Solutions to Merav Opher (2010) Problems

1. The normal of the shock is

$$\vec{n} = \frac{(\vec{B}_u \times \vec{B}_d) \times (\vec{B}_u - \vec{B}_d)}{\left|(\vec{B}_u \times \vec{B}_d) \times (\vec{B}_u - \vec{B}_d)\right|}$$

Since from the plot you can obtain all the three components of B_u and B_d, the normal can be easily found. The shock speed is:

$$V_s = \left(\frac{\rho_d \vec{u}_d - \rho_u \vec{u}_u}{\rho_d - \rho_u}\right) \cdot \vec{n}$$

With the shock Theta_Bn can be found. Alfven mach number, sonic mach number as well as the compression are easy and can be obtained directly using the graphs.

2. Considering a shock coordinate system where the parallel axis || is along the normal of the shock, $\hat{n} = \cos(\theta_1)\hat{R} + \sin(\theta_1)\cos(\phi_1)\hat{T} + \sin(\theta_1)\sin(\phi_1)\hat{N}$, where θ_1 and ϕ_1 are the angles between the normal **n** and the radial upstream velocity V_1. θ_1 is defined as the angle measured between **n** and **R.** ϕ_1 is the angle between the projection of n on the (T, N) plan and T (see Supplementary figure 1). The perpendicular axis are taken as $\hat{n}_{\perp 1} = \hat{T} \times \hat{n} / \left\|\hat{T} \times \hat{n}\right\|$, and $\hat{n}_{\perp 2} = \hat{n} \times \hat{n}_{\perp 1} / \left\|\hat{n} \times \hat{n}_{\perp 1}\right\|$.

The three-dimensional Rankine-Hugoniot conditions in this shock coordinate system (after a transformation to the deHoffman -Teller frame) are:

$$\begin{aligned} v_{2\|} &= \frac{v_{1\|}}{r} \\ v_{2\perp 1} &= v_{1\perp 1} \\ v_{2\perp 2} &= v_{1\perp 2} + \frac{B_{1\|2}}{B_{1\|}} \frac{(r-1) v_{1\|} v_{A1\|}^2}{\left(v_{1\|}^2 - r v_{A1\|}^2\right)} \end{aligned} \qquad \text{Eqs. (1-3)}$$

where the index “1” and “2” refer respectively, to the upstream and downstream quantities; V_{A1} is the upstream Alfven speed. In the RTN frame Eqs. (1-3) become:

$$V_{2R}=\left(\frac{V_1\cos(\theta_1)-V_S}{r}\right)\cos(\theta_1)+\frac{V_1\sin^2(\theta_1)\sin^2(\phi_1)}{\alpha_1^2}$$
$$-\left(-\frac{V_1\cos(\theta_1)\sin(\theta_1)\cos(\phi_1)}{\alpha_1}+\tan(\chi_1)\frac{(r-1)(V_1\cos(\theta_1)-V_S)(V_{A1}\cos(\chi_1))^2}{\left((V_1\cos(\theta_1)-V_S)^2-r(V_{A1}\cos(\chi_1))^2\right)}\right)\frac{\cos(\theta_1)\sin(\theta_1)\cos(\phi_1)}{\alpha_1}$$

$$V_{2T}=\left(\frac{V_1\cos(\theta_1)-V_S}{r}\right)\sin(\theta_1)\sin(\phi_1)+$$
$$+\left(-\frac{V_1\cos(\theta_1)\sin(\theta_1)\cos(\phi_1)}{\alpha_1}+\tan(\chi_1)\frac{(r-1)(V_1\cos(\theta_1)-V_S)(V_{A1}\cos(\chi_1))^2}{\left((V_1\cos(\theta_1)-V_S)^2-r(V_{A1}\cos(\chi_1))^2\right)}\right)\alpha_1$$

