Turbulence in Space Plasmas

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Why Study Turbulence

- Turbulence is just about the most fundamental and most ubiquitous physics on Earth.
  - Seen in every <u>naturally occurring</u> fluid that is disturbed
  - Responsible for atmospheric weather
  - Reason refrigerators work (and heaters)
  - Reason internal combustion engines work

- Has become a code-word for “disturbance”, “complexity”, and “nonlinearity”, but it is much more.

- Is the process by which a fluid (or gas) attempts to self-organize its energy.

Good References:

- *Space Physics* – May-Britt Kallenrode, Springer 2001. (~ $70)
- *Turbulence* -- Uriel Frisch, Cambridge University Press 1995. (~ $35 to $55)
- *Magnetohydrodynamic Turbulence* – Dieter Biskamp, Cambridge 2003. (~ $110)
- *The Solar Wind as a Turbulence Laboratory* -- Bruno and Carbone, on line @ Living Reviews in Solar Physics (free)

Overview

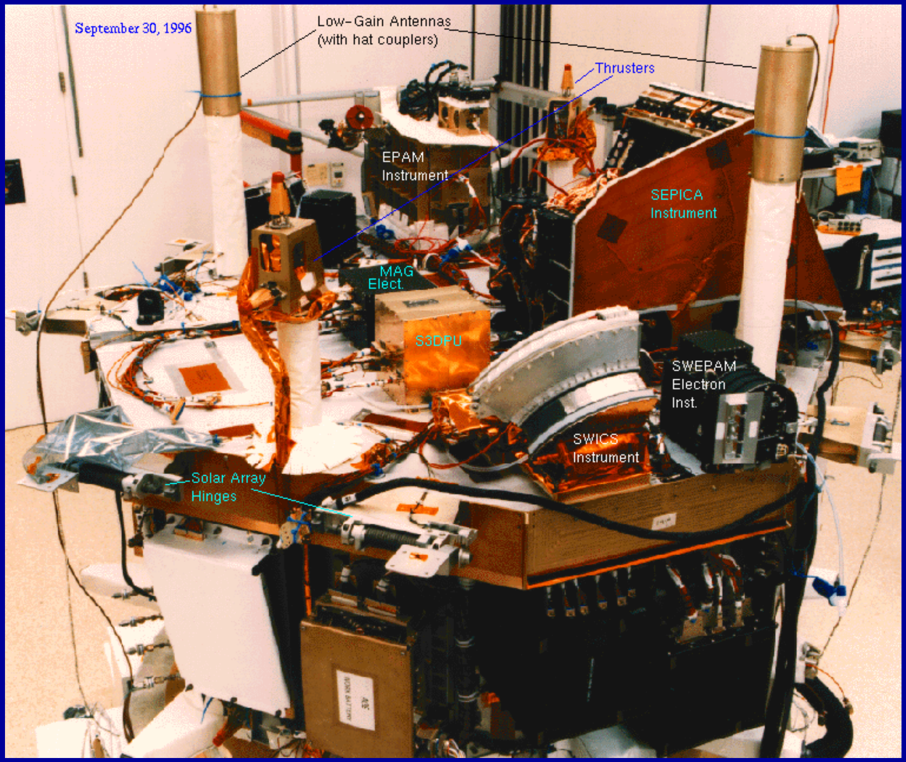
- We will:
  - Apply basic idea of turbulence theory to the solar wind.
  - Find that the interplanetary spectrum contains signatures of solar activity and in situ dynamics.
  - Supports a cascade of energy from large-scales to small where dissipation heats the background plasma.
  - Try to explain the observed heating of the solar wind. (…and with it, maybe the acceleration that produces the wind.)
  - Find evidence for fundamental anisotropies that structure the turbulent cascade.
  - Develop some models and basic idea.

- We will not:
  - Be too rigorous or work too hard!

Before There Were Spacecraft…

- There were comet tails.
- Something blows the tails of comets away from sun.
- Light pressure cannot explain it.
- Sometimes the tails become disconnected.

MAG Delivery on ACE



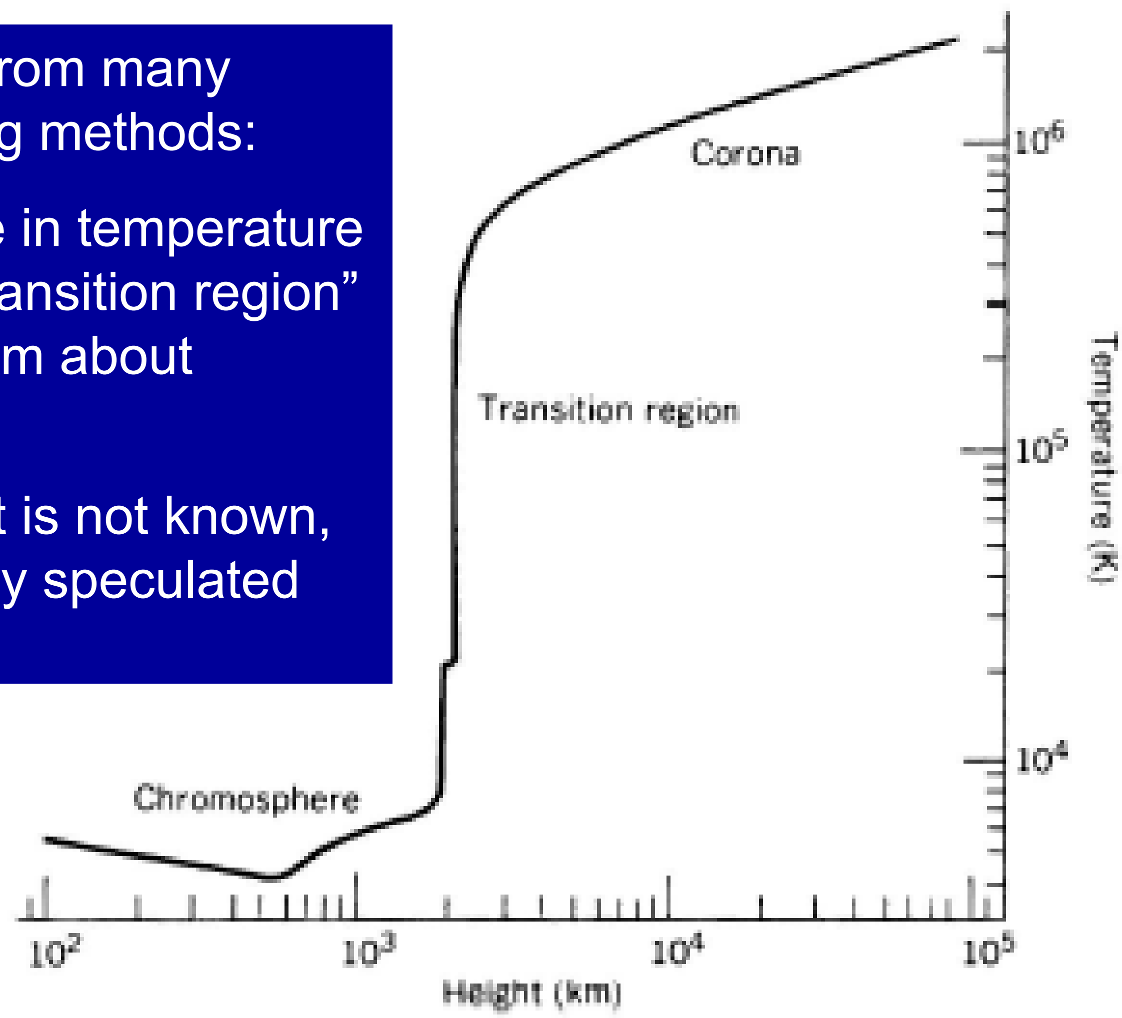
The Magnetic Field Experiment (MAG) measures the weak magnetic fields of interplanetary space, providing necessary information to interpret the thermal and charged particle measurements along with understanding the magnetospheric response to transient events.

Origin of the Solar Wind

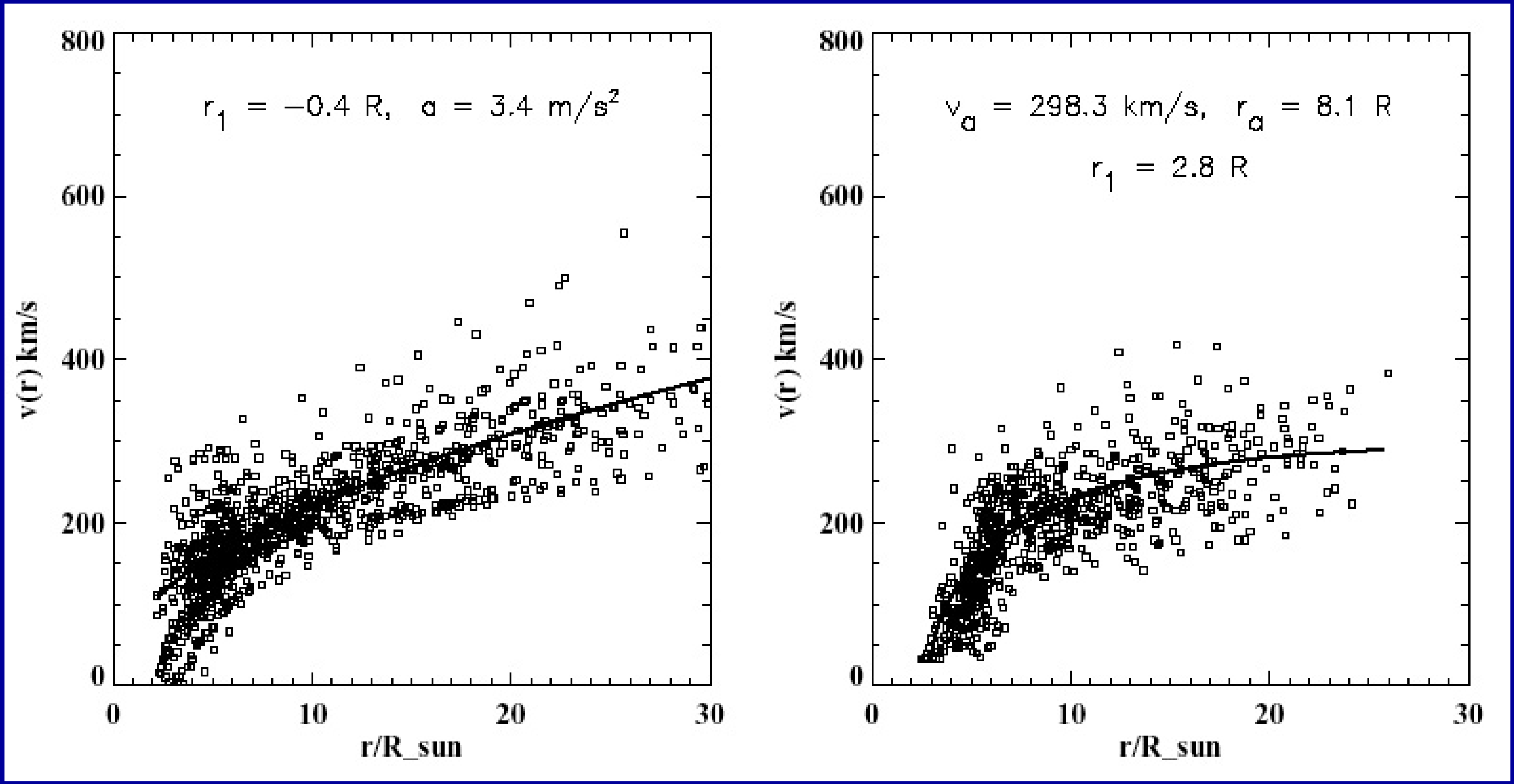
Model drawn from many remote sensing methods:

Note rapid rise in temperature in so-called “transition region” around 2000 km about photosphere.

Source of heat is not known, although widely speculated upon.



Solar Wind Acceleration



Scatter plots of velocity as a function of distance for the emitted plasma puffs and solid line showing best fits to radial profile. These plots show the expected outflow that Solar Probe will encounter during its perihelion pass (Sheeley et al., 1997).

1 AU Conditions (Near Earth)

Solar wind speed, V_{SW}, varies from 250 to 800 km/s.

(R$_{Earth}$ = 6371 km, an 8 second transit at high wind speed.)

At least 2 transients have been observed at 2000 km/s at 1 AU!

Proton density varies, but averages about 10 p^+/cm^3.

We are unable to achieve these densities on the surface of the Earth.

Proton temperature averages about 10^5 K. (V_{th} ~ 50 km/s for ions.)

Magnetic clouds are colder (1/10'th).

Interplanetary magnetic field averages 6 to 8 nT and 45° from radial direction.

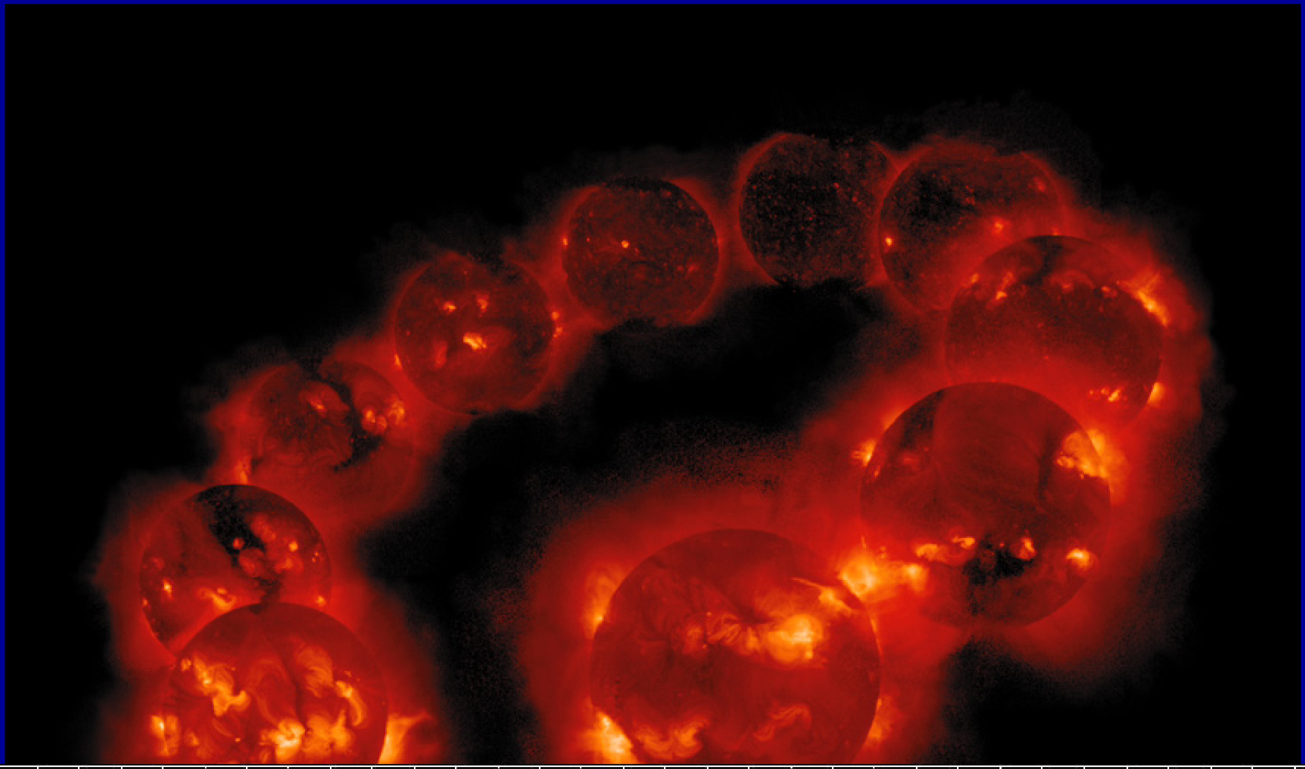
(B_{Earth} = 5.7 x 10^4 nT at Boulder, Colorado.)

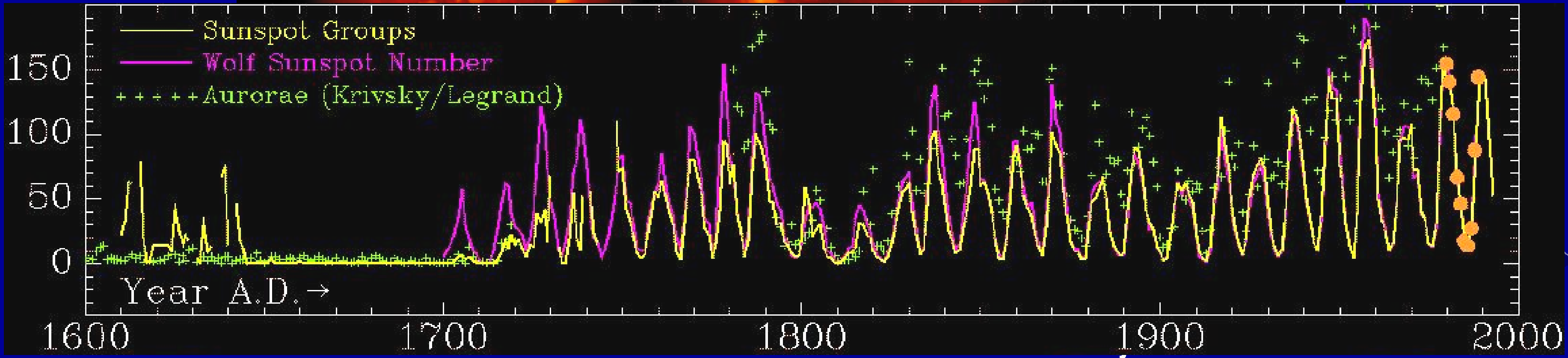
IMF can be just about anything at any time.

Have reached 55 nT during ACE era and << 1 nT.

Heavy ions (He^{++}, He^+, $Fe^{?+}$, $C^{?+}$, $O^{?+}$, etc.) provide clues to origins.

