Hybrid Simulations: Numerical Details and Current Applications

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…and numerous collaborators

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CENTER FOR INTEGRATED SPACE WEATHER MODELLING

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3. Example Simulations
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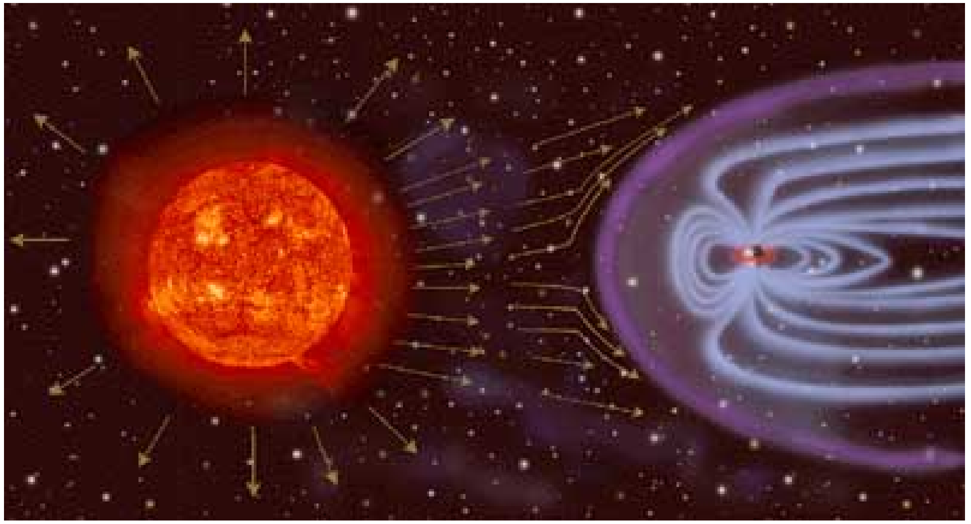
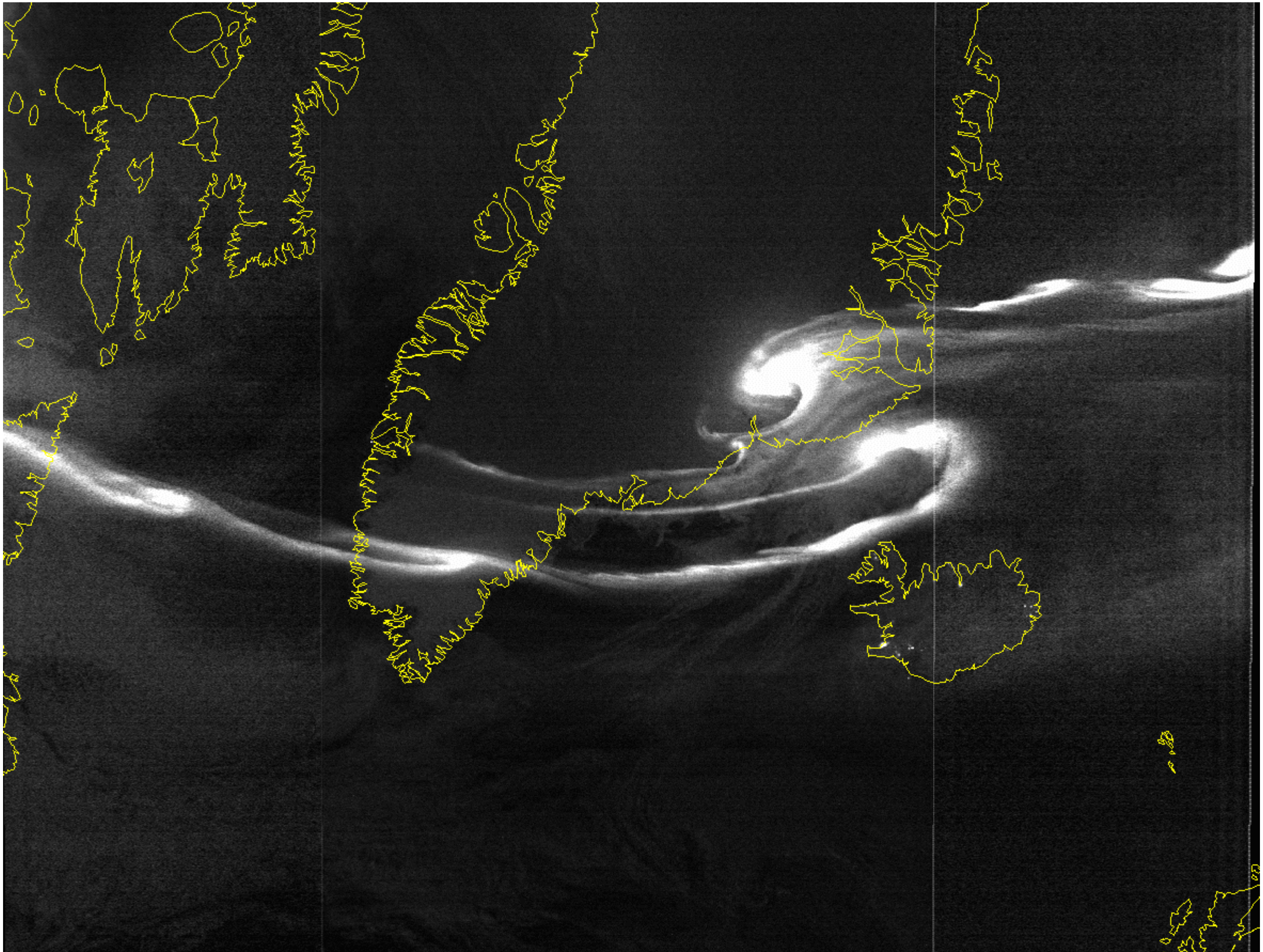
1. Heliospheric Plasmas, Solar Wind-Magnetosphere Interaction: Kinetic Physics

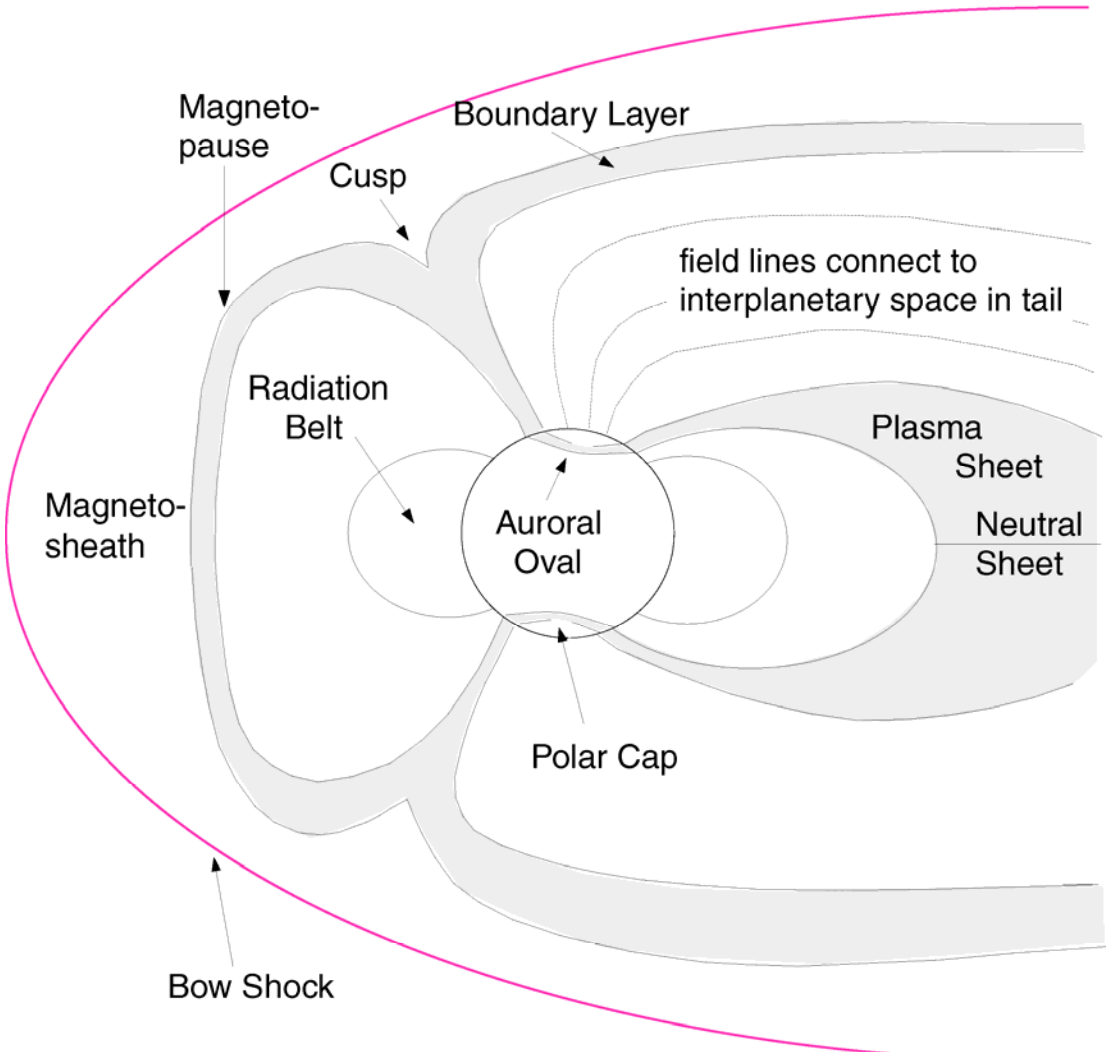


- Plasma is (mostly) magnetized
- Plasma is (mostly) collisionless

Why a kinetic approach?



Physics of the Interaction: Regions – Overview



- not to scale -

Why a kinetic approach?

Thermal and other Plasma Properties

* Temperature anisotropy (sheath, tail)
* Turbulence (upstream, sheath, solar wind, corona)
* Specific heat ratios (sheath, MP, solar wind, corona)
* Energetic particles (shocks, magnetosphere, tail)
* Unmagnetized ions (current sheets)
* Heat flux (solar wind, current sheets, boundary layers)

Why a kinetic approach?

