Planetary Dynamos & Magnetospheres – Problem Answers

1 – What if…?

A. How would the magnetosphere change if you moved the Earth to 10 AU?
Solar wind density decreases by $1/r^2$ - factor of 100. Assume solar wind speed roughly constant. So the size of the magnetosphere increases by a factor or $(100)^{1/6} = 2.15$.

B. What would happen if you doubled Earth's dipole magnetic field strength? Keeping Earth at 1 AU.
Both the sizes of magnetosphere and fraction dominated by plasmasphere increases by $2^{1/6} = 1.12$ - only 12%

C. What would happen if Earth spun x3 faster? i.e.1 day = 8 hours?
Size of magnetosphere does not change (obviously, perhaps) but the fraction of the magnetosphere that rotates - the plasmasphere - increases by $3^{1/2} = 1.73$. This would make the magnetosphere nearly all plasmasphere – comparable to Saturn.

D. What would happen to the magnetosphere if Jupiter was moved to 0.1 AU?
Simplest analysis: solar wind density (and pressure) decrease by $(0.1/5)^2 = 1/2500$ so the size of the magnetosphere would decrease by $(2500)^{1/6} \sim 3.7$ to ~20 Rj.
BUT – the solar photon flux would increase by 2500 which would strongly heat and ionize the atmosphere. Furthermore, a planet that close to the Sun is likely tidally locked to the Sun which would mean slow rotation. Would magnetospheric motions then be driven by thermospheric/ionospheric winds?

2 - Order of Magnitude Estimates

A. Information is transferred along magnetic field lines by Alfven waves. Estimate the time an Alfven wave takes to "tell" the ionosphere that reconnection has happened on the dayside magnetopause. Do this for both Earth and either Mercury or Jupiter.
Note – "order of magnitude" means ±1 in the exponent, at most one significant figure – we are not designing a spacecraft mission to land on Mars here!

$$V_A\ (km/s) = 20\ B(nT) / [n(cm^{-3})\ A_i(amu)]^{1/2}$$

As B increases closer to the planet, so VA increases (the density also increases – but less so, and it comes in only as the square root). So, we will take the minimum B to get the maximum travel time

$$Tmp\text{-}ionos = Rmp / V_A$$

Rmp = distance from center of planet to the magnetopause (often in units of planetary radii)

	MERCURY	***EARTH***	***JUPITER***
Rmp	*1.4 R_M*	*10 R_E*	*80 R_J*
Bo	*300 nT*	*3 x 10^4 nT*	*4 x 10^5 nT*
Bo/Rmp^3	*100 nT*	*30 nT*	*1 nT (10 nT more realistic)*
n	*50 cm^{-3}*	*10 cm^{-3}*	*0.1 cm^{-3}*
Ai	*1 amu*	*1 amu*	*10 amu*
V_A	*300 km/s*	*200 km/s*	*200 km/s*
Tmp-ionos	*10 sec*	*300 sec*	*8 hours!*

SO – there's a HUGE range in timescales for these magnetospheres to respond to reconnection at the magnetopause.

Note – this is the time for the ionosphere in the polar cap to "perceive" that reconnection has occurred. Some might think of this as the time for the E=-Vsw x Bsw to propagate down to the ionosphere – or for stresses imposed by the solar wind to be imposed on the ionosphere.

B. A fluxtube reconnects on the dayside magnetopause. Estimate the time it takes the end of the fluxtube that is in the magnetosheath to travel from the dayside magnetopause to the distance of the X-line in the magnetotail – say, 5 R_M for Mercury, 100 R_E for Earth, 200 R_J for Jupiter.
Note that the fluxtube is moving at the magnetosheath speed. By the time the flow has moved from the reconnection point near the sub-solar point to the terminator it has accelerated back up to close to the solar wind speed – so I took V=300 km/s. T=(Rmp+Ltail)/V

	MERCURY	*EARTH*	*JUPITER*
Rplanet	*2500 km*	*6400 km*	*72,000 km*
Rmp+Ltail	*1.4+5~6* R_M	*10+100~100* R_E	*80+200~300* R_J
Tmp-xline	*15000/300~50 sec ~ minute*	$6 \times 10^5/300 \sim 2000$ *s ~hour*	$7 \times 10^4 \times 300/300 \sim 7 \times 10^4$ *~day*

Again, a huge range in values – suggesting a more rapid response the smaller the planet.