Solar Activity 1992-1999



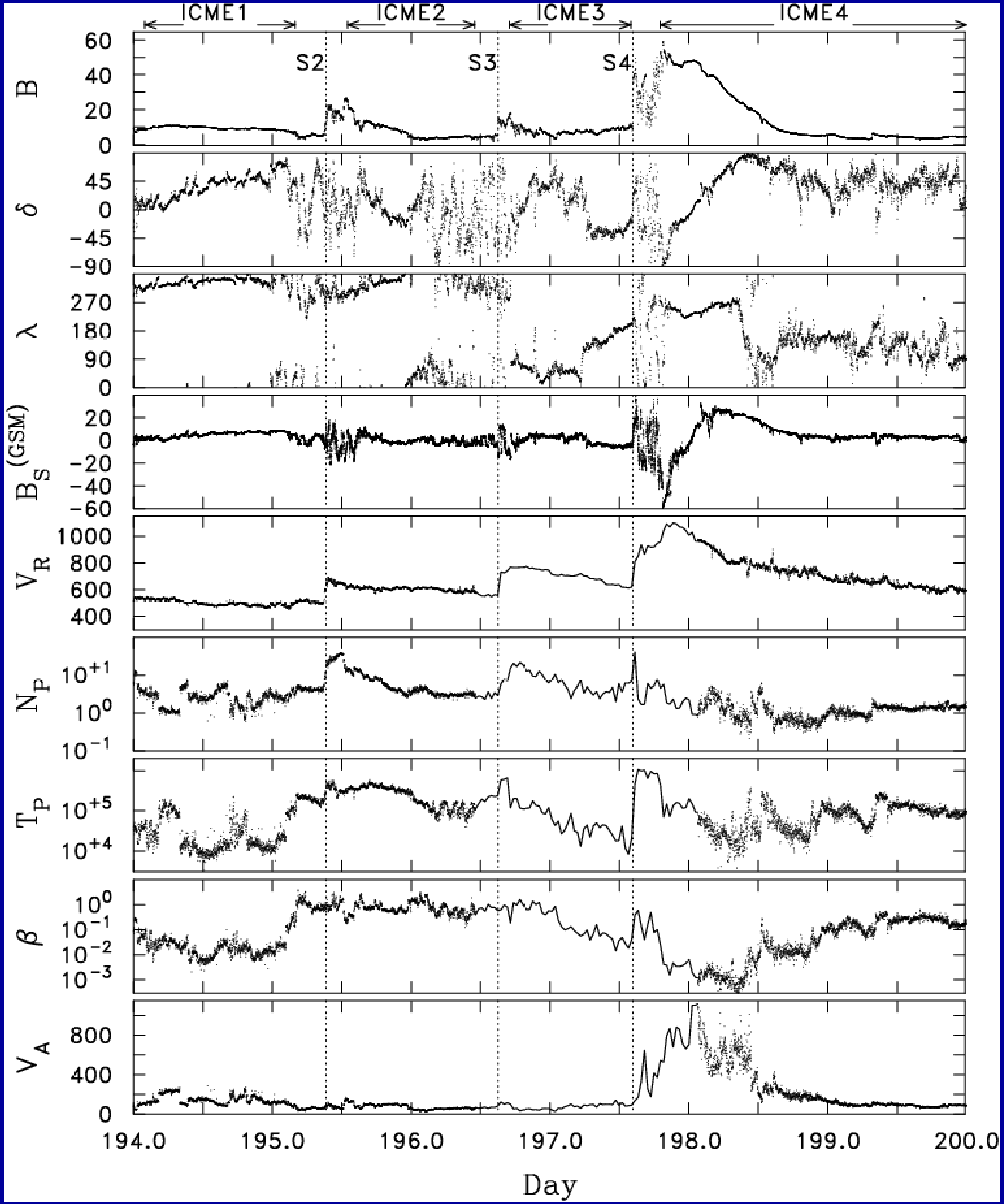


Transient Arrival of Bastille Day 2000

Over a week of extensive solar activity 3 shocks and associated ejecta arrived at Earth.

Each resulted in energetic particle enhancements to varying degrees.

Each resulted in magneto-spheric storm activity to varying degrees.



Solar Energetic Particles

Solar flares on Bastille Day 2000 marked the eruption of coronal magnetic fields and associated plasma.

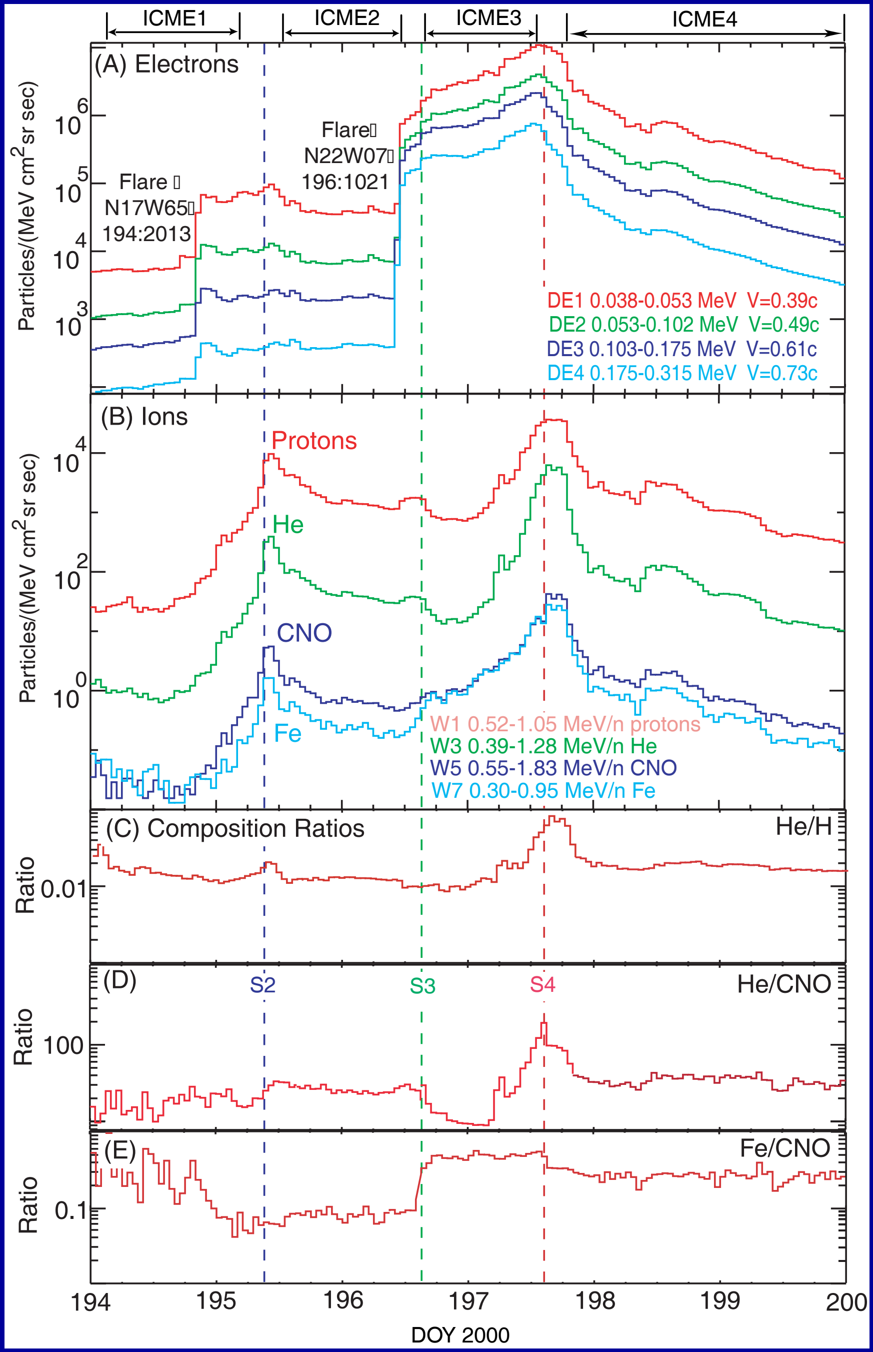
This ejection resulted in the acceleration of ions and electrons via mechanisms still under study.

ACE observed the energetic particles arrived at Earth several days ahead of the ejecta.

Orbiting spacecraft were damaged or destroyed.

Loss of electrical power was threatened.

Astronauts were endangered.

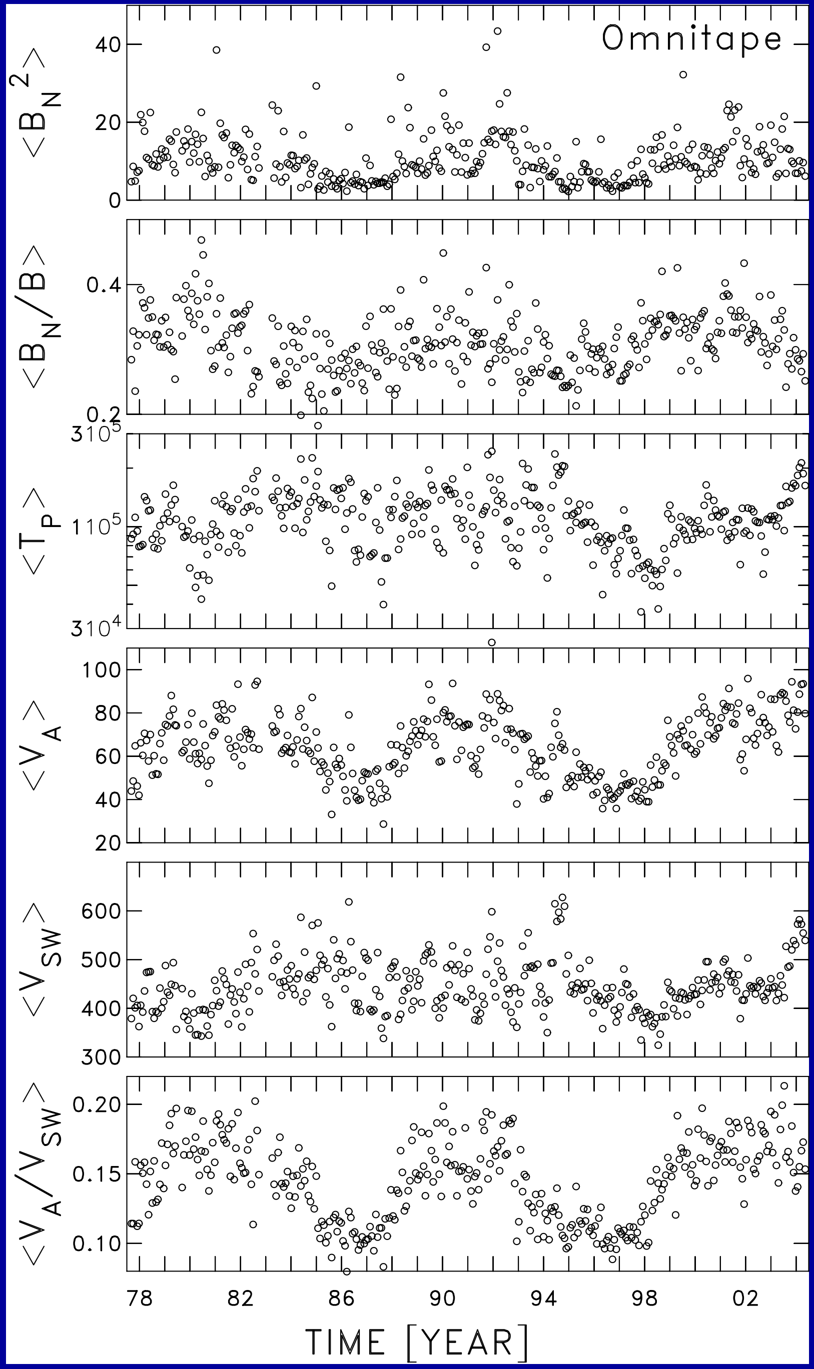


Solar Wind Variability

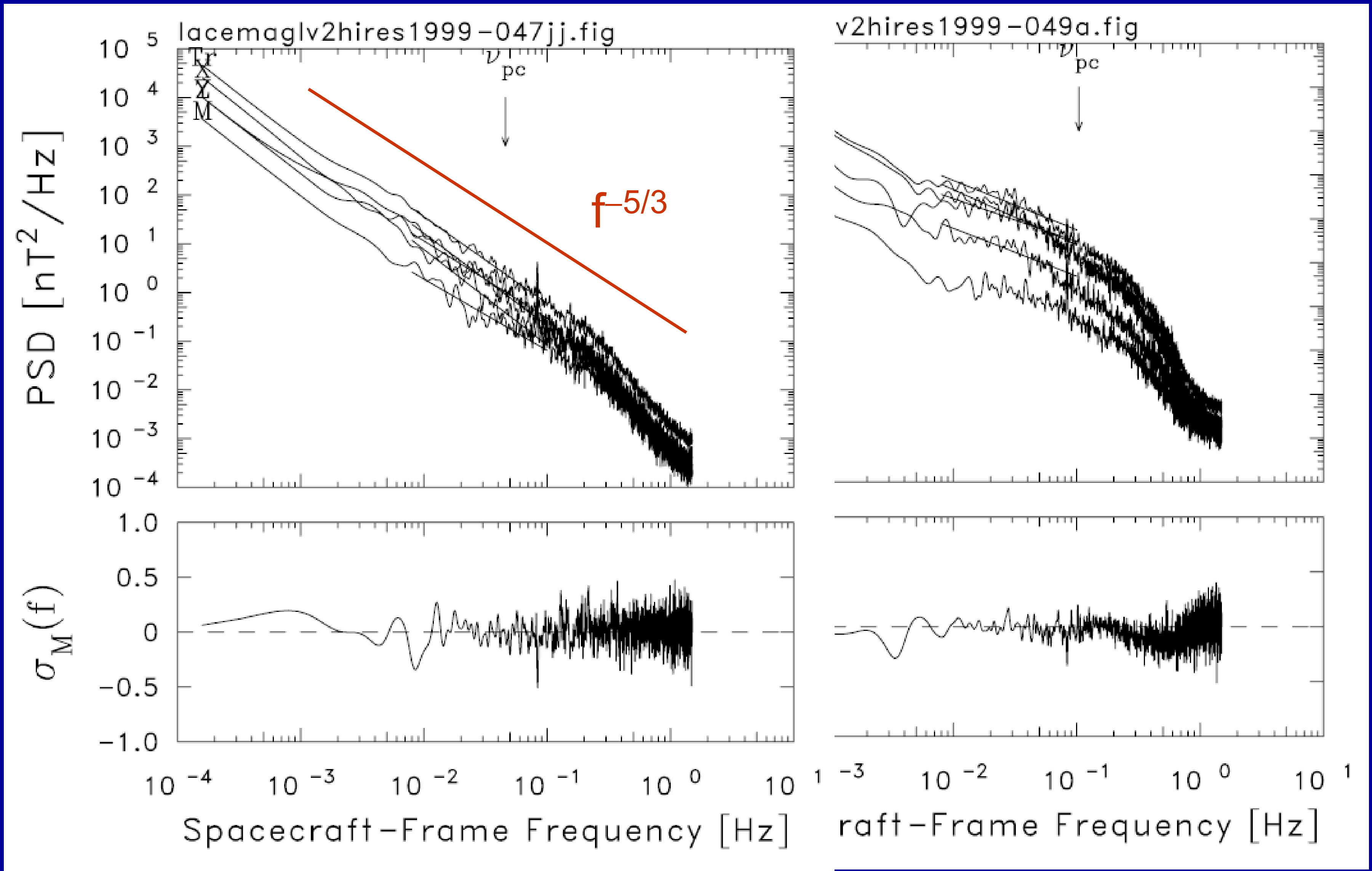
While we will see there is all sorts of variability from one hour to the next, there is also a systematic variability tied to the solar cycle.

This reflects the Sun's changing state with high-speed wind sources moving to new latitudes and multiple wind sources interacting.

This variability propagates into the outer heliosphere, forms merged regions of plasma that alter the propagation of galactic cosmic rays Earthward, and effects the acceleration of energetic particles within the heliosphere.



Analysis: Day 49 of 1999



So what's happening in all these cases?

Today let's focus on two related observations:

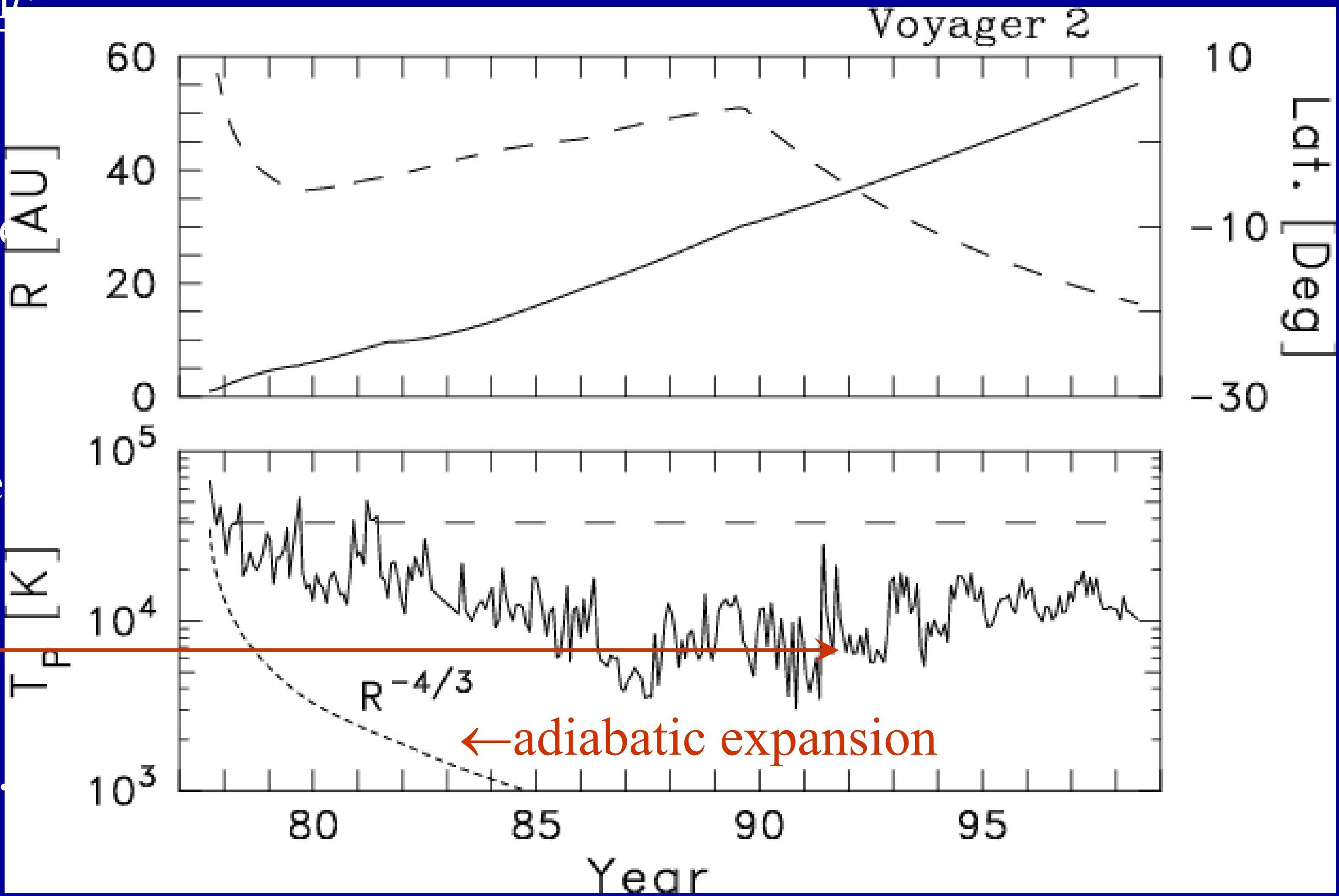
There are fluctuations and there is heating!