Signatures and Coupling

* Field-aligned currents (magnetosphere, tail)
* Energetic ions (shocks, cusp, tail, ring current)
* Heat flux (sheath, MP, solar wind)
* Poynting flux (magnetosphere, tail)
* Small spatial/temporal scales (micro physics/all)

Physics of the Interaction: Time Scales (1AU/MS)

gyro frequency: $\Omega_{ci} = eB / (mc)$

$\tau_{ci} \sim 0.5$ to 10 s

plasma frequency: $\omega_{pi} = (4\pi ne^2/m)^{1/2}$

$\omega_{pi} / \Omega_{ci} \sim 100$ to $10{,}000$

→ Electrons on much faster time scales

Physics of the Interaction: Spatial Scales (1AU/MS)

gyro radius: $\rho = v_{th} / \Omega_{ci}$

$\rho_i \sim 20$ to 200 km

for $T_e \sim T_i \rightarrow \rho_e \sim \rho_i / 40$

inertial length: $\lambda = c/\omega_{pi}$ (skin depth)

$(\rho_i / \lambda_i)^2 = \beta_i \sim 0.1$ to 5

→ Electrons on much smaller spatial scales

2. Computational Models

Computational Models:
Fluid Versus Kinetic Approach

• Idealized particle motion, moments, Maxwell's equations, closure relations → fluid models:

MHD

• General particle motion, Maxwell's equations, self-consistent wave-particle interaction, few idealizations → kinetic models:

Vlasov and Particle Codes

Computational Models: Strengths and Weaknesses

- MHD codes:

  - very successful
  - early-on large-scale
  - many time steps
  - good conserv. properties
  - "easily" parallized
  - "easy" non-uniform grids

- Particle Codes:

  - historically, successful on smaller scales
  - can address kinetic instabilities & waves, anisotropies, energization, thermalization, boundary layers, mass- and energy transport

Computational Models: Types of Kinetic Codes

- explicit or implicit codes
- relativistic, e-m full-particle codes
- electrostatic codes
- Vlasov codes vs. PIC codes
- Darwin codes
- hybrid codes (kinetic ions, electron fluid)

Hybrid Codes: Algorithms

- Early codes:

  Auer et al., 1962, 1971; Forslund & Friedberg 1971; Chodura, 1975; Sgro & Nielson 1976

- Leroy et al. (1981): 1-D implicit

  Swift & Lee, 1983; Hewett, 1980

- Harned (1982): predictor-corrector

  Winske & Quest, 1986; Brecht & Thomas, 1988

- Fujimoto (1989): velocity extrapolation
- Horowitz (1989): iteration

Hybrid Codes: Algorithms

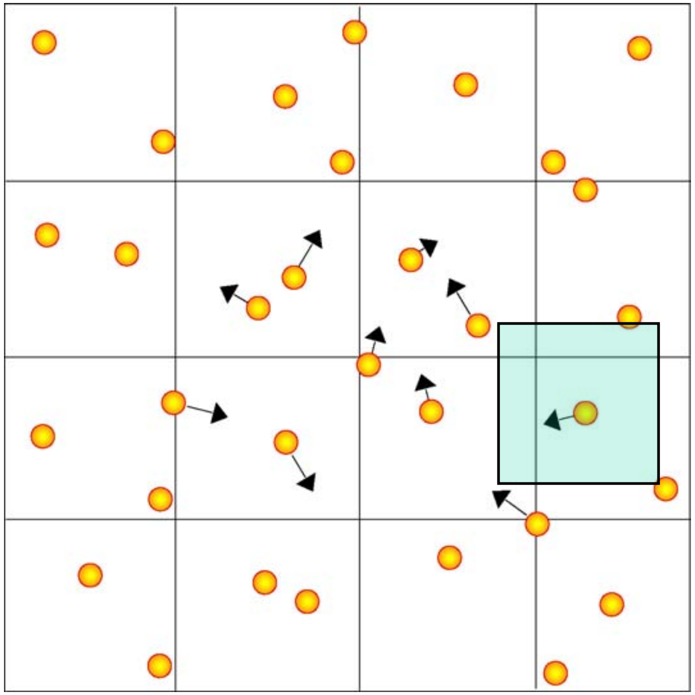
Some recent codes:

One-Pass (Omidi and Winske, 1995; Fujimoto, Thomas)

Moment Method (Quest, 1983; Matthews, 1994)

Improved Predictor-Corrector (efficient +substepping, Krauss-Varban, 2005)

Hybrid Codes: PIC Method



- “Finite size” particles, follow motion
- Collect & interpolate moments onto grid
- Solve e.m. fields on grid

Hybrid Codes: Equations

- Electrons:

  - massless, quasi-neutral fluid

    $en_e = q_i n_i$

  - momentum equation

    $(d/dt)\, n_e m_e \mathbf{v}_e = 0 = -en_e\, (\mathbf{E} + \mathbf{v}_e \times \mathbf{B}/c) - \nabla \bullet \mathbf{P}_e$

  - closure relation model:

    scalar pressure with const. T_e, or adiabatic,
    or pressure tensor

Hybrid Codes: Equations

- Ions:

  - particle advance

    $m_i \, (\partial / \partial t) \, \mathbf{v}_i = q_i \, (\mathbf{E} + \mathbf{v}_i \, \mathbf{B}/c)$

    $(\partial / \partial t) \, \mathbf{x}_i = \mathbf{v}_i$

    → leapfrog

  - collect moments (charge density, current) on grid

Hybrid Codes: Equations

- Electromagnetic fields

  - Faraday's law

$$(\partial / \partial t)\, \mathbf{B} = -c\ \nabla \times \mathbf{E}$$

  - Ampere's law

$$\nabla \times \mathbf{B} = 4\pi\, \mathbf{J}/c = 4\pi\, q_i n_i\, (\mathbf{v}_i - \mathbf{v}_e)\, /c$$

  - Electric field from electron momentum equation

$$\mathbf{E} = -\mathbf{v}_i \times \mathbf{B}\,/c\ - \nabla \mathbf{p}_e\, /(q_i n_i)\ - \mathbf{B} \times (\nabla \times \mathbf{B})\, /(4\pi\, q_i n_i)$$

→ State equation for E, time-advance for B, plus leapfrog means: information is not necessarily available at points in time when needed

Normalization

- spatial scale: c/ω_{pi}
- velocity: c
- temporal scale: ω_{pi}^{-1} (in code), Ω_{ci}^{-1} (input/output)
- B: "B_o" and $\omega_{pi} / \Omega_{ci}$
- → E: $v_A B_o$ and $(\omega_{pi} / \Omega_{ci})^2$
- temperature: "β" – for fictitious species of unit density in unit field
- density: "n_o"

→ With this normalization, simulation becomes independent of *actual* value of $\omega_{pi} / \Omega_{ci}$

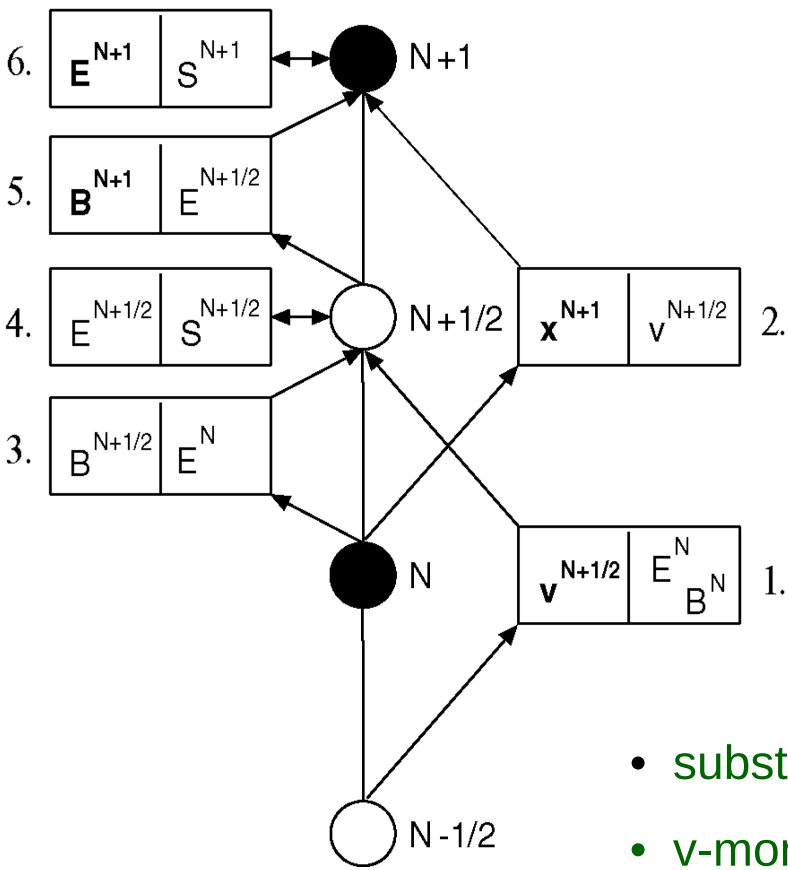
Popular Hybrid Code Variations

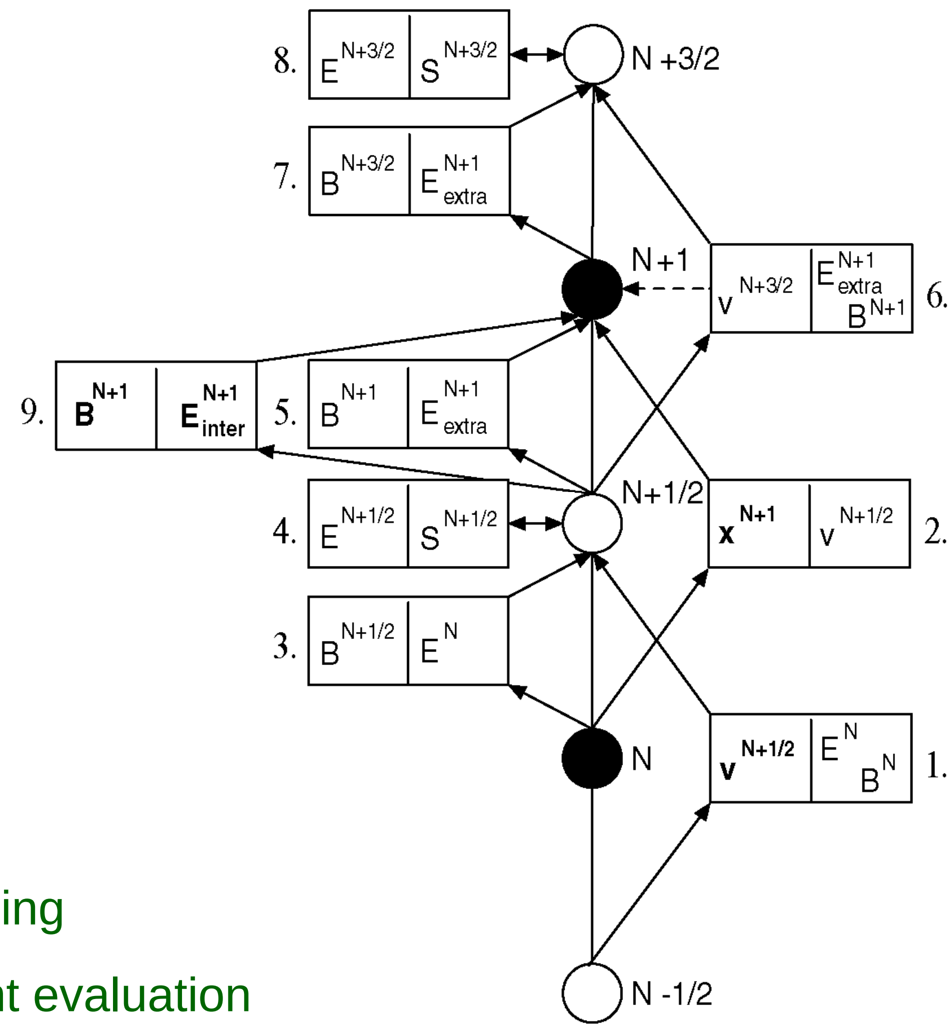
- one-pass
- CAM-CL (moment method)
- predictor-corrector
- other variations (electron energy equation, finite electron mass, electron pressure tensor)

