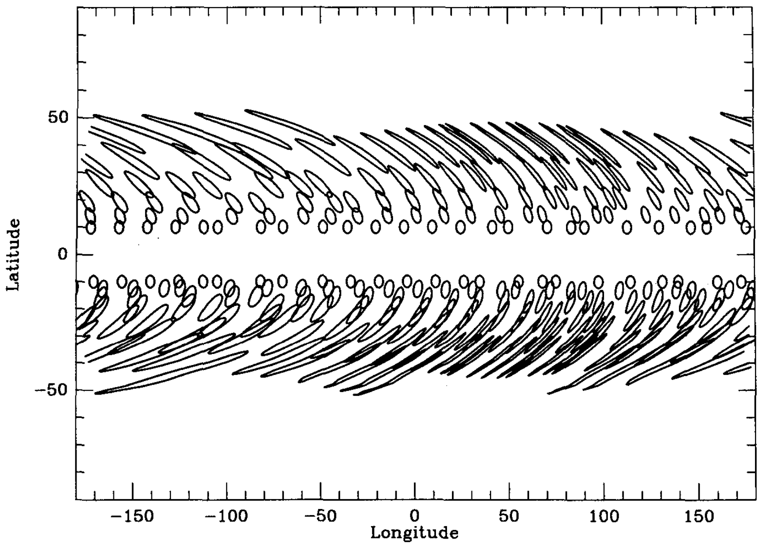
Longuet-Higgins, Trans. Roy. Soc. London, Ser. A, 1968

Waves on Other Planets

- Wave features are seen in the permanent cloud cover surrounding Venus
  - Kelvin waves
  - Rossby waves
- Planetary wave features are also seen in the atmosphere of Jupiter

Waves in the Venus Atmosphere

Smith et al., J. Atmos. Sci., 1993



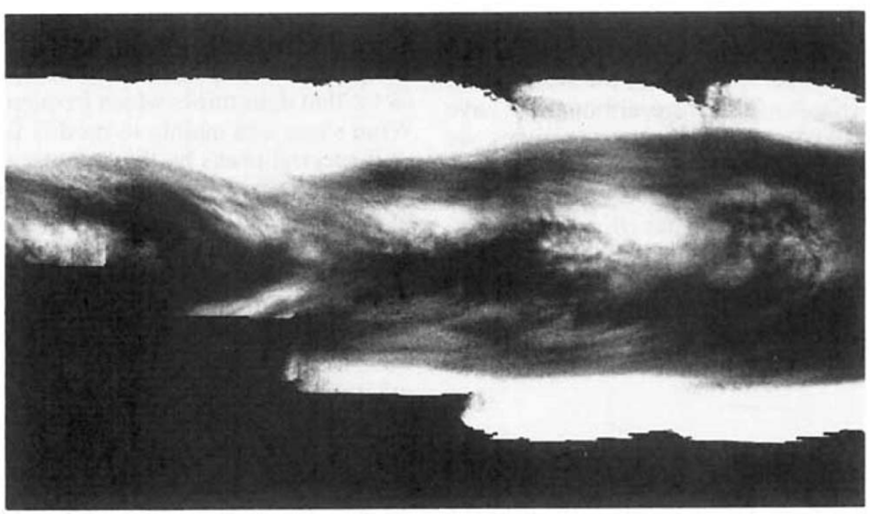


FIG. 19. Simple cylindrical projection of a set of *Galileo* images taken as the cloud system rotated under the spacecraft.