Heating in the Voyager Data

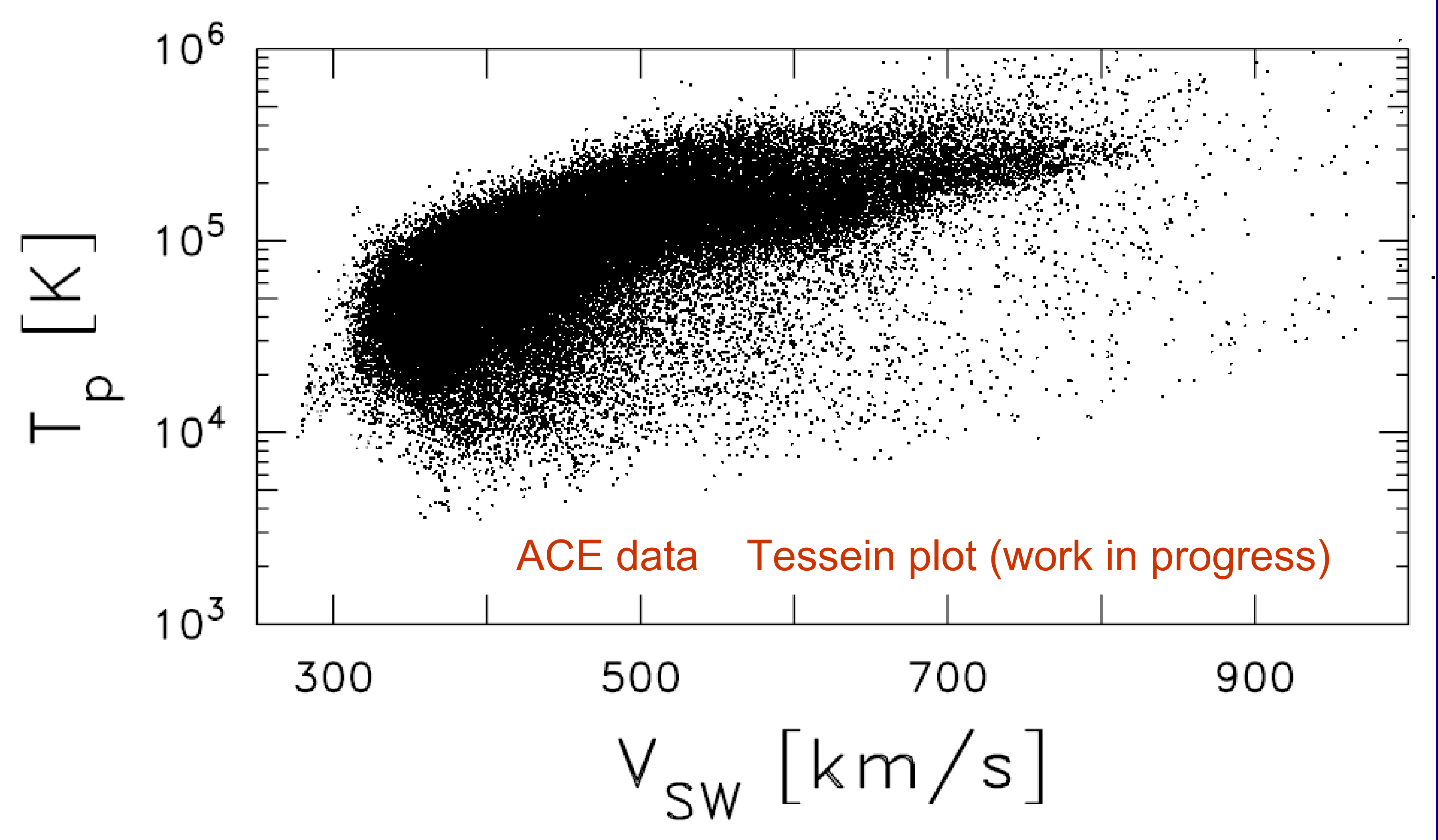
Proton temperature observed by Voyager is hotter than adiabatic expansion predicts. Schwartz et al., *JGR*, **86**, 541 (1981) showed spectrum must be dynamic

Proton heating continues to > 70 AU where transient sources (shear shocks, etc.) are largely gone

Only pickup ions from interstellar neutrals are left.



Heating Rates at 1 AU



Formalisms of Plasma Physics

$$\nabla \bullet \mathbf{E}(\mathbf{x},t) = 4\pi\rho$$

$$\nabla \bullet \mathbf{B}(\mathbf{x},t) = 0$$

$$\nabla \times \mathbf{E}(\mathbf{x},t) = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B}(\mathbf{x},t) = \frac{4\pi}{c}\mathbf{J}(\mathbf{x},t) + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \equiv \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

Maxwell's Equations governing Electro-Magnetic Fields

$$\frac{\partial f_s(\mathbf{x},\mathbf{v},t)}{\partial t} + \mathbf{v} \bullet \nabla f_s + \sum_s \frac{q_s}{m_s}\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) \bullet \nabla f_s = 0$$

Vlasov Equation

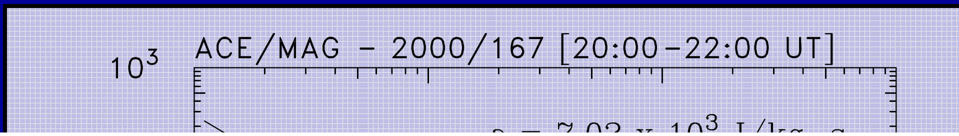
(Collisionless Boltzman's Equation)

$$\rho(\mathbf{x},t) \equiv \sum_s q_s \int f d\mathbf{v}$$

$$\mathbf{J}(\mathbf{x},t) \equiv \sum_s q_s \int \mathbf{v} f d\mathbf{v}$$

Charge density, currents, etc. are computed from the distribution of particles for all species

Magnetohydrodynamics (MHD)



There is a mathematical formalism that (we believe) describes the

$$\rho_M\left(\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\bullet\nabla)\mathbf{V}\right)=-\nabla P+\frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}+\nu\nabla^2\mathbf{V}$$

$$\frac{\partial \rho_M}{\partial t}+\nabla\bullet(\rho_M\mathbf{V})=0 \quad \text{or} \quad \nabla\bullet\mathbf{V}=0$$

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times(\mathbf{V}\times\mathbf{B})+\frac{c^2}{4\pi\sigma}\nabla^2\mathbf{B}$$

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J}$$

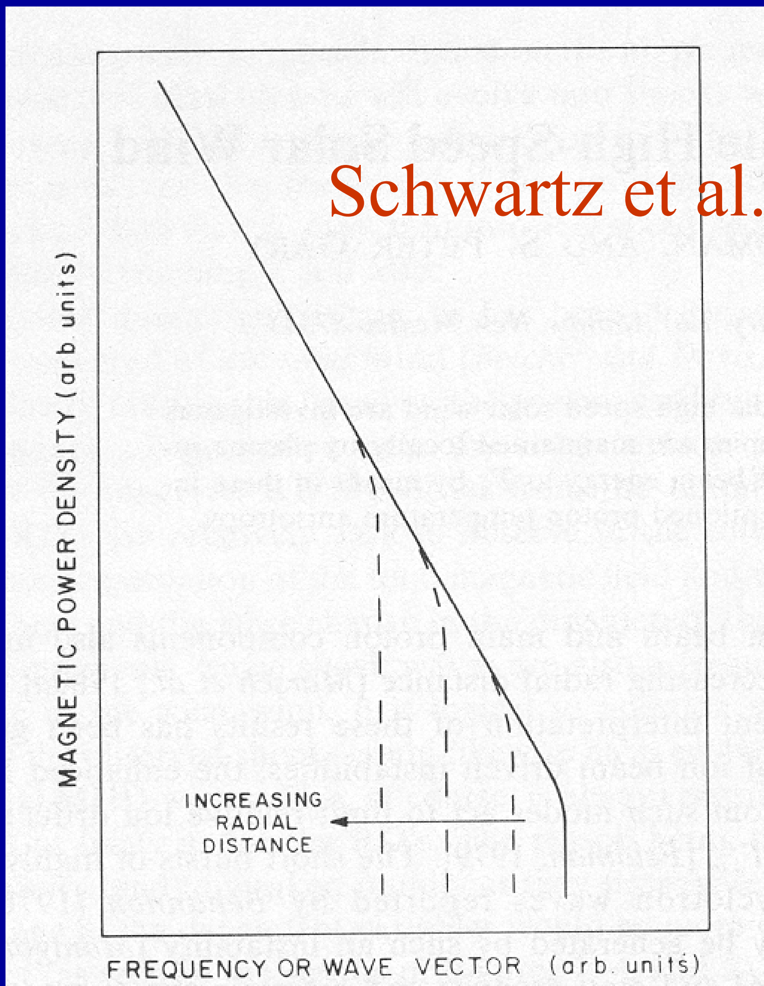
$$\nabla\equiv\hat{\mathbf{x}}\frac{\partial}{\partial x}+\hat{\mathbf{y}}\frac{\partial}{\partial y}+\hat{\mathbf{z}}\frac{\partial}{\partial z}$$

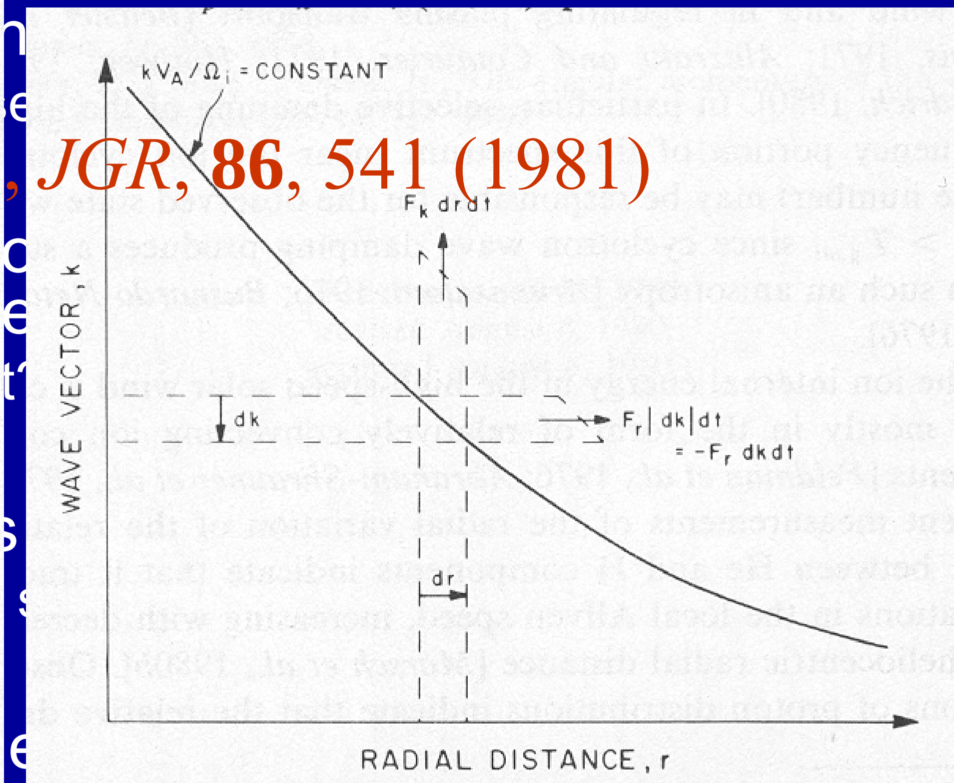
Low-Frequency Waves

- Alfven waves:
  - Transverse magnetic and velocity fluctuations
  - No density compressions
  - $\omega = \mathbf{k} \cdot \mathbf{B_0} / \sqrt{(4\pi\rho)}$
- Fast Mode waves:
  - Some field-aligned **V** and **B** fluctuations
  - Some compression for off-axis propagation
  - $\omega = k B_0 / \sqrt{(4\pi\rho)}$

What Heats the Solar Wind?

Schwartz et al., *JGR*, **86**, 541 (1981)



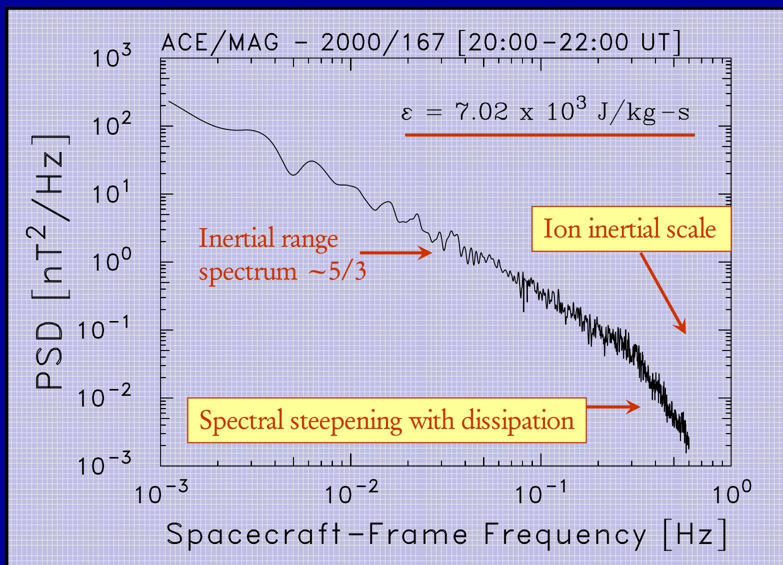


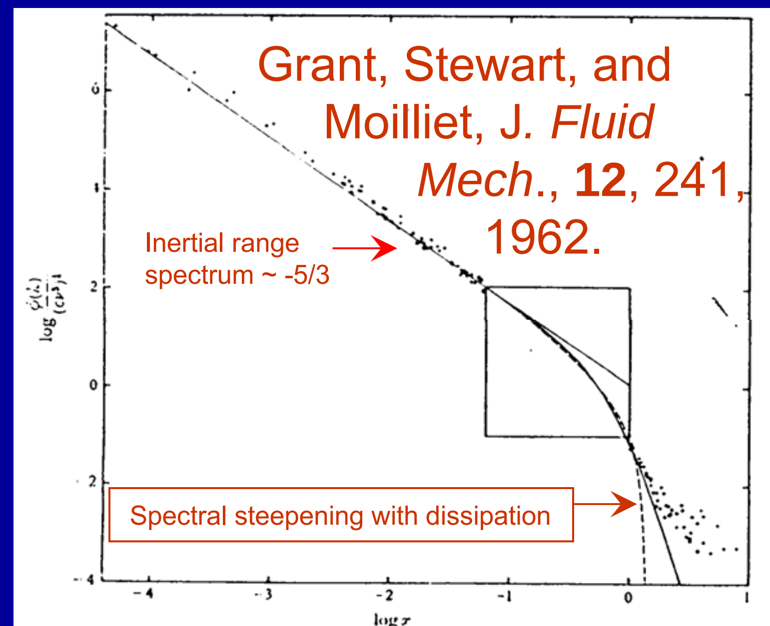
...trals / newborn pickup ions in the outer

...ation energy in the outer heliosphere

A Suggestive Association

- Magnetohydrodynamics
- Hydrodynamics





$$\rho_M\left(\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\bullet\nabla)\mathbf{V}\right)=-\nabla P+\frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}+\nu\nabla^2\mathbf{V}$$

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$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times(\mathbf{V}\times\mathbf{B})+\frac{c^2}{4\pi\sigma}\nabla^2\mathbf{B}$$

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J} \qquad \nabla\equiv\hat{\mathbf{x}}\frac{\partial}{\partial x}+\hat{\mathbf{y}}\frac{\partial}{\partial y}+\hat{\mathbf{z}}\frac{\partial}{\partial z}$$

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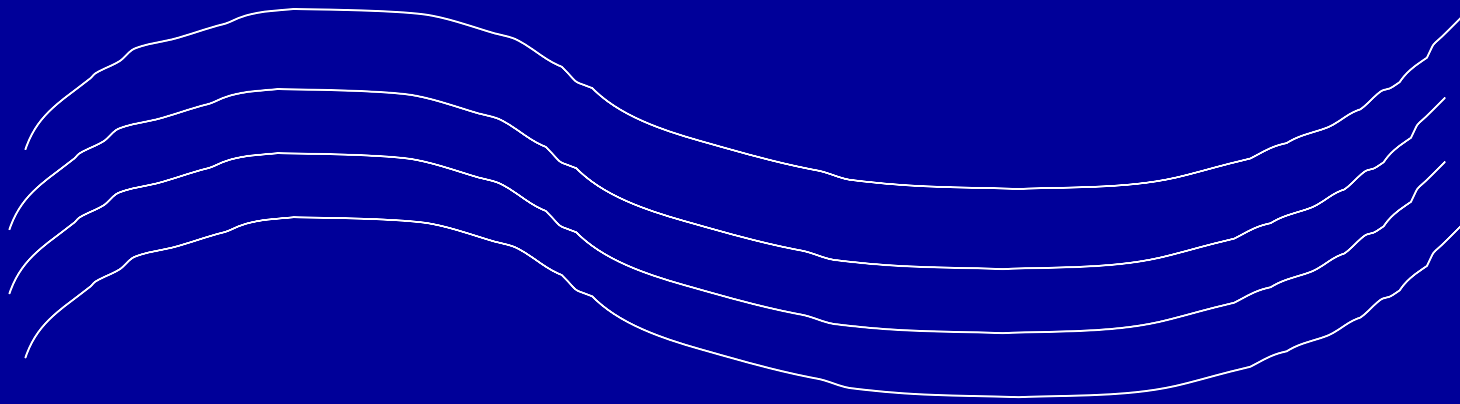
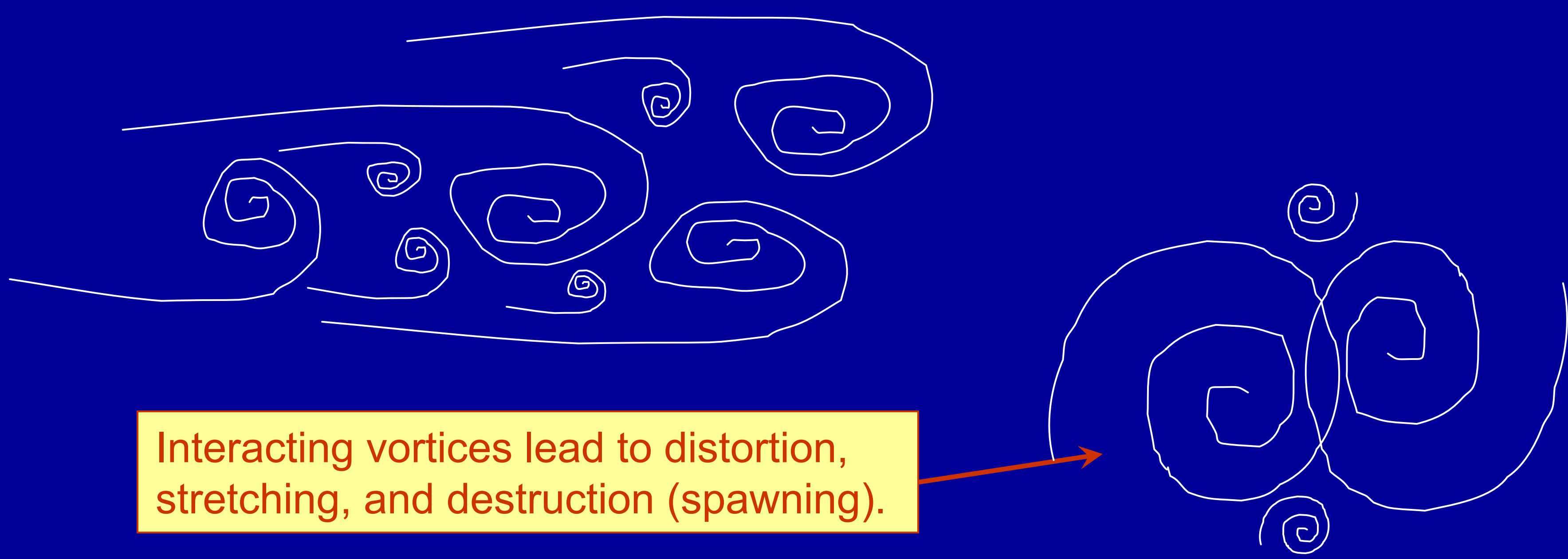
$$\frac{\partial \rho_M}{\partial t}+\nabla\bullet(\rho_M\mathbf{V})=0 \quad \text{or} \quad \nabla\bullet\mathbf{V}=0$$

One Example of Turbulence

Contributed by Pablo Dimitric of the Bartol Research Inst.