→ Codes are distinct in the way they deal with the fact that E, B, v, and n are not available at the same time(s)

Flow Charts:

Simple Explicit Method vs. Predictor-Corrector





- substepping
- v-moment evaluation

Moment Methods (CAM-CL)

- Use moment method to advance unknown velocity or current density ½ step ahead
- Faster than additional particle push required in P-C
- Collect appropriate moments and apply a separate equation of motion
- CAM-CL:

  - current density → easier to include multiple species

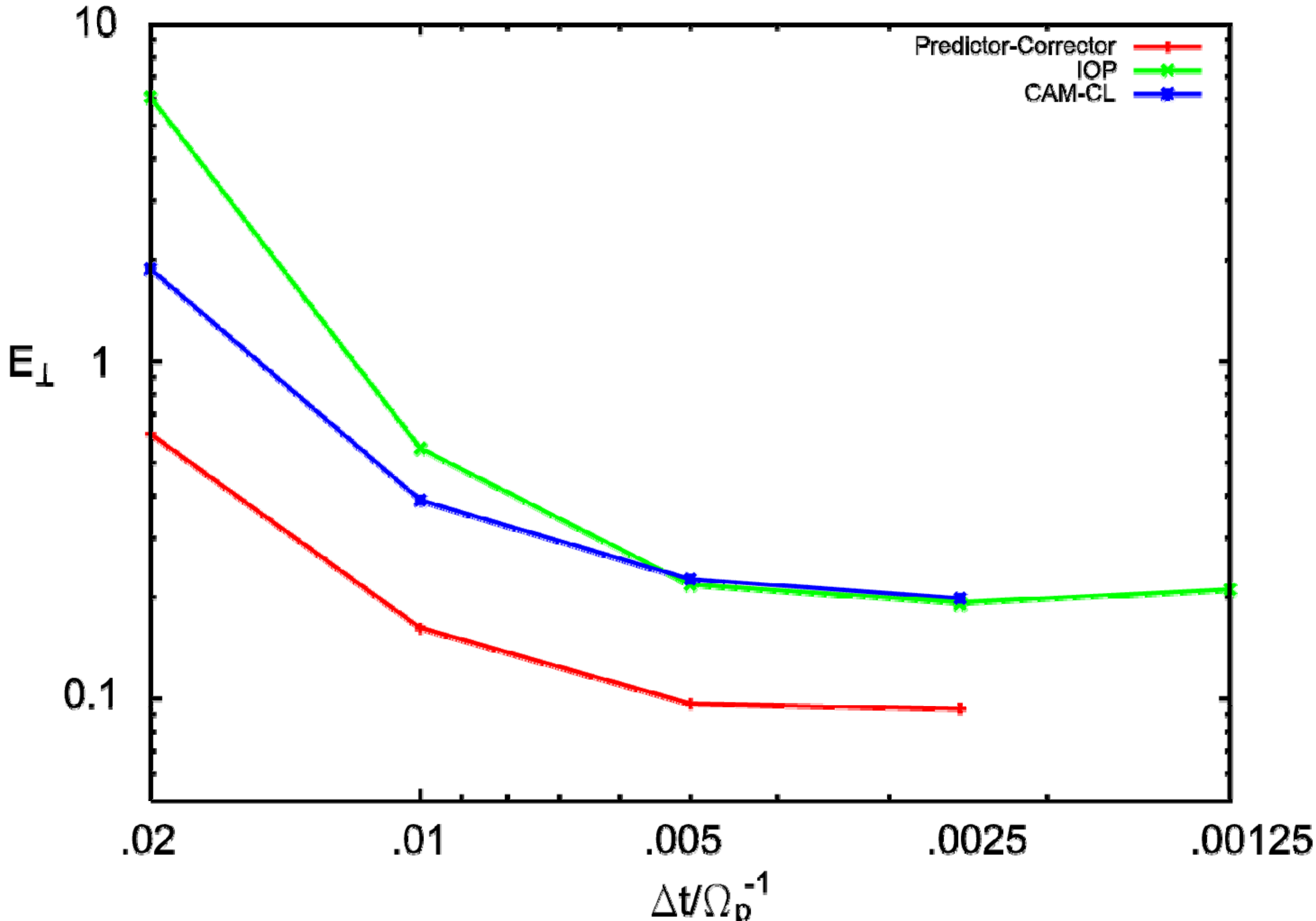
  - advective term absent (included via time centering)

  - no ion pressure tensor required

Matthews, 1994

Numerical Properties:

Drifting Plasma Regions with Anti-Parallel Fields



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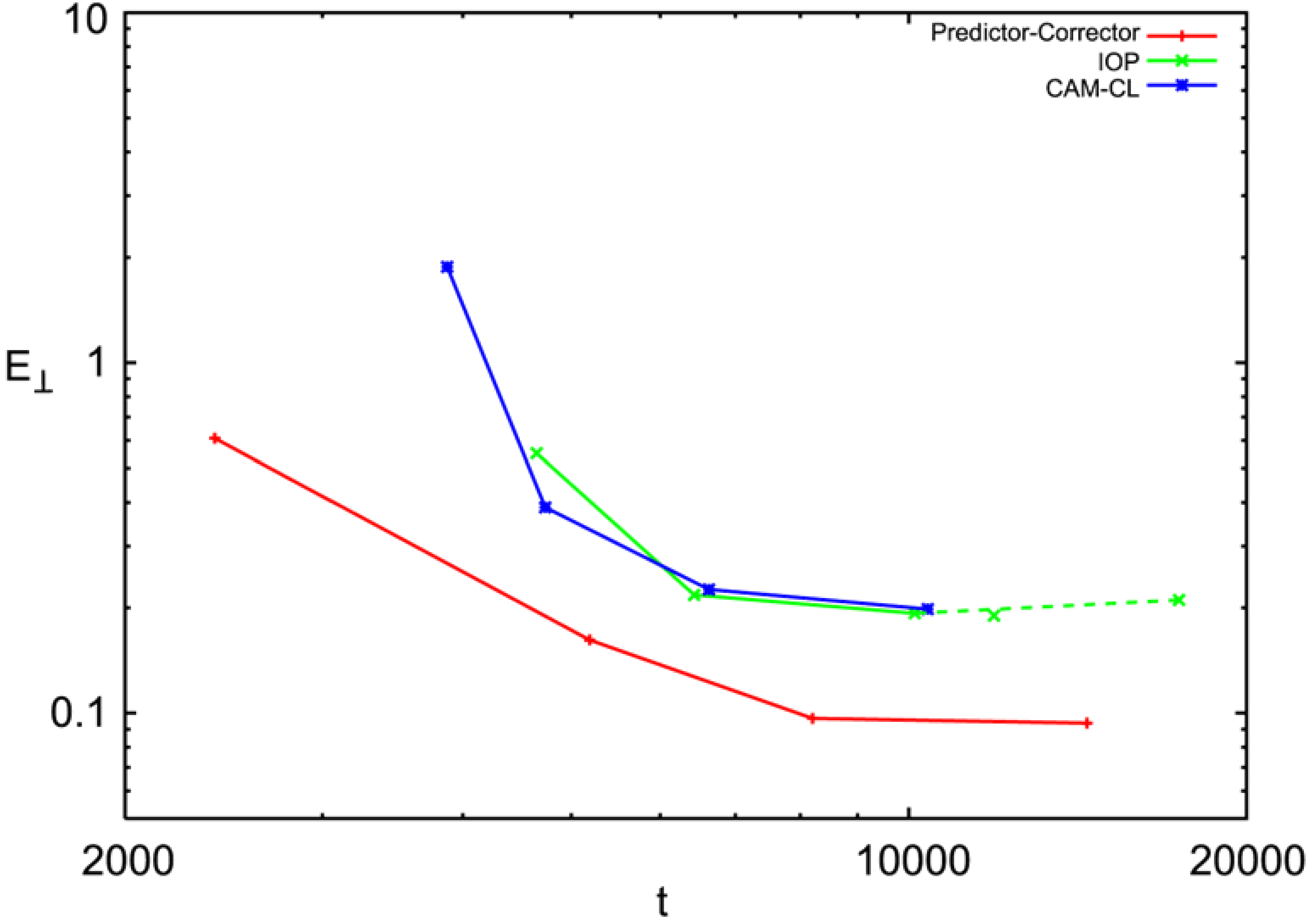
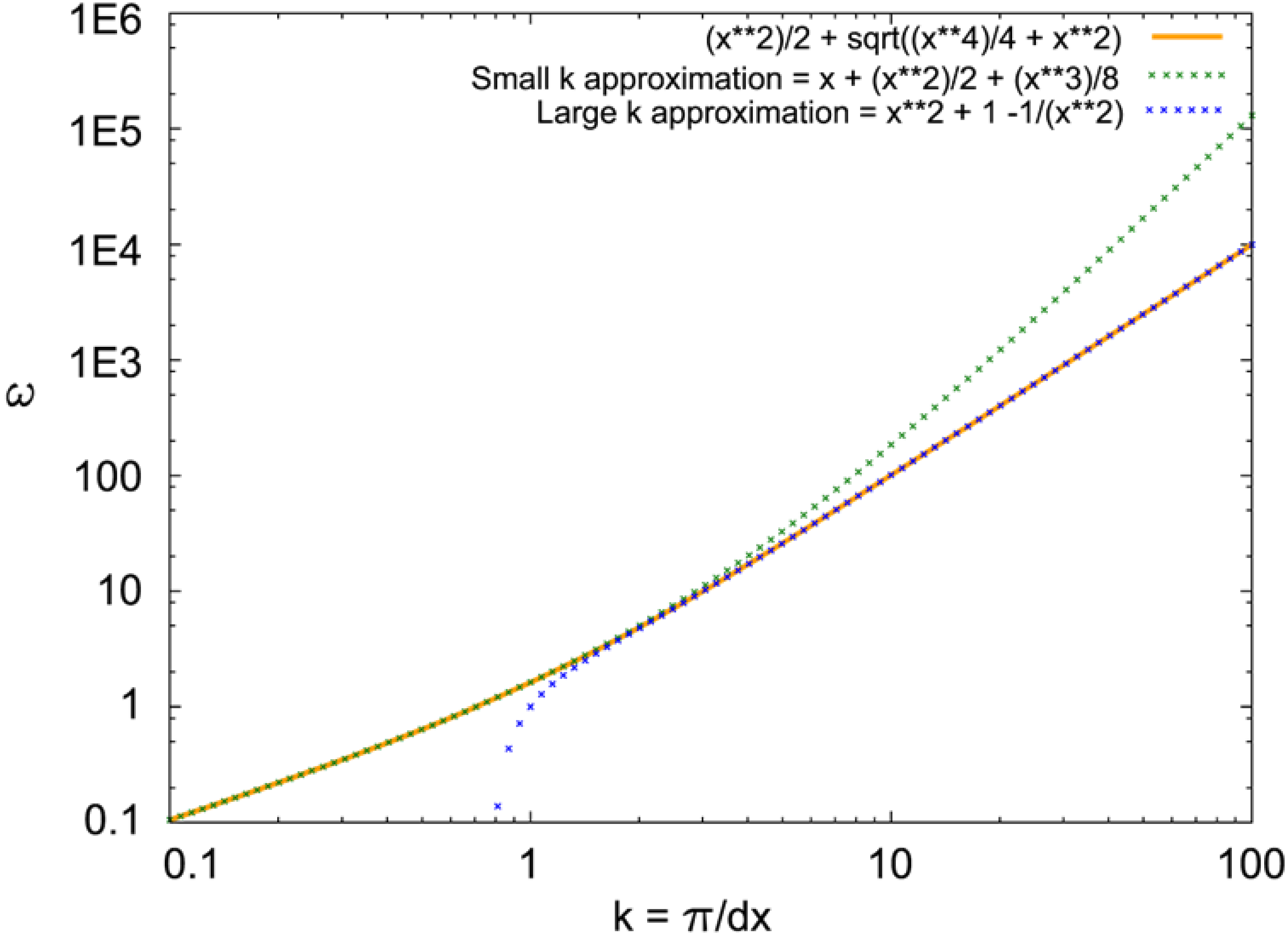