Eqs.(4-6)

$$V_{2N}=\left(\frac{V_1\cos(\theta_1)-V_S}{r}\right)\sin(\theta_1)\sin(\phi_1)-\frac{V_1\cos(\theta_1)\sin(\theta_1)\sin(\phi_1)}{\alpha_1^2}$$
$$\left(-\frac{V_1\cos(\theta_1)\sin(\theta_1)\cos(\phi_1)}{\alpha_1}+\tan(\chi_1)\frac{(r-1)(V_1\cos(\theta_1)-V_S)(V_{A1}\cos(\chi_1))^2}{\left((V_1\cos(\theta_1)-V_S)^2-r(V_{A1}\cos(\chi_1))^2\right)}\right)\frac{\sin^2(\theta_1)\cos(\phi_1)\sin(\phi_1)}{\alpha_1}$$

$\alpha_1=\sqrt{\sin^2(\theta_1)\sin^2(\phi_1)+\cos^2(\theta_1)}$, χ_1 is the angle between B_1 and the shock normal: $\cos(\chi_1)=\sin(\theta_1)\cos(\phi_1)$ and r is the compression ratio, $r=\rho_2/\rho_1$.

3. The Termination Shock at the time of the Voyager 2 crossing had, on average, for $\theta_1=15^\circ$, $\phi_1=165^\circ$.

4. The students should sketch the main MHD variables for a tangential discontinuity (heliopause) (so total pressure should be constant). They should also be able to sketch the jump at the termination shock. They should be able to compare Voyager 1 and 2 and qualitatively see that the magnetic pressure at Voyager 2 will be stronger therefore the heliopause will be closer to the sun than Voyager 1.

Shocks, Homework

5. a. The hydrodynamic jump conditions for plasma, $\gamma = 5/3$, are

$$M_2^2 = \frac{M_1^2+3}{5M_1^2-1} \rightarrow \frac{1}{5} \quad (1)$$

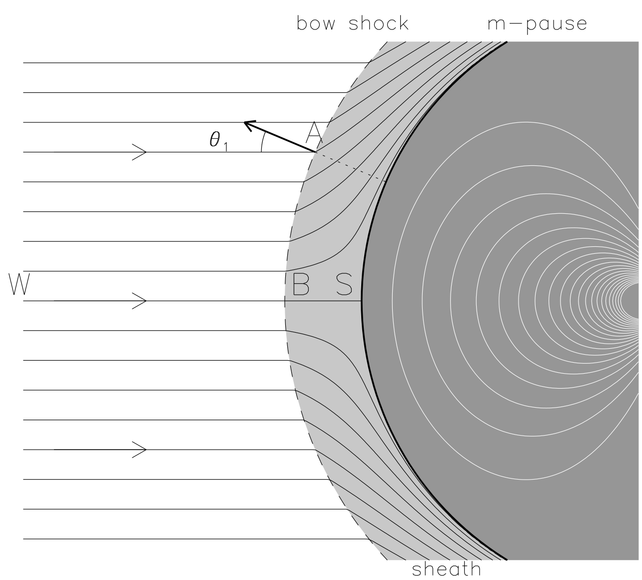
$$\frac{\rho_2}{\rho_1} = \frac{v_{n,1}}{v_{n,2}} = \frac{4}{1+3/M_1^2} \rightarrow 4 \ , \quad (2)$$

where limits are for $M_1 \rightarrow \infty$. We can use these to write

$$\frac{p_2}{\rho_2} = \frac{v_2^2}{\gamma M_2^2} \rightarrow 3\,v_2^2 \rightarrow \frac{3}{16}\,v_1^2 = \frac{3}{16}\,v_{sw}^2 \ . \quad (3)$$

This gives the temperature

$$T_B = \frac{m_p}{2k_{\rm B}}\frac{p_B}{\rho_B} \simeq \frac{3m_p}{32k_{\rm B}}\,v_{sw}^2 \ . \quad (4)$$



A simplified of the day-side magnetosphere. Flow streamlines are shown as solid curves originating at the Sun, far to the left. The bow shock is a dashed arc and the magnetopause is a thick solid arc. The magnetosphere proper is dark grey, with some white magnetic field lines shown. The magnetosheath is the lighter greay region between the bow shock and the magnetopause.