Courtesy of G. Schubert

Stationary Waves

- Quasi-geostrophic dynamics
- Vertical propagation
  - Charney-Drazin theory

Stationary Waves
Quasi-Geostrophic Dynamics

- **Motion nondivergent except when coupled to planetary vorticity**
- **Stream function is geostrophic**

$$\psi = \varphi / f$$

- **Dynamics governed by quasi-geostrophic potential vorticity equation**
  - **Potential vorticity includes the effects of divergence on vorticity change (conserved for barotropic divergent flow)**
- **Linearized quasi-geostrophic potential vorticity equation governs propagation of extratropical waves through zonal mean flows**

Stationary Waves

Vertical Propagation (Charney-Drazin Theory)

Vertical Wavenumber for constant background winds

$$m^2 = \frac{N^2}{f_o^2}\left[\frac{\beta}{\bar{u} - c} - (k^2 + l^2)\right] - \frac{1}{4H^2}$$

For m^2 <0 waves attenuate with altitude

For $c=0$ (stationary waves)

Only long waves can propagate through strong winter westerlies

No waves can propagate through summer easterlies

Stationary Waves
Sources

- Excited mainly by flow over large continental mountain areas
- Stronger in the northern hemisphere
- Strongest waves are the very long waves (s=1-3)

Wave Transports

- **Transports originate in the nonlinear advective (convective) terms**

$$\partial \rho A / \partial t = -\nabla \rho \mathbf{u} \cdot A + \ldots$$

$$\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}', \;\; A = \overline{A} + A' \rightarrow$$

$$\partial \overline{\rho} \overline{A} / \partial t = -\nabla \cdot \overline{\rho} \overline{\mathbf{u}' A'} + \ldots$$

$$\overline{\rho} \overline{\mathbf{u}' A'} = \text{Wave flux}$$

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Wave Transports

- Planetary waves transport heat momentum and constituents (e.g., ozone) latitudinally
- They play an significant role in the heat, momentum and ozone budgets
- The transports may force changes (sometimes dramatic) in the mean state when waves are
  - Transient
  - Nonconservative
  - Nonlinear
  - Encounter a critical level

Transports

Role in sudden Stratospheric Warmings

- **Sudden Warmings are forced by a sudden amplification of stationary waves in the high latitude winter stratosphere**
- **Decelerates polar westerlies and waves transport heat northward**
- **Circumpolar vortex breaks down and winds change from westerlies to easterlies**
- **Warming near the pole can exceed 50K in the stratosphere**

Sudden Stratospheric Warming

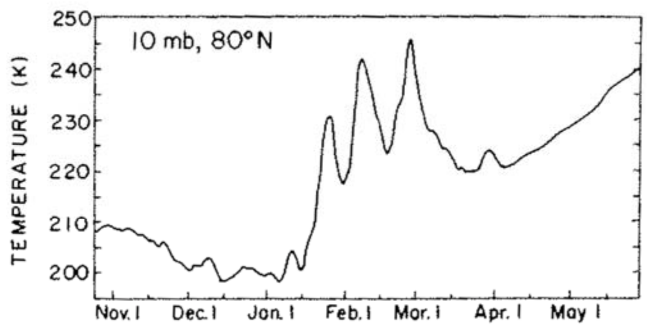
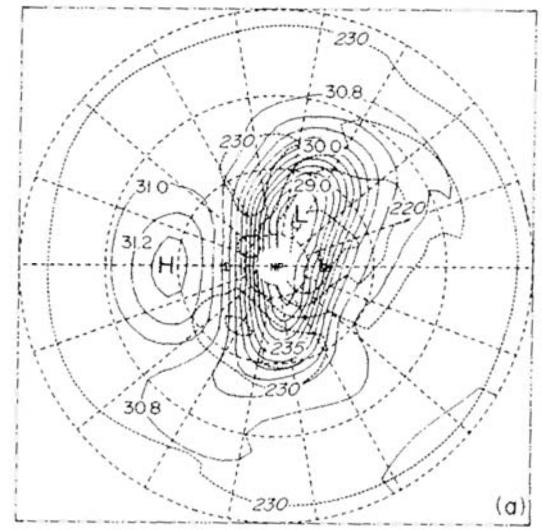
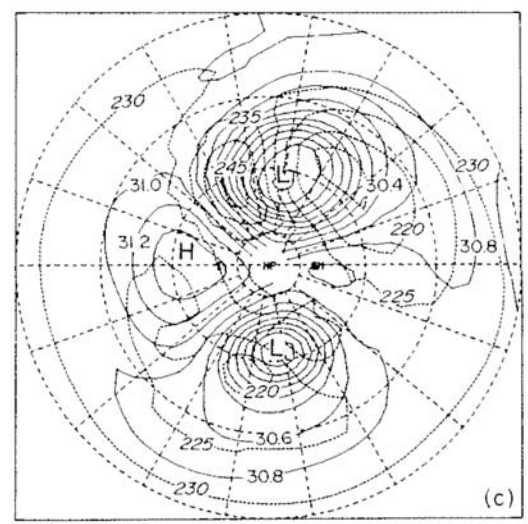