Numerical Properties:
Dispersion Relation of Parallel Whistlers



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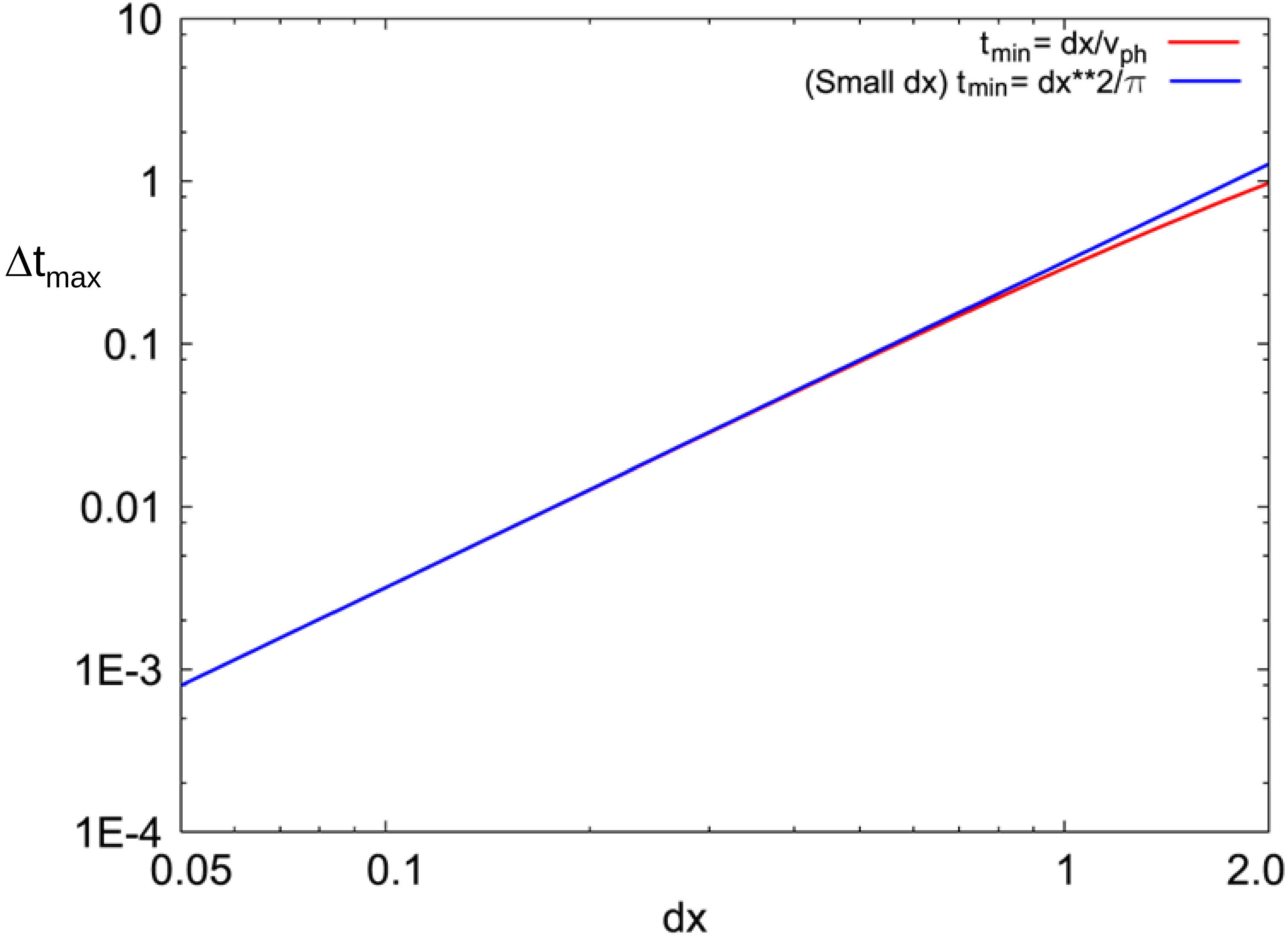
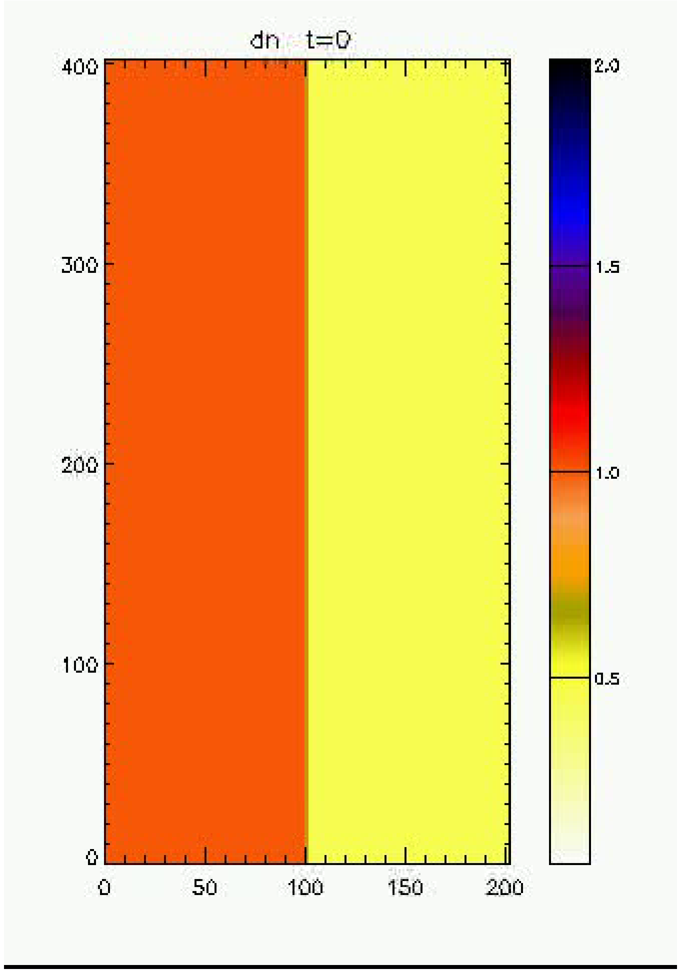


CFL-Condition Example: Solar Wind Reconnection



Example: Solar Wind Reconnection

v_{ph} = omega/k ; $k = \pi/\Delta x = 15.7$ ($\Delta x = \Delta y = 0.2$)

→ $\Delta t_{max} = \Delta x/v_{ph} = \Delta x^2/\pi \sim 0.013$

Low density regions:

(a) unlimited, $n \sim 0.05\ n_o$ → marginally unstable at $\Delta t=0.01$ and 20 substeps

(b) artificially limited to $n > 0.1\ n_o$ → stable at 0.01/8 = 0.00125

... substepping of more than 8-16 rarely useful

Some Examples in Detail:

All examples run on (fast, 64-bit AMD) single CPU!

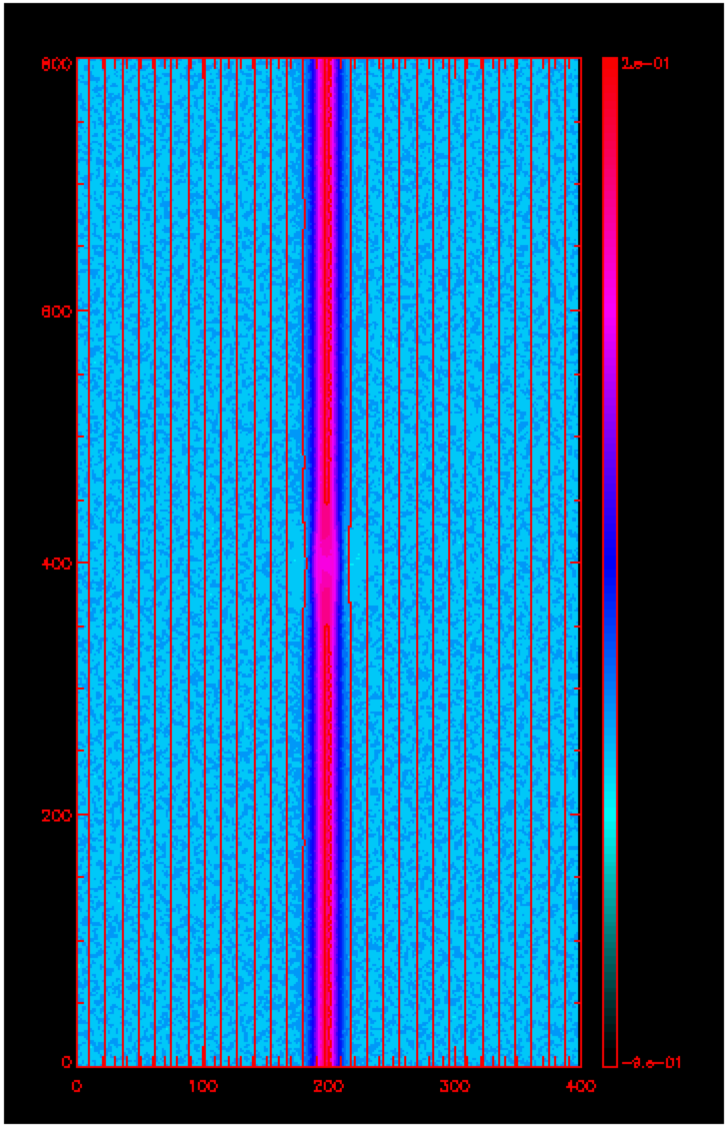
- Thin current sheets and reconnection in the magnetotail, the solar wind, and the low corona