Hydrodynamic Turbulence: Laminar vs. Turbulent Flow





Navier-Stokes Equation

Mass Density

Fluid Pressure

Dissipation of Energy

$$\rho_M\left(\frac{\partial V}{\partial t}+(V\bullet\nabla)V\right)=-\nabla P+\nu\nabla^2 V$$

$$\nabla\bullet V=0$$

Convective derivative (time variability following the flow)

Incompressibility

(constant mass)

Energy Conservation

$$\int V \bullet \left[\rho_M \left(\frac{\partial V}{\partial t} + (V \bullet \nabla) V \right) = -\nabla P + \nu \nabla^2 V \right]$$

$$V \bullet \frac{\partial V}{\partial t} \rightarrow \frac{1}{2} \frac{\partial V^2}{\partial t} = \frac{\partial E}{\partial t}$$

Energy Conserving

$$V \bullet (V \bullet \nabla) V \rightarrow V \bullet \left[\frac{1}{2} \nabla V^2 - \frac{1}{2} V \times \left(\nabla \times V \right) \right]$$

0

$$V \bullet \left(\nabla V^2 \right) = \nabla \bullet \left(V^2 V \right) - V^2 \left(\nabla \bullet V \right)$$

0 0

Wave Vector Dynamics

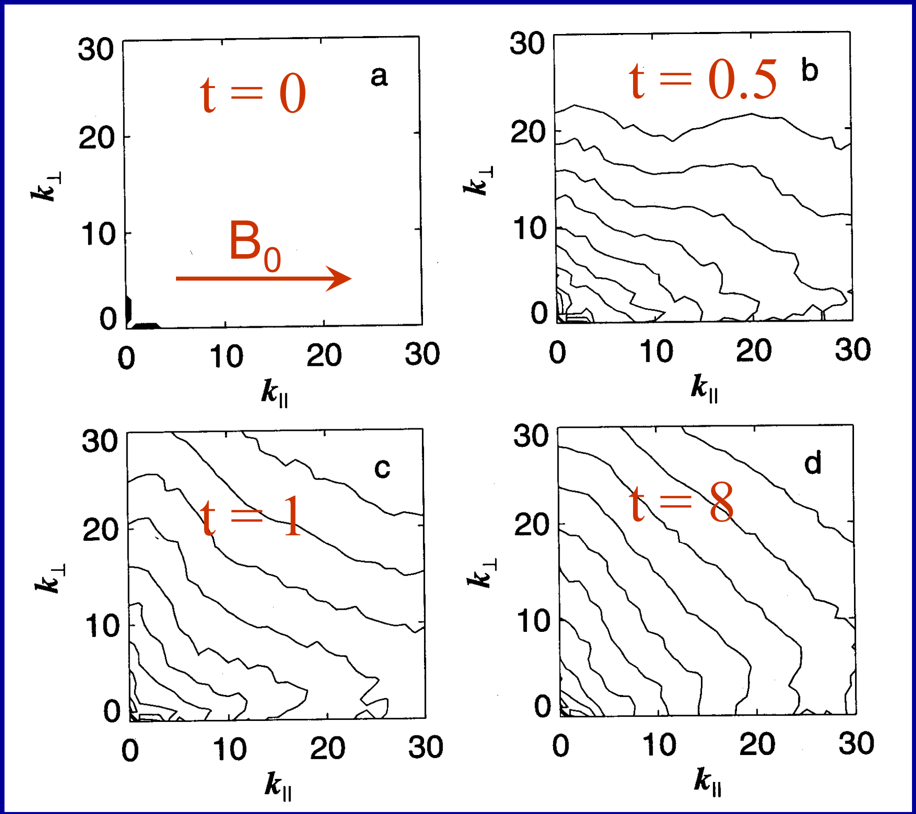
$$\rho_M\left(\frac{\partial V}{\partial t}+(V\bullet\nabla)V\right)=-\nabla P+\nu\nabla^2 V$$

If $V(x)\equiv\sum_k V_k e^{ik\cdot x}$

Dissipation at large wave numbers

$$\rho_M\left(\frac{\partial V_k}{\partial t}+\sum_{k=k_1+k_2} V_{k_1}\bullet ik_2 V_{k_2}\right)=-ikP_k-\nu k^2 V_k$$

Evolution of $E_B \rightarrow k_\perp$



Ghosh et al.,

J. Geophys. Res., **103**, 23,691, 1998.

J. Geophys. Res., **103**, 23,705, 1998.

In a 2D MHD simulation:

with DC magnetic field,

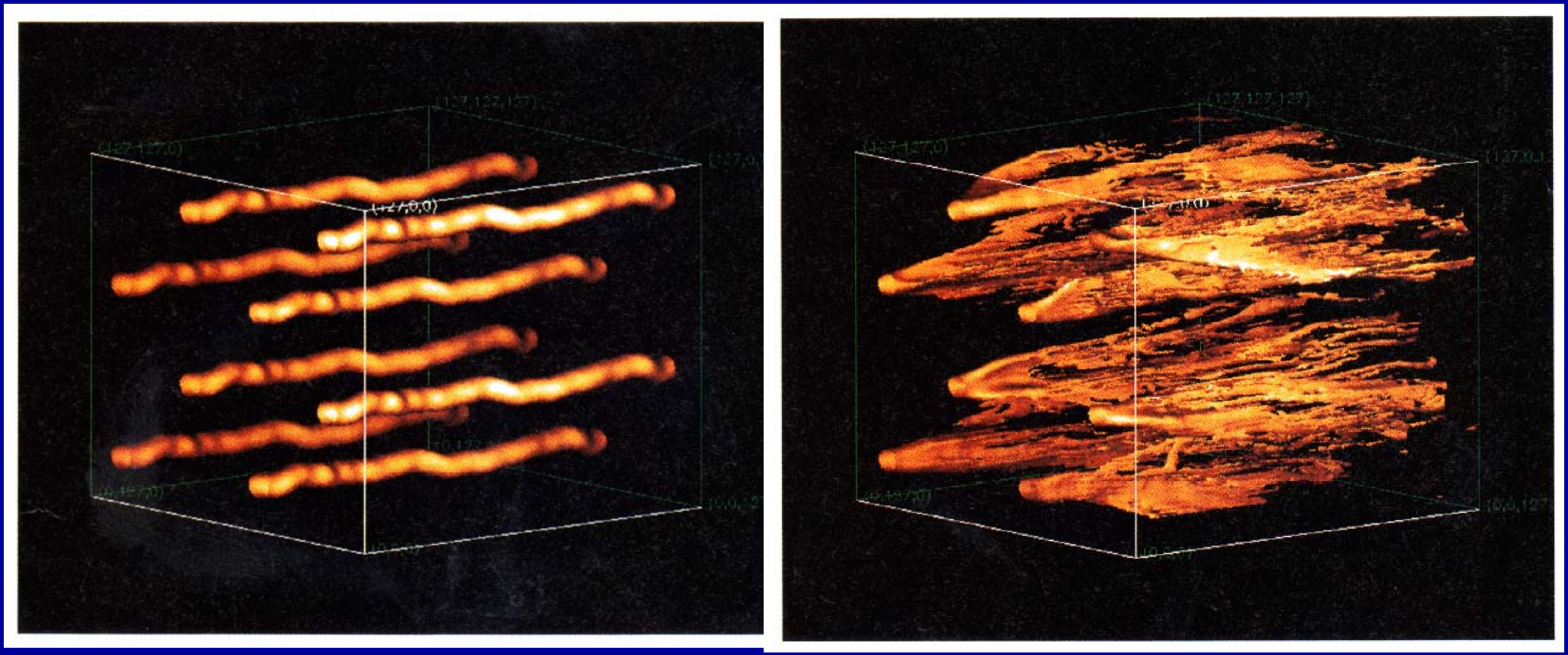
energy is placed in a few k,

background noise,

energy moves to larger wave vectors and moves away from the mean field direction.

Initial input of energy at scales incapable of dissipation evolves toward scales where dissipation can occur.

Diverging Field Lines



The 2D component (right) leads to perpendicular spatial variation so that field lines and the energetic particles on them diverge.

Bieber et al., *J. Geophys. Res.*, **101**, 2511-2522, 1996.

Kolmogorov's First Hypothesis of Similarity

This means that the inertial and dissipation ranges can be represented as :

$$E(k) = \varepsilon^{2/3} k^{-5/3} F\left(\eta k\right)$$

where $F\left(\eta k\right)$ is a universal function.

$F\left(\eta k\right) \rightarrow C_K \cong 1.6$ in the inertial range.

The value $C_K \cong 1.6$ has been confirmed by experiment :

Sreenivasan, K.R., Phys. Fluids, 7(11), 2778 (1995)

It also means that $\tau \sim L / \delta V$.

What is Kolmogorov Saying?

Large-scale fluctuations (eddy's, waves, shears, ejecta, shocks, whatever) contain a lot of energy, but direct dissipation of that energy is slow (except maybe shocks).

The turbulent inertial range cascade converts energy of the large-scale objects into smaller scales until dissipation becomes important.

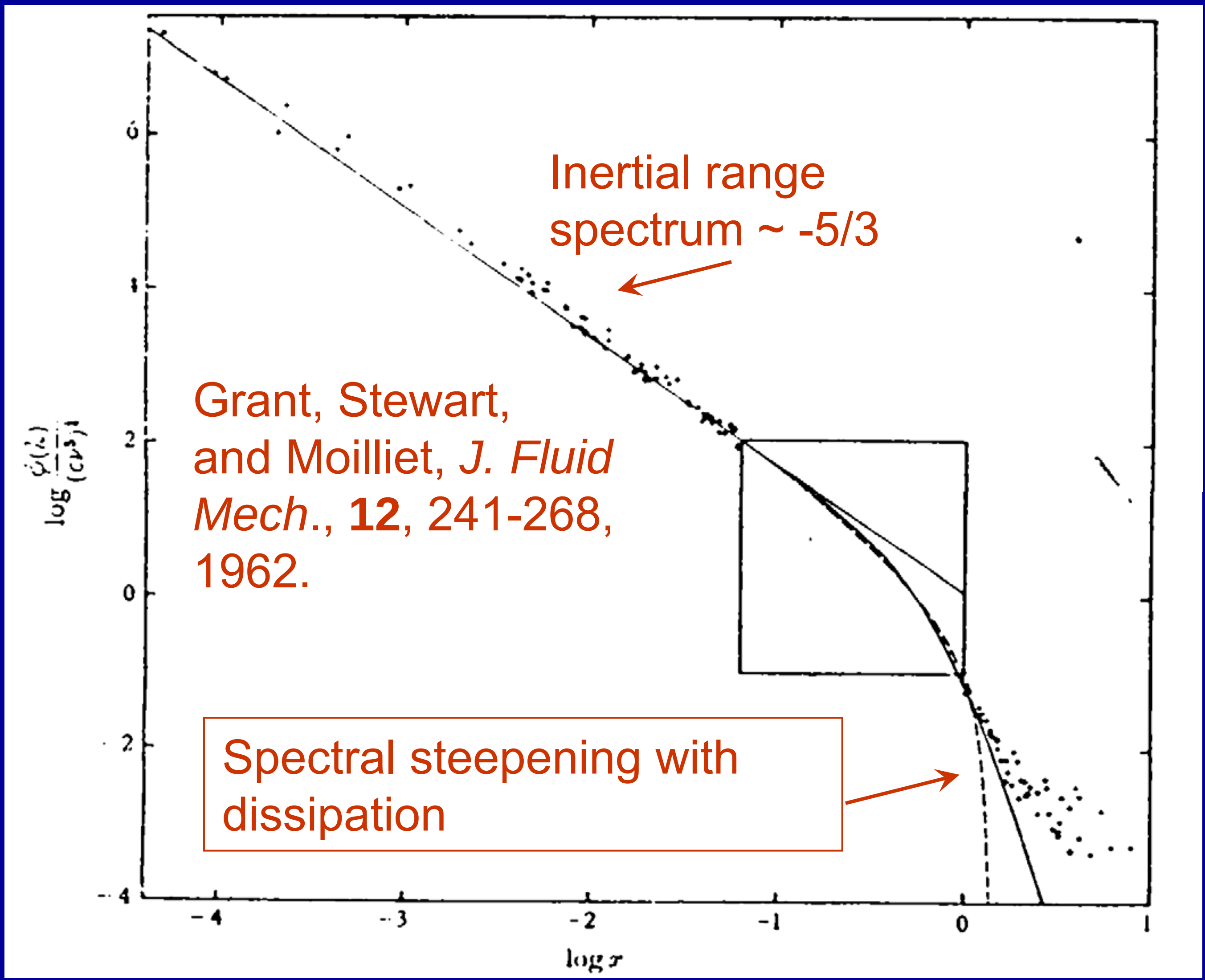
In this manner, the large-scale "structure" of the flow can heat the thermal particles of the fluid.

This occurs within the fluid description!

Does this apply to the solar wind and other space plasmas?

It appears that dissipation in the solar wind occurs *outside* the fluid description, which complicates and changes the problem.

