PLANETARY DYNAMOS: PS SOLUTIONS

$\boxed{1}$ (a) $2\rho \underline{\Omega} \times \underline{u} \approx \underline{J} \times \underline{B}$

$\Rightarrow \quad 2\rho \Omega U \sim \frac{1}{\mu_0} B^2/L$ (used Ampere's Law).

$\Rightarrow \quad B \sim \left(2\rho \Omega \mu_0 U L\right)^{1/2}$

(b) using $Re_M = \sigma \mu_0 U L \Rightarrow$

$$B \sim \left(2\rho \Omega Re_M/\sigma\right)^{1/2}$$

(c) $Re_M = 1000$

$\sigma = 10^6$ S/m

$\rho = 10^4$ Kg/m^3

Earth: $\Omega = 7.3 \times 10^{-5}\ s^{-1}$, Mercury: $1.2 \times 10^{-6}\ s^{-1}$

$B_{earth} \sim \left(2 \cdot 10^4 \cdot 7.3 \cdot 10^{-5} \cdot 10^3/10^6\right)^{1/2}$
$\sim \left(1.5 \times 10^{-3}\right)^{1/2}$
$\sim 4 \times 10^{-2}$ T

$B_{Mercury} \sim \left(2 \cdot 10^4 \cdot 1.2 \times 10^{-6} \cdot 10^3/10^6\right)^{1/2}$
$\sim \left(2.4 \times 10^{-5}\right)^{1/2}$
$\sim 5 \times 10^{-3}$ T

(d) For Earth $\quad r_{CMB} \sim 0.5\, r_{surf}$

$\Rightarrow \quad B_{surf} \sim \left(\frac{r_{CMB}}{r_{surf}}\right)^3 B_{CMB}$
$\sim (0.5)^3 \cdot (4 \cdot \times 10^{-2})$
$\sim 5 \times 10^{-3}$ T
$= 5 \times 10^6$ nT

Actual $B_{surf} \sim 6 \times 10^4$ nT

For Mercury: $r_{CMB} \sim 0.75\, r_{surf}$

$$\Rightarrow B_{surf} \sim (0.75)^3 B_{CMB}$$
$$\sim (0.4)(5\times10^{-3})$$
$$\sim 2\times10^{-3}\ T$$
$$= 2\times10^{6}\ nT$$

actual $B_{surf} \sim 2\times10^{2}\, nT$

<u>Discussion:</u>

Earth: ~ 2 orders of magnitude off
Mercury: ~ 4 orders of magnitude off.

BUT: We didn't take into account the fact that not all of the B in the core reaches the surface.

B in the core is made up of 2 parts:

$$\underset{\rightarrow}{B} = \underset{\rightarrow}{B_T} + \underset{\rightarrow}{B_P}$$

toroidal B   poloidal B.

$$= \nabla\times T\hat{r} + \nabla\times\nabla\times P\hat{r}$$

- only B_P leaves the core.

- $B_P \sim 0.1\, B_T$ (this might gain an order of magnitude in our estimates. Maybe OK for Earth, Mercury still 3 o.o.M. off.

$\boxed{2}$ (a) $$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

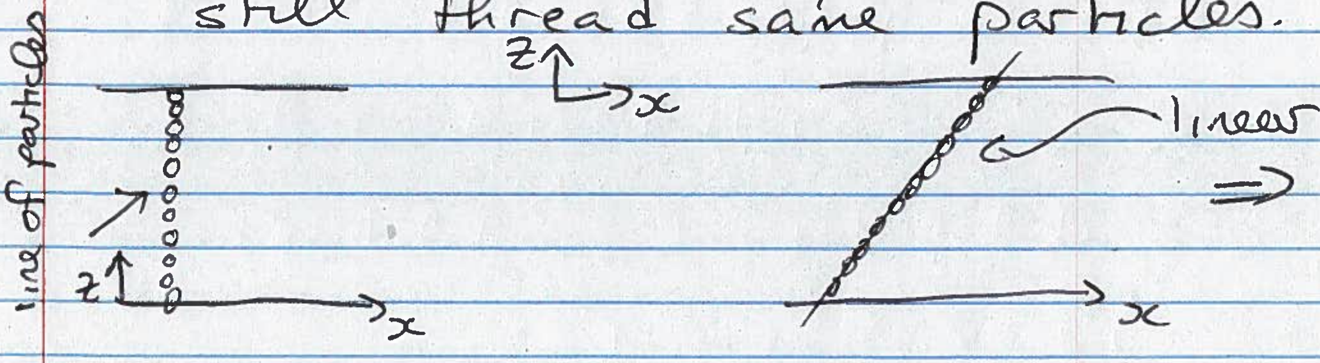
$$\frac{T}{\tau} \sim \alpha \frac{T}{L^2}$$