Fig. 6.1. Variation of zonal-mean temperature at 10 mb, 80°N, from October 1978 through May 1979, derived from LIMS data. [After Gille and Lyjak (1984).]



Feb 17



Feb 21

Fig. 6.3. Polar stereographic charts of 10 mb height (solid curves; contour interval 0.2 km) and temperature (dashed curves; contour interval 5 K) from LIMS data for the following days in 1979; (a) February 17, (b) February 19, (c) February 21, (d) February 26, (e) March 1, and (f) March 5. "GM" designates the Greenwich Meridian, which extends horizontally toward the right from the North Pole (NP). Latitude circles are shown at 20° intervals, with the outermost circle at 20°N. (*Figure continues.*)

Andrews et al., Middle Atmosphere Dynamics, 1987

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The Coldest Place in the Earth's Atmosphere is the Summer Polar Mesopause

Effect of Dissipating Waves

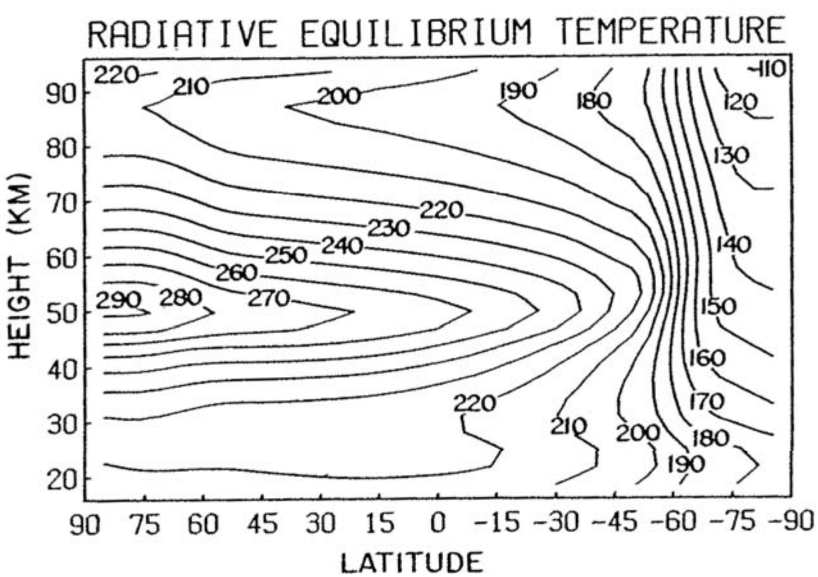


FIG. 3. Derived radiative equilibrium temperature distribution at solstice. Winter hemisphere on right.