Verification of Kolmogorov Prediction

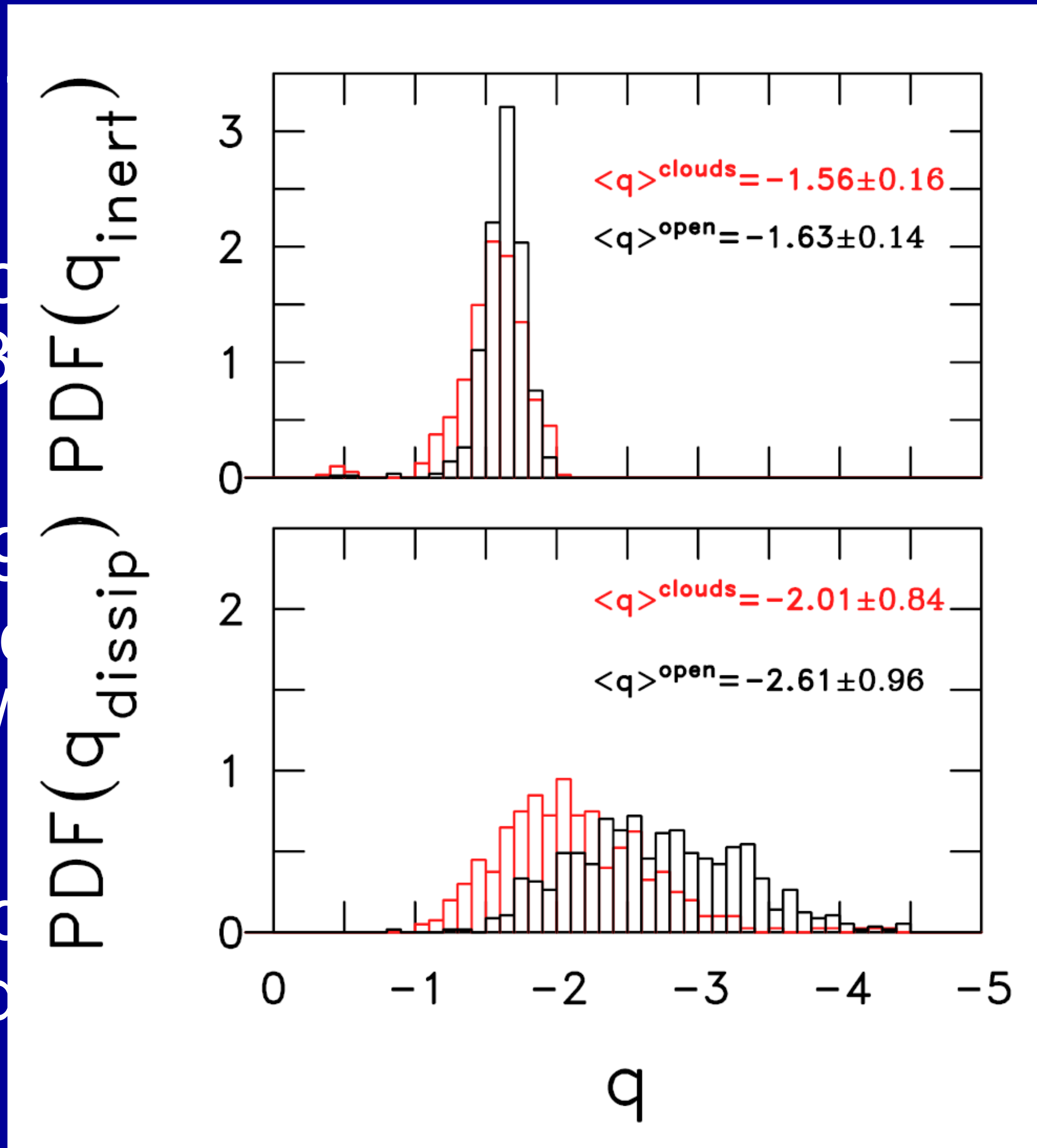


Predictions

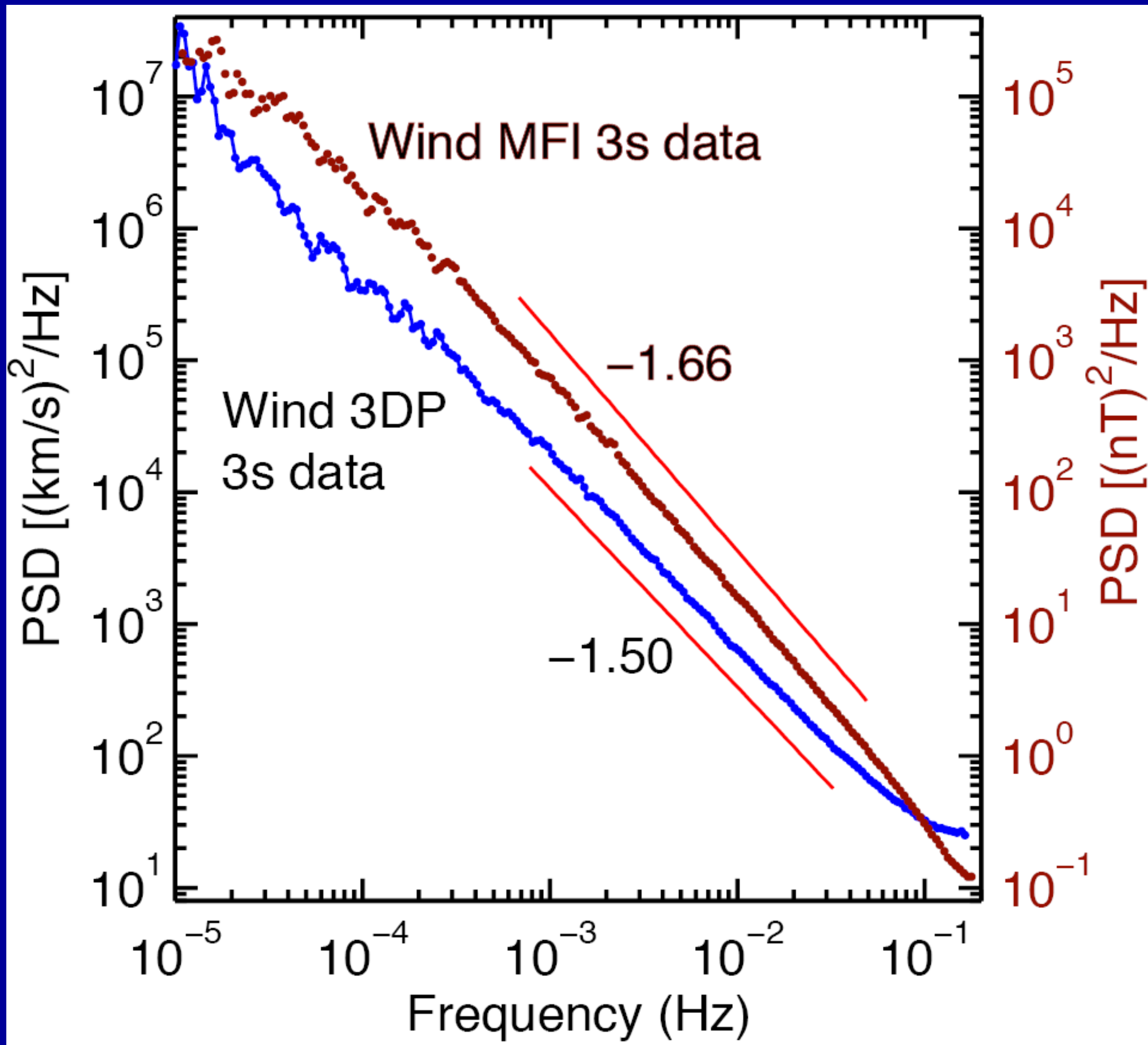
- Kolmogorov c
predict a –5/3

- Kraichnan (19
propagation t
predicts a –3/

- Goldreich and
power law ind

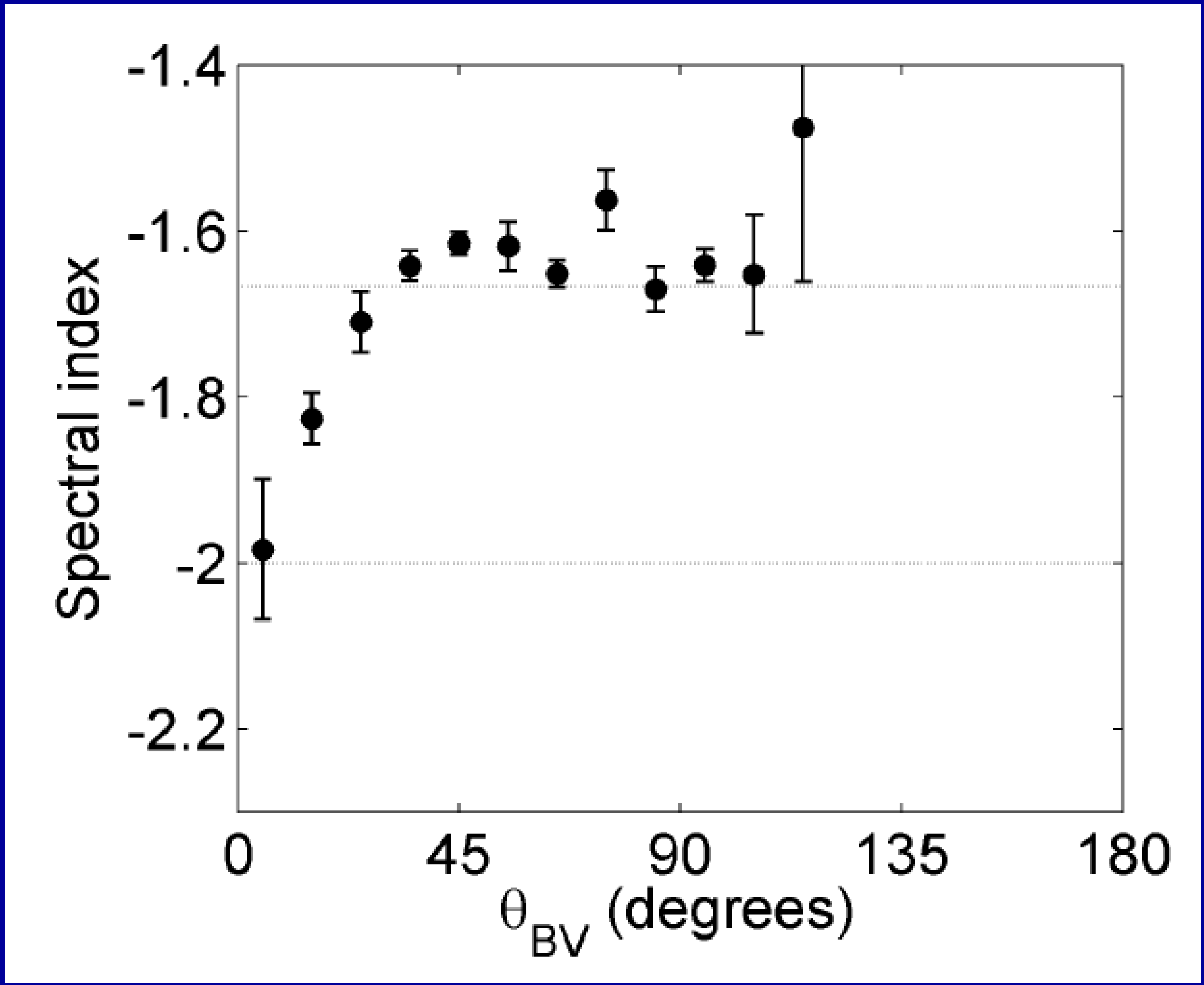


V and B Spectra are Different



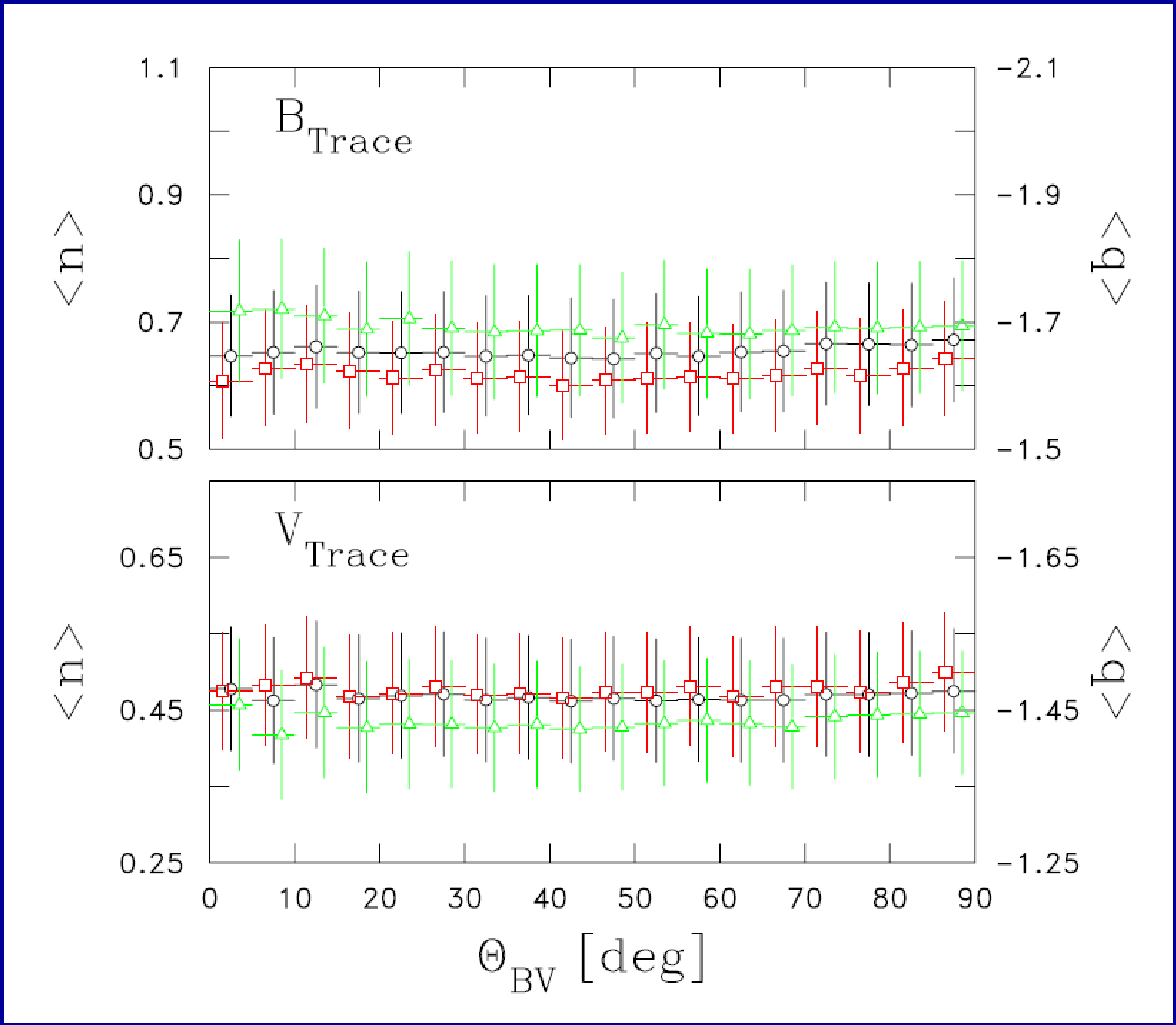
Podesta et al., *J. Geophys. Res.*, **111**, A10109, 2006.

High-Latitude Θ_{BV} Result



Horbury et al., unpublished.

10 Years of ACE Observations



Tessein et al., unpublished.

Do the concepts of hydrodynamic turbulence apply to MHD?

-- How do we apply and extend them?

Does energy move about ergodically in MHD?

-- Or, is the mean field a stabilizing influence?

Are there reproducible spectral predictions?

-- And do they resemble observations?

Are the isotropic theories of hydrodynamics appropriate?

-- If not, then what?

Is there a predictable rate of energy dissipation?

-- And does it agree with observations?

Is this an important process, or solution to one problem?

-- Will the same physics accelerate the solar wind?

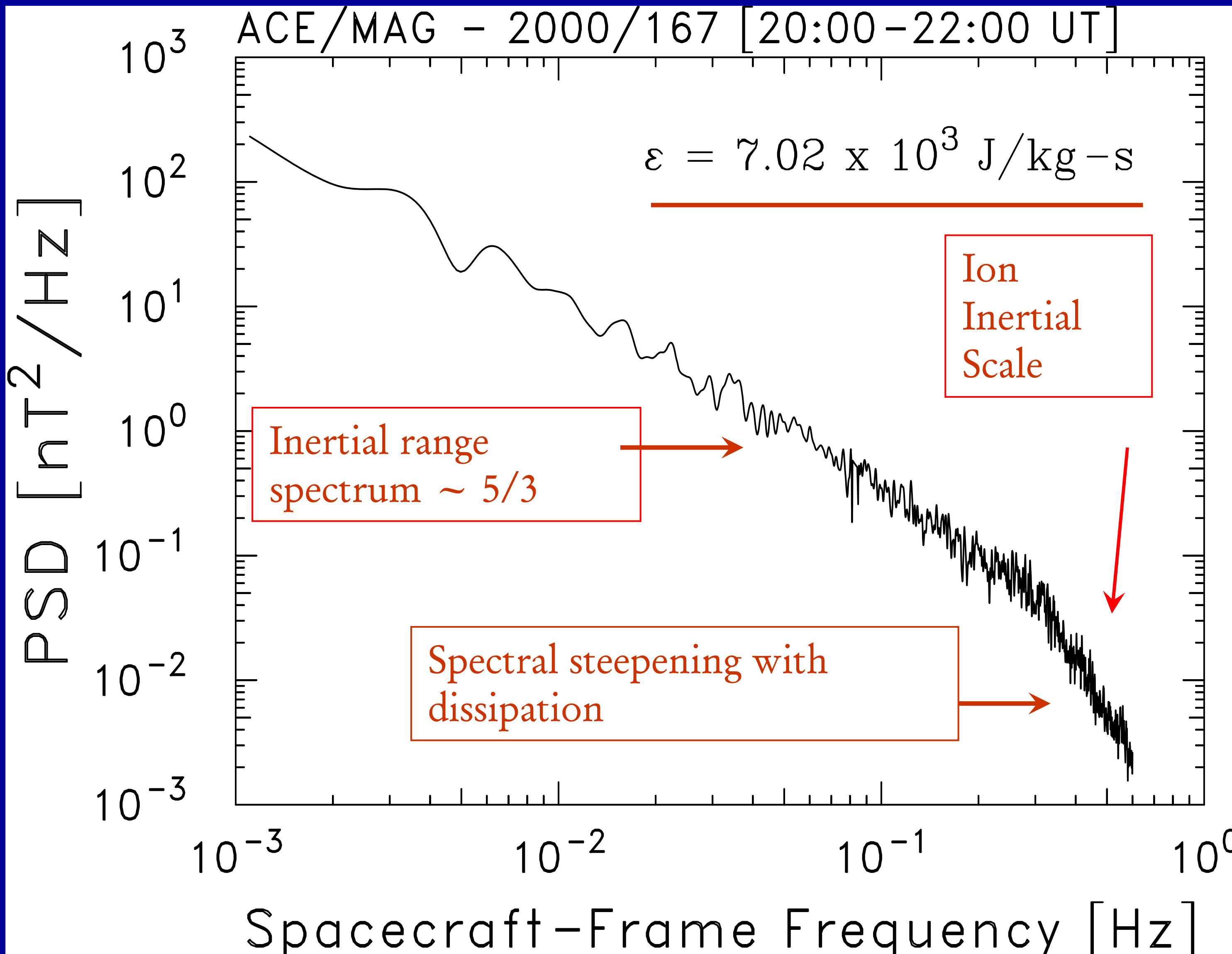
A Model for Interplanetary Turbulence

- Large-scale disturbances (shocks, ejecta, heliospheric current sheets, stream interactions) provide energy to drive the turbulent cascade.

- Intermediate-scale fluctuations form an inertial range to transport energy to the smallest scales.

- The small-scale fluctuations form a dissipation range where the (single) fluid approximation breaks down and energy is dissipated into heat.

Interplanetary Magnetic Spectrum



$C_K \varepsilon^{2/3} k^{-5/3}$

here

≈ 1.6

f^{-3}

"dissipation range"

$$\frac{dZ^2}{dr} = -\frac{A'}{r} Z^2 - \frac{\alpha}{U} \frac{Z^3}{\lambda} + \frac{E_{PI}}{U}$$

$$\frac{d\lambda}{dr} = -\frac{C'}{r} \lambda + \frac{\beta}{U} Z - \frac{\beta}{U} \frac{\lambda}{Z^2} E_{PI}$$

$$\frac{dT}{dr} = -\frac{4}{3} \frac{T}{r} + \frac{1}{3} \frac{m_p}{k_B} \frac{\alpha}{U} \frac{Z^3}{\lambda}$$

Test of the Matthaeus/Isenberg Merger

Predicted wave energy is good.

Predicted T_P is good until ~ 30 AU, but then prediction runs high.

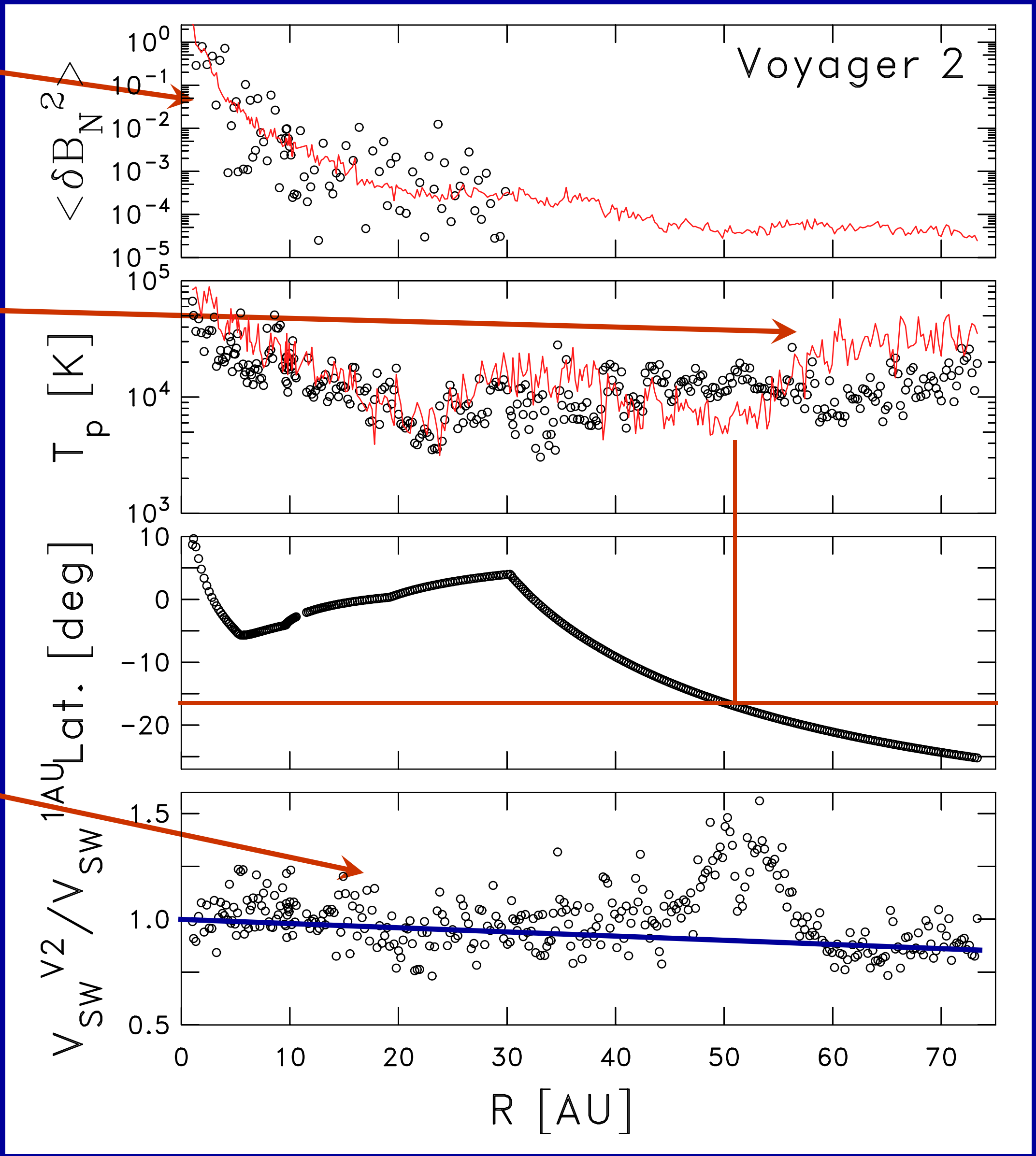
Clearly T_P too high beyond 55 AU.