$$\Rightarrow \tau \sim \frac{L^2}{\alpha}$$

(b) $$\frac{\tau_1}{\tau_2} = \left(\frac{L_1}{L_2}\right)^2 = \left(\frac{10^2}{10^3}\right)^2 = 10^{-2}$$

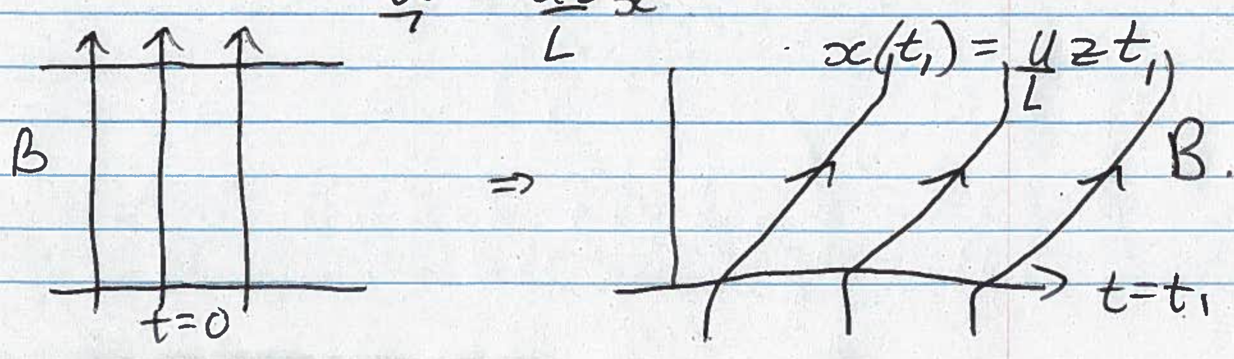
$\Rightarrow$ 100 km body cools ~ 100x faster.

$\boxed{3}$ (a) Key point: If fluid is perfectly conducting, then magnetic fields frozen into fluid. So take a vertical profile of fluid parcels @ $t=0$ & see where they are @ a later time. B field must still thread same particles.



$t=0.$ $\xrightarrow{\quad}$ $t = t_1$

$$\underline{u} = \frac{uz}{L}\hat{x}$$



(b) $$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B})$$
$$= \nabla \times \left(\frac{u}{L} z \hat{x} \times B_0 \hat{z} \right)$$
$$= \nabla \times \left(-\frac{u}{L} z B_0 \hat{y} \right)$$
$$= \frac{uB_0}{L} \hat{x}$$

(c) $$\frac{\partial \underline{B}^i}{\partial t} = \frac{uB_0}{L} \hat{x}$$
$$\Rightarrow \underline{B}^i(t) = \frac{uB_0}{L} t \hat{x}$$

original field.

$$\Rightarrow \underline{B}_{tot}(t) = \frac{uB_0}{L} t \hat{x} + B_0 \hat{z}$$

(d) $$\nabla \times \left(+ \frac{uz}{L} \hat{x} \times \frac{uB_0 t}{L} \hat{x} \right)$$
$= 0$ $\Rightarrow$ no new field generation.

$$\Rightarrow \underline{B}_{tot}(t) = \frac{uB_0}{L} t \hat{x} + B_0 \hat{z}$$

(e) solution ↑ looks like (a) sketch.