Common theme: high resolution (separation of scales) and/or low ion beta require very small cell size

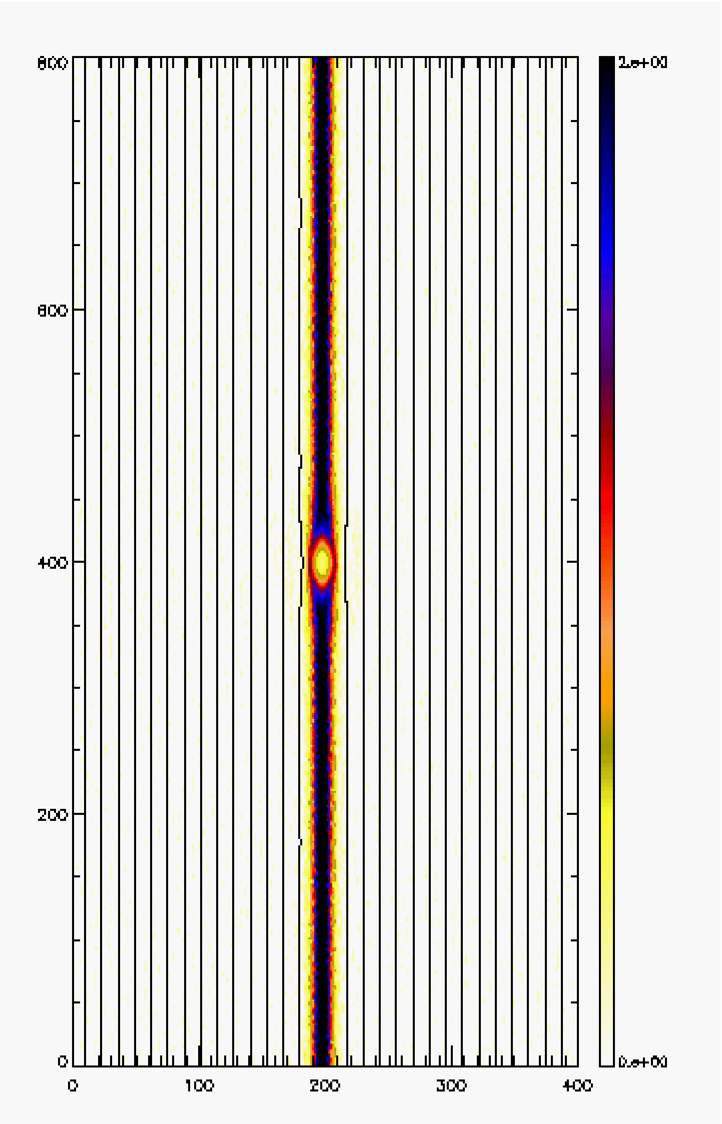
- Interplanetary shocks and SEPs

- Global simulations of the magnetosphere

Generic High-Resolution Reconnection



- density -



- current -

Interplanetary Shocks

and

Solar Energetic Particles

- numerical considerations -

Interplanetary Shocks and SEPs

- Black-box Models and Source Description

- Role of Simulations in SEP models

Reames, 1999

428 PARTICLE ACCELERATION AT THE SUN AND IN THE HELIOSPHERE

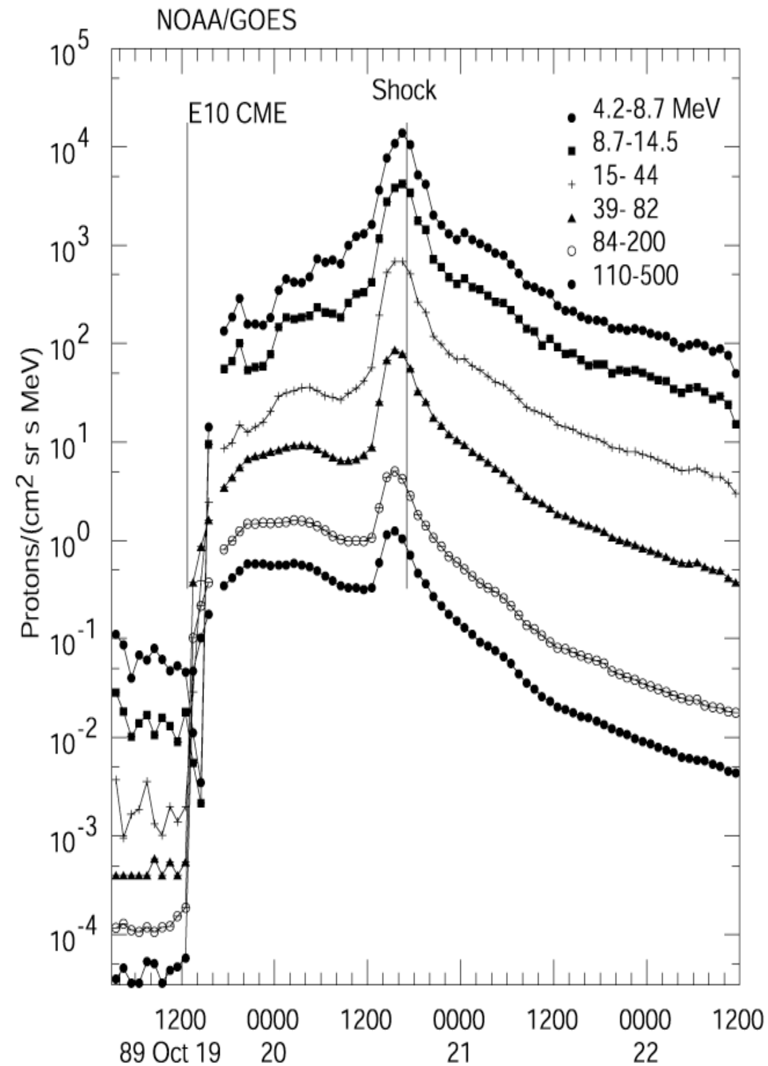
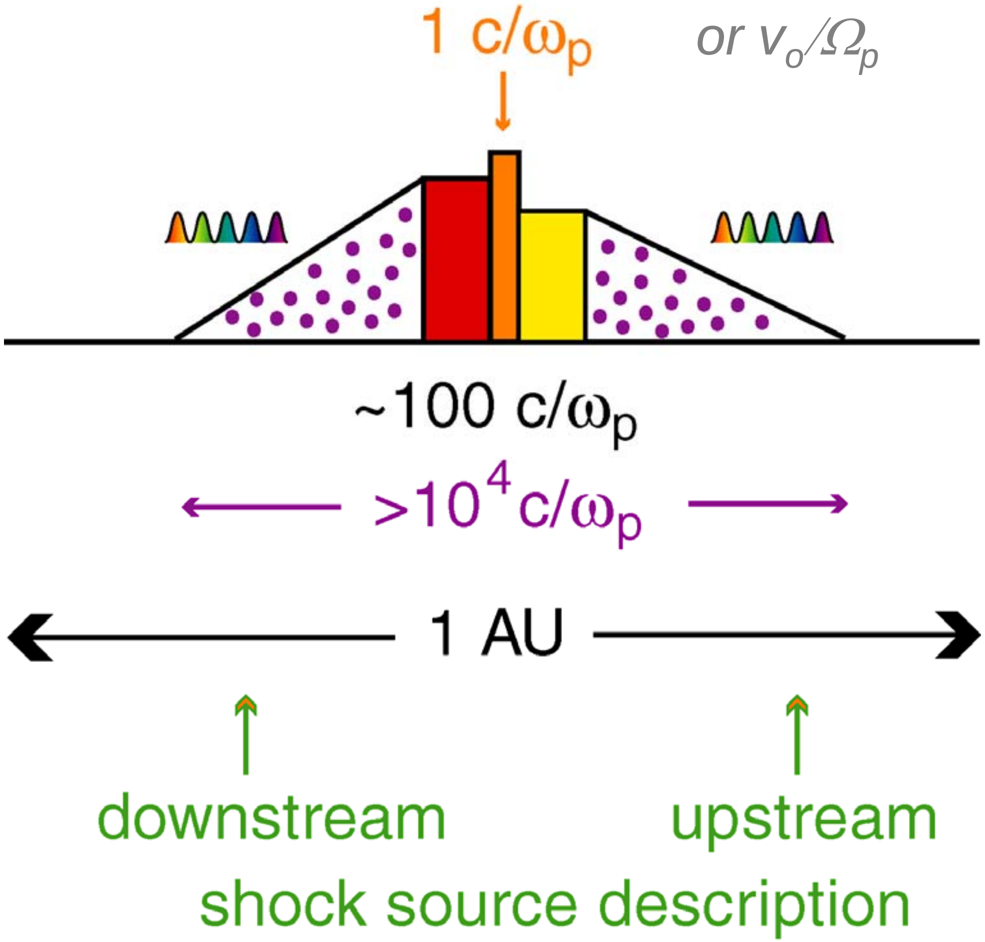


Figure 3.2. Intensity-time pro les at different energies for the large 1989 October 19 event show at time pro les with intensity peaks near the time of shock passage even at very high energies at 1 AU.

SEP Shock Sources:



Scales and Extrapolation

Conservative estimate:

Assume target energy of 1MeV.

Convected gyro radius in 6nT B-field 10^5 km ~ 10^3 c/ω_p

Need several resonant λ in system in 1 direction

→ e.g., 10,000 x 500 c/ω_p (assuming 2-D).

