Problems on Cosmic Rays for Heliophysics Summer School

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1. Approximate the solar wind as a spherically symmetric, purely radial, flow at constant speed, $\mathbf{U} = V_w \mathbf{e}_r$. Suppose solar energetic particles as a function of momentum p are continuously injected, also spherically symmetrically, from the solar atmosphere at radius r_0 at a constant rate $Ap^{-\gamma}$. Solve the Parker transport equation to determine the dependence of the energetic particles on energy and radius $r>r_0$, if the radial diffusion coefficient κ_{rr} is a constant, independent of radius and momentum.

2. Using the expression for the average drift velocity of the guiding centers of an isotropic distribution of energetic particles in the presence of a spatially varying magnetic field $\mathbf{B(r)}$, $\mathbf{V_d} = (p\ c\ w)/(3q)\ \mathrm{curl}\ (\mathbf{B(r)}/B^2$, find the drift velocity in a Parker spiral magnetic field.

3. Consider a steady, spherically symmetric heliosphere with solar-wind velocity V_w, and where the galactic cosmic-ray intensity is $f(p) = f_\infty(p)$ at some large radius r_1. If the radial cosmic-ray diffusion coefficient $\kappa_{rr}(r,p)$ is large enough that the dimensionless parameter $\eta = r V_w/ \kappa_{rr} << 1$, find an approximation to the intensity for $r < r_1$, to first order in η.

4. Suppose that a planar shock is propagating steadily, in the x-direction, through a collisionless plasma and interacting with energetic particles which satisfy Parker's transport equation. Consider the interaction of energetic charged particles with a planar, time-steady shock with density jump r_{sh} , propagating in the x-direction in a uniform plasma with shock speed V_{sh}. Work in the coordinate frame moving with the shock, and put the shock at a fixed position x=0.

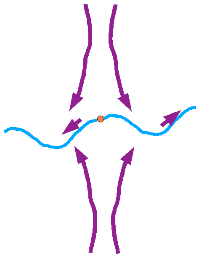
   Far upstream there is a population of energetic particles, for which the distribution function is $f(p) = \alpha p^{-\gamma}$. Neglect any particles originating elsewhere determine the resulting particle distribution. Work in the coordinate frame in which the shock is at rest, and at x = 0.

5. Consider again a planar shock propagating in the x-direction, but let the distribution function of the energetic particles depend on the coordinate y, along the shock face, and let there be a magnetic field **B** which is at an angle θ_{bn} to the normal to the shock (x-direction). Obtain the jump condition for the distribution function f at the shock and show that the jump condition at the shock depends on the variation of the intensity along the shock face.

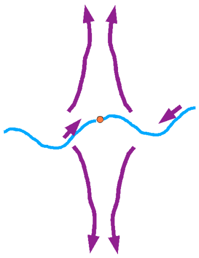
6. Consider energetic particles which satisfy Parker's equation in a time-steady, spherically symmetric heliosphere where the solar wind flows radially outward from the Sun at a constant speed V_w from an inner radius R_0 to a termination shock at an outer radius $R_{sh} >> R_0$, beyond which it it drops to the value $V_2 = V_w/r_{sh}$ (r_{sh} >1 and <4) and continues to flow radially beyond the shock, with a speed which decreases as $1/r^2$ out to the outer boundary at $r = R_b$. Suppose that the energetic-particle diffusion coefficient κ_{rr} is independent of energy and varies as r.

a. Consider the case where there is no source of particles at the shock and instead there is an intensity $Ap^{-\gamma}$ imposed at the outer boundary $r = R_b$. Find the solution for the intensity inside of the boundary.
b. Assuming that he only source of particles is at low values of momentum p concentrated at the shock. The outer boundary is at $r = R_b$,. Find the solution.
c. If you are able to, plot the two solutions for reasonable parameters.

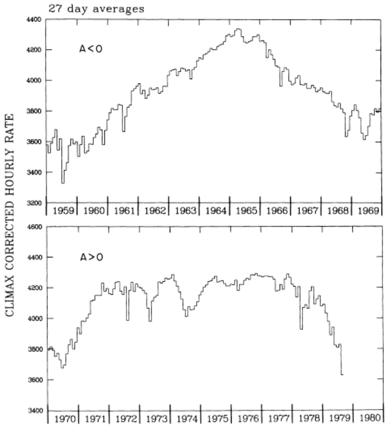
7. For several years around each sunspot minimum, the interplanetary magnetic field field is approximately an ideal Parker spiral with the field in one direction in the northern solar hemisphere and the opposite direction in the southern hemisphere, with the sign change occurring abruptly at the heliospheric current sheet, near the equator. The magnetic-field directions change sign at each sunspot maximum. Cosmic-ray ions with hundreds of MeV energies drift rapidly in the spiral field. The drift directions are indicated schematically below. In the left figure, for the current sign of the fields, the particles drift inward over the heliospheric poles, whereas in the years around the last sunspot minimum, they drifted inward along the current sheet, as indicated in the right figure below. At sunspot minimum the current sheet is quite flat, and as time progresses away from minimum, the oscillation or warp of the current sheet increases. The observed time variation of cosmic rays is given to the far right for two sunspot cycles. A>0 corresponds to northern magnetic field out, and A<0 to the opposite configuration.



Northern Field outward



Northern Field inward



a. Assuming that the increasing warp of the current sheet away from sunspot minimum (the center of each intensity plot), discuss how this might be explained qualitatively in terms of the drifts.
b. In the above picture, the intensity minima at the edges of each plot, near each sunspot maximum are caused by the increased solar activity near sunspot maximum. At the Maunder minimum, in the 17th century, there were few sunspots, and the cosmic-ray intensity was somewhat higher for several sunspot cycles. It did not show as pronounced a

11-year cycle, and the 22-year variation was relatively more pronounced. Discuss a possible interpretation of this effect in terms of drifts.