Neutral ion density at ∞ is 0.1 cm^{-3}.

Slowing of solar wind can be used to obtain estimate for neutral ion density.

Wang and Richardson get 0.09 and 0.05 in two papers.

Smith et al, *Astrophys. J.*, **638**, 508-517, 2006.



Large-Scale Wave Vector Anisotropy

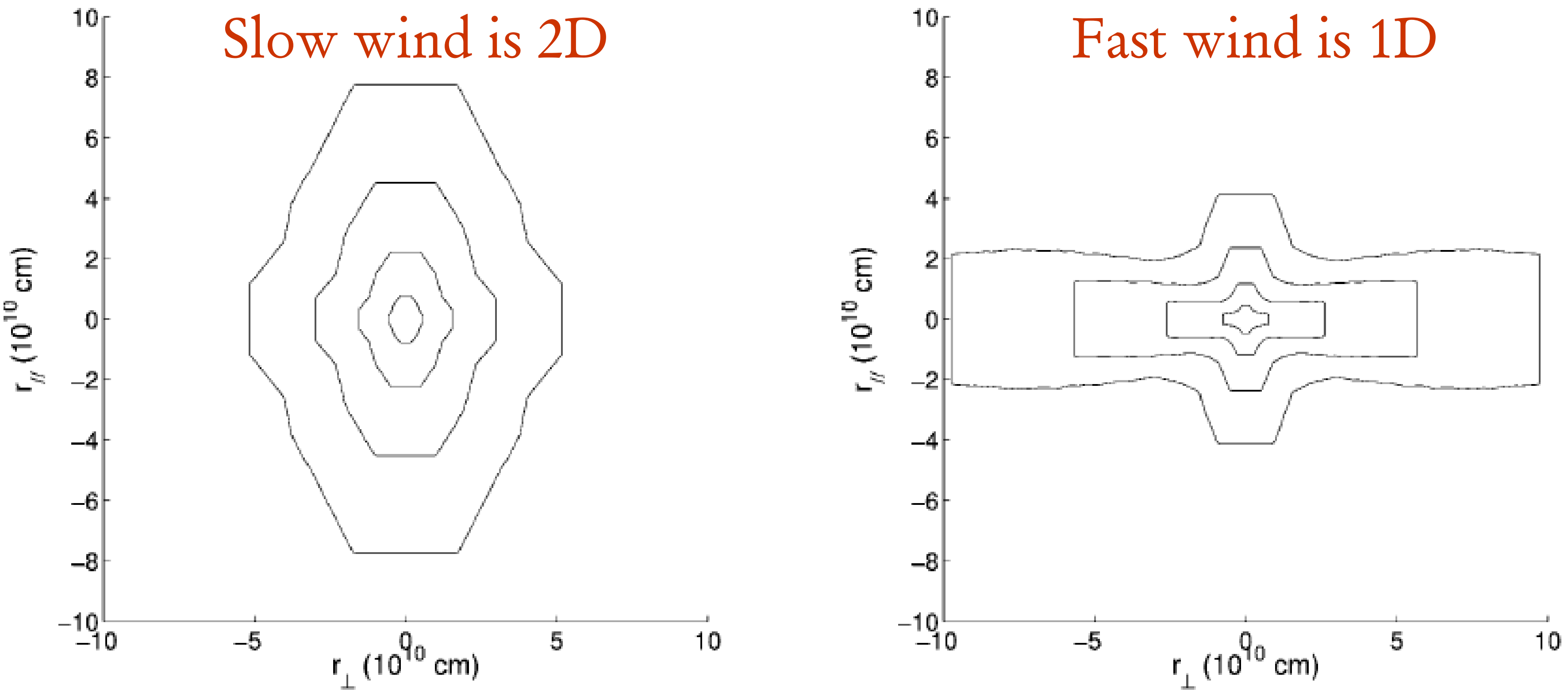
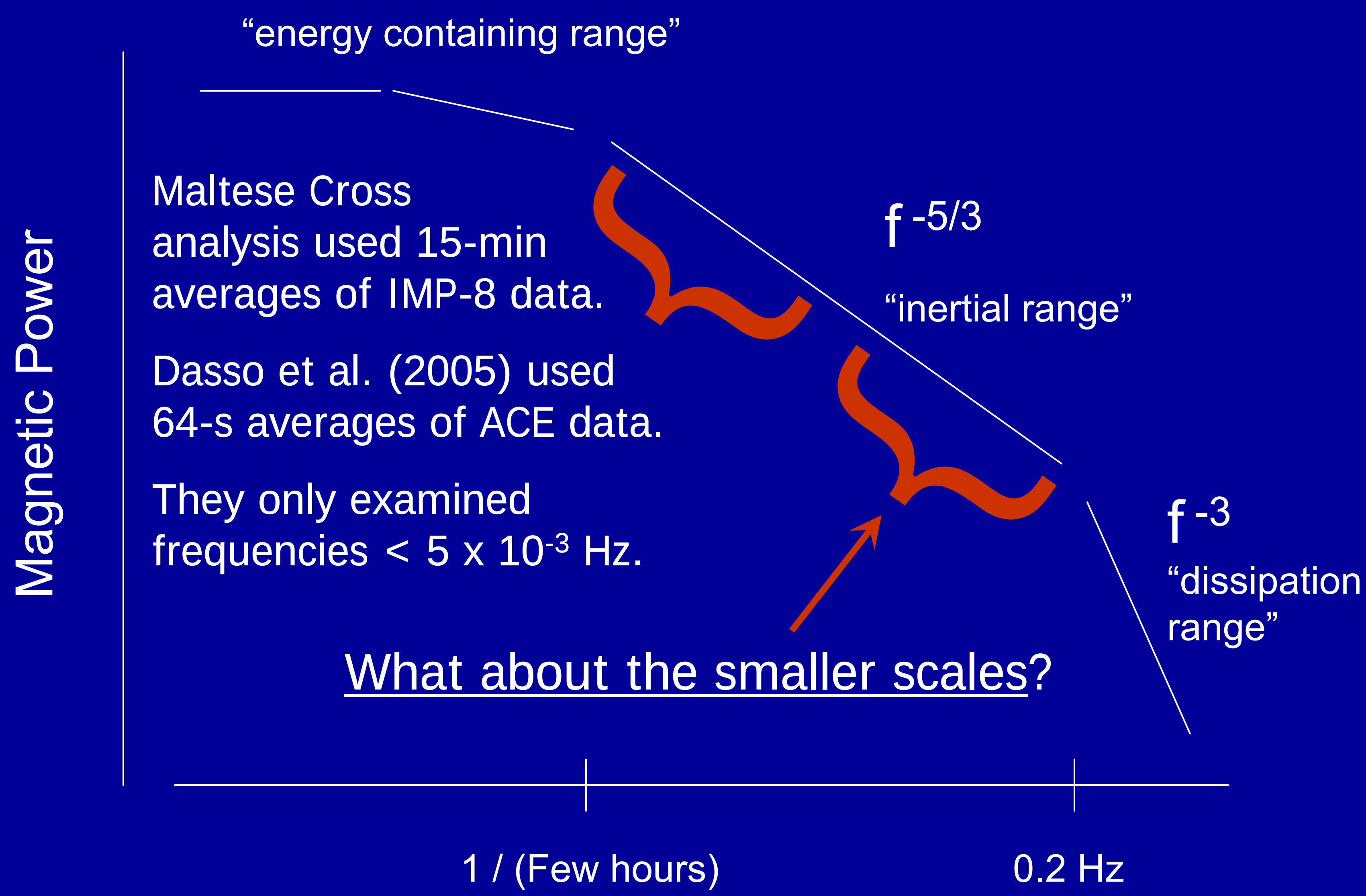


FIG. 1.—Level contours for $R_{bb}(r)$. *Left*, slow solar wind (V_{sw} < 400 km s^{-1}); *right*, fast solar wind (V_{sw} > 500 km s^{-1}). (See text.) Levels are at 1200, 1400, 1600, and 1800 km^2 s^{-2}.

Dasso et al., *ApJ*, **635**, L181-184, 2005.

Kolmogorov Spectrum of Interplanetary Fluctuations



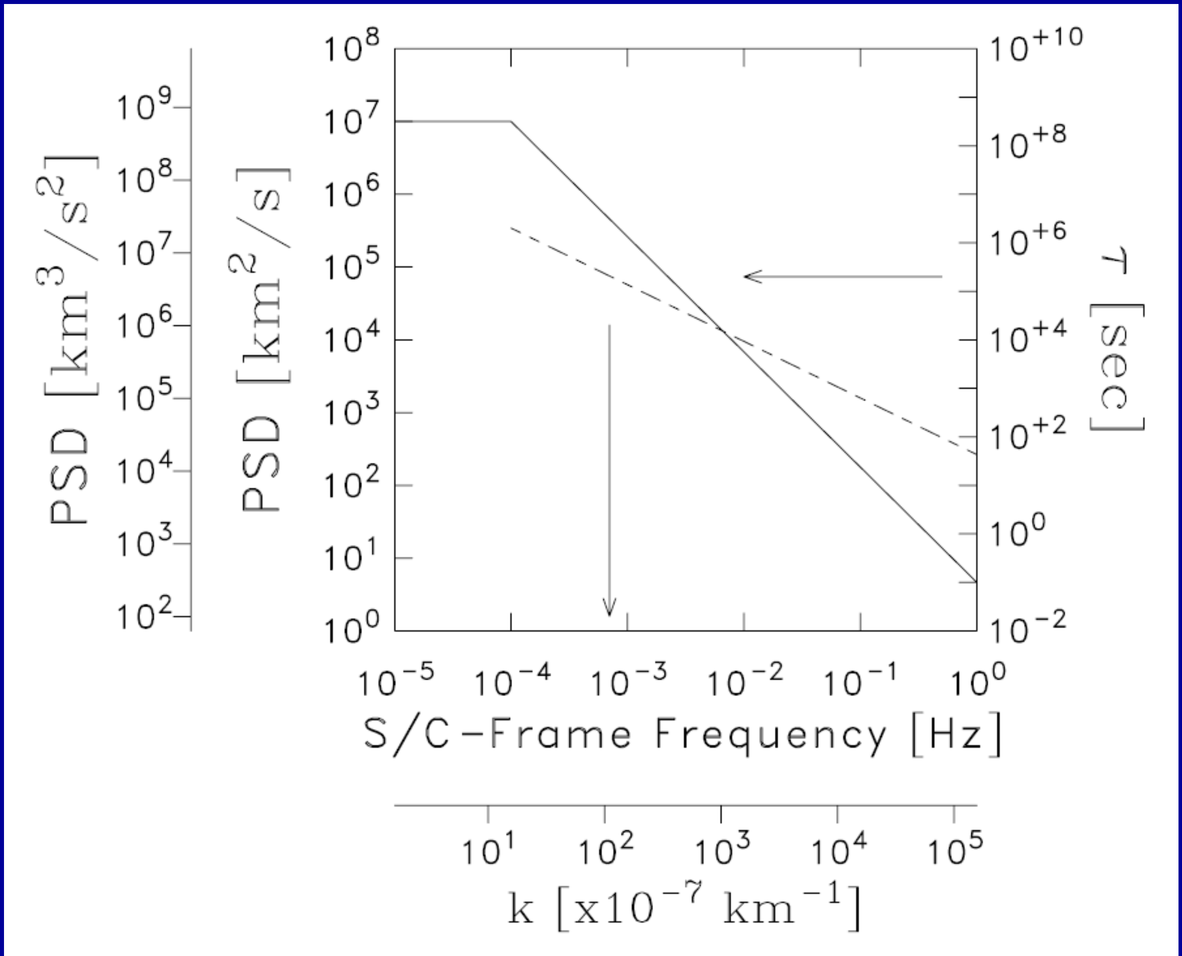
Eddy Lifetime

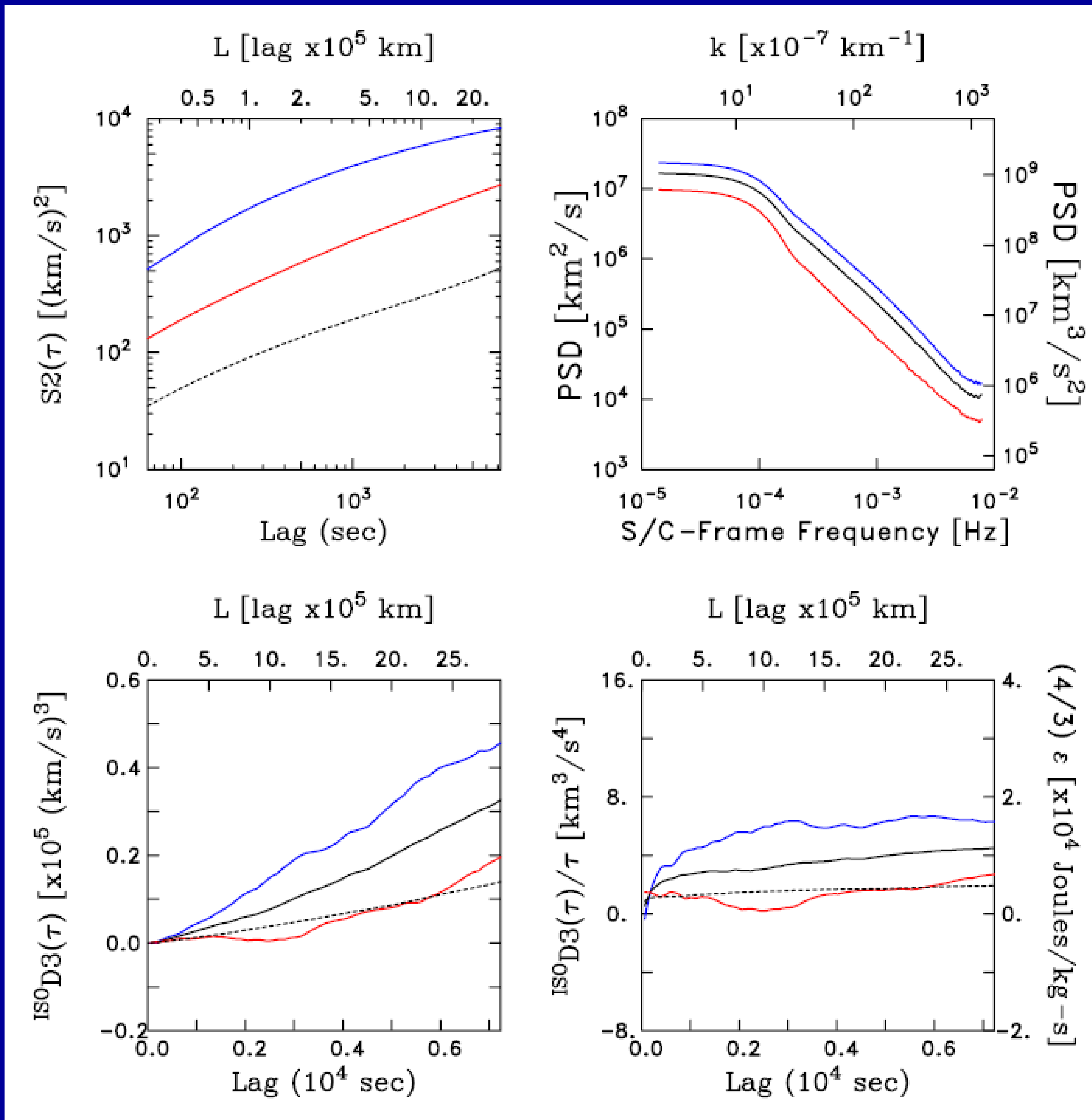
A turbulent eddy gives up its energy in ~1 turnover time.

$E_L \sim \delta V^2 + \delta B^2$

$\tau \sim L / \delta V$

f_{sc} > ~1 mHz have shorter lifetimes than the transit time to 1 AU and are generated in situ!





We can apply these ideas to the solar wind (ACE) assuming various geometries.

For now, let's assume isotropy:

We find more power in outward propagating fluctuations.

The energy cascades at:
$\varepsilon = 6 \times 10^3$ J/kg-s
and outward propagation cascades more aggressively.