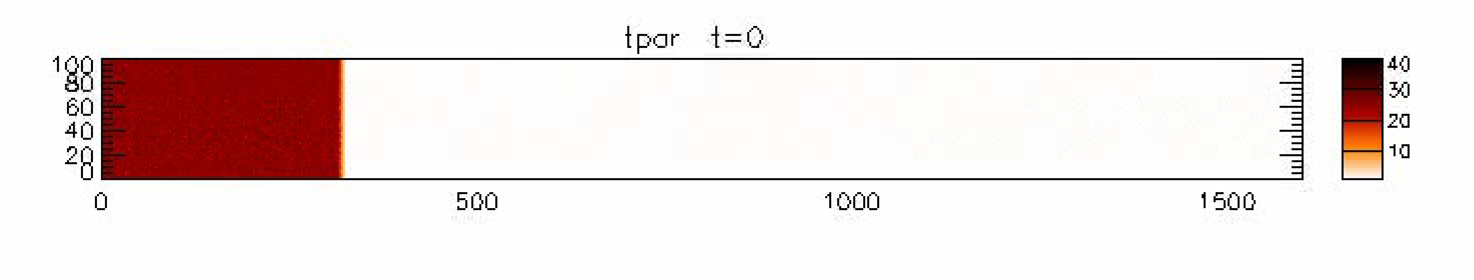
Typical time step 0.01 Ω_p^{-1}, $2.5\cdot10^6$ pp/s / CPU

1 hour of real time (~transit time at $M_A = 5$)

→ 5 days on 40 CPUs

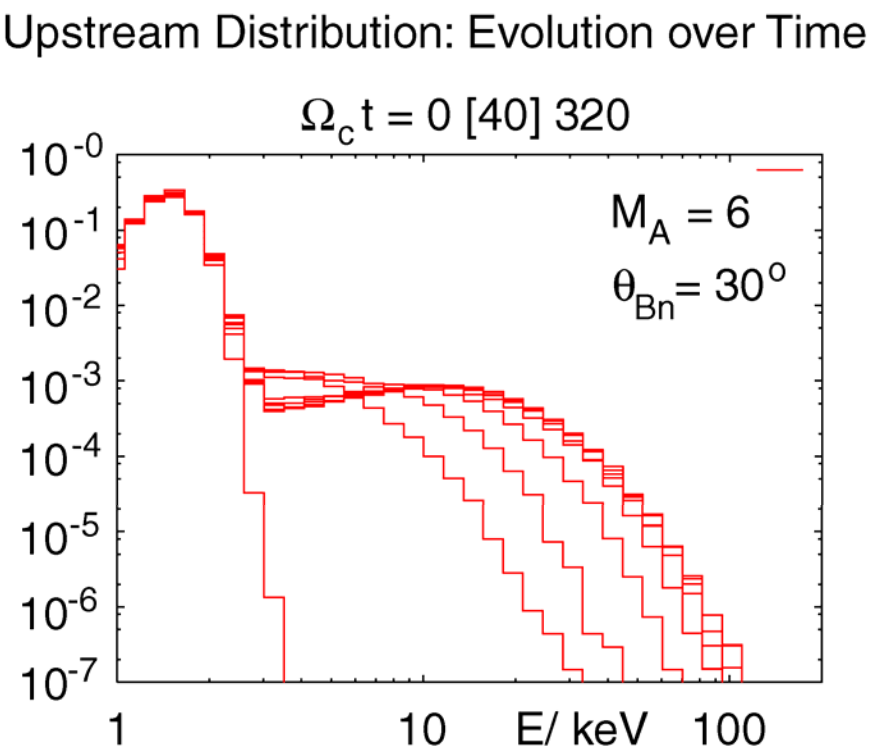
1. power-law → extrapolation
2. quasi-linear estimate too restrictive
3. energetic tail (seed particles) can be described by separate population