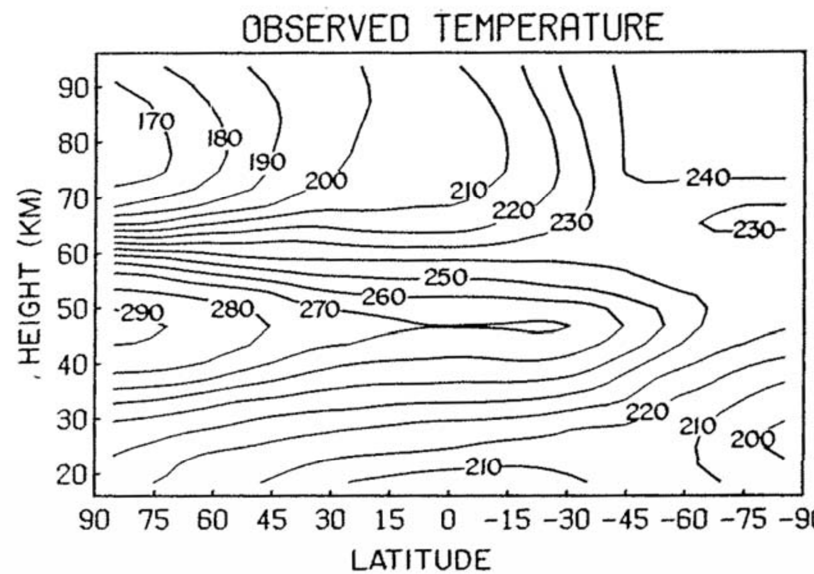
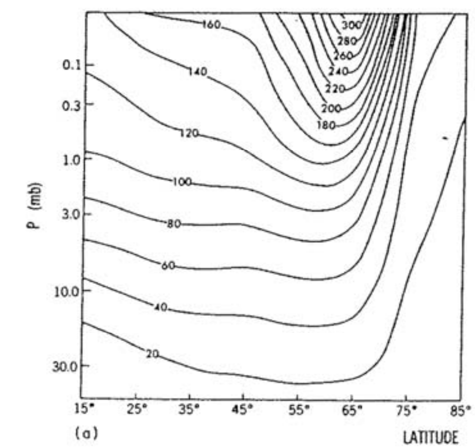


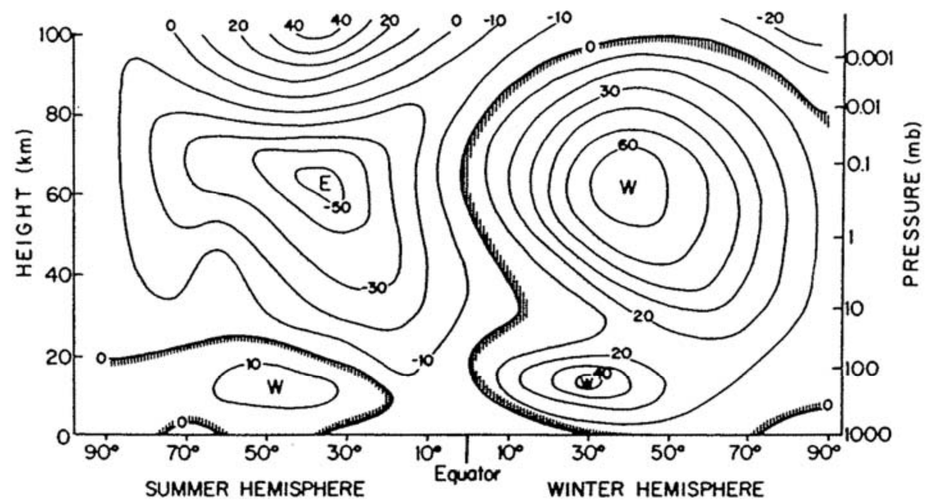
FIG. 4. Observed zonal mean temperature distribution at solstice (after Murgatroyd, 1969). Winter hemisphere on right.

Deceleration of the Stratospheric Jets

- **Wave drag decelerates the stratospheric winds**
- **Dominant contributor is thought to be small-scale gravity waves**
- **Causes the jets to close**
- **Cooling over summer pole is implied by thermal wind relation**



$$\frac{\partial \overline{u}}{\partial z} = -\frac{R\partial T}{f\partial y}$$



Positive shear (decreasing westward winds) implies colder temperatures to the north