The Dissipation Range (Really New Territory)

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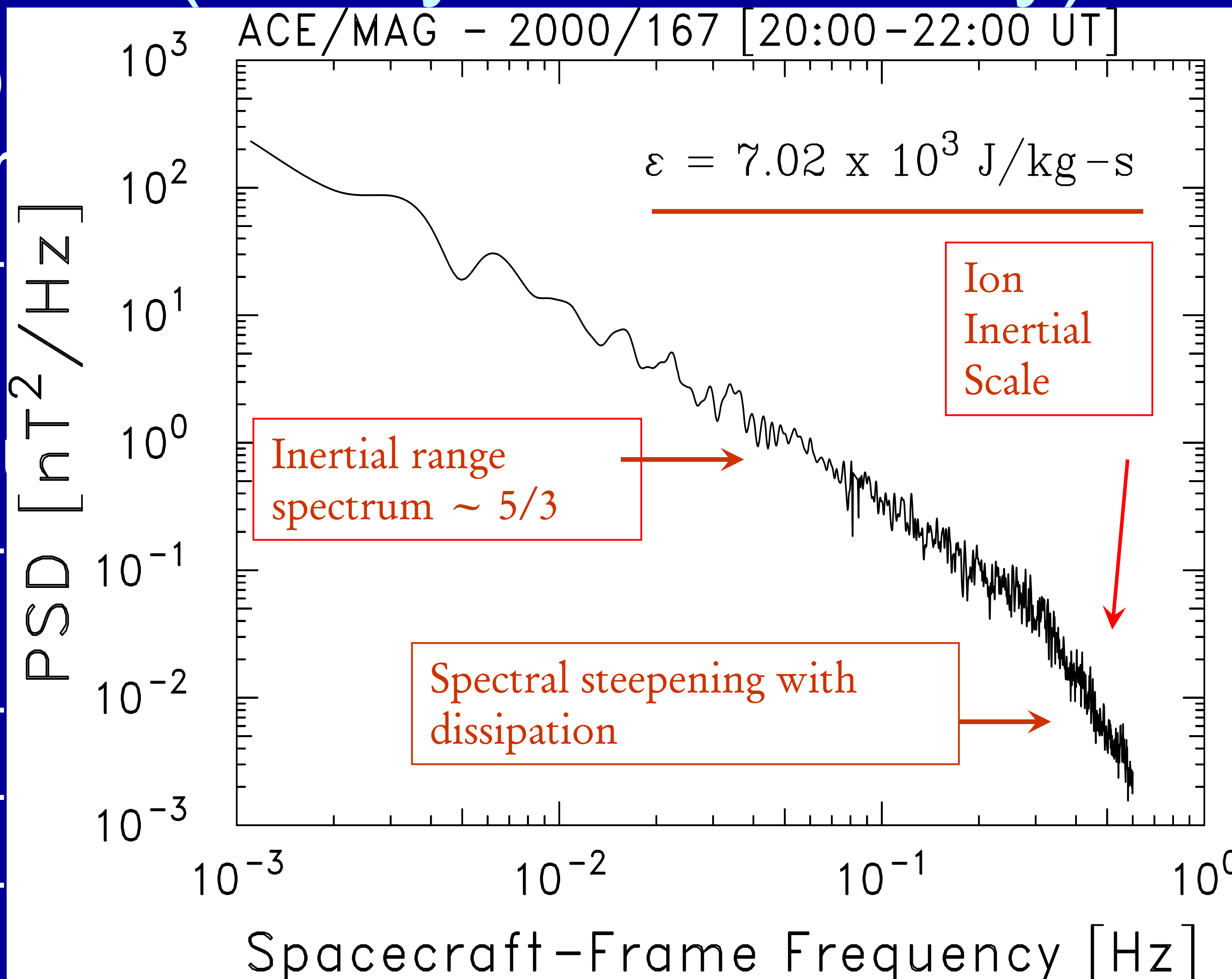


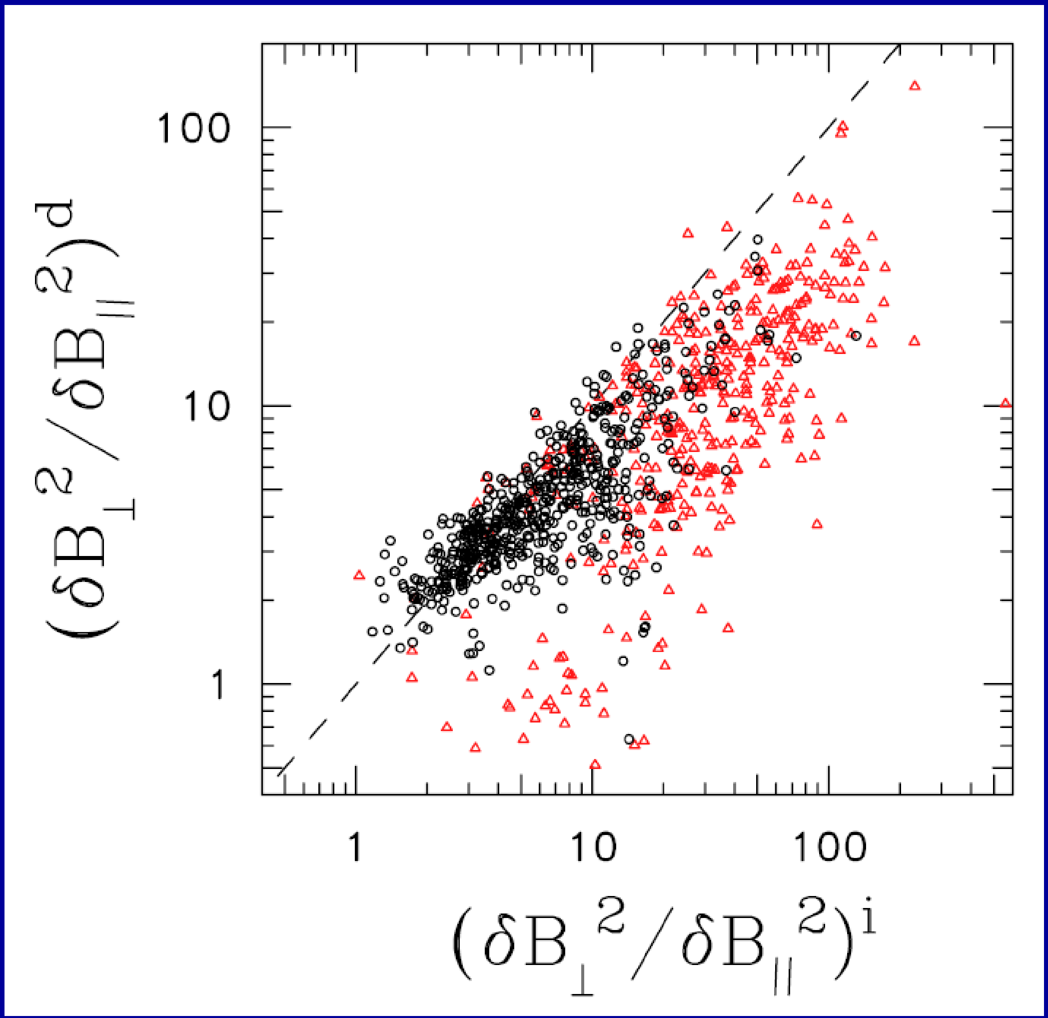
Dissipation Range Characteristics

- Less 2D than in the inertial range.
  - Same for all wind conditions.

- Cyclotron damping provides ~ ½ the damping.
  - Mild polarization, but not total.
  - Other ½ does not depend on polarization.

- Range of spectral values.
  - Steepness from f^{-2} to f^{-5}.
    (Smith et al., *ApJ Lett*., **645**, L85-L88, 2006).
  - Extends beyond 200 Hz.
    (Denskat et al., *J. Geophys*., **54**, 60-67, 1983).

Dissipation Range Fluctuations are More Compressive than the Inertial Range



Hamilton et al., JGR, submitted, 2007.

In mean field coordinates we can compute the variance anisotropy of the magnetic fluctuations in the inertial and dissipation ranges.

The anisotropy in the inertial range is consistently greater than in the dissipation range.

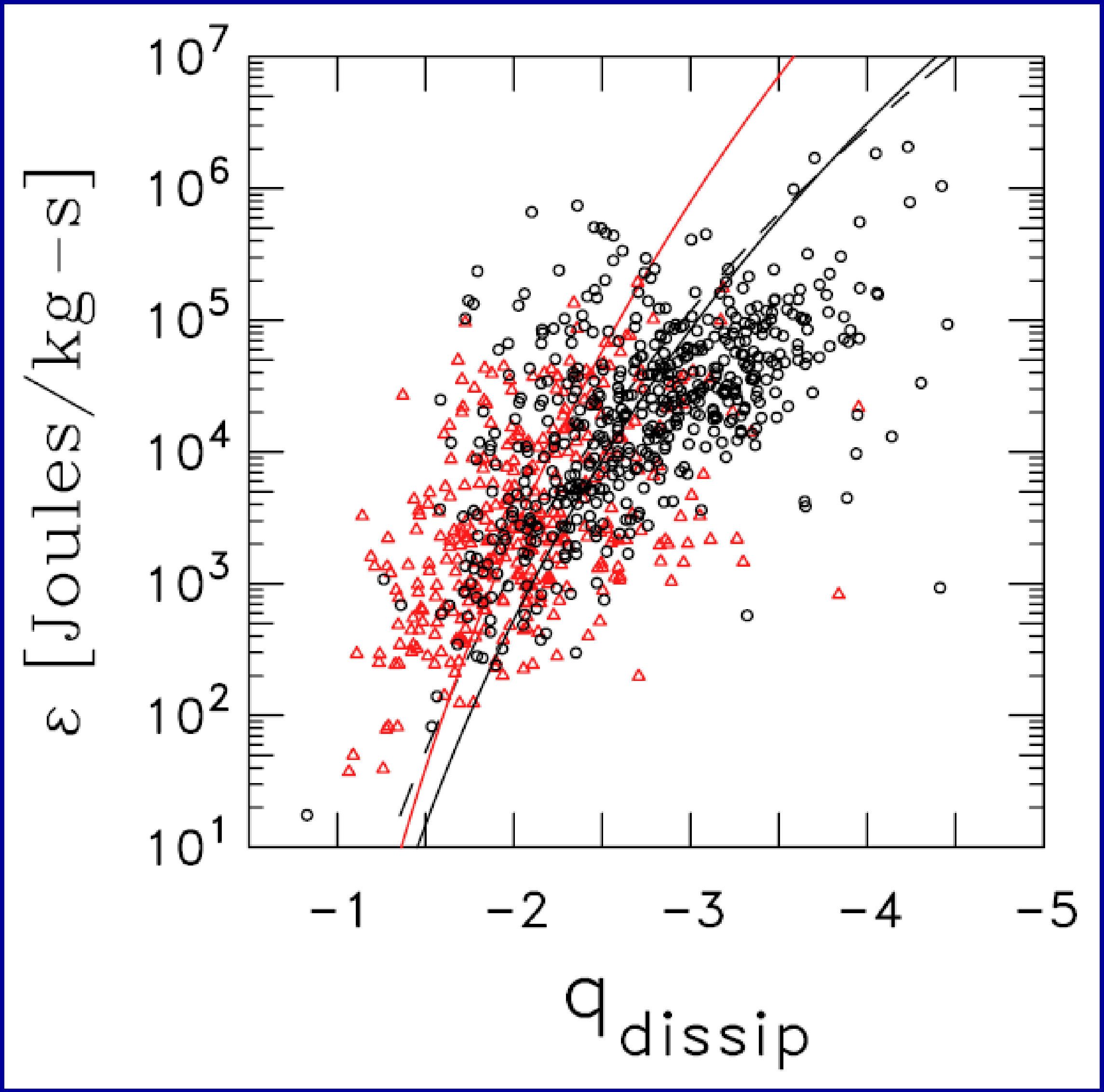
This means the dissipation range is more compressive since magnetic fluctuations parallel to the mean field are correlated with density fluctuations.

Dissipation Range Scales with ε

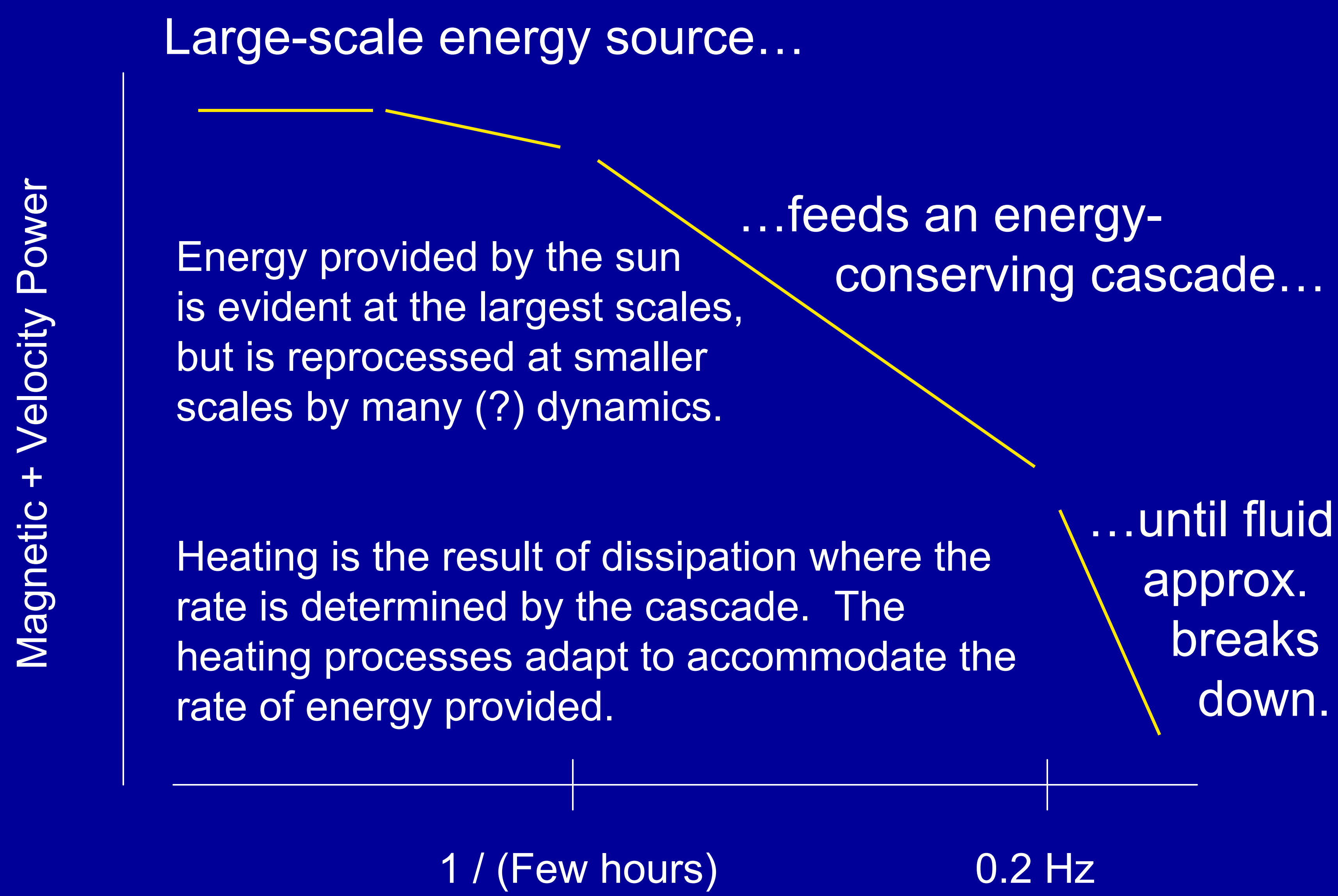
The higher the inertial range spectrum, the greater the cascade rate, the steeper the dissipation range spectrum will be.

This seems to say that the harder you “stir” the fluid, the more aggressively you dissipate the energy.

From Smith et al., ApJL, 645, L85--L88, 2006.



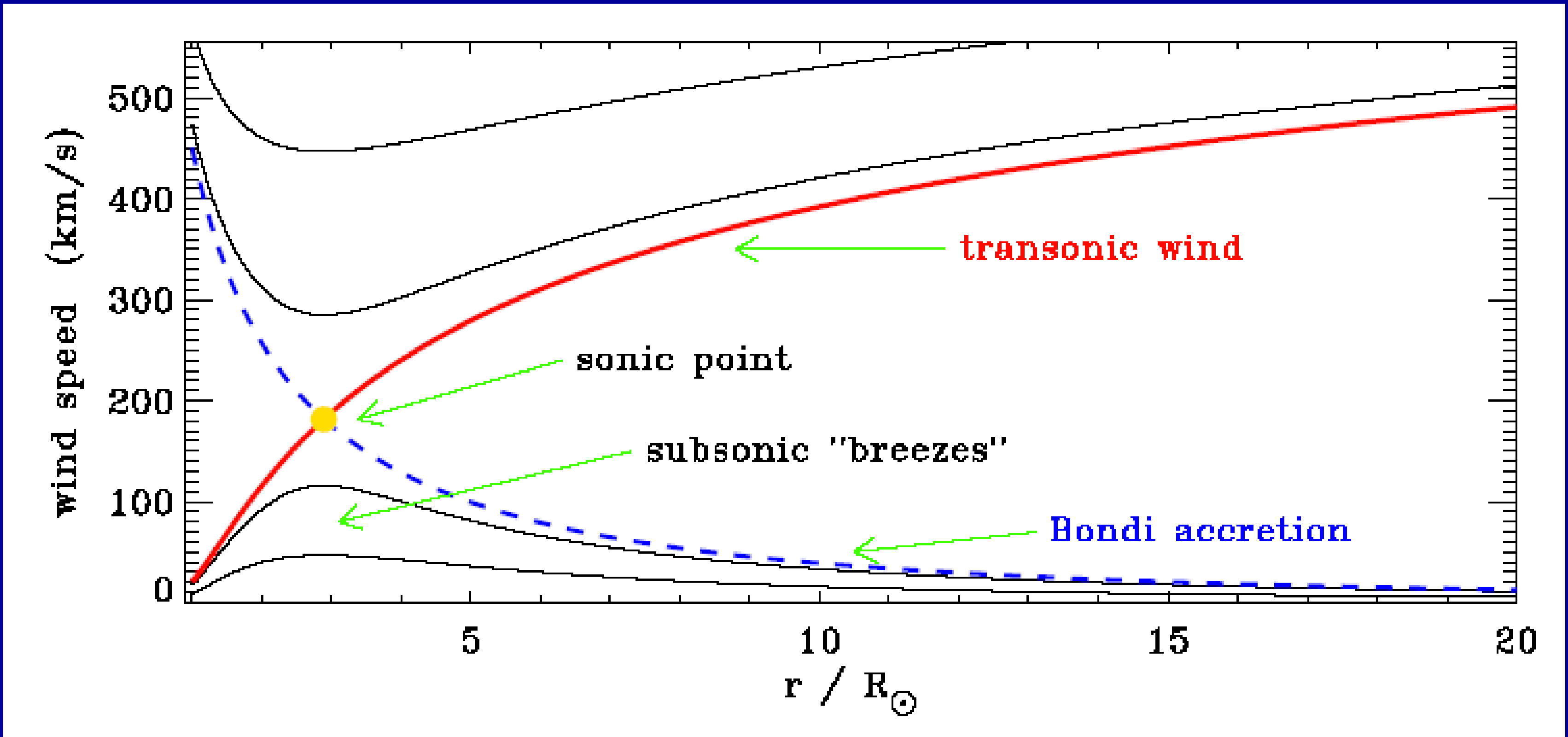
Summary



Large-scale energy source…
…feeds an energy-conserving cascade…
…until fluid approx. breaks down.
Energy provided by the sun is evident at the largest scales, but is reprocessed at smaller scales by many (?) dynamics.
Heating is the result of dissipation where the rate is determined by the cascade. The heating processes adapt to accommodate the rate of energy provided.
Magnetic + Velocity Power
1 / (Few hours)
0.2 Hz