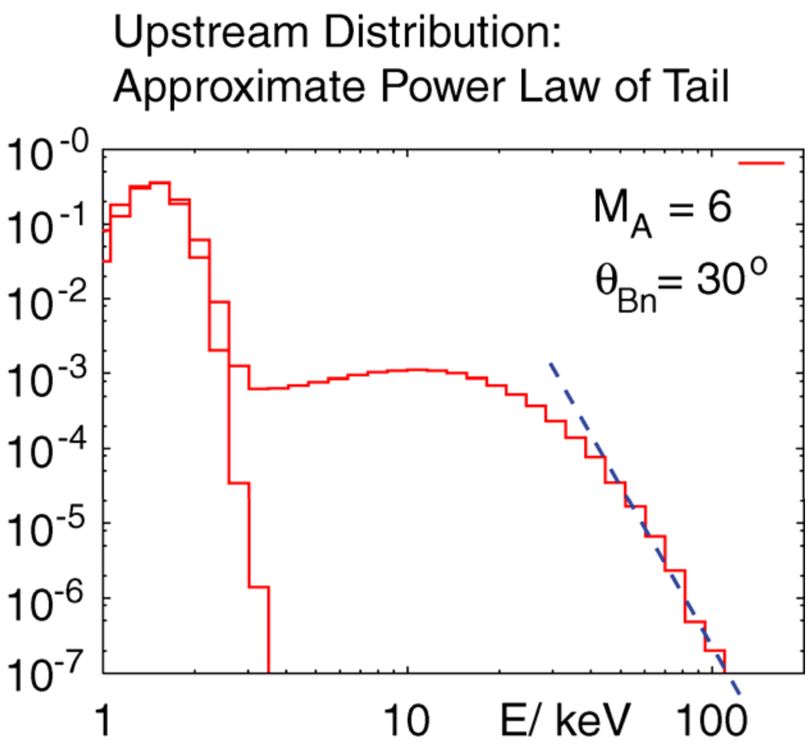
Shock Set-up and Overview



$T_{||}$ and B_z ; M_A = 6.0, θ_{Bn} = 30^o

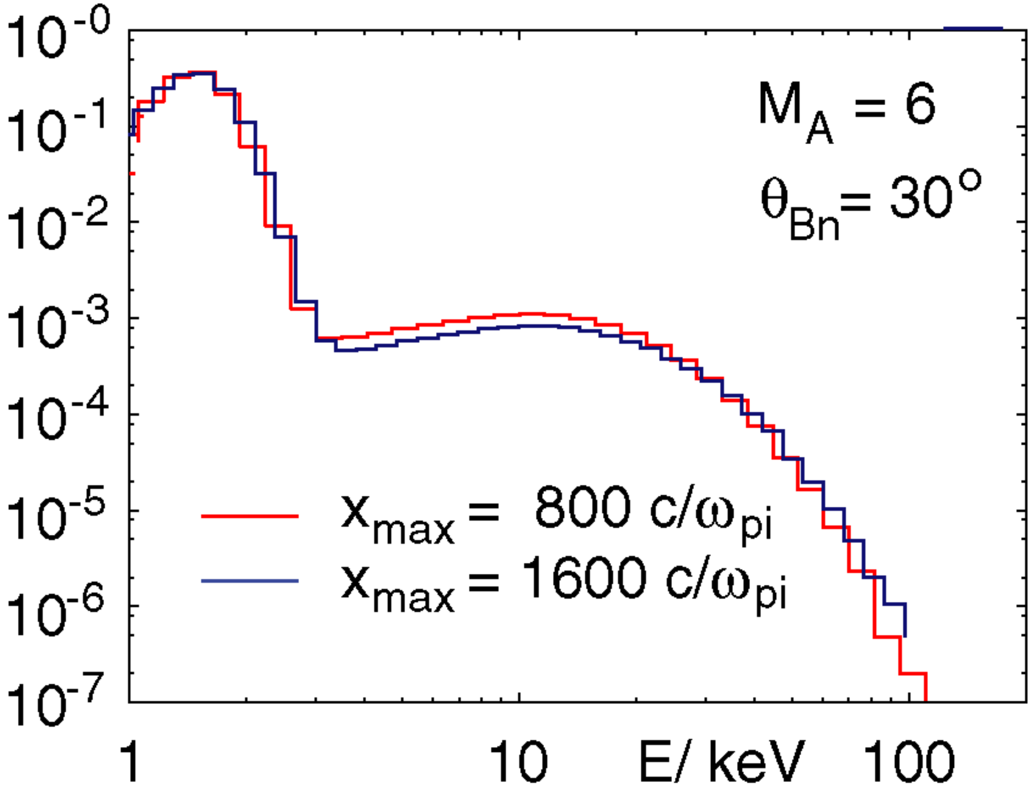
Ion Distributions: quasi-parallel case



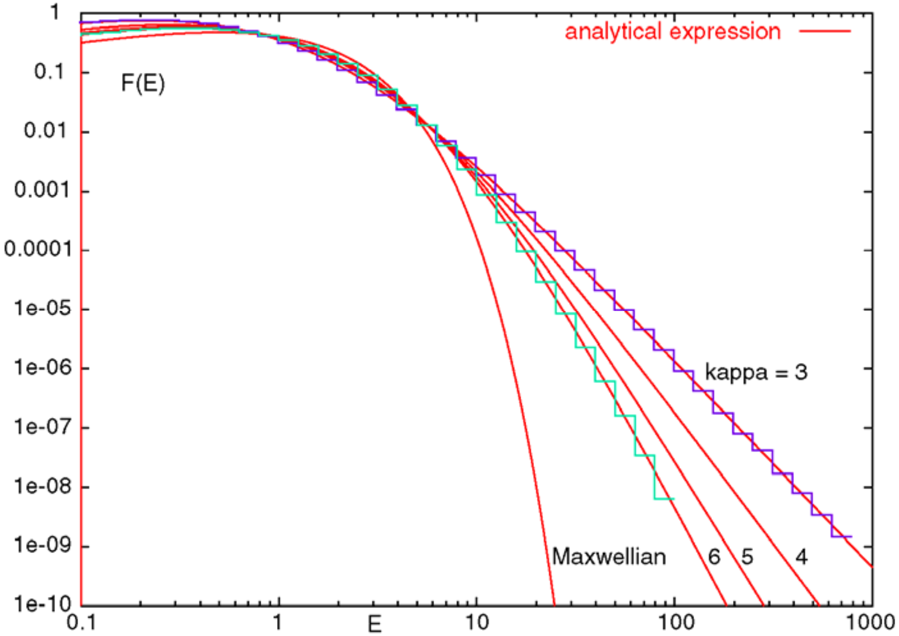


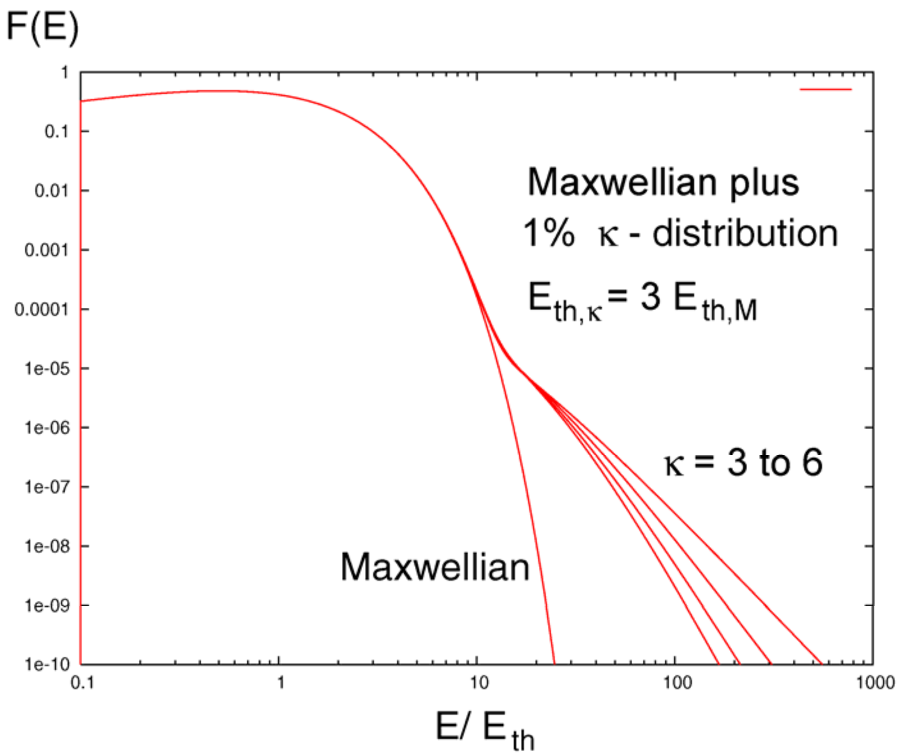
Ion Distributions: quasi-parallel case

Convergence with simulation domain size



Modeling Tail/ Seed Population with κ- Distribution





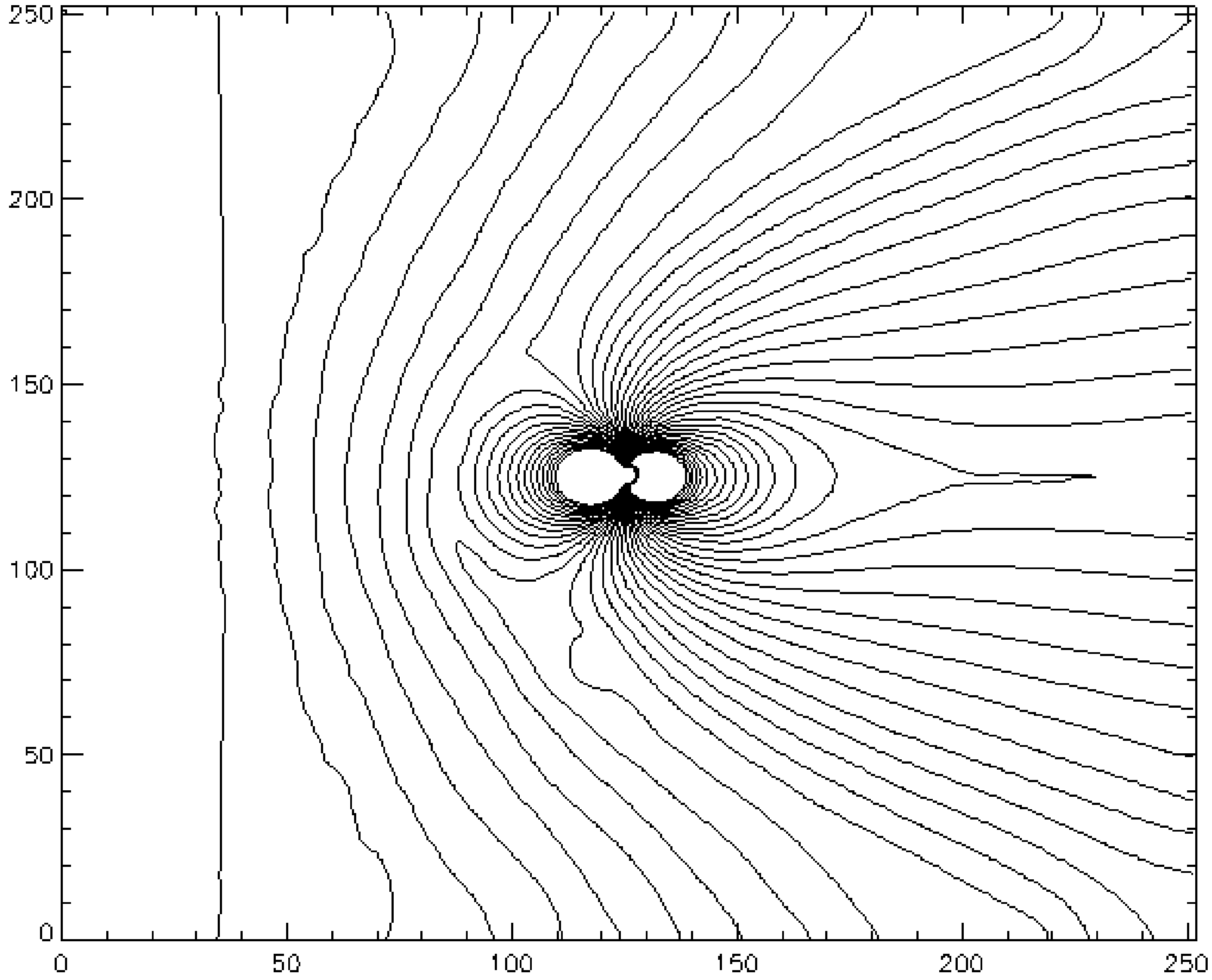
- *reaches higher energies, provides better statistics in wing*
- *combined Maxwellian / κ-distribution can model actual solar wind superthermal ions*

Bow Shock Simulations

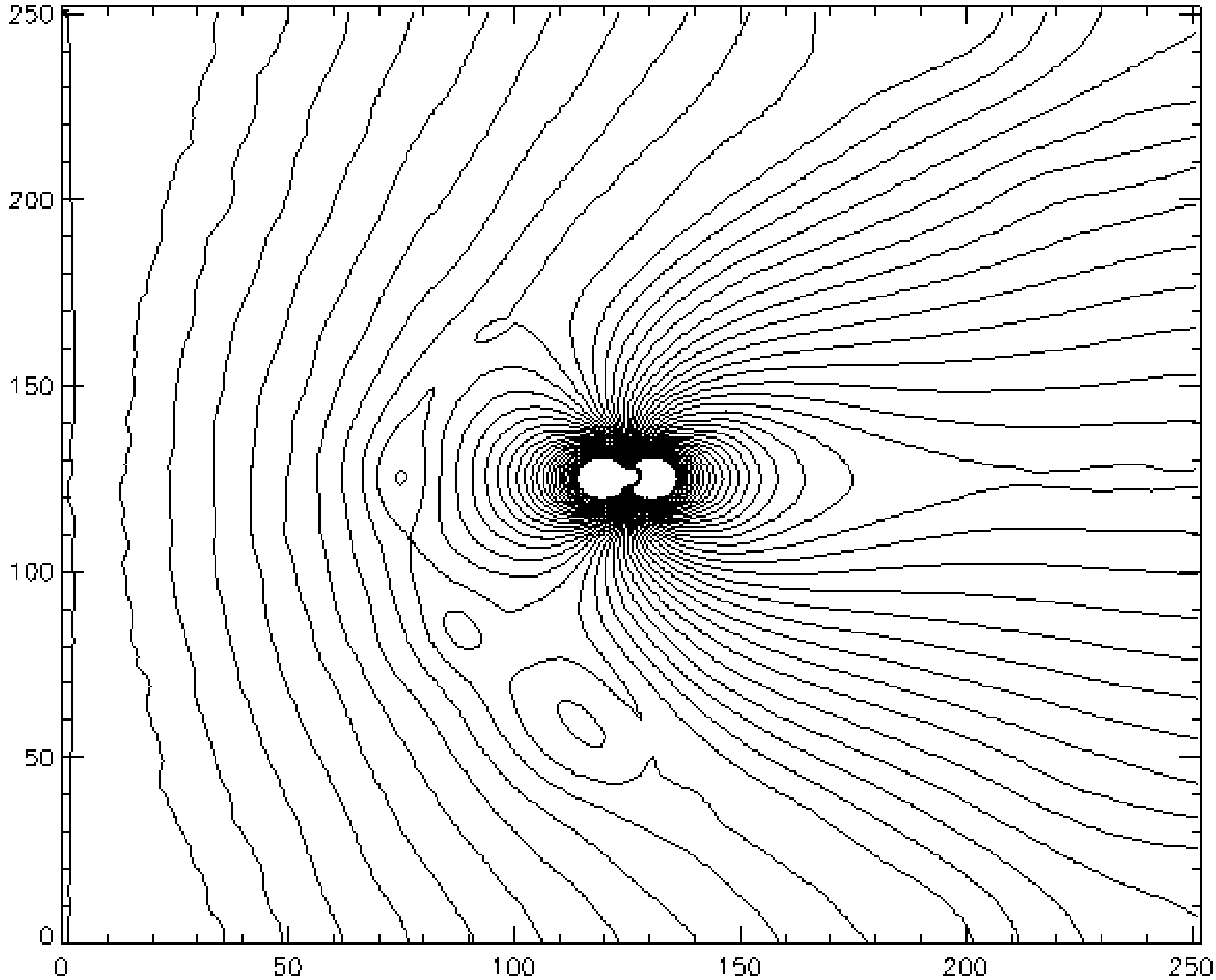
here: effect of resistivity model(s)

why add resistivity?