b. Using eq. (3) gives

$$\tfrac{1}{2}v_B^2 = \frac{1}{6}\frac{p_B}{\rho_B} \ , \quad (5)$$

and thus

$$\frac{p_B}{\rho_B} + \frac{2}{5}\,\tfrac{1}{2}v_B^2 = \frac{16}{15}\frac{p_B}{\rho_B} = \frac{p_S}{\rho_S} \ . \quad (6)$$

From this we find

$$T_S = \frac{16}{15}T_B = \frac{m_p}{10k_{\rm B}}\,v_{sw}^2 = 1.2\times10^5\,{\rm K}\left(\frac{v_{sw}}{100\,{\rm km/s}}\right)^2 \ . \quad (7)$$

Taking $v_{sw} = 800$ km/s gives $T_S = 7.7\times10^6$ K.

c. It is clear from eq. (2) that

$$\rho_B \;=\; 4\,\rho_{sw} \quad . \tag{8}$$

Adiabatic compression leads to the ratio

$$\frac{\rho_S}{\rho_B} \;=\; \left(\frac{T_S}{T_B}\right)^{1/(\gamma-1)} \;=\; \left(\frac{T_S}{T_B}\right)^{3/2} \;=\; \left(\frac{16}{15}\right)^{3/2} \;=\; 1.102 \quad . \tag{9}$$

From this we find

$$\rho_S \;\simeq\; \left(\frac{16}{15}\right)^{3/2} 4\,\rho_{sw} \;=\; 4.407\,\rho_{sw} \quad . \tag{10}$$

d. The flow angle is defined

$$\tan\theta_j \;=\; \frac{v_{t,j}}{v_{n,j}} \tag{11}$$

where v_t, the flow tangent to the shock, is the same on both sides. This leads to

$$\tan\theta_2 \;=\; \frac{v_t}{v_2} \;\simeq\; 4\,\frac{v_t}{v_1} \;=\; 4\tan\theta_1 \quad . \tag{12}$$

The deflection across the shock is

$$\Delta\theta \;=\; \theta_2 - \theta_1 \;\simeq\; 4\,\theta_1 - \theta_1 \;=\; 3\theta_1 \quad , \tag{13}$$

after using the small angle approximation to replace $\tan\theta_j \simeq \theta_j$.

e. The post-shock velocity vector is

$$\mathbf{v}_2 \;=\; v_{n,2}\hat{\mathbf{n}} + v_{t,2}\hat{\mathbf{t}} \;=\; v_{n,2}\,[\,\hat{\mathbf{n}} + \tan\theta_2\hat{\mathbf{t}}\,] \;=\; v_{n,2}\,[\,\hat{\mathbf{n}} + 4\tan\theta_1\hat{\mathbf{t}}\,] \tag{14}$$

where $\hat{\mathbf{n}}$ is the shock normal and $\hat{\mathbf{t}}$ a tangent vector perpendicular to it. The square magnitude is therefore

$$|\mathbf{v}_2|^2 \;=\; v_{n,2}^2\,[\,1 + 16\tan^2\theta_1\,] \quad . \tag{15}$$

Compared to the local sound speed this is

$$\frac{|\mathbf{v}_2|^2}{c_{s,2}^2} \;=\; \frac{v_{n,2}^2}{c_{s,2}^2}\,[\,1 + 16\tan^2\theta_1\,] \;=\; M_2^2\,[\,1 + 16\tan^2\theta_1\,] \;=\; \frac{1}{5}[\,1 + 16\tan^2\theta_1\,] \quad ,$$

using the hypersonic limit from eq. (1). Setting this to exceed one gives the requirement

$$\tan\theta_1 \;>\; \tfrac{1}{2} \quad , \quad \theta_1 > \tan^{-1}(1/2) \;=\; 26.5^\circ \quad . \tag{16}$$