C. The other end of the above recently-reconnected fluxtube is in the ionosphere. If the ionospheric end of the fluxtube traverses the polar cap in the same time as the "free" end traverses down the tail to the X-line, what sort of speed is implied for the anti-sunward flow in the ionosphere? Again, both Earth and either Mercury or Jupiter, please.

For the polar cap we take the diameter to be about Rplanet/2

	MERCURY	*EARTH*	*JUPITER*
Rpc	*2500/2 ~1500 km*	*6400/2 ~3200 km*	*72,000/2 ~36,000 km*
Tmp-xline	*50 s*	*2000 s*	7×10^4 *s*
Vpc	*30 km/s*	*1.5 km/s*	*0.5 km/s*

The astute student will note that the high speed at Mercury would not be feasible if Mercury had a thick atmosphere/ionosphere.

So – try generalizing here – For Earth we are assuming that the solar wind pushes the ionosphere around. what if the ionosphere is a very good conductor? What if the ionosphere is a very poor conductor? . What if the ionosphere has more momentum than the solar wind?

3 - Length of Magnetotail

The polar cap is the region of open field lines – lines that are attached to the Earth at one end and are swept along by the solar wind at the other. Observations at high latitudes suggest that the Earth's polar cap boundary is at the magnetic latitude of about 78°

A. What is the total magnetic flux through the Earth's polar cap?
Estimate of polar cap area Apc ~ $\pi (R_E \sin 12°)^2 \sim 0.14\, R_E^2 \sim 5.7 \times 10^{12}\ m^2$
(you can do the integral more carefully if you want – but you get the same number to 2 sig figs)
Multiplying by the polar field ~2 x Bo=2 x 3 x 10^{-5} *nT*
We then get $3.4 \times 10^8\ T\, m^2 = 340$ *Mwb*
Net flux is, of course, zero since north and south poles are of opposite polarity.

B. What is the total magnetic flux through the region of open field lines on the magnetopause?

Conservation of flux means that the flux through the magnetopause must be the same, on average, as the flux out of the polar caps.

C. Making some simple assumptions, calculate the area of the region of open field lines on the magnetopause.
For the flux to be constant, then BA=constant so Amp=Apc (Bpc/Bmp).
Taking Apc~0.14 R_E^2 *and Bmp~5nT, then Amp~0.14 x 2Bo/5nT=0.14x 30,000/5* R_E^2 *~1500* R_E^2

D. Observations of the ionosphere indicated that there is a total potential drop of about 65kV. Estimate the width of the region of open field lines on the magnetopause.
Taking 65kV~E_{SW} *x W(= Polar cap width) ~ V@mp x B@mp x W (at the magnetopause)*
W~65kV / 2mV/m ~ 3 x 10^7 *m ~ 5*R_E

E. Now estimate the length of the magnetotail.
Area of magnetopause that is open = Length x Width ~1500 R_E^2
L~1500/5~300 R_E

F. The distance to the Moon is 60 R_E. How does this compare with the length of the magnetotail? How often does the Moon spend in the magnetotail?
L~300 R_E*~ 5* a_{moon}
Width of tail ~30 R_E *so the time that the Moon spends in the magnetotail is Tmoon x 30* R_E */(2 π 60* R_E*)~29 days/4 π ~2.7 days. Small fraction <10*th *of orbit.*

G. Estimate the fraction of the whole magnetopause that is threaded by open field lines.
Area of magnetopause ~ $2\pi (15R_E)^2 + L\, 2\pi (15R_E)$ *~30,000* R_E^2
So, the fraction that is open is 2x1500/30,000 ~10%

4 – Getting a feel for the Earth's magnetic field

A. How fast are the flows in the Earth's geodynamo? Could you walk/run/drive that speed?
Speeds at the core-mantle boundary of about 10 km/yr. Probably a little faster than snail's pace. But there's lots of momentum in the flow!

B. How fast is the Earth's north pole moving? Or south – are they the same? Could you walk/run/drive that speed?

http://en.wikipedia.org/wiki/North_Magnetic_Pole
Says 55-60 km/year (from Canada towards Russia). That's fast – but still slower than walking pace.
The south pole is barely moving.
Note that the N & S poles are not opposite – by a lot – 530km, in fact.

C. How much stronger/weaker is the field at the top of the Earth's core than the surface?

For a dipole, the field strength drops as $1/R^3$*. So, using Rcore/Rp~0.55, we can estimate the dipolar component to be about 6 times stronger. But the high-order multipoles are much stronger so that the field is considerably stronger in the core. Anyone find how much from Volume III, Ch. 7? I could not.*