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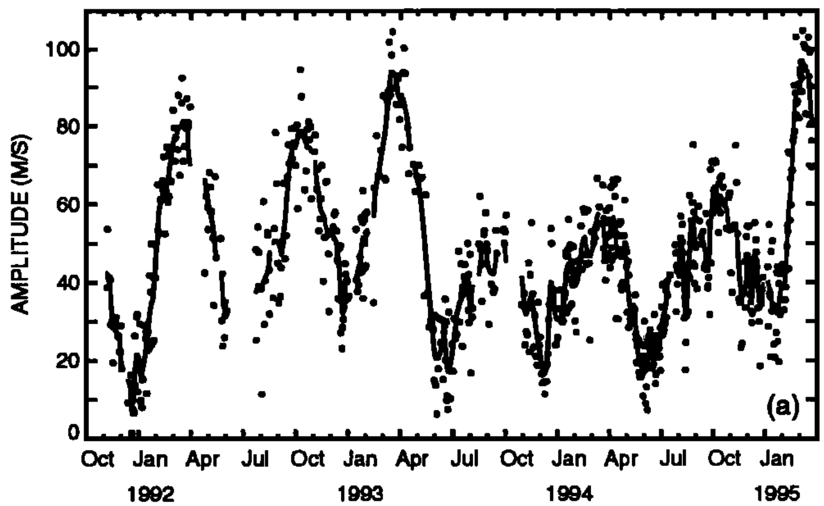
Waves in the Ionosphere-Thermosphere System

- Tides
  - Periodic response to astronomical forcing
  - Diurnal period and its harmonics (principally semidiurnal)
  - Forced by absorption of solar radiation by atmospheric constituents (O_3, H_2O, O_2, …)
  - Lower thermosphere diurnal variation dominated by tides forced in troposphere and stratosphere
  - Middle and upper thermosphere dominated by tides forced *in situ*

Waves in the I-T System (Cont.)

- **Gravity waves**
  - **Excited in lower atmosphere and auroral zone**
- **Traveling Planetary waves**
  - **Upward extensions of waves in mesosphere**
  - **Possibly excited by geomagnetic storms**

Tides in Lower Thermosphere



Daily estimates of diurnal tidal amplitude from HRDI data for an altitude of 95 km

Burrage et al., GRL, 1995

Planetary Waves in the Ionosphere

Two-Day-Wave

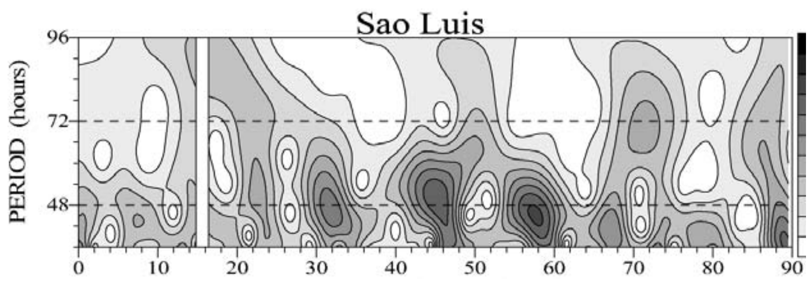


Figure 11. Wavelet spectra calculated from the hourly *foF2* data for the period range 36–96 hours for the seven ionosonde stations.