f. The radial and poloidal velocities are

$$v_r \;=\; \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} \;=\; 2\frac{A}{R_{mp}^2}\left(\frac{r^2}{R_{mp}^2} - \frac{R_{mp}^3}{r^3}\right)\cos\theta \quad , \tag{17}$$

$$v_\theta \;=\; -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} \;=\; -\frac{A}{R_{mp}^2}\left(4\frac{r^2}{R_{mp}^2} + \frac{R_{mp}^3}{r^3}\right)\sin\theta \quad , \tag{18}$$

so evidently A is negative. The angle of the downstream flow relative to the shock normal, $\hat{\mathbf{n}} = \hat{\mathbf{r}}$, at $r = R_{bs}$, just inside the bow shock, is

$$\tan\theta_2 \;=\; -\frac{v_\theta}{v_r} \;=\; \frac{4(R_{bs}/R_{mp})^3 + (R_{mp}/R_{bs})^2}{2(R_{bs}/R_{mp})^3 - 2(R_{mp}/R_{bs})^2}\tan\theta \quad . \tag{19}$$

According to part d. this must be $\tan\theta_2 = 4\tan\theta_1$, and $\theta_1 = \theta$, to polar angle, since the flow is horizontal outside the shock. This leads to the relation

$$\frac{4(R_{bs}/R_{mp})^5 + 1}{2(R_{bs}/R_{mp})^5 - 2} \;=\; 4 \quad , \tag{20}$$

and therefore

$$R_{bs} \;=\; \left(\frac{9}{4}\right)^{1/5} R_{mp} \;=\; 1.18\,R_{mp} \quad . \tag{21}$$

This result is often cast in terms of the "stand-off" distance between the bow shock and magnetopause:

$$\Delta \;=\; R_{bs} - R_{mp} \;=\; 0.18\,R_{mp} \quad , \quad M_{sw} \to \infty \quad . \tag{22}$$

The simple model proposed above, a super-sonic flow encountering a spherical obstacle of radius R, is one for which there has been much study. Laboratory experiments with different Mach numbers and different fluids (i.e. differing γ) have led to empirical relations of the form

$$\frac{\Delta}{R} \;\simeq\; \alpha\,\frac{\rho_1}{\rho_2} \quad , \tag{23}$$

with values $\alpha \simeq 0.78$ (Seiff, *NASA Tech. Pub.* 24, [1962]). Numerical solutions of fully compressible hydrodynamics yield $\alpha \simeq 1.1$ (Spreiter, Summers & Alkse, *Planet. Space Sci.* **14** 223 [1966]), which has been subsequently used to predict the stand-off distance of the actual bow shock, at least for cases with high solar wind Mach number (see Farris & Russell, *JGR* **99**, 17681 [1994]). The simplified calculation we have performed here has found a lower value, $\alpha = 0.18 \times 4 = 0.704$, due mostly to the departure of actual post-shock flow from the incompressible solution we assumed. It is not, however, due to our assumption of a spherical magnetopause. As shown in part e., the sheath flow becomes supersonic beyond an angle of $\theta = 26^\circ$. Inside this small region there is little difference between a sphere and the actual magnetopuase. Differences outside that region can affect neither the subsonic solution nor the stand-off distance.

6. a. The velocity difference is equal to v_f, and at high Mach number this difference is

$$v_f \;=\; v_1 \;-\; v_2 \;=\; \left(1-\frac{v_2}{v_1}\right)v_1 \;\simeq\; \tfrac{3}{4}\,v_1 \quad . \tag{24}$$

Since the pre-shock material is at rest relative to the Sun, the shock moves downward at speed v_1

$$v_s \;=\; v_1 \;=\; \tfrac{4}{3}\,v_f \quad . \tag{25}$$

b. The post-shock velocity, in the shock frame, is

$$v_2 \;=\; \tfrac{1}{4}\,v_1 \;=\; \tfrac{1}{3}\,v_f \quad . \tag{26}$$

Using the fact that $v_2/c_{s,2} = M_2 = 1/\sqrt{5}$ we find that

$$\frac{1}{\sqrt{5}} \;=\; \tfrac{1}{3}\,\frac{v_f}{c_{s,f}} \qquad \Longrightarrow \qquad c_{s,f} = \frac{\sqrt{5}}{3}\,v_f \tag{27}$$