Extra Slides

Solar Wind Flow Near Sun



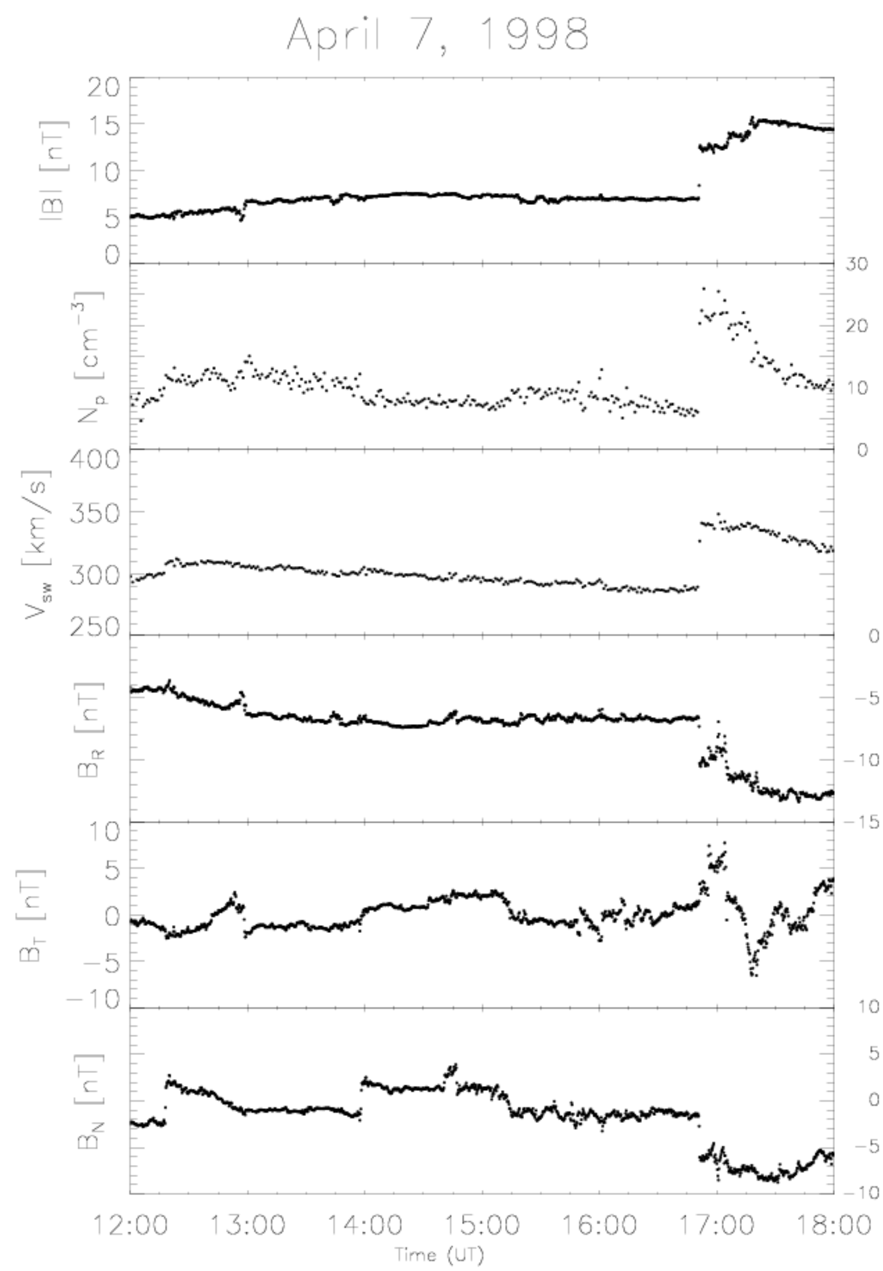
Interplanetary Shocks
(the real particle accelerators)

When fast-moving wind overtakes slower wind with a relative speed greater than the sound (Alfven) speeds, a shock is formed.

This is similar to the shock in front of a supersonic plane, except it is also magnetic.

The fluctuations associated with the density jump reflects energetic particles and pushes them to higher energies.

This produces a population of energetic charged particles in association with the shock.



Temperature Gradients R < 1 AU

In the range 0.3 < R < 1.0 AU, Helios observations demonstrate the following:

For $V_{SW} < 300$ km/s, $T \sim R^{-1.3 \pm 0.13}$

$300 < V_{SW} < 400$ km/s, $T \sim R^{-1.2 \pm 0.09}$

$400 < V_{SW} < 500$ km/s, $T \sim R^{-1.0 \pm 0.10}$

$500 < V_{SW} < 600$ km/s, $T \sim R^{-0.8 \pm 0.10}$

$600 < V_{SW} < 700$ km/s, $T \sim R^{-0.8 \pm 0.09}$

$700 < V_{SW} < 800$ km/s, $T \sim R^{-0.8 \pm 0.17}$

Adiabatic expansion yields $T \sim R^{-4/3}$.

Low speed wind expands without in situ heating!?

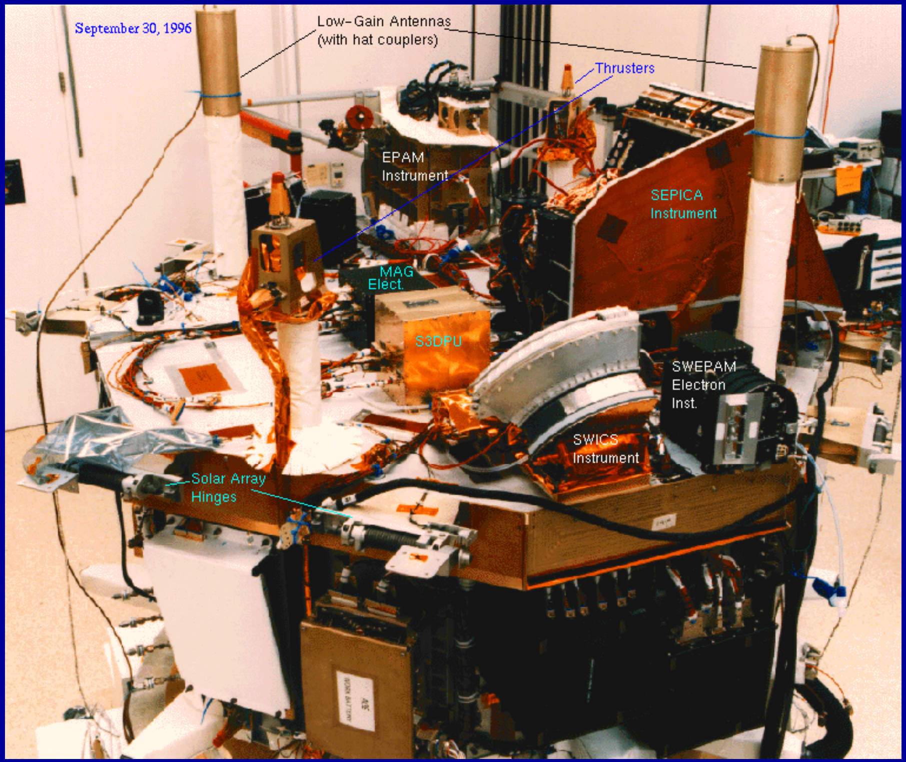
High speed wind is heated as it expands.

Why is this? It seems distinct from the high-latitude observations.

We can look to explain this through theory and link it to observations of the dissipation range and inferred spectral cascade rates.

CME Interacting with Earth

MAG Delivery on ACE



The Magnetic Field Experiment (MAG) measures the weak magnetic fields of interplanetary space, providing necessary information to interpret the thermal and charged particle measurements along with understanding the magnetospheric response to transient events.

Why Study Turbulence

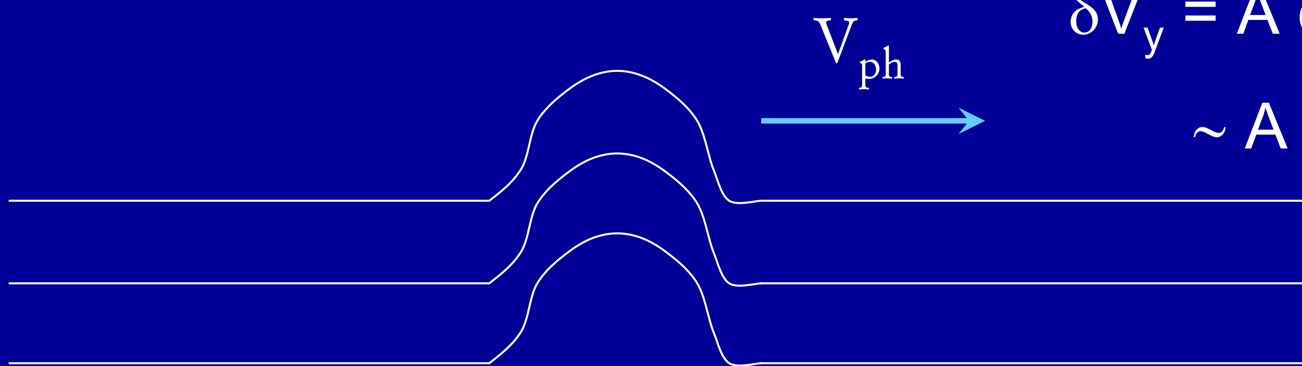
- Turbulence is just about the most fundamental and most ubiquitous physics on Earth.
  - Seen in every <u>naturally occurring</u> fluid that is disturbed
  - Responsible for atmospheric weather
  - Reason refrigerators work (and heaters)
  - Reason why planes can fly

- Has become a code-word for “disturbance”, “complexity”, and “nonlinearity”, but it is much more.

- Is the process by which a fluid (or gas) attempts to self-organize its energy.

Alfven Waves

(One Paradigm for Interplanetary Fluctuations)



$$\delta V_y = A\, e^{i(k \cdot x - \omega t)}$$

$$\sim A \cos(k \cdot x - \omega t)$$

Magnetic field lines "hold" charged particles.

Can be placed in "tension" (have energy associated with configuration and a rest state).

A "disturbance" or perturbation attempts to return to rest, but mass loading gives inertia.

So a magnetic field line with charged particles is analogous to a guitar string – once plucked it oscillates with a characteristic frequency and a wave propagates.

The Universe Creates Disorder

- It’s a thermodynamic law!
  - Whatever order is created, there are physical processes seeking to destroy it.
- A wave will not propagate forever.
  - It will be damped away to heat the fluid.
- Or, it will spawn other fluctuations, and they will spawn others, and in time dissipation will win.
  - This is turbulence in the traditional view.

Motivation

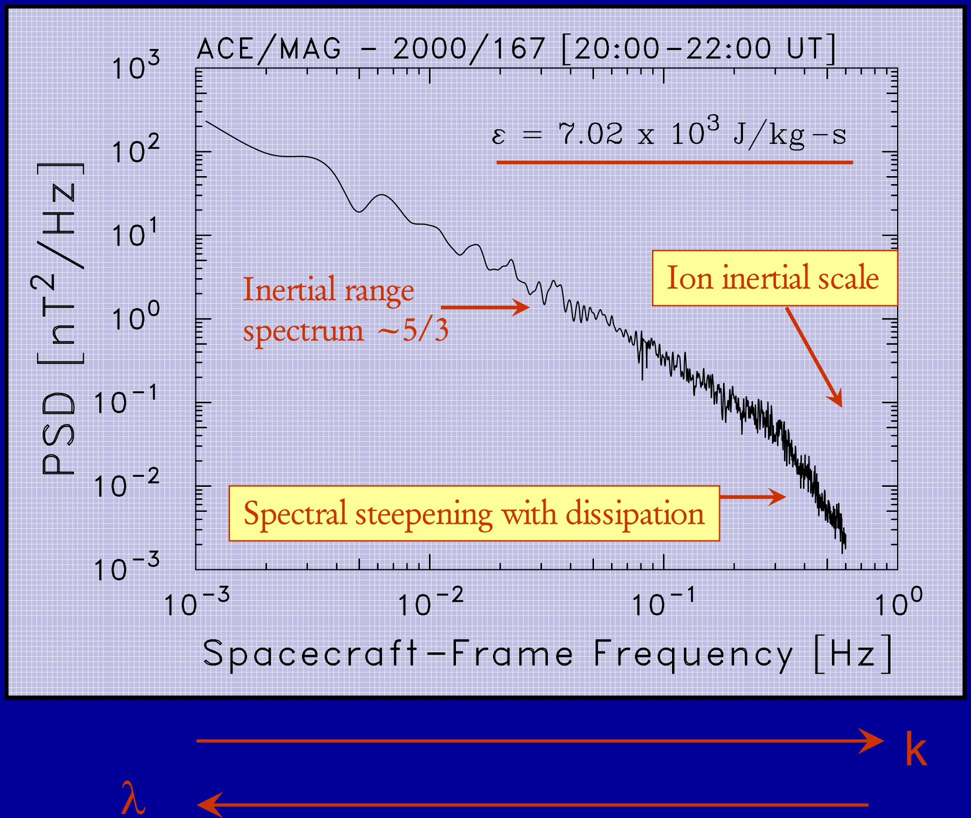
- Solar wind fluctuations are observed to exhibit an $f^{-5/3}$ spectral form in the range from a few hours to a few seconds.

Figure (right) shows the magnetic power spectrum over 2+ decades in frequency and steepening to form the dissipation range.

The -5/3 spectrum extends down to about 10^{-5} Hz.

Because the wind speed is so great, we believe that temporal measurements (Hz) are equivalent to spatial measurements (km^{-1}):

$\nu = \mathbf{k} \cdot \mathbf{V}_{sw}/2\pi$ and $k=2\pi/\lambda$.



Navier-Stokes Fourier Transformed

$$\rho_M\left(\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\bullet\nabla)\mathbf{V}\right)=-\nabla P+\nu\nabla^2\mathbf{V}$$

$$\nabla\bullet\mathbf{V}=0$$

Fourier Transform in x:

$$V_k=\left(\frac{1}{2\pi}\right)^3\int V(x)e^{ik\cdot x}dx \text{ and } V(x)=\sum V_k e^{-ik\cdot x}$$

$$\rho_M\left(\frac{\partial \mathbf{V_k}}{\partial t}+\sum_{\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2}\left(\mathbf{V}_{\mathbf{k}_1}\bullet\mathbf{k}_2\right)\mathbf{V}_{\mathbf{k}_2}\right)=\mathbf{k}P-\nu k^2\mathbf{V}$$

$$\mathbf{k}\bullet\mathbf{V_k}=0$$

Kolmogorov’s Theory

Turbulent interactions are local in wavenumber space.

Therefore, P(k) depends on k, but not on other k’s.

Interactions are energy-conserving.

Therefore, P(k) depends on ε (energy dissipation rate).

Simple dimensional analysis:

$$V^2\,dk \quad = \quad \varepsilon^\eta \quad k^q$$

$$[L/T]^2\,L = ([L/T]^2 / T)^\eta \quad [1/L]^q$$

L: $3 = 2\eta - q$

T: $-2 = -3\eta$ → $\eta = 2/3$ and $q = -5/3$

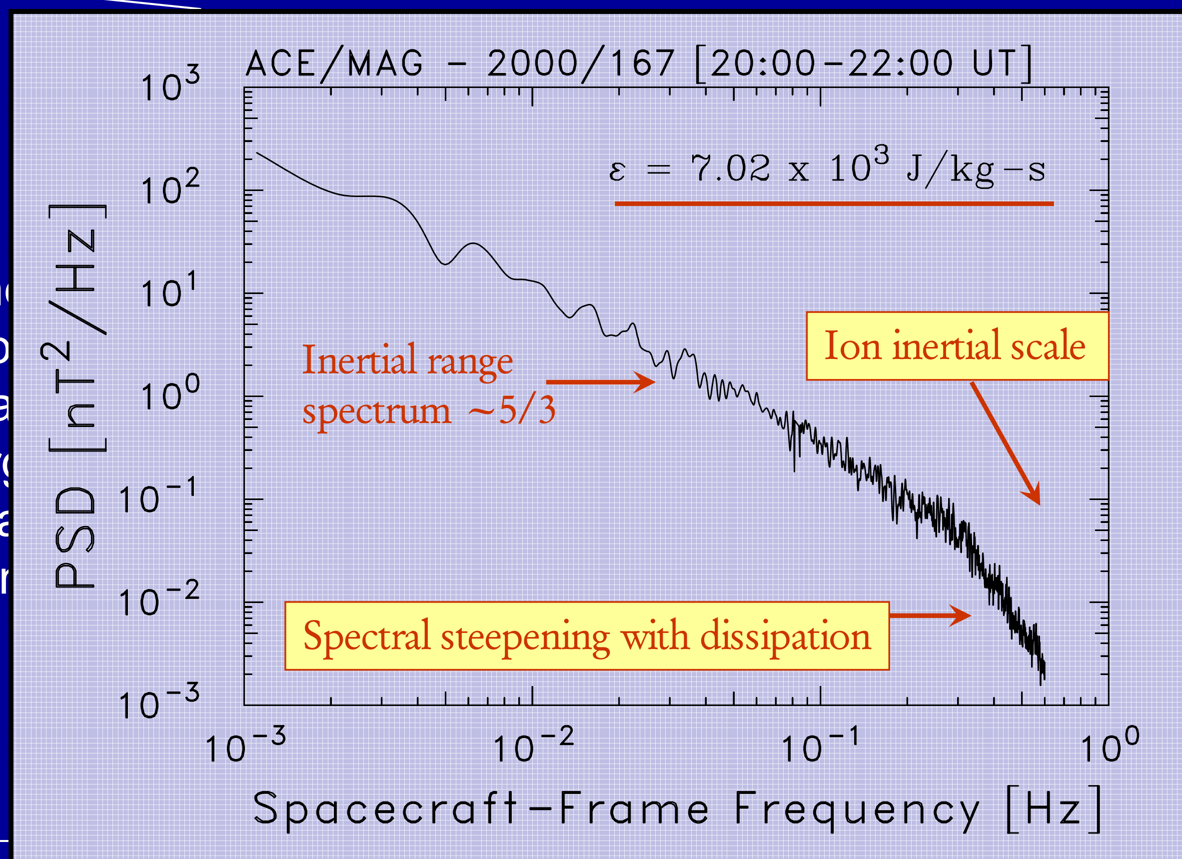
$$\boxed{P_k = C_K \varepsilon^{2/3} k^{-5/3} \quad \text{where} \quad C_K \approx 1.6}$$

Spectrum of Interplanetary Fluctuations

f^{-1} "energy containing range"

Magnetic Power

The
pip
ma
larg
sca
car



Few hours

0.5 Hz

Absolute Equilibrium Ensemble

Kolmogorov Spectrum of Interplanetary Fluctuations

To apply the Kolmogorov formula [Leamon et al. (1999)]:

1. Fit the measured spectrum to obtain "weight" for the result
   - Not all spectra are -5/3! I assume they are!
2. Use fit power at whatever frequency (I use $\sim$10 mHz)
3. Convert $P(f) \rightarrow P(k)$ using V_{SW}
4. Convert $\delta B^2 \rightarrow \delta V^2$ using V_A via ($\delta V^2 = \delta B^2/4\pi\rho$)
5. Allow for unmeasured velocity spectrum ($R_A = ½$)
6. Convert 1-D unidirectional spectrum into omnidirectional spectrum

$$\varepsilon = (2\pi/V_{SW}) \left[(1+R_A)\,(5/3)\,P_f^B\,(V_A/B_0)^2 / C_K\right]^{3/2} f^{5/2}$$

$$P_k = C_K \varepsilon^{2/3} k^{-5/3}$$

where

$$C_K \approx 1.6$$