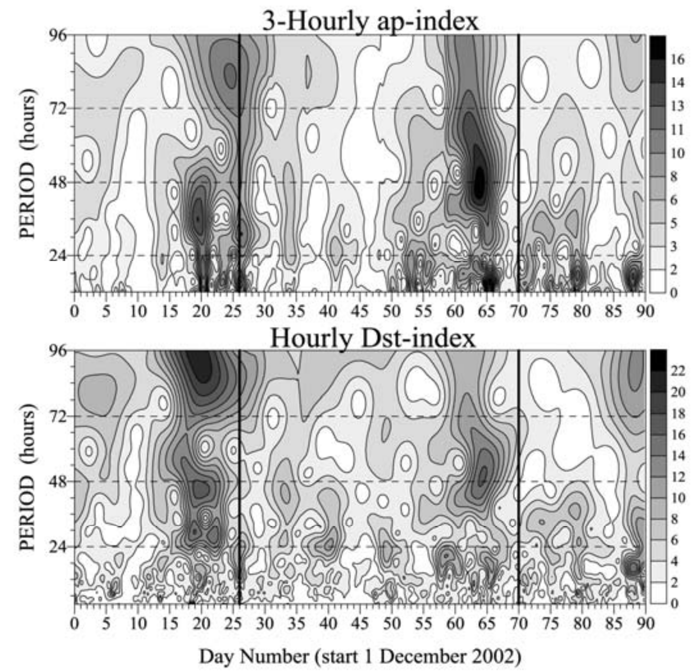


Figure 4. Wavelet spectra of the 3-hourly geomagnetic a_p-index (upper plot) and the hourly equatorial D_{st}-index (bottom plot) for the period of December 2002 to February 2003. The vertical thick lines on both plots outline the time period when the 2-day wave event is present in the MLT region.

Pancheva, JGR, 2006

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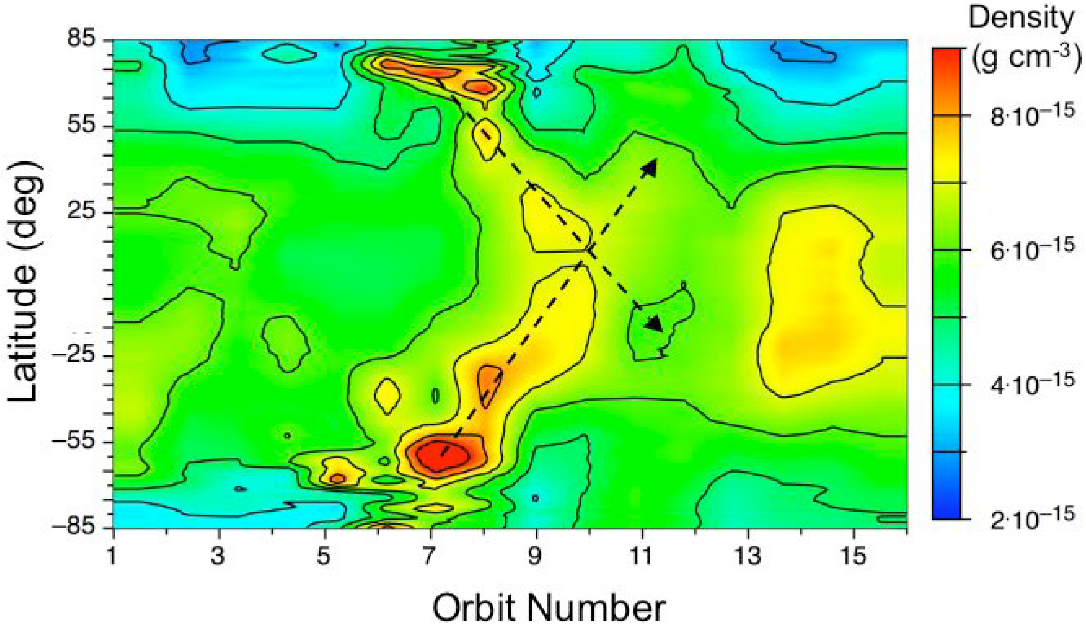
Gravity Waves in the I-T



Traveling Atmospheric Disturbances seen in density near 400 km measured by the accelerometer on the CHAMP satellite in connection with a geomagnetic disturbance. The disturbances appear to penetrate into opposite hemispheres from their origins.

Forbes, J. Met. Soc. Japan, 2007

Summary

- **Traveling free waves are normal modes of Laplace's Tidal Equation**
- **Free traveling planetary waves are prominent features of the stratosphere**
- **Quasi-stationary waves dominate mid-latitude long-wave field during winter**
  - **Transport heat, momentum and ozone**
  - **Induce large changes in the mean state**
    - **Sudden mid-winter Stratospheric Warmings**
    - **Maintain summer polar mesopause out of radiative equilibrium**
    - **Long-term (semi-annual, quasi-biennial) oscillations**
- **Waves in the I-T system achieve large amplitudes and respond to geomagnetic energy sources**