The downward flows is therefore related to the post-shock temperature

$$T_f \;=\; \frac{3}{5}\,\frac{\bar{m}}{k_{\rm B}}\,c_{s,2}^2 \;=\; \frac{1}{3}\,\frac{\bar{m}}{k_{\rm B}}\,v_f^2 \;=\; 2\times10^7\,{\rm K}\,\left(\frac{v_f}{100\,{\rm km/s}}\right)^2 \quad . \tag{28}$$

c. After reflection there is a new shock propagating upward into the flare plasma. The velocity difference is the same, $v_1 - v_2 = v_f$, but the pre-shock plasma of this shock is the post-shock plasma from the incident shock: the flaring plasma. This is not, however, a high Mach-number shock so

$$\begin{aligned} v_f \;&=\; v_1 \;-\; v_2 \;=\; \left(1-\frac{v_2}{v_1}\right)v_1 \;=\; \left(1-\frac{2M_1^2+6}{8M_1^2}\right)v_1 \\ &=\; c_{s,f}\,M_1\left(\tfrac{3}{4}-\tfrac{3}{4}M_1^{-2}\right) \;=\; \tfrac{3}{4}\,c_{s,f}\,(M_1-M_1^{-1}) \quad . \end{aligned} \tag{29}$$

The results of the previous section therefore lead to the condition

$$\tfrac{3}{4}\,(M_1-M_1^{-1}) \;=\; \frac{v_f}{c_{s,f}} \;=\; \frac{3}{\sqrt{5}} \quad . \tag{30}$$

This is a quadratic equations whose positive solution is $M_1 = \sqrt{5}$. This is the Mach number of the reflected shock.

d. The reflected shock enhances the pressure by

$$\frac{p_2}{p_1} \;=\; \frac{5M_1^2-1}{4} \;=\; 6 \quad , \tag{31}$$

above the pressure following the first, hypersonic, shock. This pressure is

$$\tilde{p}_2 \;=\; \tfrac{3}{4}\,\tilde{\rho}_1\tilde{v}_1^2 \;=\; \tfrac{4}{3}\,\rho_0\,v_f^2 \quad , \tag{32}$$

where ρ_0 is the pre-flare coronal density. The the final chromospheric pressure is

$$p_{\rm chr.} \;=\; 6\times\tfrac{4}{3}\,\rho_c\,v_f^2 \;=\; 8\,\rho_0\,v_f^2 \quad . \tag{33}$$

e. The transmitted shock raises the pressure by

$$\frac{p_2}{p_1} \;=\; \frac{8\,\rho_0\,v_f^2}{p_0} \;\simeq\; \tfrac{5}{4}\,M_1^2 \quad , \tag{34}$$

where p_0 and ρ_0 are pre-flare coronal values; the pre-flare chromosphere is at the same pressure as the pre-flare corona since they are separated by a *contact discontinuity*. Using the fact that $p_0/\rho_0 = (3/5)c_{s,0}^2$ this becomes

$$\frac{40v_f^2}{3c_{s,0}^2} \;=\; \tfrac{5}{4}\,M_1^2 \quad , \tag{35}$$

from which we see that $M_1 = \sqrt{32/3}(v_f/c_{s,0})$. It does not depend on r.

f. Since the pre-flare chromosphere is colder than the pre-flare corona by $1/r$, its sound speed is $c_{s,0}/\sqrt{r}$. This is the sound speed by which the pre-shock speed, v_1, is divided to form M_1. The velocity difference is then

$$\frac{v_1 - v_2}{c_{s,0}/\sqrt{r}} \;=\; \frac{3}{4}\,\frac{v_1}{c_{s,0}/\sqrt{r}} \;=\; \tfrac{3}{4}\,M_1 \;=\; \sqrt{6}\,\frac{v_f}{c_{s,0}} \quad . \tag{36}$$

This shows that the velocity jump across the transmitted shock is

$$v_1 - v_2 \;=\; \sqrt{\frac{6}{r}}\,v_f \quad . \tag{37}$$

Since the pre-shock material is at rest this is the downward velocity of the transmitted shock. In order to neglect this velocity in the reflected shock we need $\sqrt{6/r} \ll 1$.