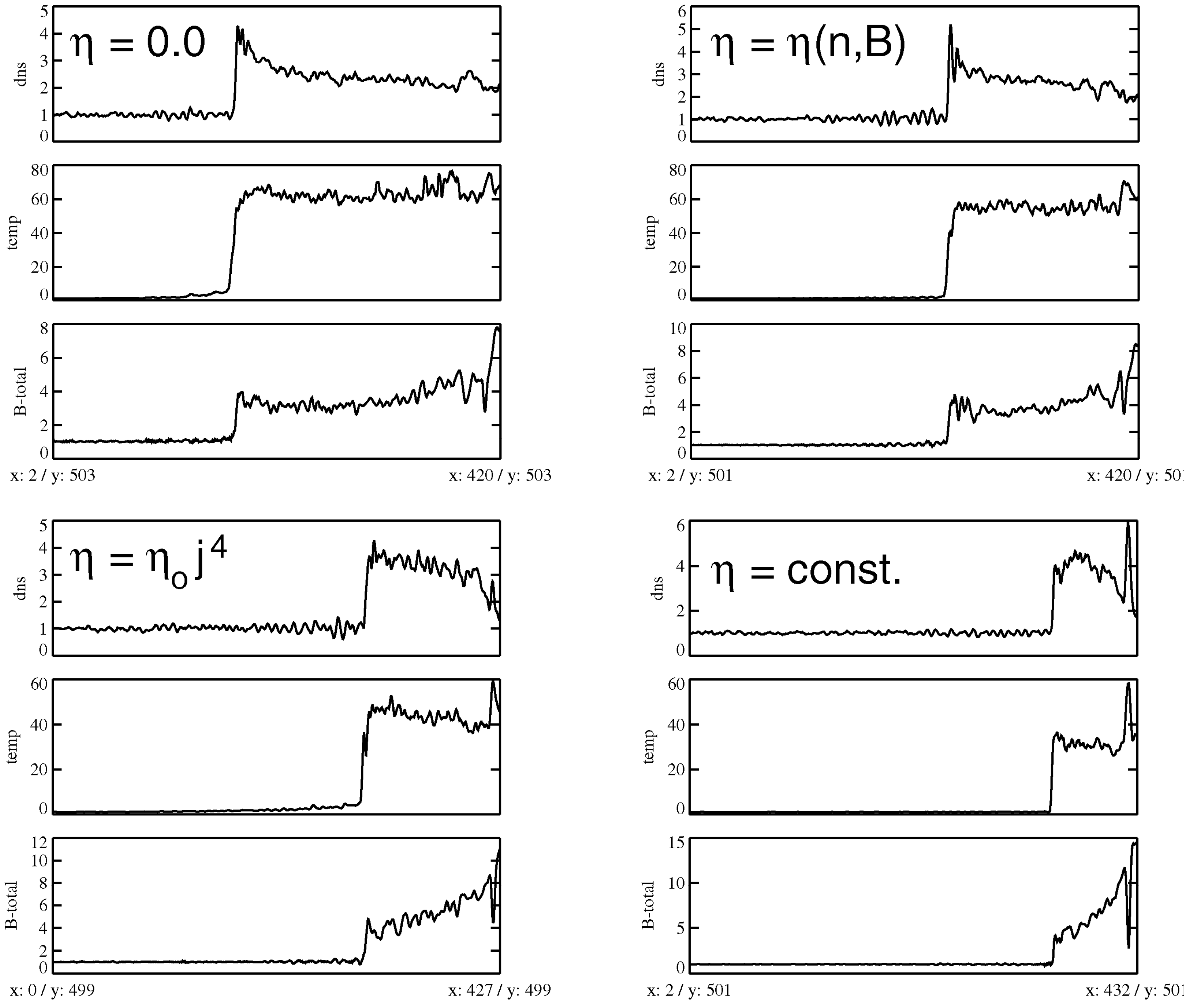
constant resistivity

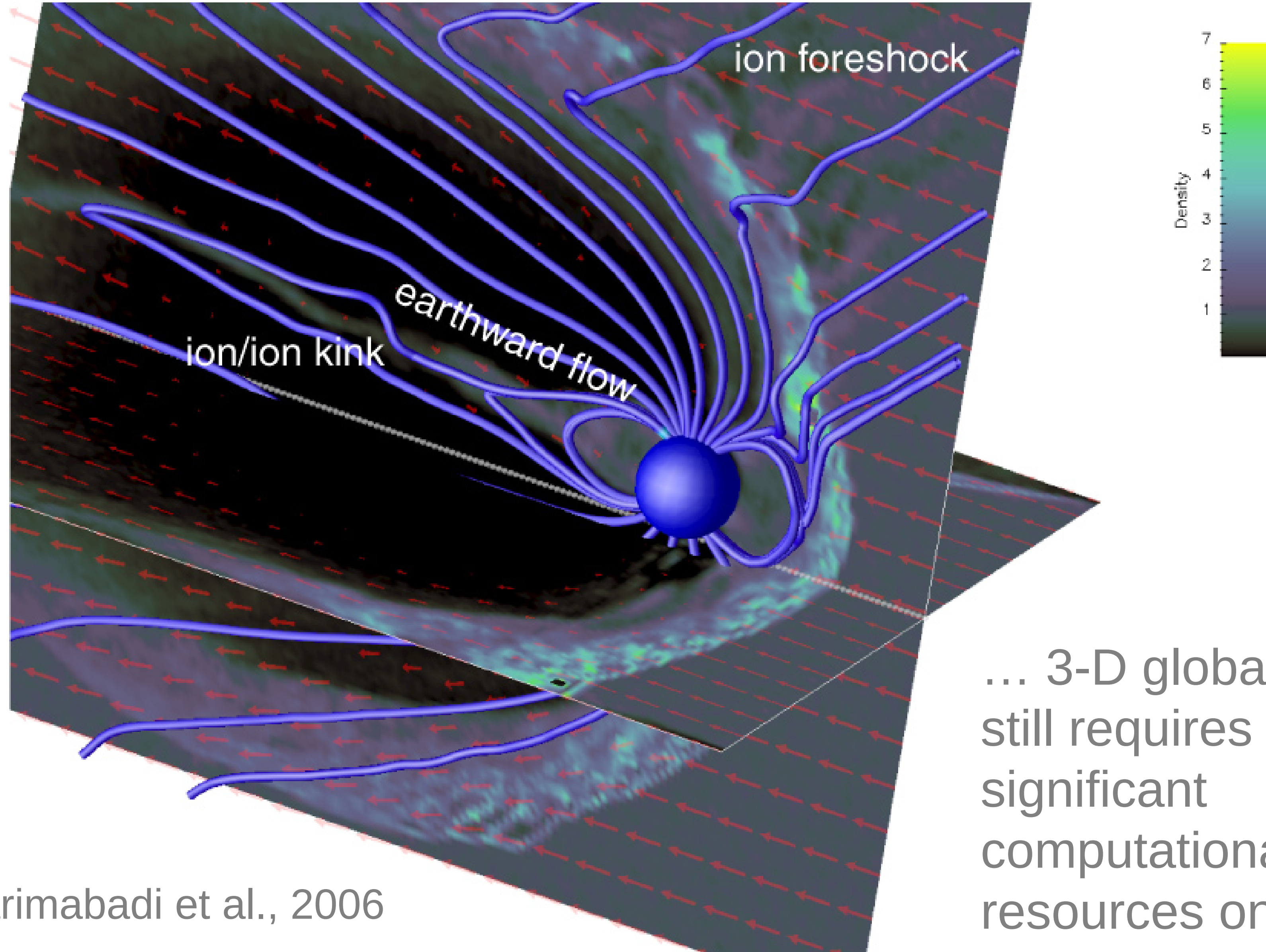


parameter-dependent resistivity



Cuts from Upstream to Magnetopause

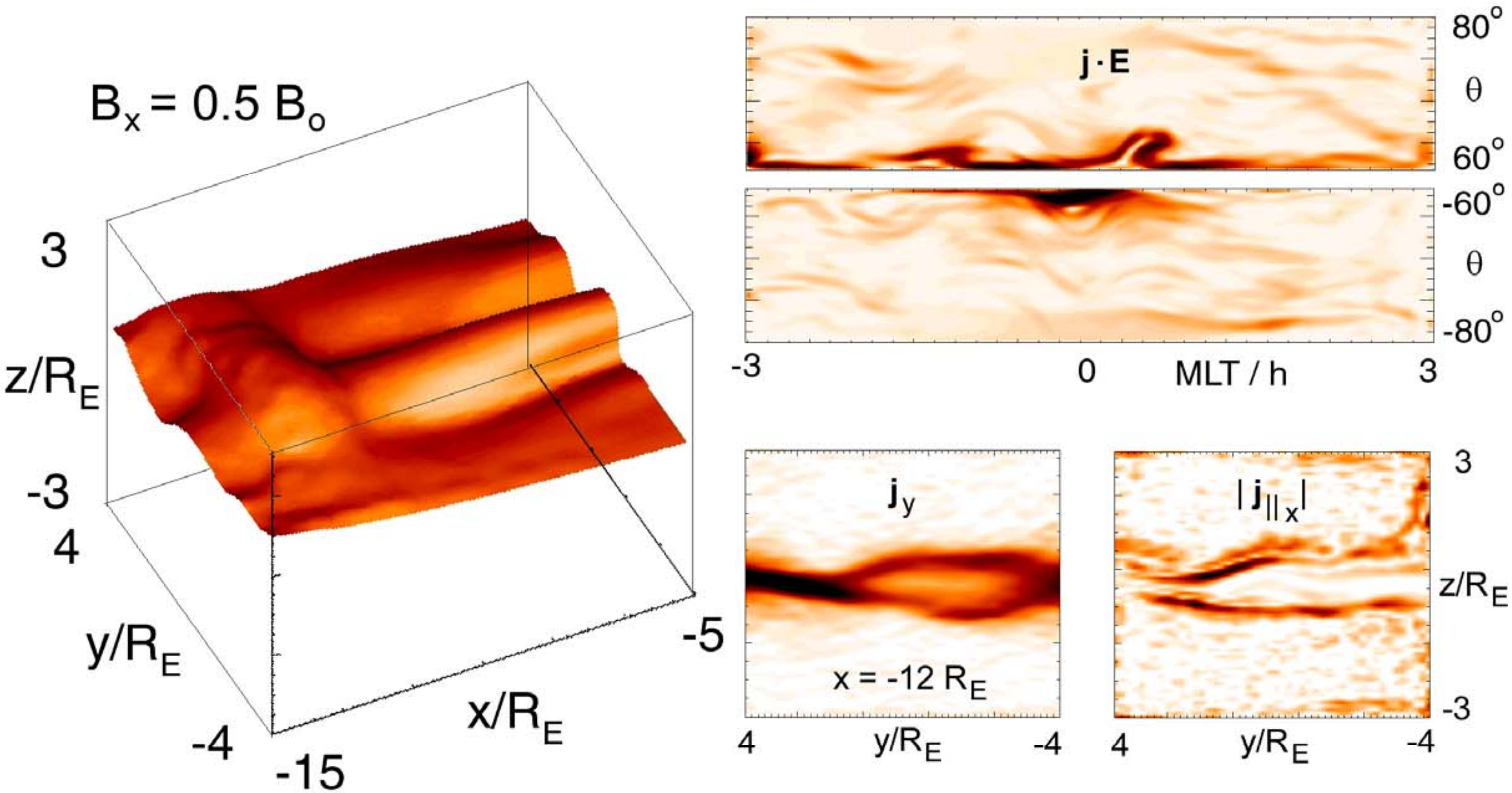




Karimabadi et al., 2006

… 3-D global still requires significant computational resources on large clusters…

→ large-scale, but localized 3-D simulations: magnetotail and ionosphere



Other Numerical Details…

- loss of cache memory correlation/ particle sorting
- energetic particles: Courant condition
- formulation of equilibria/ initialization
- boundary conditions
- low noise / linear methods
- inclusion of dipole field etc.
- parallel codes / domain decomposition
- diagnostics

Summary

- Hybrid simulations are being used successfully for a large range of topics from local to global 3-D.
- While much current research is done on parallel supercomputers, many significant problems, also in 3-D, can be addressed on single CPUs.
- Various modern versions of the Hybrid code converge well with time step, and give comparable results in most circumstances.
- Some versions are more diffusive.
- The predictor-corrector code is often the best method for challenging situations.

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