	17:25UT (first)	17:49UT (second)
$n\,[\mathrm{cm}^{-3}]$	11.62	4.25
v_R [km/s]	349.5	295.9
v_T [km/s]	64.80	79.14
v_N [km/s]	37.98	59.75
B_R [nT]	0.027	1.000
B_T [nT]	-1.759	-0.908
B_N [nT]	-2.931	-1.474

7. a. A shock always *compresses* the plasma so the post-shock region will have the higher density — the **second observation is pre-shock**. This is the second region to pass the spacecraft, and therefore lies at smaller radius than the post-shock region. The shock normal $\hat{\mathbf{n}}$ must point *toward* the pre-shock region so $\hat{\mathbf{n}}\cdot\hat{\mathbf{r}} < 0$. This is therefore a *reverse shock*.

   b. Taking the dot product with $\hat{\mathbf{n}}$ shows

$$\hat{\mathbf{n}}\cdot[\![\mathbf{B}]\!] \;=\; [\![\hat{\mathbf{n}}\cdot\mathbf{B}]\!] \;=\; [\![B_n]\!] \;=\; 0 \quad , \tag{38}$$

   from the jump condition following from $\nabla\cdot\mathbf{B} = 0$. This shows that $[\![\mathbf{B}]\!]$ is orthogonal to $\hat{\mathbf{n}}$.

   c. The jump in velocity is

$$[\![\mathbf{v}]\!] \;=\; \hat{\mathbf{n}}[\![v_n]\!] \;+\; [\![\mathbf{v}_\perp]\!] \;=\; \hat{\mathbf{n}}[\![v_n]\!] + \frac{B_n}{4\pi\rho v_n}[\![\mathbf{B}_\perp]\!] \;=\; \hat{\mathbf{n}}[\![v_n]\!] + \frac{B_n}{4\pi\rho v_n}[\![\mathbf{B}]\!] \quad . \tag{39}$$

   In the special case that $[\![v_n]\!] = 0$, the two jumps $[\![\mathbf{v}]\!]$ and $[\![\mathbf{B}]\!]$, lie along the same line (they are parallel or anti-parallel). This condition is equivalent to $[\![\rho]\!] = 0$, which is a rotational discontinuity. This does no apply here since $n_{p1} \neq n_{p2}$. Thus the velocity jump is a sum of two perpendicular components and lies in the plane spanned by them: $[\![\mathbf{B}]\!]$ and $\hat{\mathbf{n}}$.

   d. First we compute the differences directly

$$\Delta\mathbf{B} = (-0.97, -0.85, -1.46)\,\mu\mathrm{G} \qquad , \qquad \Delta\mathbf{v} = (53.57, -14.34, -21.77)\,\mathrm{km/s}$$

   whose magnitudes are

$$|\Delta\mathbf{B}| = 1.95\,\mu\mathrm{G} \quad , \quad |\Delta\mathbf{v}| = 59.58\,\mathrm{km/s}$$

   We can extract from $\Delta\mathbf{v}$ that component perpendicular to $\Delta\mathbf{B}$

$$\begin{aligned}\mathbf{w} \;&=\; \Delta\mathbf{v} - \frac{\Delta\mathbf{v}\cdot\Delta\mathbf{B}}{|\Delta\mathbf{B}|^2}\Delta\mathbf{B} \\ &=\; \Delta\mathbf{v} - \frac{-8.21}{3.79}\Delta\mathbf{B} \;=\; (51.47, -16.18, -24.92)\,\mathrm{km/s}\end{aligned} \tag{40}$$

   The normal vector $\hat{\mathbf{n}}$ must have unit magnitude and a negative radial component

$$\hat{\mathbf{n}} \;=\; -\frac{\mathbf{w}}{|\mathbf{w}|} \;=\; (-0.866, 0.272, 0.419)$$

e. The dot product of the normal vector with the magnetic fields gives the angles

$$\cos\theta_1 \;=\; \frac{\mathbf{B}_1\cdot\hat{\mathbf{n}}}{|\mathbf{B}_1|} \;=\; \frac{-1.73}{2.00} \;=\; -\,0.866 \quad , \qquad \theta_1 \;=\; 150.000^\circ$$

$$\cos\theta_2 \;=\; \frac{\mathbf{B}_2\cdot\hat{\mathbf{n}}}{|\mathbf{B}_2|} \;=\; \frac{-1.73}{3.42} \;=\; -\,0.507 \quad , \qquad \theta_2 \;=\; 120.438^\circ$$

f. From the expressions we find

$$M_{A2}^2 \;=\; \frac{n_1}{n_2} M_{A1}^2$$

and

$$(M_{A1}^2-1)\tan\theta_1 \;=\; (M_{A2}^2-1)\tan\theta_2 \;=\; \Big(\frac{n_1}{n_2}M_{A1}^2-1\Big)\tan\theta_2$$

Solving this gives

$$M_{A1} \;=\; \sqrt{\frac{\tan\theta_1-\tan\theta_2}{\tan\theta_1-(n_1/n_2)\tan\theta_2}}$$

$$M_{A1}=5.000 \quad , \qquad M_{A2}=3.024$$

g. We have several pieces of evidence that this is a **fast shock**. We could have noted early on that $|\mathbf{B}_2| > |\mathbf{B}_1|$. Later it was evident that $\mathbf{B}_2$ had deflected *away* from the shock normal (although it appears that $\theta_2 < \theta_1$ because the field is directed backwards). Finally, we wee that the Mach numbers are ordered

$$1 \;<\; M_{A2} \;<\; M_{A1} \;\;, \tag{41}$$

which is a prerequisite for the *fast shock*.

h. The Alfvén Mach number refers to the normal component of the upstream velocity $v_{1,n}$ *in the reference frame of the shock*. If we denote the shock speed v_s the inflow speed is therefore

$$v'_{n,1} \;=\; v_{n,1}-v_s \;=\; -\,M_{A1}\,v_{A1,n} \;\;, \tag{42}$$

since the inflow is always in the direction opposite $\hat{\mathbf{n}}$. The shock speed is therefore

$$v_s \;=\; v_{n,1} \;+\; M_{A1}\,v_{A1,n} \;\;, \tag{43}$$

where $v_{1,n}$ is the component of the velocity, in the spacecrtaft frame,

$$v_{n,1} \;=\; \hat{\mathbf{n}}\cdot\mathbf{v}_1 \;=\; -\,2.10\times10^7\,\text{cm/s} \;=\; -\,210\,\text{km/s} \;\;.$$

The normal component of the Alfvén velocity is

$$v_{An,1} \;=\; \frac{|\mathbf{B}_1\cdot\hat{\mathbf{n}}|}{\sqrt{4\pi n_1 m_p}} \;=\; \frac{1.73\times10^{-5}G}{9.44\times10^{-12}} \;=\; 1.83\times10^6\,\text{cm/s} \;=\; 18.3\,\text{km/s} \;\;.$$

This gives the shock velocity in the reference frame of the spacecraft

$$v_s \;=\; (\,-210 \;+\; 5.00\times18.3\,)\,\text{km/s} \;=\; -\,118\,\text{km/s} \;\;, \tag{44}$$

The shock propagates in the direction of the shock normal giving

$$\mathbf{v}_s \;=\; \hat{\mathbf{n}}\,v_s \;=\; (103,\,-32.2,\,-49.7)\,\text{km/s} \;\;. \tag{45}$$

The shock is therefore moving *outward* from the Sun, even though it is a *reverse shock*.